Part I
Commodity Market Dynamics

A proper understanding of commodity markets should start with analysis of the first kind of information at hand, i.e. the historical evolution of prices and returns on these assets over time. This first section aims at providing the reader with the most important insights to be gained from these data series: what are the main stylized facts one should be aware of when investing money in these assets? Which characteristics do they have when compared to the usual asset classes? How do they interact with each other and, more importantly, with the basic building blocks of a traditional asset allocation? Financial econometrics has now provided us with the necessary tools to answer these questions, and we will apply them in a systematic way to help us build a list of the most interesting features. The attention of academics has been increasingly focused in recent years on the understanding of the potential risks and patterns observed in commodity markets. To address the problem any investor is faced with, this section steps into this recent evolution and will be mainly devoted to measuring regularities in commodity markets by using a large dataset of commodity indices. We will follow a thorough analysis of commodity returns, both from an individual and from a cross-asset perspective. In the meantime, Part I of the book tackles three different types of problem that investors are confronted with.

The primary focus of our investigation in Part I of the book is to help the reader obtain an increased understanding of the formation of returns on commodities, both from an individual and from a cross-asset perspective. Recent books such as Ilmanen (2011) have put massive efforts into the listing of the salient features of excess returns, as they are the reason why investors would increase their exposure to any risk factor. This investigation of the past has one purpose: improving the ability of investors to estimate expected returns of this asset class. Studies like Gorton and Rouwenhorst (2005a) or Erb and Campbell (2006) geared investors toward commodities mainly by emphasizing the 'equity-like' performance over the period they consider, as well as the strong diversification impact of adding portfolios of commodities to standard assets in a global portfolio. Part I of the book aims to build on their work, by improving it in two directions. First, by using a more recent dataset that incorporates the 2008 crisis, we confront their findings to this major event and confirm or not their original findings. Second, we use a large set of new econometric tools as a magnifying glass to provide the reader with a more detailed analysis of these returns than previous studies. We tackle, for example, the two key aspects of commodities that are the forecasting power of the term structure of futures and the existence of a momentum effect in commodities.

Beyond the essential theme of expected returns, a second topic is the measure of the risk exposure that any investor has to deal with when investing in commodity markets. Building on '*What every investor should know about commodities*' by Kat and Oomen (2007a; 2007b), there are a couple of stylized facts that an investor should keep in mind when entering commodity markets: for instance the nature of volatility patterns in commodity markets, the jump activity and its impact on upcoming returns, and the behavior of correlations among commodities and with other assets. These three elements aim at helping the reader become familiar with the complex mechanism of returns on this asset class. A continued comparison to standard assets will enable readers that are familiar with such assets to get a faster grasp on the salient features of commodities.

A final aspect that is essential from a diversified portfolio perspective is the nature of the relationships between commodities and the assets traditionally included in a balanced portfolio. This aspect should matter both to hedge fund managers and to those in charge of deciding the nature of portfolio mixes to be used in pension funds. Two aspects here are rarely disentangled: first, the instantaneous cross-sectional dependency as measured by correlation is a key aspect of those cross-asset linkages. Such correlations vary through time, as illustrated in Longin and Solnik (2001) or Ang and Chen (2002). Diversification by itself is usually looked for in this part of the linkage measurement issue. Second, dynamic linkages are also important; that is, the way shocks from an asset can spread across other assets. The 2008–2011 Euro crisis is a very good example of how such shocks can spread in markets and endanger the whole financial system. Measures of such dynamics have been proposed in the literature, as in Diebold and Yilmaz (2012). We apply this approach to commodities, and bring forward evidence that commodities are not as insensitive to such shocks as one would assume.

PLAN OF PART I

This first part of the book is divided into two chapters. Chapter 1 covers the individual dynamics of commodities, investigating first expected returns on commodities from different angles, before turning to risk metrics. On expected returns, our investigations are focused on the forecasting power of the term structure of commodity futures and on the momentum of commodities. The section dedicated to risk questions the existence of leverage effects in commodities, before turning to an analysis of the jump activity in commodities. Chapter 2 investigates the cross-asset linkages both within commodities, and between commodities and standard assets. This analysis is performed from two different perspectives: first, we analyze the factors implicit in the cross-section of returns on commodities and standard assets, before considering the dynamic spillovers that are potentially found in such datasets. The first approach is thus somewhat an analysis of static linkages, whereas the second one can be seen as an analysis of the dynamics of these returns.

1 Individual Dynamics: From Trends to Risks

Starting with the asset-by-asset investigation of commodity returns, the salient features under our assessment will be first the nature and persistence of returns on commodities, moving next to the analysis of higher order moments – that is volatility, asymmetry and extreme events.

One of the first attempts to try to bring together cross-asset conclusions regarding commodities can be found in Kat and Oomen (2007a). Investigating between 22 and 29 commodities over the period 1965–2005 (when such data is available), they reach the following empirical conclusions:

1. First of all – and consistent with the results of Erb and Campbell (2006) – individual commodities do not provide investors with a risk premium on average. This conclusion has to be differentiated from the basket of commodities case: Gorton and Rouwenhorst (2005a) show how such a risk premium is associated to a basket of equally-weighted commodities by using the Commodity Research Bureau dataset covering the 1959–2005 period and including 36 commodities.
2. The persistence in commodities is found to be important: a positive or a negative shock to commodity prices usually has long-lasting effects, unlike equities and bonds. This is an essential feature for trend-following investment strategies.
3. The volatility of commodities is not found to be excessive when compared to the volatility of equities over the period under consideration.
4. They also find a limited asymmetry of returns in their dataset: the skewness of commodity returns is usually found to be close to zero.
5. Finally, one of the key properties of commodities is the frequency at which extreme events occur. Kurtosis being a natural way to measure such a tail event activity, they find excess kurtosis for most of the market under the scope of their investigation.

This list of empirical features seems, however, to be somewhat specific to the period covered by each dataset. More recently and by using various kinds of continuous time models encompassing time-varying volatility and jumps in the returns and volatility dynamics, Brooks and Prokopczuk (2011) studied in a more quantitative way the law of motion of commodities' returns. Their empirical findings show that jumps are an essential building block of the underlying data-generating process of such markets. The frequency of appearance and the size of the jumps in returns are found to be very different from one market to another. Finally, the correlation between returns and their volatility is found to a have a sign that is specific to each market: for example, a large negative return in the crude oil price should trigger a surge in its volatility that is larger than in the case of a similar but positive return. Such a pattern does not hold in the case of gold, silver and soybean, following Brooks and Prokopczuk (2011). This goes against the fifth conclusion from Kat and Oomen (2007a; 2007b), but the period covered by both studies is quite different.

Two additional aspects should be mentioned here.

1. First, as for any financial market, commodity markets are affected by time-varying volatility. This stylized fact has been investigated in many research articles such as Serletis (1994), Ng and Pirrong (1996), Haigh and Holt (2002), Pindyck (2004), Sadorsky (2006), Alizadeh *et al*. (2008) and Wang *et al.* (2008). Most of them use various specifications close to the Generalized Autoregressive Conditional Heteroskedastic (GARCH) model initially presented in Engle (1982) and Bollerslev (1986). Bernard *et al*. (2008) present results regarding the aluminum market. Whereas these contributions were based on discrete time models, continuous time finance also focused on the addition of stochastic volatility to the basic model by Schwartz (1997), as presented in Geman and Nguyen (2005) and Trolle and Schwartz (2009).
2. Second, the tail and jump issues that seem to be so important in the literature drove many attempts to build models combining time-varying volatility, persistence through the convenience yield and jumps. Deaton and Laroque (1992) found empirical evidence that agricultural prices are agitated by jumps, while Duffie *et al*. (1995) also reported fat tails found in the dynamics of returns on commodities. Pindyck (2001) finds jumps both in the commodity prices and in the inventory levels. This triggered numerous theoretical contributions based on the continuous time finance models proposed in Brennan and Schwartz (1985), Gibson and Schwartz (1990), Schwartz (1997) and Schwartz and Smith (2000). An application to agricultural markets can be found in Sorensen (2002), and to natural gas in Manoliu and Tompaidis (2002). Hilliard and Reis (1998) wrote one of the first articles adding jumps to the model by Schwartz (1997). Deng (1999) brings jumps, mean-reversion and stochastic volatility together. Casassus and Collin-Dufresne (2005) also include explicitly discontinuous jumps in their model. Liu and Tang (2011) relate the convenience yield with its volatility. Dempster *et al*. (2010) propose a continuous time model that encompasses both short- and long-term jumps, highlighting how these aspects are important to the pricing of options on commodity futures.

We now turn to the analysis of descriptive statistics computed over a set of 22 commodities and of four sub-indexes from the Goldman Sachs Commodity Index (GSCI) universe. Table 1.1 presents the annualized returns over 1995–2012, as well as the volatilities, skewness, kurtosis, minimum and maximum returns, and the estimated autoregresssive parameters of an AR(1). We compare the results obtained for commodities to those obtained for other asset classes such as equities, currencies and interest rates over the same period. The main conclusions from Table 1.1 are:

– As explained in Kat and Oomen (2007a), the realized return and the volatility profile of commodities are very similar to what equities are capable of. The average return on the four GSCI sub-indexes ranges from −1.8% for agriculture to 9.6% for precious metals. This is very similar to the range of −4.1% (Nikkei) to 16.6% (Bovespa) found in our equity sample. The returns on commodities have been positive over the 1995–2012 period for most of the commodities, as well as for the sub-periods considered in the table. This is, however, not true for the agricultural products over the 1995–2003 period: during this period, the return on sugar was −10.4% for example. Positive returns have also been delivered by the various equity indices presented in the table, but for the Eurostoxx 50 case from 2003 to 2012. There is an ongoing debate about the existence of a risk premium in commodities that would be similar to what can be found in equities: for a large majority of them at least,

Table 1.1 Descriptive statistics on commodity, stock, currencies and rates

		Ann. Returns (%)			Volatility (%)					Extremes (%)		
		Total	1995–2003	2004–2012	Total	1995–2003	2004–2012	Skewness	Kurtosis	Min.	Max.	AR
Commodities	Gold	0.089	−0.002	0.188	0.168	0.13	0.199	0.067	7.352	−0.072	0.102	−0.017
	Silver	0.112	0.005	0.232	0.301	0.215	0.368	−1.119	10.509	−0.204	0.132	−0.022
	Platinum	0.079	0.063	0.095	0.224	0.21	0.237	−0.379	5.498	−0.097	0.1	0.042*
	Aluminum	0.002	−0.04	0.046	0.206	0.153	0.248	−0.325	2.753	−0.082	0.059	−0.026
	Copper	0.061	−0.06	0.198	0.268	0.185	0.331	−0.213	4.536	−0.104	0.119	−0.044*
	Nickel	0.04	0.005	0.077	0.363	0.285	0.428	−0.186	3.903	−0.181	0.131	0.009
	Zinc	0.033	−0.041	0.114	0.288	0.176	0.367	−0.3	3.517	−0.121	0.091	−0.025
	Lead	0.069	−0.037	0.186	0.312	0.187	0.4	−0.262	3.961	−0.128	0.127	0.055*
	WTI	0.109	0.061	0.16	0.358	0.346	0.37	−0.094	4.475	−0.165	0.219	−0.008
	Brent	0.122	0.054	0.196	0.341	0.334	0.347	−0.144	3.19	−0.144	0.129	−0.042*
	Gasoil	0.117	0.051	0.188	0.32	0.318	0.322	−0.165	2.661	−0.157	0.112	0.004
	Natural Gas	0.018	0.129	−0.082	0.548	0.566	0.529	0.388	5.717	−0.222	0.346	−0.016
	Heating Oil	0.113	0.05	0.179	0.347	0.344	0.349	−0.246	2.335	−0.14	0.103	−0.031*
	Corn	0.061	0.005	0.119	0.287	0.245	0.324	−0.71	15.701	−0.284	0.092	0.036*
	Wheat	0.03	−0.007	0.067	0.314	0.256	0.362	0.21	2.269	−0.1	0.11	0.002
	Coffee	0.004	−0.095	0.114	0.386	0.444	0.318	0.075	4.984	−0.15	0.212	−0.011
	Sugar	0.018	−0.104	0.158	0.35	0.326	0.372	−0.236	3.549	−0.154	0.143	−0.002
	Cocoa	0.032	0.03	0.034	0.311	0.298	0.324	−0.065	2.407	−0.1	0.107	−0.001
	Cotton	0.001	−0.029	0.032	0.297	0.274	0.319	−1.362	26.364	−0.34	0.09	0.028
	Soybean	0.059	0.006	0.114	0.249	0.208	0.284	−0.42	3.17	−0.099	0.065	−0.009
	Rice	0.05	0.015	0.085	0.278	0.284	0.273	0.279	19.437	−0.219	0.255	0.047*
	GSCI Agri.	−0.018	−0.042	0.006	0.197	0.146	0.237	−0.119	2.696	−0.075	0.072	0.024
	GSCI Energy	0.079	0.151	0.012	0.319	0.307	0.331	−0.223	2.197	−0.144	0.098	−0.019
	GSCI Ind. Metals	0.049	−0.031	0.136	0.226	0.147	0.284	−0.28	3.154	−0.09	0.076	−0.041*
	GSCI Prec. Metals	0.096	0.016	0.183	0.178	0.13	0.216	−0.143	6.363	−0.082	0.088	0.013
Equities	Dow Jones	0.074	0.109	0.04	0.191	0.186	0.196	−0.154	7.394	−0.082	0.105	−0.062*
	S&P 500	0.066	0.095	0.038	0.203	0.191	0.214	−0.233	7.578	−0.095	0.11	−0.07*
	Nasdaq	0.085	0.108	0.062	0.27	0.304	0.232	−0.054	4.729	−0.102	0.133	−0.021
	Canadian TSX	0.064	0.07	0.059	0.182	0.165	0.199	−0.704	8.949	−0.098	0.094	0.003
	Mexico IPC	0.177	0.148	0.206	0.255	0.285	0.222	0.052	6.181	−0.143	0.122	0.092*
	Brazil BOVESPA	0.166	0.158	0.174	0.364	0.419	0.301	0.458	12.751	−0.172	0.288	0.03
	Euro Stoxx	0.033	0.084	−0.016	0.236	0.239	0.233	−0.052	4.553	−0.082	0.104	−0.009
	FTSE	0.037	0.037	0.037	0.195	0.19	0.2	−0.155	5.69	−0.093	0.094	−0.022
	CAC 40	0.031	0.062	0.001	0.238	0.242	0.235	−0.016	4.422	−0.095	0.106	−0.013

(continued)

Table 1.1 (*Continued*)

		Ann. Returns (%)			Volatility (%)					Extremes (%)		
		Total	1995–2003	2004–2012	Total	1995–2003	2004–2012	Skewness	Kurtosis	Min.	Max.	AR
	DAX	0.07	0.064	0.077	0.248	0.266	0.229	−0.12	4.126	−0.089	0.108	−0.009
	IBEX	0.048	0.097	0.001	0.237	0.237	0.238	−0.023	5.139	−0.096	0.135	0.022
	MIB	−0.035	0.039	−0.105	0.251	0.239	0.262	−0.096	4.278	−0.086	0.109	0.005
	AEX	0.029	0.069	−0.01	0.238	0.25	0.226	−0.132	5.594	−0.096	0.1	0
	OMX	0.078	0.09	0.066	0.252	0.262	0.242	0.083	3.33	−0.085	0.11	−0.009
	SMI	0.05	0.085	0.016	0.199	0.209	0.188	−0.081	5.478	−0.081	0.108	0.033*
	NIKKEI	−0.041	−0.072	−0.009	0.246	0.241	0.252	−0.272	5.669	−0.121	0.132	−0.037*
	HANG SENG	0.059	0.039	0.079	0.279	0.289	0.267	0.093	9.271	−0.147	0.172	0
	ASX	0.051	0.065	0.037	0.16	0.131	0.184	−0.484	6.115	−0.087	0.057	−0.029
Currencies	Euro	0.004	−0.009	0.016	0.099	0.096	0.102	0.128	1.822	−0.026	0.039	0.005
	Canadian Dollar	−0.02	−0.001	−0.038	0.084	0.056	0.105	−0.162	5.984	−0.058	0.028	0
	Japanese Yen	−0.013	0.018	−0.043	0.111	0.117	0.105	−0.479	4.692	−0.063	0.034	−0.003
	Australian Dollar	0.017	−0.021	0.056	0.127	0.101	0.148	−0.68	12.189	−0.087	0.069	−0.018
	Hong Kong Dollar	0	0.001	−0.001	0.005	0.003	0.006	−2.711	44.815	−0.006	0.003	−0.051*
	Singapore Dollar	0.078	0.146	0.014	0.359	0.356	0.362	−0.889	12.402	−0.301	0.133	0.009
	New Zealand Dollar	0.014	−0.012	0.041	0.129	0.104	0.15	−0.3	5.343	−0.063	0.065	0.018
	British Pound	0.002	0.001	0.003	0.09	0.077	0.101	−0.068	4.26	−0.035	0.052	0.014
	Swiss Franc	−0.021	0.007	−0.049	0.113	0.11	0.116	0.224	8.122	−0.054	0.091	−0.028
	Swedish Krona	−0.006	0.014	−0.025	0.119	0.1	0.135	−0.242	3.583	−0.065	0.031	−0.023
	Norwegian Krone	−0.01	0.012	−0.031	0.12	0.097	0.14	−0.234	11.1	−0.094	0.082	−0.025
	Indian Rupee	0.03	0.048	0.014	0.056	0.041	0.067	0.274	10.529	−0.03	0.034	0.061*
	Vietnamese Dong	0.037	0.037	0.038	0.04	0.041	0.039	9.875	228.613	−0.04	0.065	−0.117*
	Brazilian Real	0.048	0.158	−0.049	0.158	0.149	0.166	0.671	18.853	−0.119	0.114	0.01
	Mexican Peso	0.057	0.098	0.019	0.143	0.169	0.11	−0.926	99.598	−0.207	0.137	−0.143*
	Polish Zloty	0.016	0.059	−0.026	0.133	0.096	0.162	0.159	5.154	−0.069	0.048	0.027
Rates	US 2	−0.43	−0.663	−0.197	0.545	0.308	0.706	−0.025	10.397	−0.301	0.303	−0.078*
	US 5	−0.406	−0.506	−0.305	0.387	0.241	0.491	−0.163	7.377	−0.23	0.146	−0.055*
	US 10	−0.342	−0.389	−0.294	0.263	0.191	0.319	−0.224	6.533	−0.171	0.105	−0.025
	US 30	−0.275	−0.307	−0.244	0.195	0.142	0.237	−0.255	6.212	−0.114	0.08	−0.006
	German 2	−0.372	−0.468	−0.274	0.531	0.2	0.724	−0.577	31.917	−0.409	0.409	−0.025
	German 5	−0.389	−0.457	−0.32	0.32	0.181	0.414	−0.364	12.079	−0.197	0.164	0.031*
	German 10	−0.345	−0.398	−0.293	0.203	0.136	0.254	−0.072	8.856	−0.136	0.113	0.045*
	German 30	−0.325	−0.35	−0.299	0.161	0.117	0.196	0.015	4.995	−0.076	0.064	0.084*

we find a positive annualized return over the three types of periods considered here. On this debate, see Kat and Oomen (2007a), Gorton and Rouwenhorst (2005b) and Erb and Campbell (2006).[1]

- Commodities are supposed to exhibit a volatility that is larger than those of the usual equity index. On this point, our figures agree with those from Kat and Oomen (2007a) – and despite the inclusion of the 2008 crisis in our sample we do not find that commodities' volatility is higher than equities'. On average, annualized commodity volatility ranges around 30%. Three singular cases must, however, be distinguished from the others: coffee (38.6% of annualized volatility), sugar (35%) and heating oil (54.8%). Beyond these cases, the rest of the figures look very similar to stock indices for emerging or developed equities.
- The skewness figures presented in Table 1.1 should help the reader gain some intuition about the potential asymmetries in the distributions of returns on commodities. Two conclusions arise from those figures. First, the sign of the skewness depends on the type of commodity considered: while in the case of gold (0.067) and wheat it is positive (0.21), the skewness associated with cotton is large and negative (−1.362). Equity indexes conversely are primarily affected by negative skewness, but for a couple of emerging markets such as Brazil and China. For example, the S&P 500 has a negative skewness over 1995–2012 that is equal to −0.233. A similar case can be made out of the interest rate figures: the skewness obtained from the variations of the 5-year rate is equal to −0.577. When considering the results obtained from the foreign exchange rates, we obtain a picture that is very close to what is obtained from the commodity dataset: the skewness can take various signs. For example the Australian Dollar vs. the US Dollar has a skewness equal to −0.479, whereas the Euro vs. US Dollar has a skewness equal to 0.128. The US Dollar vs. the Polish Zloty has a skewness equal to 0.159, whereas the US Dollar vs. the Mexican Peso has a skewness equal to −0.926. In this respect, the commodities – considered as an asset class – appear closer to the currencies than to any other asset classes presented here. A second conclusion from this table is related to the scale of the skewness value: despite a few extreme values, the absolute value obtained from the commodities looks very similar to what is obtained from any other asset class. In this respect, the asymmetry of commodities is very close in terms of magnitude to the rest of the financial markets. The main difference here is that the sign of the asymmetry looks asset-specific.
- Turning to the kurtosis analysis, two conclusions again should be drawn from the table. First, when considering individual commodities, we find large kurtosis. This is in line with the previously quoted articles such as Kat and Oomen (2007a) emphasizing that the main difference between commodities and the rest of the asset classes lies in the extreme events found in the variations of the prices of raw materials. Their kurtosis ranges between 2.269 for coffee and 26.364 for cotton. On average each of these kurtose are higher than 3, the threshold to be reached for the empirical distribution to depart from the thin tails obtained from a Gaussian distribution. The magnitude of these kurtose is broadly speaking in line with the figures obtained on the equity side, yet with a higher degree of heterogeneity. In this respect, it is again closer to the currency markets for which we obtain high variations in kurtosis from one currency to the other. The magnitude of the kurtosis obtained with the basket of currencies considered here is, however, much higher than the one obtained from

[1] Finding a positive return for most of the commodities over the period considered here is not proof that commodity holders receive a risk premium for being long of such markets. Part II of this book will cast light on the possible macroeconomic fundamentals explaining positive or negative performances of such markets.

the commodity dataset. The second conclusion from the kurtosis computations is reached when comparing the results obtained from individual commodities and from the baskets of GSCI indices: the kurtosis associated to the latter is, on average, lower than the one computed from part of its components. For example, the WTI has a kurtosis equal to 4.475 whereas the GSCI Energy sub-index has a kurtosis equal to 2.197. A similar pattern is obtained from the GSCI Agricultural sub-index: its kurtosis is equal to 2.696 when cotton has a kurtosis equal to 26.364, and rice a kurtosis equal to 19.437. This has to be related to one of the key stylized facts about commodities: the weak correlation between them, even amongst a given commodity sector. We will discuss figures around this issue later.

– A last point must be mentioned when analyzing the basics of returns on commodities: following Kat and Oomen (2007a) and a very prolific literature that we will detail later, commodities are known to be affected by a high degree of persistence. In other words, commodities are known to exhibit sharp trends that were one of the reasons for the development of the well known trend-following industry that tries to benefit from trends in financial markets. A first way to gauge these persistent trends is to estimate a regression of the following type:

$$r_t^i = \phi_0 + \phi_1 r_{t-1}^i + \epsilon_t^i, \tag{1.1}$$

where r_t^i is the daily logarithmic return on the commodity i, ϵ_t^i is a random disturbance with an expectation equal to 0 and standard deviation equal to σ^i. ϕ_0 and ϕ_1 are real-valued parameters that can be estimated by Ordinary Least Squares (OLS).[2] The last column of Table 1.1 presents such estimates along with an asterisk for each parameter significantly different from zero. Out of the 22 estimates, only eight are different from zero. To observe persistent trends, we need to have ϕ_1^i positive: this is only the case for platinum, lead, corn and rice. These numbers are obtained by using daily returns that are thus less persistent than weekly or monthly returns. Still, when comparing these results to those obtained in the case of other assets, we have trouble finding sharply different conclusions. In the case of equity, we find two significant and positive parameters (Mexico IPC and SMI) and three negative and statistically significant ones (Dow Jones, S&P 500 and Nikkei) out of the 18 indices considered here. A similar picture is obtained in the currency case. The case of interest rates is a bit different: for these series, we have five out of eight series that yield significant estimates. From these preliminary estimates, we fail to find a picture as striking as the one presented by Kat and Oomen (2007a): over the past 15 years, there is limited evidence of a higher persistence in commodities than in other asset classes.

This preliminary analysis casts light on the key aspects we are going to focus on in the coming pages: the nature and the number of trends in commodity markets, the origin of the asymmetry in returns on commodities and finally the jump activity in commodities. These seem to be the aspects for which our preliminary analysis pointed out differences between commodities and the usual asset classes. The next section deals with the complex relationships between returns on commodities and the term structure of futures. This question has been the center of much of the academic attention over the past 30 years. We revisit this problem, as it is one of the keys to forecasting returns on raw materials. We move then to an extensive trend analysis in commodities, of the asymmetry in returns, and finally of the tail activity observed over the past 20 years.

[2] We refer any reader interested in these time series models to Box *et al*. (2008).

1.1 BACKWARDATION, CONTANGO AND COMMODITY RISK PREMIUM

Beyond the themes that will be analyzed in the coming pages, a large part of the academic literature has been devoted to the understanding of the existence of a slope in commodity futures. Basically, futures are financial contracts that entitle the buyer (respectively the seller) to buy (sell) a given amount of a certain asset at a price that is set in advance for a given maturity. Unlike options, which allow holders to exercise or not the contract, futures involve a commitment to deliver or to buy the underlying asset. These futures contracts are actively used by commodity traders – and their clients – either to hedge future flows or to speculate over the future stance of a given market. In the case of equities, this slope is solely driven by the risk-free rate through arbitrage arguments. The case of commodities is unclear: commodity futures are bought both by producers and buyers of such products to hedge their natural exposure to market fluctuations. For example, when an oil-producing company wants to hedge – i.e. wipe out the risk in its balance sheet that is purely related to the fluctuation of oil prices: its exposure – it can decide to sell futures six months in advance in order to know exactly at what price it will be able to sell its planned production in the future. On the other side of the market, a company that needs to secure the price of its buying of raw products can decide to buy such futures. Depending on the balance of hedgers – buyers and sellers – the slope of the term structure of futures would be upward or downward. When this slope is upward, market participants say that the market is in a contangoed position. Conversely, when the term structure of future prices is downward sloping, the market is said to be in backwardation.

Although this problem is of little relevance for investing in commodities,[3] it still matters from a financial economics point of view. What is more, when the trading of commodities involves the actual delivery of the underlying asset, this term structure of commodities implies some sort of a ‘risk premium’; that is, the fact that it is possible to buy, for example, a given amount of raw product for a future price that will be below the actual spot price on the day of the settlement of such futures. Several theories have tried to explain the existence of such a slope. Keynes (1930) developed a theory of ‘normal backwardation’: in a world where risk-averse commodity-producing companies are the main market participants, their need to hedge price risk should drive future prices lower. By doing so, the future price of commodities should be structurally lower than their spot prices, and such markets should be regularly backwarded. A side effect of this theory is that by buying futures and selling the spot asset, an investor would be able to generate a profit: this potential profit is usually regarded as a ‘commodity risk premium’. However, as shown in Table 1.2, such an average pattern simply does not exist: different commodities have different slopes, and through time a given commodity can either be backwarded or contangoed. This table presents the results obtained when computing $s(t,T)^i$ the future curve’s slope for asset i:

$$s(t,T)^i = \frac{F(t,T)^i}{S(t)^i} - 1, \tag{1.2}$$

where $S(t)^i$ is the spot price at time t for commodity i and $F(t,T)^i$ the corresponding future with a residual maturity equal to $T-t$. We use three different generic futures contracts,

[3] When a private or an institutional investor wishes to have an exposure to the commodity universe through futures, this regularly requires rolling the position from a future with a maturity that turned out to be short to a longer dated future. As pointed out in Gorton and Rouwenhorst (2005a), this rolling procedure of futures has, by construction, no impact on the performance of an investment in commodities provided that the net amount of this investment remains unchanged.

Table 1.2 Average difference between the 3, 6 and 9 month futures and the spot price of commodities expressed as percentages of the spot price

	All sample			1995–2003			2003–2012		
	3M slope	6M slope	9M slope	3M slope	6M slope	9M slope	3M slope	6M slope	9M slope
Aluminum	0.37*	0.66*	0.84*	0.32*	0.58*	0.68*	0.41*	0.74*	1*
Brent	−0.08*	−0.23*	−0.44*	−0.67*	−1.36*	−2.03*	0.52*	0.89*	1.15*
Cocoa	1.2*	2.27*	3.28*	1.56*	2.95*	4.37*	0.84*	1.58*	2.2*
Coffee	1.5*	2.97*	4.36*	0.68*	1.65*	2.69*	2.31*	4.3*	6.04*
Copper	−0.09*	−0.21*	−0.36*	0	−0.09*	−0.19*	−0.18*	−0.33*	−0.53*
Corn	2.32*	4*	5.16*	1.8*	3.39*	4.59*	2.84*	4.61*	5.73*
Cotton	1.82*	3.19*	5.14*	1.98*	3.42*	4.57*	1.67*	2.95*	5.7*
Gasoil	0.05*	0.11*	0.18*	−0.18*	−0.33*	−0.47*	0.28*	0.55*	0.82*
Gold	0.25*	0.24*	0.42*	0.29*	0.7*	0.65*	0.2*	−0.22	0.19*
Heating Oil	0.19*	0.29*	0.33*	−0.24*	−0.42*	−0.61*	0.61*	1.01*	1.27*
Natural Gas	1.93*	3.32*	4.37*	0.56*	0.79*	0.96*	3.29*	5.85*	7.78*
Nickel	−0.12*	−0.29*	−0.61*	−0.08*	−0.17*	−0.5*	−0.16*	−0.4*	−0.72*
Rice	2.05*	3.8*	5.23*	2.34*	4.55*	6.45*	1.77*	3.05*	4.01*
Silver	0.26*	−0.05	1.1*	0.32*	0.66*	1.21*	0.2*	−0.76*	1*
Soybean	0.2*	0.09	−0.16	0.11*	1.00E-02	−0.1	0.29*	0.17*	−0.22
Sugar	−0.21*	−0.67*	−0.95*	−1.49*	−2.54*	−2.89*	1.06*	1.2*	1*
Wheat	2.62*	4.14*	5.36*	2.14*	3.5*	4.67*	3.1*	4.77*	6.04*
WTI	−0.06	−0.22*	−0.42*	−0.83*	−1.6*	−2.31*	0.71*	1.17*	1.47*

Note: An asterisk indicates that the average is statistically different from zeros at a 5% risk level.

ranging from 3 to 9 months by periods of three months. This provides us with a dataset of slopes expressed in terms of percentage increases over the spot price for three maturities: 3, 6 and 9 months. By doing so, we can bring some statistics not only around the 3 month slope as is generally the case, but also check whether the sign of the slope is consistent across maturities.

This table confirms previous results: commodities are both affected by backwardation and contango. For example, aluminum exhibits on average an upward sloping future curve: every 3 months of maturity increase leads on average to a future price higher by 0.3% over the period considered here (1995–2012). Conversely, Brent is typically a commodity for which the future slope is negative: 3 months of additional maturity lead to a future price lower by 0.1 to 0.2%. Out of the 18 commodities reported here, only 5 of them have been backwarded on average over the period. Consistent with that, Kolb (1992) investigated 29 commodity futures, finding that there is no ‘normal backwardation’. Bodie and Rosansky (1980) ended up with a similar conclusion. What is more, over the full period, the sign of the slope is consistent across the three selected maturities. One of the only exceptions is silver over the 2003–2012 period: its 6-month slope is significantly negative (−0.76%) whereas its 9-month slope is significantly positive (1%). Finally, the sign of the futures slope can change depending on the period: for example, heating oil has a positive slope over the 1995–2003 period, and a negative one over the subsequent period. This holds across all maturities of the futures on heating oil considered here. A similar case can be made with sugar.

A natural way out of this conundrum is to assume that commodity producers are not the only hedgers intervening in such markets. Cootner (1960) and Deaves and Krinsky (1995) have formulated the 'hedging pressure hypothesis': depending on whether hedgers are net long or net short, this slope of the term structure can either be negative or positive. For example, Bessembinder (1992) found that over the 1967–1989 period, the return on futures was influenced by the net position of hedgers. With this theory, there is a commodity risk premium, and its sign depends on the net hedging pressures: when producers are dominant, the risk premium is positive, as buying futures and selling the spot asset should deliver a positive return to the holder. Conversely, when commodity consumers are the main hedgers, the risk premium should be negative overall. The evidence presented in Table 1.2 is somewhat more consistent with the conclusions of this theory, as it makes it possible to have both upward and backward sloping futures curves.

Finally, a third theory attempts to explain the existence of such a slope. The 'theory of storage' links the level and cost of commodity inventories to the shape of the futures curve. We owe this theory to Kaldor (1939) and Brennan (1991): it tries to explain why inventories are observed in periods of downward-sloping futures curves, as such a pattern implies a future spot price that should be lower than the current level and therefore a lower nominal value of the inventories held. Holding inventories helps in handling the varying demand: disruptions on the production chain would have a limited impact on the ability to meet the global demand. This stock buffer improves somewhat the comfort of the commodity producer, hence generating a 'convenience yield'. However, by doing so, the producer has now to face a market risk, linked to the fluctuations of the market price of its commodity. Such a risk is higher when storage is low: for such a case, the convenience yield should be very important and the term structure of futures downward sloping so as to provide the inventories holder with a positive risk premium. Conversely, when inventories are high and the convenience yield is therefore low, the term structure of futures should be upward sloping, merely reflecting the interest rates paid when borrowing cash to build the storage space and the actual cost of storage. Gorton *et al*. (2012) provide an empirical assessment of the impact of inventories over 31 commodity futures curves: as they point out, accessing such a dataset is difficult, especially over extended periods such as theirs.[4] They conclude that inventories have a strong explanatory power over the 'basis' of many commodities; that is, the difference between the first future and the spot price of each commodity. Inventories seem to robustly predict the sign and magnitude of risk premium in commodities, whereas the net position of traders – that measures where the hedging pressure is – has limited – if any – explanatory power. This long-standing debate is, however, still in discussion: here, the length and depth of datasets matters.

Beyond the potential explanations of such a phenomenon, there is one interesting question to be raised and answered here: a large part of the literature expects that the risk premium earned from holding commodities can be explained by the slope of these futures curves. Let $r^i_{(t,t+h)}$ be the return realized over the t to $t+h$ period by holding asset i. Table 1.3 displays the following correlations:

$$\mathrm{cor}(r^i_{(t,t+h)}, s(t,T)^i). \tag{1.3}$$

In the case of Table 1.3, h is equal to 3 months.[5] When there should be a relation between the term structure of futures and the expected returns on a given commodity, this relation should

[4] They study commodity risk premium over the 1969–2006 period, therefore limiting the impact of shorter datasets on the estimation results.

[5] The results presented here are, however, weakly dependent over the choice of this period. With this 3-month period, we simply put a larger emphasis on the stylized fact we are trying to measure.

Table 1.3 Correlation between rolling 6-month returns and the 3 to 9 month slopes

	3M slope	6M slope	9M slope
Aluminum	0	−0.03	−0.01
Brent	0.04*	0.05*	0.05*
Cocoa	0.03*	0	−0.01
Coffee	0.09*	0.1*	0.09*
Copper	0.05*	0.05*	0.05*
Corn	0.32*	0.28*	0.18*
Cotton	0.23*	0.27*	0.76*
Gasoil	0.12*	0.13*	0.14*
Gold	−0.18*	0.04*	0.03*
Heating Oil	0.14*	0.16*	0.19*
Natural Gas	0.28*	0.38*	0.4*
Nickel	0.06*	0.05*	0.02
Rice	0.45*	0.42*	0.36*
Silver	−0.02	0.01	0.23*
Soybean	0.34*	0.38*	0.36*
Sugar	0.29*	0.44*	0.36*
Wheat	0.14*	0.12*	0.09*
WTI	0.1*	0.09*	0.09*

be negative: a negative slope implies a positive return on average obtained from buying the future and selling the spot asset. From Table 1.3, we get the impression that this correlation is, however, more positive than negative. We obtain a negative correlation only in four cases, and only one of them is significantly different from zero. For the rest of the cases, this correlation is significantly positive, implying that a positive slope forecasts a positive return on commodities. What is more, the scale of this correlation ranges between 0 and 0.2 for most of the cases, which is rather low for a correlation. There are, however, four commodities for which this correlation is higher: corn, cotton, soybean and sugar. Beyond them, the correlation remains weak but significant. Hence, the commodity risk premium is poorly explained by the term structure of futures, and the sign of the relationship goes against the theory that the risk premium is negatively correlated to the slope of futures.

Cochrane and Piazzesi (2005) found that the term structure of futures has a forecasting power over the realized future variation of the underlying interest rates. By regressing those realized variations over a basket of futures with various time to maturities, they found 'tent-shaped' coefficients across futures' maturities. Combining these futures through these coefficients, they obtained a new factor that explains one third of one-year ahead excess returns. Their finding has been confirmed in Kessler and Scherer (2009) and Sekkel (2011) for non-US markets. One way to reconcile the relationship between the slope of the term structure of futures and the realized performance of the spot asset is to run a regression similar to Cochrane and Piazzesi (2005). Within this approach, the slope is assumed to contain elements that forecast future returns. The slope would therefore be driven in part by financial market participants' expectations. By using previous notation, we run the following regression:

$$r^i_{(t,t+h)} = \alpha^i_0 + \sum_{j=1}^{3} \beta^i_j s(t, T_j)^i + \epsilon(t)^i, \tag{1.4}$$

with $\epsilon(t)^i$ being a centered disturbance with volatility σ_ϵ and T_j for $j = 1, 2, 3$ being the various maturities for the slopes considered here. Here, we consider various h, from 1 week to 6 months. Given the overlapping nature of our sample, the asymptotic volatility for the OLS estimates of the previous regression has to be modified. Following Cochrane and Piazzesi (2005), we rely on a Newey–West approach to the robust estimation of those volatilities. Results are presented in Table 1.4 along with R^2. Figures 1.1, 1.2, 1.3 and 1.4 chart the β_j across futures maturities obtained with the 3-month returns, when both the realized returns on the spot asset and the slopes of the future curve have been scaled to make the parameters comparable across commodities. Our results confirm that of Cochrane and Piazzesi (2005) in terms of interest rate futures: in the case of agriculture and energy, we find tent-shaped β_j^i across maturities.[6] Most of the estimated parameters are found to be significantly different from zero at a 5% risk level. In the 3-month case, those regressions come with an R^2 that is greater than 0.1 for 6 of the 18 cases considered here, confirming that the slope of the futures curve contains information that can explain the commodity risk premium. From these results, it appears that the relation between the commodity risk premium is more complicated than the previous theories predicted. Interestingly, despite the non-financial aspect of such assets, we still find properties that are consistent with what is usually found for the standard assets. By comparing the results obtained for various h, we cast light on the dependency of our results upon the period over which the returns are computed. From the analysis of Table 1.4, when increasing the period over which the returns are computed, we obtain a growing explanatory power of this simple regression. For example, in the case of cotton, the R^2 associated with this regression is equal to 0.06 in the 1 week returns case, and to 0.663 in the 6-month case. Hence, following these regressions, the slope of the term structure of commodity futures can incorporate information that helps predict the commodity risk premium. Figures 1.1, 1.2, 1.3 and 1.4 clearly display tent-shaped parameters across maturities. The forecasting power of this regression does not seem to be as important as the one obtained in the bond market case. However, for some of the markets investigated here, we obtain an R^2 that can reach 0.6, as in the case of cotton. However, the variability of this R^2 is higher than in the bond case. Such results tend to show that the term structure of futures is a variable of interest to investors, as it expresses at least partly participants' expectations – as for other purely financial assets.

1.2 UNDERSTANDING COMMODITIES' MOMENTA

This section will focus on the measurement of trends in commodities. Trends are one of the backbones of the quantitative fund management industry: 'trend followers' are funds whose main strategy is to invest in assets with positive trends. These trend-following strategies all started under the label of Commodity Trading Advisors (CTAs), investing primarily in commodities. The name has remained, but the scope of investment possibilities has increased, extending to futures and options written on any type of asset. Still, commodities may have been the birthplace of the trend-following industry. When reading the main conclusions appearing in Kat and Oomen (2007a; 2007b) regarding the persistence of trends, the consistency between these trend-following methods and the persistence found in commodities definitely makes sense. The objective of this section is to assess the nature of these trends in commodities through their measurement.

[6] Unlike Cochrane and Piazzesi (2005) who use forward rates from 2 to 10 years, here we can only rely on the most liquid short-term contracts.

Table 1.4 Regression results of the 1-week to 6-month returns over the slopes of the future curve

	1-week returns				1-month returns				3-months return				6-months return			
	3M slope	6M slope	9M slope	R^2	3M slope	6M slope	9M slope	R^2	3M slope	6M slope	9M slope	R^2	3M slope	6M slope	9M slope	R^2
Aluminum	0.003	−0.001	0	0.001	0.007*	−0.004	0	0.002	0.002	−0.009*	0.004	0.002	0.011	0.002	−0.005	0.001
Brent	−0.004	0.004	−0.001	0	−0.017	0.033*	−0.016*	0.003	−0.065*	0.061*	−0.017	0.006	0.049	−0.129*	0.078*	0.014
Cocoa	0.004*	−0.003	0.001	0.001	−0.002	0.009*	−0.005*	0.002	0.033*	−0.016*	−0.002	0.013	0.06*	−0.005	−0.02*	0.037
Coffee	−0.002	0.005*	−0.003*	0.004	−0.021*	0.035*	−0.017*	0.03	−0.039*	0.064*	−0.03*	0.041	−0.073*	0.079*	−0.025*	0.046
Copper	0.001	0.007*	−0.005*	0.003	−0.004	0.013	−0.006	0.004	−0.007	0.006	0.002	0.003	−0.067*	0.027	0.013	0.007
Corn	0.002*	0.001	−0.001*	0.012	0.008*	0.001	−0.003*	0.051	0.014*	0.011*	−0.008*	0.124	0.008*	0.02*	−0.005*	0.195
Cotton	0.002*	−0.001*	0.001*	0.061	0.007*	−0.005*	0.003*	0.205	0.006*	−0.009*	0.008*	0.624	−0.014*	−0.002*	0.012*	0.663
Gasoil	0.001	0	0	0.003	0.012	−0.012	0.007*	0.014	0.031*	−0.064*	0.039*	0.029	0.111*	−0.215*	0.114*	0.046
Gold	−0.009*	0	0	0.003	−0.04*	0*	0.002*	0.021	−0.09*	0.001*	0.002*	0.04	−0.148*	0.001*	0.001	0.051
Heating Oil	0.004	−0.006	0.003*	0.009	0.014*	−0.016*	0.009*	0.028	0.04*	−0.065*	0.037*	0.052	0.042*	−0.092*	0.054*	0.035
Natural Gas	0.001	0.003*	−0.002*	0.041	−0.004*	0.015*	−0.007*	0.132	−0.018*	0.015*	0.002*	0.174	−0.002	−0.003	0.01*	0.08
Nickel	0.006	−0.001	−0.001	0.001	0.013	0.002	−0.007*	0.002	0.076*	0.023*	−0.039*	0.019	0.133*	0.045*	−0.063*	0.032
Rice	0.002*	0	0	0.033	0.009*	−0.002*	0.001	0.124	0.011*	0.005*	−0.001	0.207	0.005*	0.001	0.009*	0.221
Silver	−0.005*	0*	0.003*	0.16	−0.002	0	0.004*	0.062	−0.02*	0	0.006*	0.053	−0.058*	0	0.007*	0.041
Soybean	0.002*	0.001	−0.001*	0.03	0.011*	0.003	−0.003*	0.109	−0.009*	0.022*	−0.004*	0.149	0.017*	−0.029*	0.03*	0.18
Sugar	0.001*	0.002*	−0.001*	0.029	−0.001	0.012*	−0.006*	0.126	−0.013*	0.029*	−0.01*	0.248	−0.028*	0.032*	0	0.297
Wheat	0.002*	0	0	0.006	0.004*	0.001	−0.001	0.016	0.006*	0.001	−0.001	0.02	0.003	0.004	−0.001	0.019
WTI	−0.007	0.014	−0.007	0.003	−0.025	0.054*	−0.028*	0.012	0.07*	−0.06*	0.019	0.012	0.157*	−0.19*	0.081*	0.015

Note: The first three columns present the estimated parameters and the final column displays the R^2 otbained with the full regression. An asterisk indicates a statistically significant relation at a 5% risk level. We used a Newey–West estimation to the parameters' volatilities.

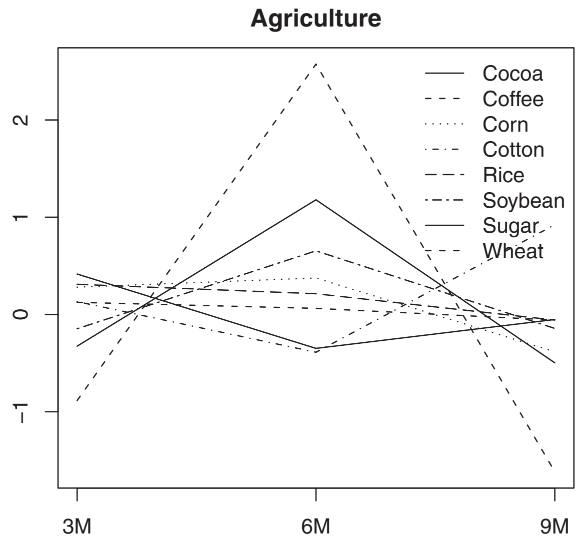

Figure 1.1 Tent-shaped regression for agriculture products

When it comes to analyzing trends in commodities, a first question must be raised and answered: should we consider working on the prices or on the returns of financial assets? Following Ghoshray (2011a), there seems to be mixed evidence of unit roots in commodity prices: returns are usually considered as they are, by construction, trend-stationary. Prices are the results of the combination of positive and negative trends: the combination of those two types of trends is not always trend-stationary. Hence the diagnostic of trend-stationary prices depends on the dataset used, as the various conclusions obtained in the literature demonstrate.

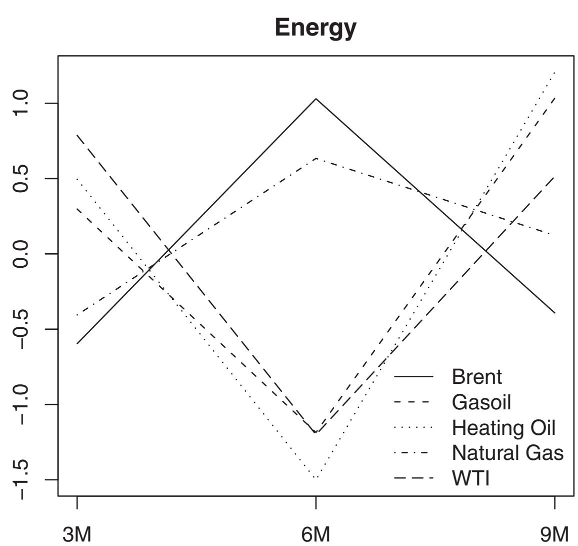

Figure 1.2 Tent-shaped regression for energy products

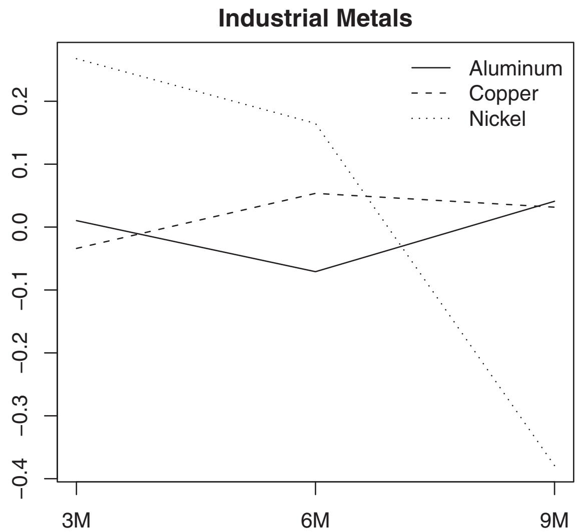

Figure 1.3 Tent-shaped regression for industrial metals

For example, Sapsford (1985), Grilli and Yang (1988), Helg (1991) and Ardeni and Wright (1992) found evidence that commodities are trend-stationary. Conversely, Cuddington and Urzua (1989), Cuddington (1992), Bleaney and Greenaway (1993) and Newbold *et al*. (2005) used empirical tests showing that commodities are difference-stationary. This question of stationarity will be investigated in Part III of this book. Here, we aim to consider commodities as any other financial asset: in this respect, we will work in this section with returns on commodities. Let p_t^i be the logarithm of the price of asset i at the closing market session at

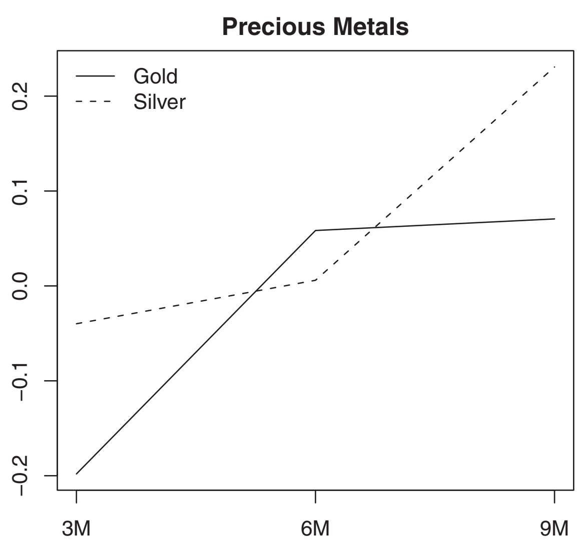

Figure 1.4 Tent-shaped regression for precious metals

time t. The return obtained from investing in asset i between the date $t-1$ and t r_t^i is therefore calculated as follows:

$$r_t^i = p_t^i - p_{t-1}^i. \tag{1.5}$$

In a similar fashion to the presentation in Ghoshray (2011a; 2011b), a trend in the asset i can be represented in the following way:

$$p_t^i = a_0^i + a_1^i t + \epsilon_t^i, \tag{1.6}$$

where ϵ_t captures the deviation from the trend. The sign of a_1^i decides the nature of the trend. Moving back to returns, this equation turns out to be

$$r_t^i = a_1^i + \epsilon_t^i - \epsilon_{t-1}^i. \tag{1.7}$$

When a_1^i is positive (respectively negative), the asset is driven by a positive (negative) trend. The first three columns of Table 1.1 displayed estimates for a_1^i across commodities from 1995 to 2012, using daily data. Obviously, the sign of long-term trends varies depending on assets. For example, from 1995 to 2003, the price of sugar seems to have been affected by a negative trend, while the price of natural gas has been affected by a positive trend. A large literature tries to assess whether a constant sign affects the price of commodities. Ghoshray (2011b) examines the Prebisch–Singer hypothesis[7] that commodity prices should be decreasing over the long run, using recent advances in unit root testing in the presence of multiple breaks. While Leon and Soto (1997) found that 17 commodities were affected by negative trends, Ghoshray (2011b) only found seven negatively trending commodities. Cuddington (1992) found that five out of 22 commodity prices had negative trend commodities over the 1900–1983 period – and that 16 of them were diagnosed as trendless. Wickham and Reinhart (1994) found a long-term negative trend that is shared by many commodities. Bleaney and Greenaway (1993) found that the sign of trends actually depends on the kind of commodities investigated. More recently, Cashin and McDermott (2002) found evidence of persistent negative trends in commodities over 140 years.

The main issue here is that these trends have a low probability of being stable: worldwide business cycles, political events or technical innovations can have various effects on these trends that are difficult to foresee. This section is not focused on the rationale behind these trends: we will discuss those points later in the book. Table 1.1 gives some hints regarding the variation of these trends. The question now is to discuss how persistent these trends are – and how many types of trends affect the dynamics of commodities.

Before turning to this, a necessary question to be answered is how useful to investors this kind of knowledge is. In a recent contribution, Miffre and Rallis (2007) present evidence that trend-following strategies perform well for commodities, consistent with previous results on the equity market. Similar findings are obtained in Fuertes *et al*. (2008). Erb and Campbell

[7] The Prebisch–Singer conjecture states that commodity prices should exhibit a persistent negative trend relative to manufactured products. Different explanations have been proposed: first, unions in the manufactured product sector are stronger than unions in the production of raw products – essentially because these products come from developing countries – implying that in return the price of manufactured products will have to be increased more often than commodities' in reaction to an increasing labor cost. A second explanation states that the sensitivity of the demand for commodities to a wage increase is lower than those of manufactured products, implying in turn that commodity prices should increase at a slower pace than manufactured products. These are the two explanations proposed respectively by Prebisch (1950) and Singer (1950). It seems that the Prebisch–Singer conjecture is also used as a loose designation for a negative long-term trend in commodities.

(2006) use a momentum strategy that invests in the best-performing commodities against the worst-performing ones. Irwin *et al*. (1997) showed how a channel trading system makes it possible to generate abnormal returns on soybean over the 1984–1988 period. Lukac *et al*. (1988) obtained similarly a positive return from various technical trading rules. Szakmary *et al*. (2010) obtained positive returns from trend-following strategies from 22 out of 28 futures on commodities. The basics of such investment approaches should push investors to hold a long position (respectively short) into commodities with a positive (negative) past performance. Despite the apparent simplicity of the approach, many issues need to be solved, such as the horizon over which to compute the past performance of each asset – that is the past momentum. There are a lot of alternative investment strategies that track this momentum effect in commodities, such as filters, moving average or channel breakouts: the trend-following methods are in fact strongly related to technical analysis. Here again, evidence of the presence of a stronger momentum in commodities is, however, not fully consistent. Marshall *et al*. (2008), for example, do not find that quantitative market timing strategies are consistently profitable over 15 commodity futures. Wang and Yu (2004) find that short-term contrarian strategies deliver a positive return to investors. The underlying issue here is, of course, to be able to estimate trends from financial returns and to discuss how stable these trends are.

Building on Equation (1.6), there are two potential origins of trends in assets:

1. Trends can be understood as a natural tendency for returns to have one particular sign, regardless of the period considered or of the frequency used to compute it. This is measured in Equation (1.6) by a_1^i.
2. Beyond that, $\eta_t = \epsilon_t - \epsilon_{t-1}$ can be affected by a positive autocorrelation – that is to say that when η_t goes through a shock, this shock will have long-lasting effects on the returns themselves. This implies that when $\eta_t > 0$, η_{t+1} stands a higher chance of being the same sign although with a more limited scale. Understanding the origin of trends requires disentangling these two potential origins.

In this section, we use time series models to estimate and characterize the trends and momenta in commodities – and we compare these results to similar estimates of standard assets.

1.2.1 Persistence of Shocks in Commodities

A first simple and intuitive approach to gather evidence that commodities exhibit persistent trends is to compute the autocorrelation of returns of order h; that is, for a given asset i:

$$\rho_h^i = \frac{cov\left(r_t^i, r_{t-h}^i\right)}{V\left(r_t^i\right)}, \tag{1.8}$$

where $cov(.,.)$ stands for covariance and $V(.)$ for variance. An asset with persistent returns will stand a greater chance of being affected by trends: when receiving a positive shock over a given period of time, an asset with positively autocorrelated returns will stand a greater chance of displaying a performance of a similar sign over the following periods.[8] We compute these autocorrelations using a dataset covering the 1995–2012 period, using closing returns at daily, weekly and monthly frequencies. Table 1.5 presents such estimates along with a

[8] On this point, see the standard forecasting procedure of an Autoregressive (AR) model as presented in Box *et al*. (2008).

Table 1.5 Autocorrelation coefficient over various assets from 1995 to 2012

	Daily			Weekly			Monthly		
	1995–2012	1995–2003	2004–2012	1995–2012	1995–2003	2004–2012	1995–2012	1995–2003	2004–2012
Gold	−0.02	−0.02	−0.02	−0.01	0.02	−0.03	−0.16*	−0.08	−0.23*
Silver	−0.02	−0.05*	−0.02	0.01	−0.02	0.02	−0.12	−0.25*	−0.09
Platinum	0.04*	0.02	0.06*	0.03	−0.03	0.07	0.2*	0.04	0.26*
Aluminum	−0.03	0.03	−0.05*	−0.03	0	−0.04	0.14*	−0.18	0.26*
Copper	−0.04*	0.04*	−0.07*	0	0.11*	−0.04	0.19*	−0.15	0.29*
Nickel	0.01	0.02	0.01	−0.01	−0.01	−0.01	0.07	−0.04	0.11
Zinc	−0.02	−0.01	−0.03	−0.06	−0.02	−0.06	0.02	−0.11	0.04
Lead	0.06*	0.05*	0.06*	−0.08*	−0.05	−0.08	0.08	−0.15	0.11
WTI	−0.01	0.03	−0.04	−0.07*	−0.05	−0.09*	0.07	−0.06	0.21*
Brent	−0.04*	−0.02	−0.07*	−0.04	−0.03	−0.06	0.17*	0.04	0.3*
Gasoil	0	0.02	−0.01	0.01	0	0.02	0.12	0.04	0.21*
Natural Gas	−0.02	−0.01	−0.02	0	0.06	−0.05	−0.02	−0.03	−0.01
Heating Oil	−0.03*	−0.03	−0.03	−0.04	−0.04	−0.04	0.08	−0.02	0.19*
Corn	0.04*	0.03	0.04	−0.07*	−0.14*	−0.04	−0.01	0.01	−0.03
Wheat	0	0.04	−0.02	−0.03	−0.06	−0.02	−0.08	−0.05	−0.1
Coffee	−0.01	−0.01	−0.01	−0.03	−0.06	0.03	−0.18*	−0.16	−0.23*
Sugar	0	0.01	−0.01	0.03	0.08	−0.01	0.1	−0.02	0.17
Cocoa	0	−0.02	0.02	0	0.02	−0.02	−0.2*	−0.14	−0.27*
Cotton	0.03	−0.02	0.06*	0.04	0.04	0.03	−0.06	−0.16	−0.01
Soybean	−0.01	0	−0.01	0.01	−0.08	0.07	0.01	0.11	−0.03
Rice	0.05*	0.03	0.06*	0.03	0	0.05	−0.12	−0.13	−0.11
GSCI Agri.	0.02	0.06*	0.01	−0.01	−0.01	−0.01	0	0	−0.01
GSCI Energy	−0.02	0	−0.03	0	0.03	−0.02	0.18*	0.08	0.27*
GSCI Ind. Metals	−0.04*	0.04*	−0.06*	−0.03	0.02	−0.05	0.19*	−0.19	0.29*
GSCI Prec. Metals	0.01	−0.01	0.02	−0.03	0	−0.04	−0.19*	−0.15	−0.23*
DOW JONES INDUS. AVG	−0.06*	0	−0.11*	−0.08*	−0.08	−0.08	0.05	−0.05	0.17
S&P 500 INDEX	−0.07*	−0.01	−0.12*	−0.08*	−0.1*	−0.06	0.11	0	0.24*
Euro Stoxx 50 Pr	−0.01	0.01	−0.03	−0.06	0.02	−0.13*	0.12	0.07	0.18
HANG SENG INDEX	0	0.02	−0.03	0.01	0.03	−0.02	0.07	0.02	0.14
EUR-USD X-RATE	0	−0.02	0.03	0.01	−0.01	0.03	0.01	0.06	−0.02
USD-CAD X-RATE	0	−0.01	0	−0.01	−0.04	0	−0.08	−0.01	−0.1
USD-JPY X-RATE	0	0.02	−0.03	−0.05	−0.03	−0.1*	0	0	−0.02
AUD-USD X-RATE	−0.02	0.01	−0.03	−0.04	−0.04	−0.05	0.05	0.02	0.06
British Pound Spot	0.01	−0.02	0.04	−0.02	0	−0.03	0.04	−0.17	0.17
Swiss Franc Spot	−0.03	−0.04*	−0.02	−0.01	−0.02	−0.01	−0.06	0.03	−0.13

Note: * denotes a statistically significant autocorrelation at a 5% risk level.

breakdown for the 1995–2003 and 2004–2012 periods. By comparing results over these various periods and frequencies, we will be able to obtain some hints about the persistence of market episodes. Should ρ_h^i be positive and significant at a 5% risk level, this provides evidence that asset i exhibits persistence. Conversely, when ρ_h^i is negative and significant, asset i has a mean-reverting behavior, rapidly correcting its trajectory in case of a large positive or negative return.

As in the previous tables, we compare the results obtained in the case of commodities to those obtained with more traditional assets. From Table 1.5, there is almost no persistence in the returns of standard assets: the only case for ρ_h^i positive and significant is obtained in the case of S&P 500 at a monthly frequency for the 2004–2012 period. The rest of the significant figures are negative, unveiling some mean-reverting properties in financial returns. Turning to commodities, the results are clearly different: we obtain a higher number of significant figures – not necessarily positive. At daily and weekly frequencies, we respectively find a statistically significant autocorrelation for 7 and 3 of the commodities in our dataset – not necessarily the same ones. Weekly returns that have a statistically significant autocorrelation have a negative sign at a weekly frequency, implying some mean-reversion properties. When breaking down the samples into subsamples, we find limited evidence of stability of persistence at weekly or daily frequencies. Now, turning to monthly data, we do find a higher number of positive autocorrelations that are different from zero. Platinum, aluminum and copper are fair examples: a positive shock of these has a greater chance of being long-lasting when compared, for example, to zinc. However, here again, the stability of such figures is rather limited.

From Table 1.5, we are able to reach two conclusions:

1. Weak evidence exists that the alleged trends in commodities come from persistent shocks. When one of these commodities goes through a shock – such as a shortage on production for various reasons – there is no statistical evidence that this shock should turn into a persistent trend.
2. Still, Table 1.1 presented figures showing that commodities have, on average, a non-zero performance, whose sign can change depending on the period. This implies that a proper way to track these unobservable trends is to use a proper time series model that would allow us to measure how commodities switch from one trend to another over the years.

Markov switching (MS hereafter), as initially presented in Hamilton (1989), makes this possible: in the next section, we will make use of it to evaluate the presence of changing trends in commodities. We will then be able to diagnose whether commodities exhibit more persistent trends than traditional assets.

1.2.2 The Nature of Momentum in Commodity Markets

Now[9] that persistence can be somewhat discarded as a source of trends in commodities, we move on to measuring trends dynamically. Trends are now considered as being made of a combination of an expected such return as a_1^i in Equation (1.7) and a corresponding volatility. This question is well known and discussed in the case of equity markets: bull, bear and range trading periods are usually disentangled. A bull market is a market phase during which expected returns are positive and volatility is low. A bear market is, on the contrary, characterized by

[9] We are thankful to Mathieu Gatumel for his valuable help on the building of this section.

a strongly negative return and a higher volatility. A range trading episode corresponds to a period for which the market has no clear trend, ranging in a low volatility environment between two levels. For an investigation of such phases, see, for example, Maheu *et al*. (2012). When it comes to commodities, there is no clearly identified evidence of the existence of bull, bear or range trading episodes, despite the obvious interest of the trend-following industry in such empirical works. These different types of trends underlie the momentum strategies of CTA funds, from which they are trying to gain a benefit. The key issue here is, of course, to be able to estimate these market phases, and above all to be able to determine how many of them are usually necessary for commodity markets. Markov switching models, as presented in Hamilton (1989), are typically the kind of time series modeling approach that makes such a task achievable. These models had significant success in the financial industry, as they made it possible to accommodate the intuition accumulated so far in terms of trends and momenta within a simple statistical model. In the coming sections, we present the MS model along with the estimation of the number of regimes required to model commodities.

1.2.2.1 A Brief Presentation of the Markov Switching Model

We provide the reader with a short presentation of Hamilton's (1989) Markov switching model. This model was initially introduced in the literature by focusing on the US business cycle. Its use to estimate the regimes[10] driving financial markets has since been developed in various articles such as Chauvet and Potter (2000), Ang and Bekaert (2002) or more recently Maheu *et al*. (2012). This time series model aims at modeling and estimating the changes in regimes that affect different kinds of economic series. It relies on the assumption that the probability of moving from one regime to another varies over time, while the transition probabilities[11] are constant. Ang and Beckaert (2002) present an asset allocation strategy based on an MS(2) model, underpinning the economic performance of such a model when compared to a single regime one. Maheu and McCurdy (2000) present a variation of an MS(2) model that provides evidence about the duration of each market cycle. Chauvet and Potter (2000) use this modeling approach to build coincident and leading indicators. However, most of these articles focus on equity markets: investigating returns on commodities by using this modeling approach should therefore provide us with very interesting insights.

We present the basic intuition of using a two-regime MS model before turning to a more general case. Let r_t be the logarithmic return on a given asset at time t, for the holding period between $t-1$ and t. Let s_t be an integer value variable that is equal to 1 (respectively 2) at time t if regime 1 (respectively 2) prevails in the economy. Given that the regime i prevails, the conditional distribution of returns is as follows:

$$r_t | s_t \sim N(\mu_i, \sigma_i). \tag{1.9}$$

The probability to be in regime 1 at time t can be written as:

$$P(s_t = 1) = P(s_t = 1 | s_{t-1} = 1) \times P(s_{t-1} = 1) + P(s_t = 1 | s_{t-1} = 2) \times P(s_{t-1} = 2). \tag{1.10}$$

[10] As explained later, a trend or a momentum in this respect can either be considered a regime, or a combination of regimes.
[11] Transition probabilities are the fixed probabilities of switching from one regime coming with certainty from another.

$P(s_t = 1|s_{t-1} = 1)$ is assumed to be constant and equal to p, and $P(s_t = 2|s_{t-1} = 1) = 1 - p$. With a similar argument, $P(s_t = 2|s_{t-1} = 2) = q$ and $P(s_t = 1|s_{t-1} = 2) = 1 - q$. These transition probabilities can be gathered into a transition matrix as follows:

$$\Pi = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}, \tag{1.11}$$

such that

$$P_t = \Pi P_{t-1}, \tag{1.12}$$

with $P_t = (P(s_t = 1), P(s_t = 2))^\top$. The parameters driving the model are thus the moments associated with asset returns for each state and the matrix Π. The usual estimation strategy is a maximum likelihood one, based on the filtering approach developed in Hamilton (1989). This two-regime case can be generalized to an n-regime one: in such a case, s_t can take integer values ranging from 1 to n, and the Π matrix becomes an $n \times n$ matrix.

It would be possible to consider that r_t follows a more sophisticated conditional model, like a switching GARCH model. Nevertheless, as presented in Aingworth *et al*. (2006), increasing the number of states in the Markov chain allows us to obtain a sequence of models that gets closer to a stochastic volatility model. In addition to that, the unconditional distribution of an MS model is a mixture of Gaussian distributions – as presented in Hamilton (1989). This distribution is consistent with fat tails and the asymmetry usually found in financial return datasets (see Bertholon *et al.*, 2007). As straightforward as our modeling approach looks, it is still consistent with the stylized facts of financial returns while being parsimonious – an essential feature when one tries to test for a higher number of regimes, given the rapidly growing estimation difficulties when this number increases. Beyond this, the weak evidence of persistence in shocks on commodities or standard assets does not make it necessary to add an autoregressive parameter to the conditional means for each state.

1.2.2.2 Testing for the Number of Regimes in an MS Model

Beyond the insight regarding the time series dynamics of returns, one of the main issues with the MS model is to select the proper number of regimes. This is all the more important as our primary interest here lies in providing evidence around the nature of trends in commodity markets. Gatumel and Ielpo (2011) present a test around this issue making it possible to estimate the number of regimes in a financial time series. A large part of the literature assumes that markets are driven by two types of trends:

1. a trend up – with a low volatility in the case of equities, and
2. a trend down – with a large volatility.

Uptrends are known as ‘bull’ markets and downtrends are ‘bear’ markets. Many articles assume that two regimes are enough to correctly capture the evolution of the main equity indices. See, for example, Henry (2009), Al-Anaswah and Wilfling (2011) or Dionne *et al*. (2011). Maheu *et al.* (2012) and Gatumel and Ielpo (2011) present evidence that in financial markets there are more regimes than just bulls and bears.

Before turning to the presentation of the test, we need to understand what can be expected from this 'more than two regimes' approach. The cross-asset empirical analysis built by Gatumel and Ielpo (2011) shows that different kinds of regimes can be obtained:

– The first kind of regime is a 'trend regime', i.e. up or down regimes. In this kind of market configuration, prices are either trending upward or downward – which is why these can be labeled trend regimes. The volatility associated with such regimes can be of various magnitudes: in the equity case, the uptrend usually comes with a low volatility, unlike the downtrend. Both these regimes last the longest of all regimes, as measured from the diagonal of the transition matrix.
– The second kind of regime is a regime that captures extreme or tail events in financial time series. The associated expectation and volatility – once annualized – are usually very large. However, such events do not last for long, being associated with a low diagonal element of the probability matrix.
– Finally, two to three regimes can have a low diagonal element in the transition probability matrix, but they can be part of a sequence of events. For example, one regime with a positive trend and one regime with a negative trend can form a sequence. Returns will go from one to another of these regimes very rapidly: prices will then rise and fall over short periods. By doing so, MS models are able to mimic a key market momentum, i.e. trendless market episodes with small-scale oscillations. Practitioners usually refer to such episodes as 'range trading' as prices evolve between a low and a high range.

These are the three types of trends found in the literature. The global estimations performed in Table 1.1 are just the result of the combination of these types of trends. The literature provides us with various tests to estimate the number of relevant regimes driving the time series. Three types of tests can be found in the literature:

1. penalized likelihood tests,
2. Kullback–Leibler distance based tests, and
3. tests based on the empirical likelihood surface.

We do not put much emphasis on these approaches, as the interested reader will find an up-to-date literature review in Gatumel and Ielpo (2011). Psaradakis and Spagnolo (2003) consider methods based on complexity-penalized likelihood criteria. Smith *et al*. (2006) present a Kullback–Leibler divergence based criterion. Finally, Gatumel and Ielpo (2011) use a density-based approach. When discriminating between one and two regime situations, both Psaradakis and Spagnolo (2003) and Smith *et al*. (2006) provide accurate results. However, when this number of regimes increases beyond three, the accuracy of the estimation of the number of regimes can drop sharply. The test proposed by Gatumel and Ielpo (2011) provides accurate estimates over the Monte Carlo tests, which is why we focus on this approach here.

Let $f_{n_1}(r_t; \hat{\theta}_{n_1})$ be the likelihood function associated with an estimated Markov-switching model with n_1 states. Let $f_{n_2}(r_t; \hat{\theta}_{n_2})$ be a similar quantity in the case of an MS model with n_2 regimes. θ_{n_i} is the vector of the parameters to be estimated by maximum likelihood in the n_i-regime case. The two specifications are compared through their associated log density computed with the estimated sample. Let $z_t^{n_1,n_2}$ be the following quantity:

$$z_t^{n_1,n_2} = \log f_{n_1}(r_t; \hat{\theta}_{n_1}) - \log f_{n_2}(r_t; \hat{\theta}_{n_2}) \tag{1.13}$$

The approach proposed here is based on the following test statistics:

$$t_{n_1,n_2} = \frac{\frac{1}{T}\sum_{t=1}^{T} z_t^{n_1,n_2}}{\hat{\sigma}_{n_1,n_2}}, \tag{1.14}$$

where T is the total number of available observations in the sample used to estimate the parameters, and $\hat{\sigma}_{n_1,n_2}$ a properly selected estimator of the standard deviation of $\frac{1}{T}\sum_{t=1}^{T} z_t^{n_1,n_2}$. Several methods have been proposed in the literature to estimate the standard deviation. We use a Newey–West (1987) estimator with different lags.

Under the null hypothesis that both models provide an equivalent fit of the returns' distribution, Theorem 1 in Amisano and Giacomini (2007) provides the asymptotic distribution of this test statistic:

$$t_{n_1,n_2} \sim N(0,1). \tag{1.15}$$

The correct number of regimes is the one that delivers the best fit from the previous statistics point of view, while being as parsimonious as possible. We retain the optimal number of regimes, $\hat{n}$, such that : $t_{\hat{n},\hat{n}_{n-1}} \geq q_\alpha$ and $t_{\hat{n}_{n+1},\hat{n}_n} < q_\alpha$, with q_α being the quantile for a given risk level of α%. Monte Carlo simulations show that this risk level should be set to 1%.

1.2.2.3 Estimation Results

We use the previous estimation scheme for the number of regimes using a dataset of daily closing returns over January 1995 to April 2012. This dataset includes various commodities, commodity indices, equity indices, currencies and interest rates. With this dataset, we first estimate the correct number of regimes necessary to adequately describe the behavior of each asset.

Figure 1.5 summarizes these results, charting the sorted number of regimes for each asset. When commodities exhibit marked trends (up or down), the test simply diagnoses that a two regime model is enough. When just two regimes alternate, a trend-following mechanism could generate abnormal returns – assuming that the persistence of each regime is high enough. When the number of regimes is higher than two to three, abnormal returns are likely to fade out, as range trading periods prevent the trend-following mechanism from capturing the correct trend.

The results reproduced present globally mixed evidence that trends in commodities are stronger than those in equities or currencies, for example. Still, we find that in the cases of WTI, heating oil, corn, wheat, cocoa and cotton, a model with three regimes is enough to capture the key trends. Among the nine series for which we find that a three regime model is accurate, eight are series from the commodity universe: on top of the six commodities previously mentioned, the GSCI indices for the agricultural and energy sectors also match this type of market behavior. In the meantime, rice should be characterized by six different regimes: behind the 5% annualized return over the 1995–2012 period presented in Table 1.6, there are many different market episodes behind this single figure.

Globally speaking, out of our subset of 21 commodities, six are characterized by two regimes, six by four regimes, eight by five regimes and just one is characterized by six regimes. The period considered must indeed have a large influence over such results, but when comparing the results presented in Gatumel and Ielpo (2011) the basic intuitions remain:

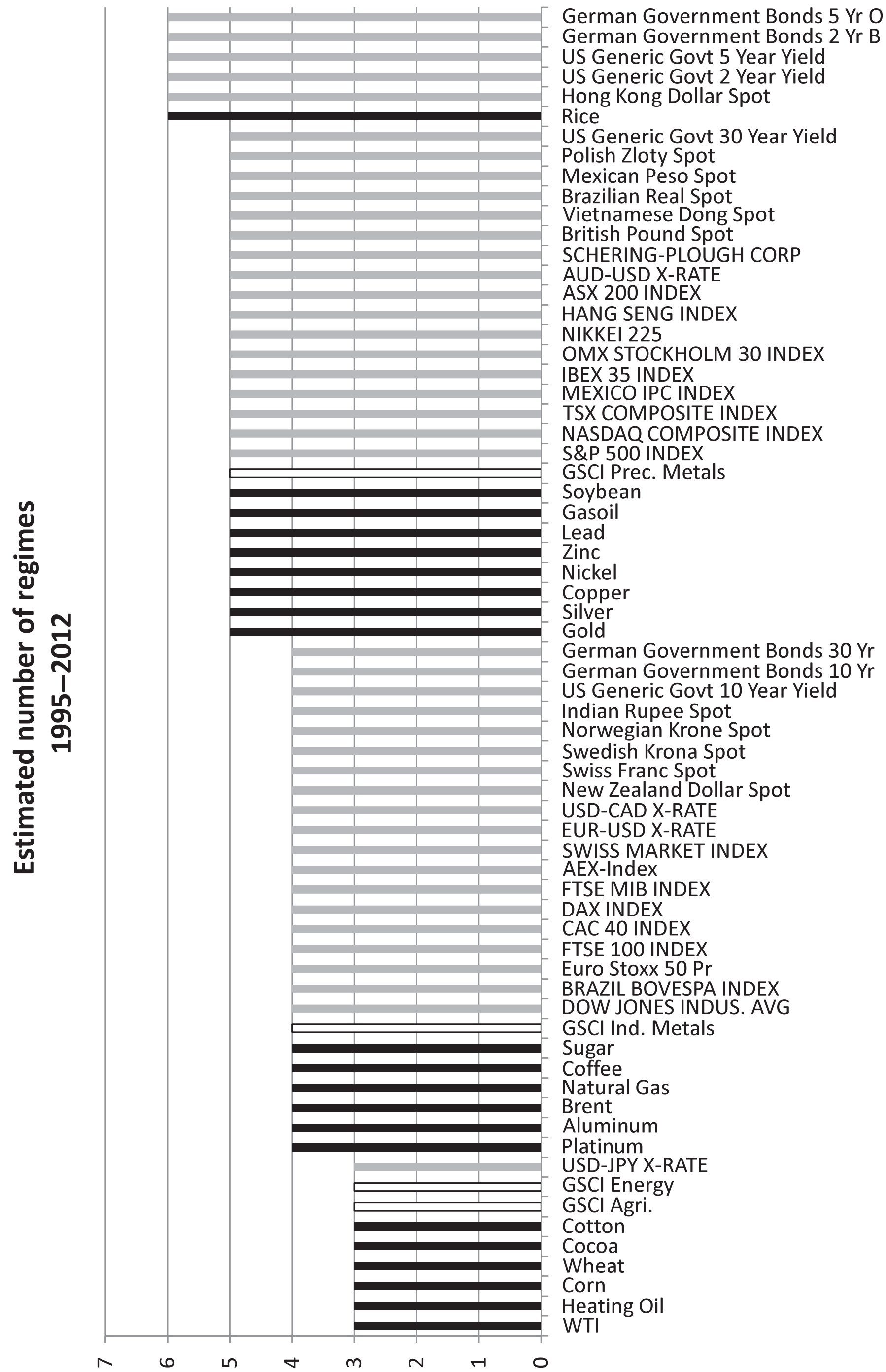

Figure 1.5 Number of regimes in commodities

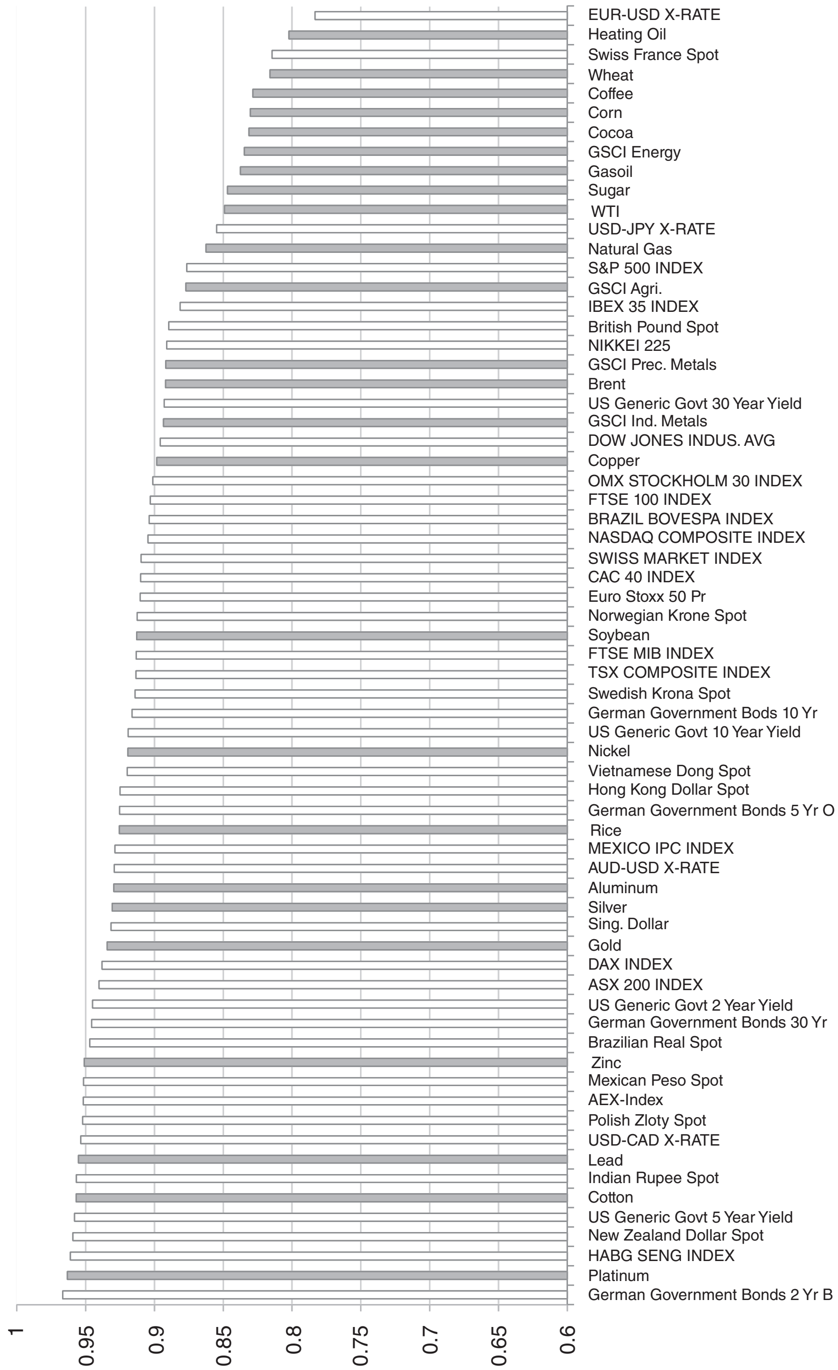

Figure 1.6 Sorted average persistence of regimes across assets

Table 1.6 Annualized expectations and volatilities across regimes

		GSCI Agri.	GSCI Energy	GSCI Ind. Metals	GSCI Prec. Metals	S&P 500	10Y US Rate
Regime 1	μ_1	0.064	−0.733	−0.050	1.962	−3.439	−0.155
	σ_1	*0.266*	*0.528*	*0.117*	*0.113*	*0.126*	*0.109*
Regime 2	μ_2	−0.075	0.507	−0.354	−0.066	0.261	−0.126
	σ_2	*0.130*	*0.253*	*0.420*	*0.075*	*0.091*	*0.275*
Regime 3	μ_3	0.079	−6.240	0.064	−1.360	2.998	0.062
	σ_3	*0.266*	*0.245*	*0.167*	*0.228*	*0.134*	*0.202*
Regime 4	μ_4			0.185	0.057	−0.315	−0.014
	σ_4			*0.237*	*0.344*	*0.423*	*0.529*
Regime 5	μ_5				−0.350	0.053	
	σ_5				*0.101*	*0.109*	

commodities are among the assets with the lowest number of regimes – that is with the strongest trends – and energy commodities typically display a low number of regimes.

When turning to the case of the commodity indices, we find three regimes for agriculture and energy, four for industrial metals and five for precious metals. Table 1.7 as well as Figures 1.7 to 1.12 provide additional details about the nature of these regimes. We detail here each of these cases and then compare them to the case of the S&P 500 and the US 10-year rate:

- As noted previously, the agriculture and the energy GSCI indices share the same number of regimes. An analysis of Table 1.7 gives us hints regarding the nature of these regimes. For both cases there are two very persistent regimes – which are regimes whose corresponding value on the diagonal of the transition matrix presented in Table 1.7 are very close to 1. Both these two persistent regimes' expectations have an opposite sign. In the case of the GSCI agricultural sub-index, regime 1 has a 6.4% annualized performance for an annualized volatility equal to 26.6%. When entering such a market regime, the probability of remaining in this kind of regime is 0.99. Regime 2 has a negative expected return (−7.5% for a volatility equal to 13%), and the probability of staying in this regime is 0.96. Thus, on average, the most common regime over the 1995–2012 period is one for which agricultural prices have been rising with a larger volatility. Drops in agricultural prices also occured, but with a lesser volatility. This is clearly the opposite of what is usually found in equities. The main risk for agricultural products comes from bad harvests which have a tendency to pull prices higher, with a greater amount of volatility. The last regime (regime 3) is thus a regime with a much weaker persistence (0.32), a higher expected return of 7.9% and a volatility equivalent to regime 1. When entering this type of regime, the probability that the market enters a downtrend – i.e. regime 2 – is estimated to be equal to 0.68. This type of regime should be regarded as a tail event: a stronger than expected return that is on the risk side of the market when prices rise. However, from Figure 1.7, such a regime never received a probability higher than 50%. Hence, it has been selected to help fit the tails of the returns on such an index.
- The structure of momenta in the GSCI energy index is very similar to what we uncovered from our estimations when using the agricultural index. The energy index is also characterized by two persistent trends, one positive with an expected return of 50.7% (and a

Table 1.7 Transition matrices

	GSCI Agri.					GSCI Energy				
	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5
Regime 1	0.99	0.01	0.00			0.95	0.05	0.00		
Regime 2	0.00	0.96	0.04			0.00	0.95	0.04		
Regime 3	0.01	0.68	0.32			0.09	0.80	0.11		
Regime 4										
Regime 5										
	GSCI Ind. Metals					GSCI Prec. Metals				
	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5
Regime 1	0.37	0.00	0.63	0.00		0.22	0.00	0.03	0.01	0.75
Regime 2	0.00	0.97	0.00	0.03		0.02	0.97	0.00	0.00	0.00
Regime 3	0.60	0.00	0.40	0.00		0.30	0.04	0.32	0.00	0.34
Regime 4	0.00	0.01	0.00	0.99		0.01	0.00	0.01	0.98	0.00
Regime 5						0.37	0.00	0.29	0.00	0.34
	S&P 500					US 10Y Rate				
	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5	Regime 1	Regime 2	Regime 3	Regime 4	Regime 5
Regime 1	0.07	0.00	0.20	0.03	0.69	0.61	0.00	0.38	0.00	
Regime 2	0.02	0.98	0.00	0.00	0.00	0.00	0.99	0.00	0.01	
Regime 3	0.06	0.07	0.15	0.00	0.72	0.55	0.00	0.45	0.00	
Regime 4	0.00	0.00	0.03	0.97	0.00	0.00	0.02	0.00	0.98	
Regime 5	0.34	0.00	0.30	0.00	0.36					

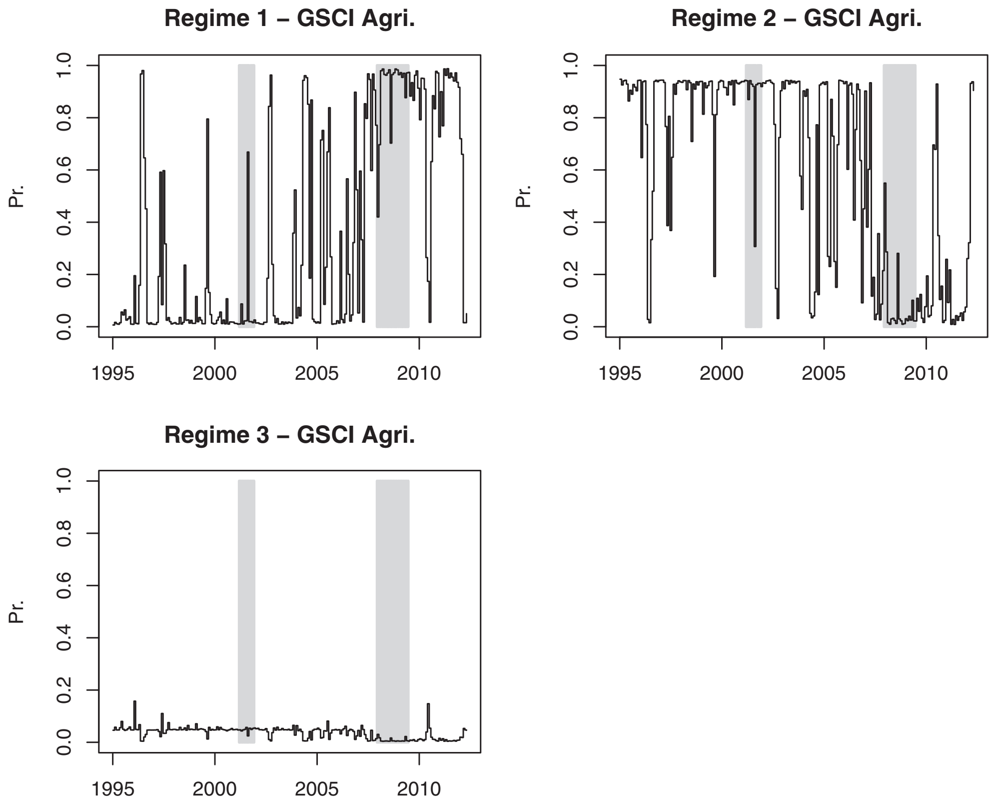

Figure 1.7 Time series of regimes in GSCI Agriculture

corresponding volatility of 25.3%) and one negative with an expected return of −73% (for a volatility of 52.8%). Contrary to agricultural markets, the regime with the highest volatility matches the negative return. From Figure 1.8, such a trend typically occured in 2008 during the sharp drop in energy prices over the third and fourth quarters of the 2008 crisis. These two regimes are highly persistent, as their corresponding probabilities on the diagonal of the transition matrix are close to 1. As in the agricultural case, a third regime is selected that accounts for a tail event: regime 3 has an expected return equal to −624%: when entering such regime on a given day, the probability of staying in this regime is equal to 11%. This is more of a one-day event, that is usually followed by a market episode of regime 2 type. As in the case of agriculture, we find here that trends are strong and of only two varieties, which in turn should imply positive and abnormal returns for the trend-following industry. Again, these results depend on the period covered in the dataset.

– Industrial metals have a structure different from trends. As in the two previous cases, there are two main persistent trends in industrial metals: one is a positive trend with an expected return of 18.5% (for a volatility of 23.7%), and one is negative with an expected return of −35.4% (for a volatility of 42%). These two trends are very persistent, as in previous cases. The largest volatility is obtained for the trend affected by a negative return. Beyond these two, regimes 1 and 3 articulate one with another: when in regime 1 (with an expected

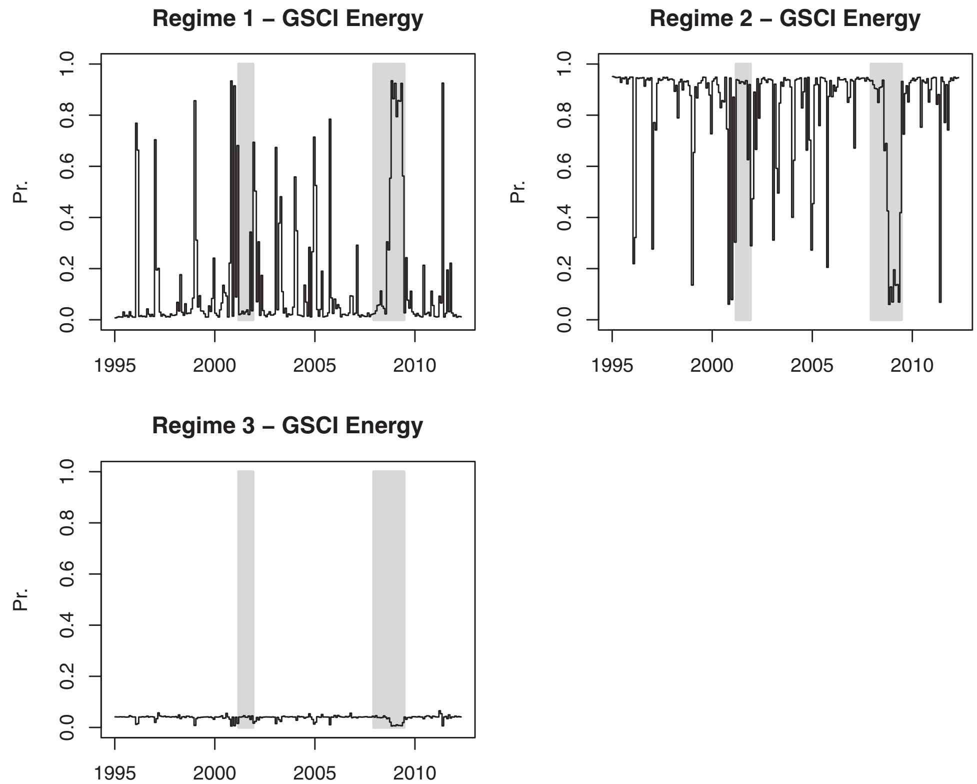

Figure 1.8 Time series of regimes in GSCI Energy

return of −5%), the market statistically stands a 60% chance of going to regime 3. Coming from regime 3, the market now stands a 40% chance of switching back to regime 1. By switching from one of these regimes to another, the MS model simply reproduces a period of range trading, with typically a low average expected return – and an even lower return for trend-following strategies, given the number of switches from one regime to the other.

– Finally, the most complex structure of trends is obtained for the commodity group celebrated for its high diversification effect (Hillier *et al.*, 2006). In the case of precious metals – as represented by the GSCI sub-index – five regimes are estimated over the 1995–2012 period. As for previous cases, two regimes with expected returns of an opposite sign are found. In the case of regime 2, the expected return is −6.6% (for a volatility of 7.5%), and in the case of regime 4, this expected return is 5.7% (for a volatility of 34.4%). From Figure 1.10, this regime matches the 2008 crisis period: the expected return is not that high but volatility is large. Beyond these two regimes, regimes 1, 3 and 5 articulate with each other to create a chain of extreme variations: their respective expected returns are 196%, −136% and −35%. Their persistence is very low: the probability when entering each of these regimes of remaining in this type of market trend is below 35%. Still, the estimated joint articulation can be read in the transition matrix in Table 1.7: regimes 1 and 5 are related, and regimes 3 and 5 as well, creating two kinds of very volatile range trading patterns. Here again, this type of configuration is a drag on the returns on trend-following strategies.

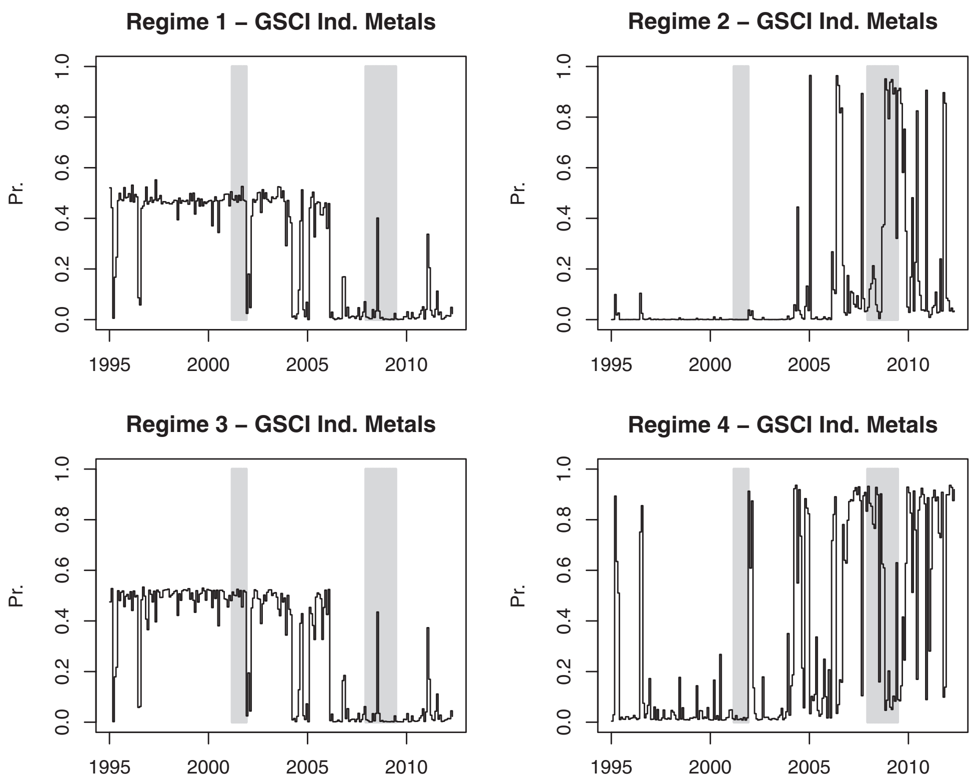

Figure 1.9 Time series of regimes in GSCI Industrial Metals

Finally, when comparing these results to those obtained for the S&P 500 and the US 10-year rate, we find similarities and key differences. First, as for every commodity, these standard assets exhibit two persistent regimes with expected returns of an opposite sign. The rest of the regimes are either used to account for extreme events or for range trading situations. There are two major differences between commodities and standard assets:

1. In the case of commodities, the average number of regimes is lower than for standard assets. This should pledge for higher returns of trend-following strategies when they are using commodities as an investment vehicle.
2. However, a second difference appears from our empirical analysis: Figure 1.6 presents the sorted average persistence of regimes; that is, the average of the diagonal of the transition matrix. When analyzing how persistent regimes are on average, we clearly see a strong heterogeneity across commodities. While gold, platinum, zinc and cotton are characterized by persistent trends, sugar, cocoa, corn and coffee exhibit the lowest degree of persistence, materializing the Erb and Campbell (2006) statement that commodities are indeed a heterogeneous asset class that can hardly be regarded as a whole.

In the next section, we discuss how these characteristics impact the performance of trend-following strategies, as these are widely used and applied to commodities in the financial industry.

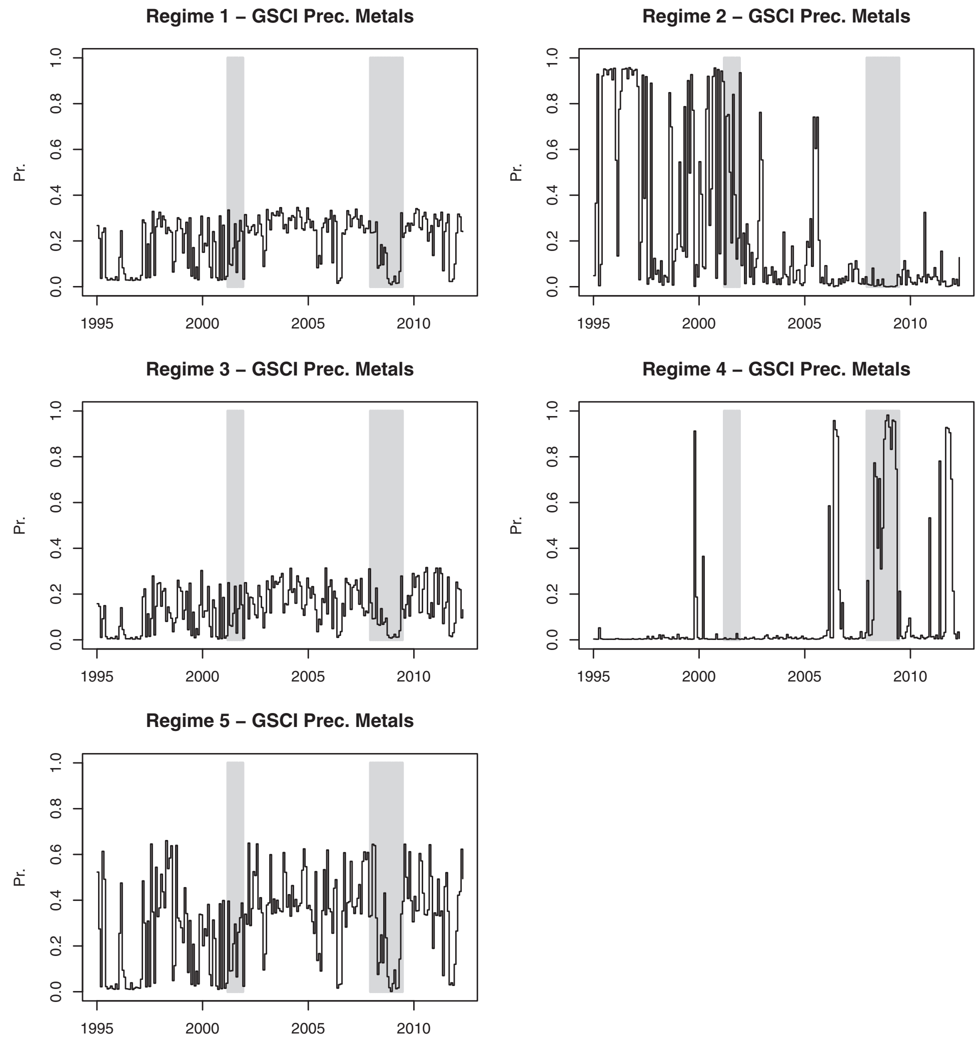

Figure 1.10 Time series of regimes in GSCI Precious Metals

1.2.3 Time Series Momentum and the Number and Nature of Regimes

Based on the previous analysis, there are only weak chances that commodities would be a more suitable asset class for trend-following strategies: the nature of trends in these markets is very close to those observed in other asset classes. Yet, a thorough analysis is required here to judge whether commodities can deliver similar risk-adjusted returns to trend-followers. We focus our research here on the concept of 'time series momentum', as developed in Moskowitz *et al.* (2012).

1.2.3.1 Time Series Momentum

Momentum strategies are one of the most well-known strategies on equity markets: they amount to overweighting stocks that were top performers in the past, and underweighting those with the lowest past returns. Such a strategy had a tendency to generate positive returns – ‘abnormal returns’ – which should be arbitraged away by market participants. This cross-sectional effect in the stock market has been largely documented in the academic literature. On this point see, for example, Jegadeesh and Titman (1993) and Asness (1994) for the US equity case and the extension to other asset classes presented in Asness *et al.* (2010).

Moskowitz *et al.* (2012) present evidence of another type of momentum effect that is closer to the typical CTA investment strategy: they find persistence in the performance of a wide variety of futures, including stocks, bonds, currencies and commodities. When a given future delivers a positive (respectively negative) performance over past months, it stands a greater chance of also having a positive (negative) performance in the upcoming months, creating persistence in the performance. It generates, in turn, again an ‘abnormal’ return when applying a trend-following strategy.[12] To show this effect, they use the following investment strategy: they propose to hold a long (respectively short) position in assets having experienced a positive (negative) performance. The return from such a strategy $r_t^{\mathrm{strat},i}$ at time t obtained from asset i is computed in the following way:[13]

$$r_t^{\mathrm{strat},i} = \mathrm{sign}(r_{t-12,t}^i)\frac{40\%}{\sigma_t^i}r_t^i, \tag{1.16}$$

where $r_{t-12,t}^i$ is the return on asset i over the past 12 months, as in their article they use monthly data. The investment strategy has a constant risk exposure, with a targeted volatility of 40%. In order to reproduce their results with our sample, we use a slightly different estimate of volatility. Instead of theirs, we use a 30-day rolling estimate: beyond the appeal of a simple and widely used estimate in the industry, our results should be seen as a robustness check of theirs. To this strategy, we add a second one, still belonging to the trend-following methodologies: a breakout investment strategy. Again, this is a simple strategy that aims at capturing trends in financial asset prices. The logic is as follows: starting from a neutral position, an investor would take a long (respectively short) position in a given asset i when this asset’s price reaches a value that is higher (lower) than the maximum over a given period of time. Once the investor enters the position, it is held until the converse signal is observed: such strategies are sometimes referred to as ‘stop-and-reverse’ strategies. Similarly to the time series momentum strategy, there is no neutral positioning once the investor is in the market. For additional development on such strategies, see, for example, the recent paper of Clare *et al.* (2012). In the same spirit as the time series momentum strategy, we consider the minimum and maximum computed over the past 12 months of data: by doing so, we hope to make the two trend-following strategies as comparable as possible.

1.2.3.2 Empirical Results

Running the previous investment strategies over our sample[14] of commodities, stock indices, currencies and rates, we obtain the results presented in Table 1.9 for the time series momentum

[12] It remains an abuse to think that such an effect can be ‘arbitraged away’ by market participants: such patterns were exploited during the 2003–2007 period by using currencies: the carry – that is the fact that a lower interest rate is paid over the funding currency than the interest received from the currency used as an investment vehicle – was the main reason for the trend on the GBP. Even though this trade seemed ‘riskless’ over this period, the reversal that occured in 2007–2008 reset most of the profits accumulated by the average of the trend-following industry. An arbitrage requires that the trade is strictly riskless, which is clearly not the case here.

[13] We follow as much as possible the notation in Moskwitz *et al.* (2012).

[14] The original dataset of daily data has been turned into a monthly dataset by sampling only the end of month prices for each market.

Table 1.8 Average Sharpe ratios by asset class

		Mean Sharpe Ratio		
		Full sample	1995–2003	2004–2012
Trend-Following	Commodities	0.30	0.33	0.30
	Equity indices	0.85	1.07	0.67
	Currencies	0.42	0.96	−0.19
	Bonds	0.31	0.43	0.17
Breakout	Commodities	0.24	0.18	0.29
	Equity indices	0.85	1.11	0.62
	Currencies	1.40	2.39	0.41
	Bonds	0.06	0.08	0.03

approach, and in Table 1.10 for the breakout strategy. In the case of the time series momentum strategy, most of the Sharpe ratios associated with this investment strategy are positive. They range from −0.297 in the case of soybean to 0.943 in the case of gold for the commodities data. For other asset classes, Sharpe ratios are quite similar in terms of scales except for a few remarkable assets, most of which are emerging currencies: in the case of the Vietnamese Dong the Sharpe ratio is equal to 9.2 and in the case of the Real, it reaches 5.9. These extraordinary figures are outliers coming from extreme movements on these markets. From this preliminary analysis, our results are quite consistent with those in Moskowitz *et al.* (2012): for most of the assets, the return on such a time series momentum strategy is positive, with an average value that is close to 0.45. The breakout strategy results are very similar to those that we have just described, confirming that both strategies are actually tracking the same market premium. When gathering data by groups of assets, we get the results presented in Table 1.8. If we had to rank asset classes along their suitability for momentum based strategies, commodities would never come first, even when slicing the full sample into two sub-periods: commodities come last, except for the 2004–2012 period. With a volatility budget equal to 40%,[15] the average expected return would reach 12% per year, which may seem to be an interesting final return and is quite low given the large risk exposure.[16] This fact is obvious from Figures 1.13 and 1.14: those figures present the sorted Sharpe ratios computed from the full sample for each asset for both methods. Most of the commodities are found in the weakest half of the sample. The average 0.30 Sharpe ratio that we obtain in the momentum strategy case is actually very close to the results obtained in Miffre and Rallis (2007).[17] They, however, found higher risk-adjusted returns: as mentioned previously, the nature of the results is dependent on the period that is covered in the empirical study. The Sharpe ratio obtained in the commodity case for the breakout strategy is slightly below those of the momentum strategy. It remains, however, of a similar magnitude. Finally, very interestingly, the average of the Sharpe ratios obtained from the commodities is the most stable over all the assets. This is especially obvious in the results obtained with the momentum strategy: in such a case, we obtain a Sharpe ratio that remains around 0.3 for the three samples considered. The diversification effect – coming from the weak

[15] That is when building a portfolio of futures with an *ex ante* volatility of 40%.

[16] A typical *ex ante* risk exposure in the hedge fund industry would be around 20%, with an expected Sharpe ratio ranging from 0.5 to 1, thus generating larger *ex ante* expected returns.

[17] They find a Sharpe ratio ranging from 0.42 to 0.57 depending on the length of the period used to compute the past average return. In their study, they allow it to vary between 1 and 12 months. See their Table 1.

Table 1.9 Returns on the time series momentum strategy of Moskowitz *et al.* (2012) across assets

	1995–2012		1995–2003		2004–2012	
	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio
Gold	0.377	0.943	0.145	0.362	0.6	1.501
Silver	0.044	0.11	−0.078	−0.195	0.187	0.467
Platinum	0.203	0.508	0.202	0.505	0.228	0.569
Aluminum	0.015	0.037	0.168	0.421	−0.11	−0.274
Copper	0.077	0.192	0.013	0.034	0.205	0.513
Nickel	0.235	0.587	0.424	1.06	0.041	0.102
Zinc	0.29	0.725	0.309	0.772	0.299	0.746
Lead	0.084	0.209	0.066	0.166	0.14	0.35
WTI	0.06	0.151	0.1	0.251	0.036	0.089
Brent	0.164	0.411	0.118	0.295	0.193	0.482
Gasoil	0.239	0.598	0.158	0.394	0.299	0.748
Natural Gas	0.04	0.1	0.062	0.155	0.018	0.045
Heating Oil	0.241	0.603	0.206	0.515	0.272	0.68
Corn	0.052	0.129	−0.026	−0.064	0.149	0.372
Wheat	−0.009	−0.022	−0.072	−0.18	0.052	0.131
Coffee	0.083	0.209	0.21	0.524	−0.041	−0.103
Sugar	0.073	0.183	−0.006	−0.015	0.135	0.338
Cocoa	0.017	0.042	0.199	0.496	−0.163	−0.408
Cotton	−0.029	−0.071	0.116	0.29	−0.164	−0.411
Soybean	−0.119	−0.297	−0.054	−0.136	−0.145	−0.363
Rice	0.096	0.24	0.273	0.682	−0.043	−0.107
GSCI Agri.	0.082	0.205	0.178	0.444	−0.002	−0.004
GSCI Energy	0.232	0.58	0.384	0.961	0.089	0.222
GSCI Ind. Metals	0.211	0.529	0.136	0.34	0.316	0.79
GSCI Prec. Metals	0.263	0.657	0.085	0.212	0.436	1.09
DOW JONES INDUS. AVG	0.3	0.749	0.289	0.722	0.312	0.781
S&P 500 INDEX	0.375	0.938	0.502	1.255	0.254	0.635
NASDAQ COMPOSITE INDEX	0.306	0.766	0.391	0.978	0.214	0.536
TSX COMPOSITE INDEX	0.305	0.762	0.285	0.714	0.334	0.834
MEXICO IPC INDEX	0.434	1.085	0.318	0.796	0.588	1.469
BRAZIL BOVESPA INDEX	0.332	0.829	0.527	1.317	0.141	0.352
Euro Stoxx 50 Pr	0.399	0.997	0.678	1.696	0.135	0.339
FTSE 100 INDEX	0.293	0.733	0.349	0.873	0.263	0.657
CAC 40 INDEX	0.397	0.992	0.554	1.385	0.259	0.647
DAX INDEX	0.417	1.043	0.508	1.271	0.322	0.805
IBEX 35 INDEX	0.316	0.789	0.414	1.035	0.244	0.61
FTSE MIB INDEX	0.353	0.884	0.452	1.13	0.242	0.605
AEX-Index	0.406	1.015	0.59	1.475	0.228	0.569
OMX STOCKHOLM 30 INDEX	0.475	1.187	0.625	1.562	0.342	0.855
SWISS MARKET INDEX	0.302	0.755	0.448	1.121	0.165	0.413
NIKKEI 225	0.175	0.436	0.212	0.529	0.161	0.402
HANG SENG INDEX	0.238	0.596	0.247	0.617	0.247	0.618
ASX 200 INDEX	0.331	0.828	0.331	0.827	0.358	0.895
EUR-USD X-RATE	0.056	0.139	0.4	1.001	−0.294	−0.736
USD-CAD X-RATE	0.194	0.484	0.256	0.64	0.124	0.31
USD-JPY X-RATE	0.155	0.387	0.311	0.778	−0.033	−0.083
AUD-USD X-RATE	0.173	0.434	0.525	1.313	−0.17	−0.426

(*continued*)

Table 1.9 (*Continued*)

	1995–2012		1995–2003		2004–2012	
	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio
Hong Kong Dollar Spot	0.542	1.356	1.498	3.745	−0.429	−1.073
Singapore Dollar	0.264	0.66	0.298	0.746	0.229	0.573
New Zealand Dollar Spot	0.237	0.593	0.407	1.018	0.08	0.199
British Pound Spot	0.051	0.128	0.241	0.602	−0.123	−0.309
Swiss Franc Spot	0.113	0.283	0.355	0.888	−0.133	−0.332
Swedish Krona Spot	0.114	0.285	0.425	1.062	−0.196	−0.489
Norwegian Krone Spot	−0.013	−0.032	0.139	0.349	−0.167	−0.418
Indian Rupee Spot	0.605	1.5125	1.314	3.285	−0.104	−0.26
Vietnamese Dong Spot	3.681	9.2025	3.87	9.675	3.49	8.725
Brazilian Real Spot	2.359	5.8975	4.643	11.607	0.084	0.21
Mexican Peso Spot	0.068	0.169	0.26	0.651	−0.127	−0.317
Polish Zloty Spot	0.124	0.311	0.428	1.07	−0.168	−0.42
US Generic Govt 2 Year Yield*	0.413	1.033	0.427	1.067	0.384	0.959
US Generic Govt 5 Year Yield	0.264	0.66	0.309	0.773	0.206	0.515
US Generic Govt 10 Year Yield	0.117	0.292	0.215	0.539	0.008	0.02
US Generic Govt 30 Year Yield	−0.08	−0.201	0.088	0.22	−0.256	−0.64
German Government Bonds 2 Yr B	0.027	0.067	−0.034	−0.084	0.116	0.291
German Government Bonds 5 Yr O	0.085	0.214	0.05	0.125	0.106	0.266
German Government Bonds 10 Yr	0.057	0.143	0.212	0.531	−0.118	−0.294
German Government Bonds 30 Yr	0.105	0.261	0.104	0.259	0.09	0.225

*The profit and losses presented here in the case of rates are computed using the approximation that the return on a long bond position is equal to the duration of the bond times the variation in rates to which we added the monthly carry.

correlation between commodity sectors as presented earlier – must be playing a major role in this result.

However, the 2008 period has been a very difficult one for the trend-following industry, with the collapse of long-dated trends from September 2008 onwards.[18] The persistence of trends should indeed be an essential feature of markets from which trend-followers extract abnormal returns. To test such an hypothesis, we ran an OLS regression of the Sharpe ratios obtained previously on key metrics from the previous section:

– The number of regimes: the higher the number of regimes, the more difficult it should be for the trend-following investment strategy of Moskowitz *et al.* (2012) to generate a consistently positive return. What is more, when the number of regimes increases, the diagonal elements in the transition matrix can have a tendency to reduce, as cross-regime dependency increases, jeopardizing the potential profits obtained from a trend-following strategy. We denote $n(i)$ the number of regimes in the case of asset i.
– Hence, the persistence of regimes is also a key element: a stable and persistent regime should be a natural support to a trend-following strategy. Using the previously estimated regimes for each asset, we use as an independent variable the average of the diagonal elements of the estimated transition matrix. The higher this variable and the higher the Sharpe ratio

[18] Some market observers may argue that these events have happened before.

Table 1.10 Returns on the breakout strategy

	1995–2012		1995–2003		2004–2012	
	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio
Gold	0.41	1.026	0.245	0.614	0.566	1.416
Silver	0.118	0.294	−0.078	−0.196	0.232	0.579
Platinum	0.191	0.477	0.118	0.296	0.27	0.675
Aluminum	0.046	0.115	0.067	0.167	0.053	0.131
Copper	0.118	0.294	0.1	0.25	0.155	0.387
Nickel	0.178	0.445	0.378	0.946	−0.011	−0.028
Zinc	0.233	0.583	0.076	0.19	0.357	0.892
Lead	0.029	0.074	0.063	0.157	0.036	0.09
WTI	0.044	0.11	0.12	0.301	−0.046	−0.116
Brent	0.055	0.137	−0.085	−0.214	0.172	0.431
Gasoil	0.157	0.394	0.082	0.205	0.22	0.549
Natural Gas	−0.123	−0.308	−0.222	−0.556	−0.033	−0.083
Heating Oil	0.171	0.428	0.131	0.328	0.214	0.536
Corn	0.058	0.146	0.003	0.008	0.133	0.333
Wheat	0.167	0.419	0.2	0.501	0.125	0.313
Coffee	0.025	0.062	−0.138	−0.346	0.159	0.397
Sugar	−0.06	−0.149	−0.169	−0.422	0.024	0.061
Cocoa	−0.032	−0.079	0.066	0.165	−0.128	−0.32
Cotton	0.029	0.073	0.09	0.225	−0.048	−0.12
Soybean	−0.2	−0.5	−0.25	−0.625	−0.151	−0.377
Rice	0.107	0.268	0.282	0.704	−0.093	−0.232
GSCI Agri.	0.067	0.168	0.164	0.411	−0.03	−0.076
GSCI Energy	0.173	0.432	0.264	0.659	0.088	0.221
GSCI Ind. Metals	0.176	0.44	0.157	0.392	0.225	0.563
GSCI Prec. Metals	0.298	0.744	0.131	0.329	0.459	1.147
DOW JONES INDUS. AVG	0.315	0.788	0.416	1.039	0.22	0.549
S&P 500 INDEX	0.365	0.912	0.532	1.331	0.212	0.531
NASDAQ COMPOSITE INDEX	0.281	0.703	0.58	1.45	−0.004	−0.01
TSX COMPOSITE INDEX	0.375	0.937	0.426	1.064	0.35	0.876
MEXICO IPC INDEX	0.391	0.978	0.296	0.739	0.509	1.274
BRAZIL BOVESPA INDEX	0.314	0.784	0.393	0.982	0.238	0.595
Euro Stoxx 50 Pr	0.408	1.019	0.687	1.717	0.143	0.358
FTSE 100 INDEX	0.277	0.692	0.291	0.728	0.24	0.599
CAC 40 INDEX	0.367	0.918	0.621	1.551	0.135	0.336
DAX INDEX	0.428	1.07	0.545	1.363	0.307	0.767
IBEX 35 INDEX	0.466	1.165	0.591	1.476	0.353	0.884
FTSE MIB INDEX	0.311	0.778	0.462	1.155	0.198	0.495
AEX-Index	0.457	1.143	0.685	1.712	0.249	0.622
OMX STOCKHOLM 30 INDEX	0.39	0.975	0.442	1.106	0.355	0.889
SWISS MARKET INDEX	0.299	0.747	0.333	0.832	0.289	0.723
NIKKEI 225	0.177	0.443	0.179	0.447	0.173	0.432
HANG SENG INDEX	0.357	0.891	0.46	1.15	0.258	0.645
ASX 200 INDEX	0.153	0.383	0.024	0.06	0.271	0.678
EUR-USD X-RATE	0.032	0.081	0.211	0.527	−0.146	−0.364
USD-CAD X-RATE	0.07	0.176	0.205	0.512	−0.04	−0.101
USD-JPY X-RATE	0.252	0.629	0.401	1.002	0.121	0.301
AUD-USD X-RATE	0.237	0.592	0.445	1.112	0.037	0.092

(*continued*)

Table 1.10 (*Continued*)

	1995–2012		1995–2003		2004–2012	
	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio	Avg. Return	Sharpe Ratio
Hong Kong Dollar Spot	0.478	1.196	1.431	3.578	−0.49	−1.225
Singapore Dollar	0.262	0.656	0.255	0.637	0.257	0.643
New Zealand Dollar Spot	0.308	0.77	0.516	1.289	0.113	0.282
British Pound Spot	0.018	0.044	0.144	0.36	−0.094	−0.235
Swiss Franc Spot	−0.064	−0.159	0.028	0.071	−0.151	−0.377
Swedish Krona Spot	0.176	0.441	0.449	1.121	−0.073	−0.182
Norwegian Krone Spot	0.025	0.063	0.22	0.55	−0.165	−0.411
Indian Rupee Spot	0.702	1.756	1.411	3.527	0.025	0.062
Vietnamese Dong Spot	3.648	9.12	3.971	9.927	3.287	8.217
Brazilian Real Spot	2.399	5.998	4.857	12.142	−0.046	−0.115
Mexican Peso Spot	0.019	0.046	0.134	0.335	−0.113	−0.282
Polish Zloty Spot	0.377	0.943	0.646	1.616	0.098	0.244
US Generic Govt 2 Year Yield*	0.336	0.84	0.359	0.897	0.311	0.776
US Generic Govt 5 Year Yield	0.224	0.559	0.28	0.7	0.159	0.398
US Generic Govt 10 Year Yield	−0.056	−0.14	−0.062	−0.154	−0.058	−0.146
US Generic Govt 30 Year Yield	−0.166	−0.416	−0.206	−0.516	−0.136	−0.339
German Government Bonds 2 Yr B	0.076	0.19	0.065	0.162	0.078	0.194
German Government Bonds 5 Yr O	−0.15	−0.374	−0.189	−0.473	−0.12	−0.301
German Government Bonds 10 Yr	−0.072	−0.18	0.039	0.096	−0.186	−0.465
German Government Bonds 30 Yr	0.012	0.03	−0.031	−0.077	0.053	0.133

*The profit and losses presented here in the case of rates are computed using the approximation that the return on a long bond position is equal to the duration of the bond times the variation in rates to which we added the monthly carry.

of a trend-following strategy we expect diagonal elements to grow when the anti-diagonal elements – that measure the probability of switches between regimes – decrease. We denote this variable $a(i)$ in the case of asset i.

– Then, very different trends in nature – that is in terms of expected return and volatility – can also endanger the profits obtained from a trend-following mechanism. When moving from a regime with a low volatility and a positive expected return to a regime with a high volatility and a negative expected return with the constant exposure of *ex ante* 40% of volatility, the realized return observed over such switches can destroy the performance of the investment strategy. We measure such heterogeneity through two variables: we denote respectively $m(i)$ and $v(i)$ the absolute difference between the minimum and the maximum expectation and volatility across the regimes of asset i. The larger this difference, the more difficult it should be for trend-following strategies to cope with such switches between regimes.
– Finally, to capture this negative premium observed in the previously mentioned plots, we add the variable $1_{\text{commo}}(i)$ that is a variable that is equal to 1 if the ith asset is a commodity and 0 if not. We add similar variables $1_{\text{equity}}(i)$ and $1_{\text{currencies}}(i)$ that are respectively equal to 1 if the ith asset belongs to the equity and currency asset classes.[19]

[19] For the OLS estimation to be performed, we need to discard one of the asset classes. Here, we refrain from introducing a variable for rates.

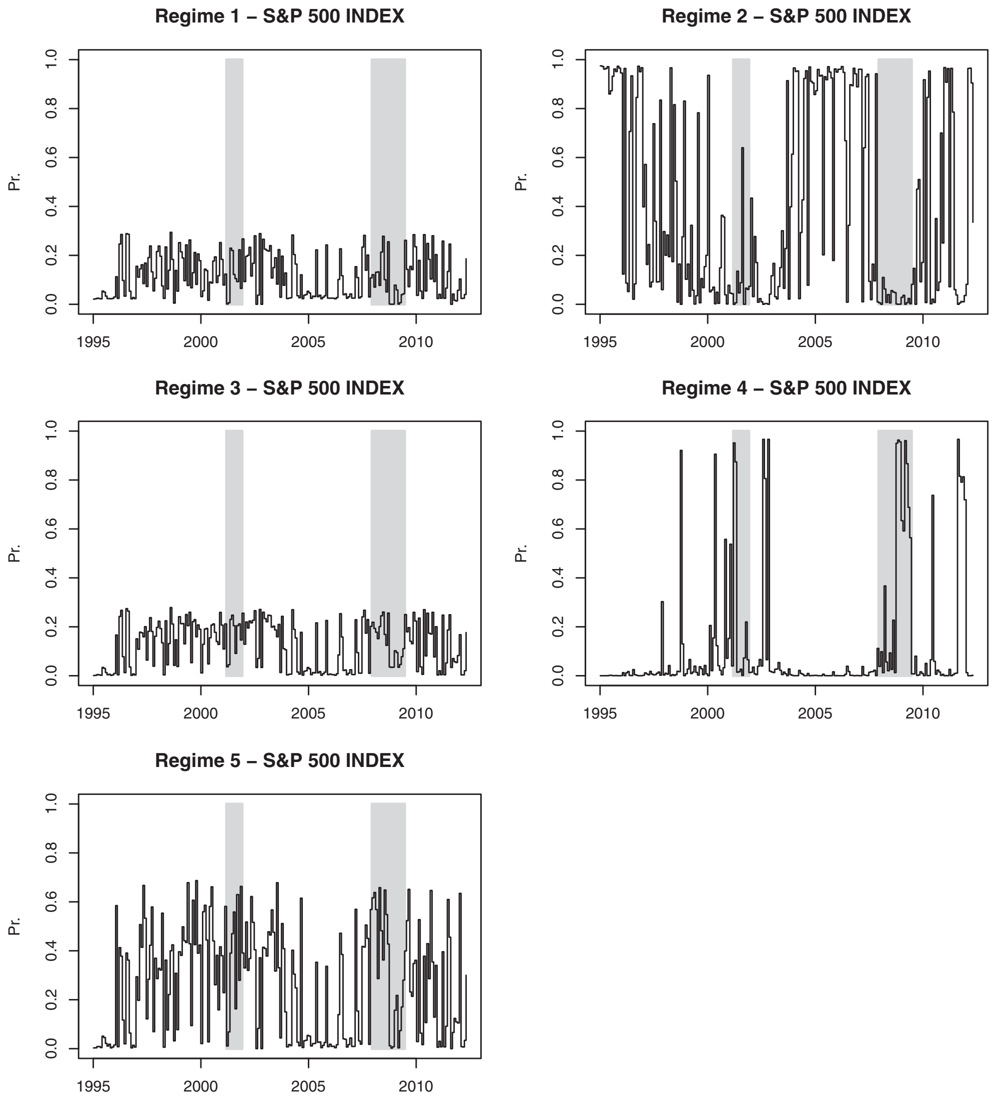

Figure 1.11 Time series of regimes in S&P 500

We ran the following regression:

$$SR(i) = \alpha + \beta_1 n(i) + \beta_2 a(i) + \beta_3 m(i) + \beta_4 v(i) + \beta_5 1_{\text{commo}}(i) + \beta_6 1_{\text{equity}}(i) + \beta_7 1_{\text{currencies}}(i) + \epsilon(i), \tag{1.17}$$

with $\epsilon(i)$ a centered disturbance. Results are available in Table 1.11. The results obtained in the momentum and in the breakout cases are different. In the time series momentum case, at a 5% risk level, there are three variables that explain statistically the Sharpe ratios obtained across strategies. The first of these variables is the number and the persistence of regimes that are positively related to the realized Sharpe ratio. Such statistical evidence supports the argument

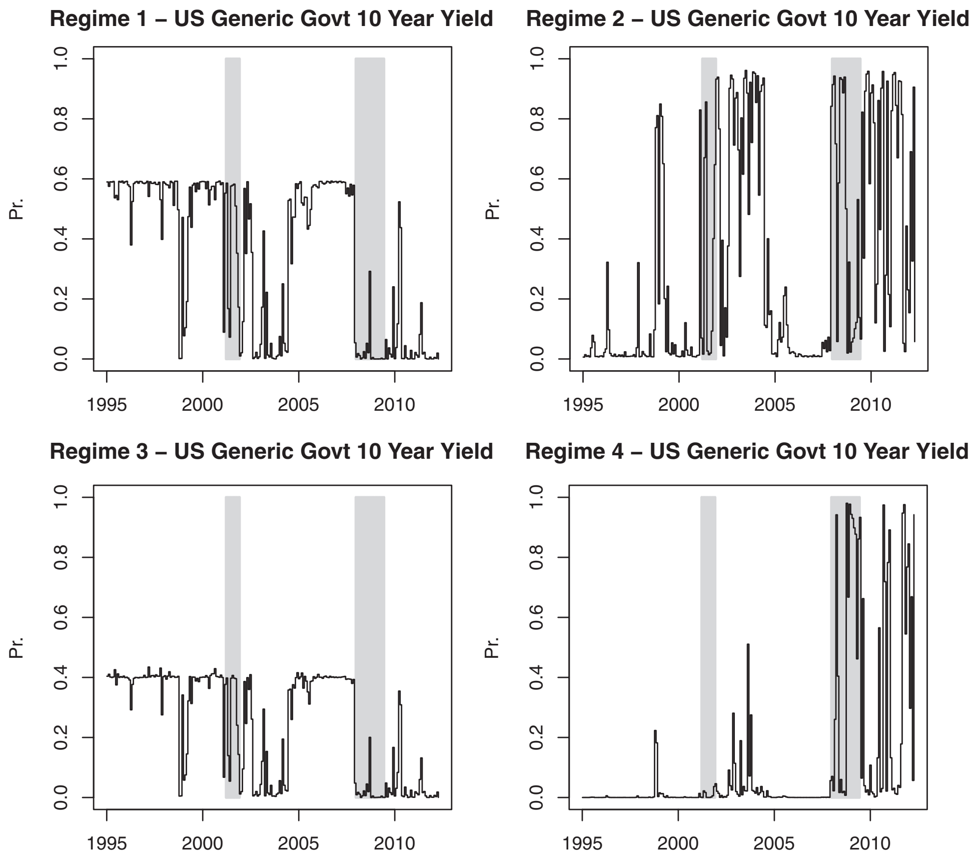

Figure 1.12 Time series of regimes in US 10Y rates

that a high number of regimes does not always threaten the performance of trend-following strategies, and that persistence of trends is essential to this industry. The third significant variable is the dummy variable for equity: equities seem to have generated an increase of the Sharpe ratio by 0.58 over the period. For the rest of the variables, we obtain the expected sign without finding statistically significant results: expectation and variance heterogeneity variables are negatively related to performance, and the persistence of regimes has a positive sign in the regression. Overall, we obtain an R^2 of 0.37, which is by usual standards quite high. Now, turning to the breakout strategy, we find three significant variables: persistence appears to have been a strong support to the performance of such a strategy. With a persistence equal to 1 – that is the maximum value for this average of probabilities – the expected Sharpe ratio from our regression should be equal to 2.13. The dummy variables for equities and currencies are significant as well, highlighting the interest of these assets for the trend-following industry. Signs of insignificant variables are also consistent with intuition, but again for the number of regimes that is positively related to the performance of the strategy.[20] The R^2 is lower than in the previous case and reaches a value of 0.19.

[20] This variable would be considered significant only at a 15% risk level.

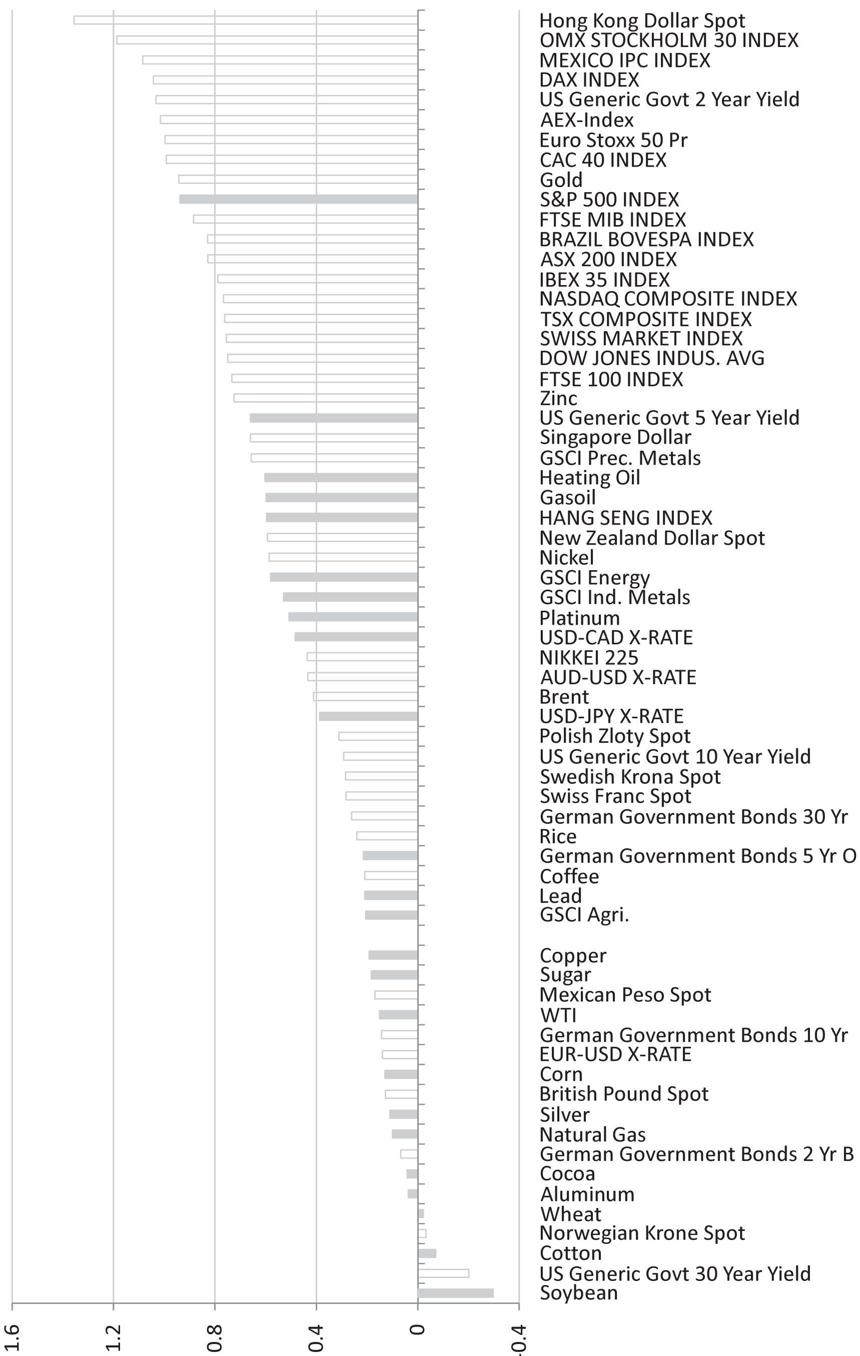

Figure 1.13 Sorted Sharpe ratios for the time series momentum strategy

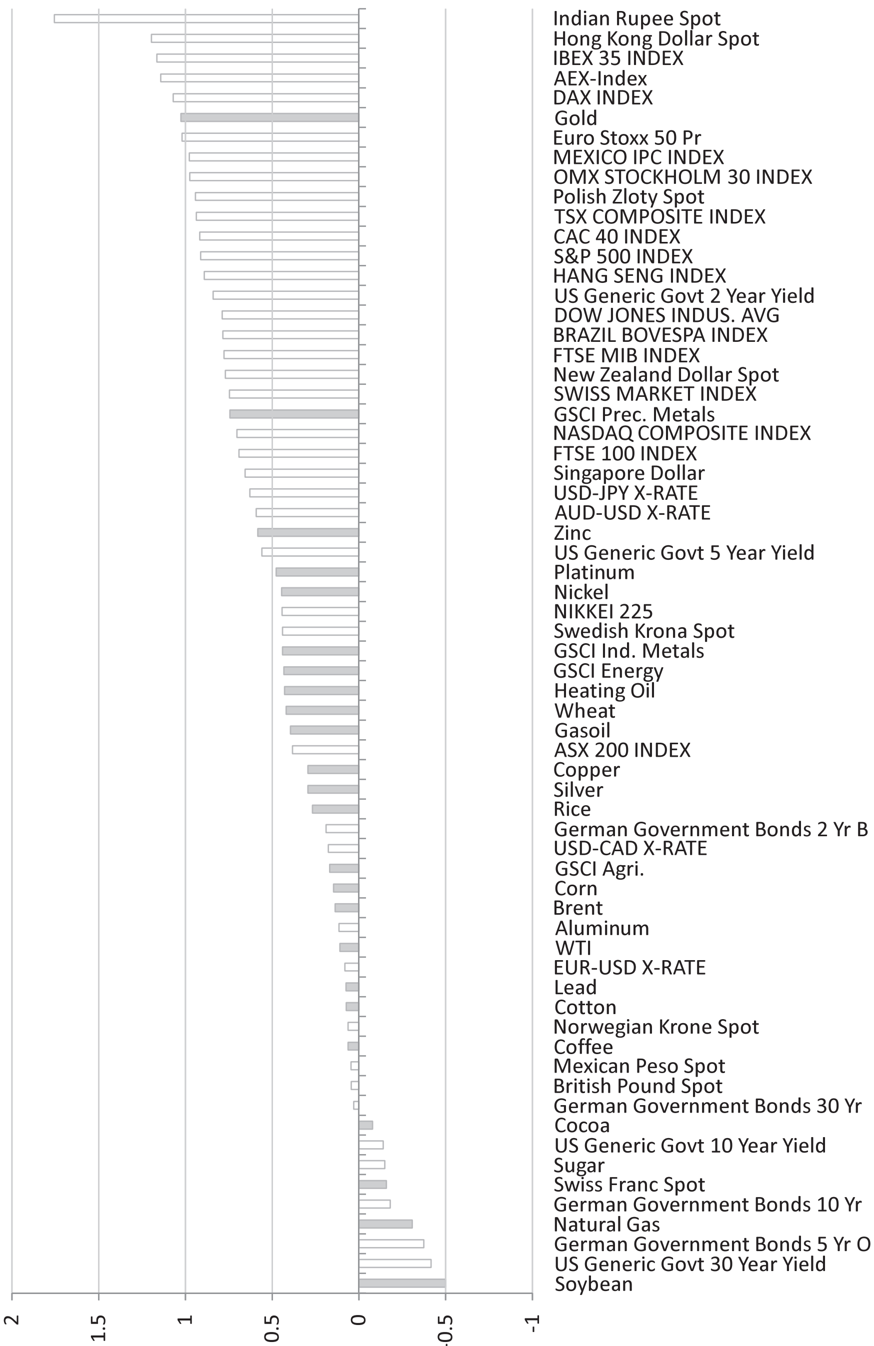

Figure 1.14 Sorted Sharpe ratios for the breakout strategy

Table 1.11 OLS regressions of the Sharpe ratio from the time series momentum strategy on explanatory factors

		Coef.	Std. Dev.	*t*-stat
Trend-following	Nb. of Regimes	0.14*	0.06	2.48
	Persistence	0.50	0.26	1.92
	Expectations heterogeneity	−0.01	0.02	−0.41
	Variance heterogeneity	−0.22	0.16	−1.31
	Is Commodity	0.11	0.14	0.77
	Is Equity	0.58*	0.14	4.04
	Is Currency	0.16	0.16	1.03
	R^2	0.37		
Breakout	Nb. of Regimes	0.2	0.13	1.53
	Persistence	2.13*	0.85	2.50
	Expectations heterogeneity	0.06	0.04	1.60
	Variance heterogeneity	−0.36	0.39	−0.93
	Is Commodity	0.38	0.34	1.14
	Is Equity	0.87*	0.34	2.57
	Is Currency	0.96*	0.36	2.62
	R^2	0.19		

Hence, the trend-following industry should be interested in commodities for the stability of the Sharpe ratio obtained with such strategies over such assets, and not for the stability of its trends. Still, as shown by the previous tables, the obtained Sharpe ratio can vary a lot depending on the considered commodity. The impact of the increasing use of commodities as investment tools by pension, hedge and mutual funds remains an open question: to what extent will those market participants increase the similarity between standard assets and commodities? This central question needs to be raised and answered by the CTA community. The impact of this economic phenomenon is also a potential explanation for the appearance of such trends. We will further discuss this point in Part II of the book. For now, we turn our attention toward risk patterns in commodities.

1.3 VOLATILITY TO RETURNS SPILLOVERS AND TAIL EVENTS IN COMMODITIES

As investors should be driven by Sharpe ratios, they should be both interested in expected returns and in the risk associated with these returns. This section investigates how risky commodities are. We first tackle the existence of returns to volatility spillovers – that is how the previously studied expected returns interact with volatility – in commodity markets before moving to the measurement of jumps in commodity markets.

1.3.1 Spillover Effects in Commodity Markets

1.3.1.1 The Leverage Effect

The relation between returns and volatility on most equity markets is known to be asymmetric: an increase in volatility that is subsequent to a negative return is larger than when a positive return occurs. This stylized fact for equities is now well documented, starting from the contribution of Black (1976). The main contributions in this respect are Christie (1982), Amihud and Mendelson (1987), Schwert (1989b), Cheung and Ng (1992), Campbell and Hentschel (1992), Damodaran (1993) and Koutmos (1998). In such markets, the usual explanation for such a pattern is related to the leverage of firms: when entering a crisis, the debt to equity ratio is high – and on average firms are leveraged – turning any negative return into an increasingly risky environment. This negative spillover effect of returns on subsequent volatilities is thus usually referred to as a leverage effect. Now, equity markets are not the only ones for which there is a positive or negative spillover from returns to volatility, and commodities are no exception. This section is dedicated to the measurement of these spillover effects across commodities.

Building on previous notation, r_t^i is the return on date t obtained from investing in asset i. r_t^i is a random variable, and follows some distribution L with expectation μ^i and a volatility that can change on every date t σ_t^i. Thus,

$$r_t^i \sim L\left(\mu^i, \sigma_t^i\right). \tag{1.18}$$

Following Gourieroux and Jasiak (2001),[21] there is a statistical spillover effect whenever the correlation between r_t^i and the variations of σ_t^i is different from zero; that is

$$\operatorname{cor}\left(r_t^i, \sigma_{t+1}^i - \sigma_t^i | \mathcal{F}_t\right), \tag{1.19}$$

where $\mathcal{F}_t$ is the filtration obtained from the information available at time t. For many of the time series models that are used in this book, $\mathcal{F}_t = \sigma\{r_{t-1}^i, r_{t-2}^i, \ldots\}$. The available information is hence made of past returns on asset i. In the case of equities, this correlation is usually found to be negative.

1.3.1.2 Literature Review

The previously cited references on the equity market show consistent evidence that the previous correlation is negative, especially in the case of US equity. Is there any leverage effect in commodity markets? Giamouridis and Tamvakis (2001) assume that there is a non-zero correlation between past returns and volatility in such markets for two reasons:

1. First, for many commodities, the level of inventories is a key variable: any shortage should create abnormal positive returns, and boost the level of volatility in the meantime.
2. Second, there is a minimum price that makes producers break even: below such prices, producers are running their business making losses. Given the implications of dropping prices, such markets stand a good chance of having a limited downside risk.

Testing this intuition over commodity indices, the authors find evidence of a positive correlation between returns and volatility for both the GSCI and the JPMCI indices over the 1996–2000 period. They confirm that this relation is negative in the case of the S&P 500. As for many articles that will be quoted here, their approach relies on the Exponential GARCH (EGARCH) model presented initially in Nelson (1991). The empirical investigation presented

[21] They state this condition in terms of covariance, but as volatilities are non-zero and positive, it changes nothing about their conclusion when stating it in terms of correlation.

in this section will be based on the same type of modeling approach. Under Nelson's (1991) model, returns are driven by the following data-generating process:

$$r_t^i = \mu^i + \sigma_t^i \epsilon_t^i \tag{1.20}$$

$$\log \sigma_t^2 = \omega + \alpha \left| \epsilon_{t-1}^i \right| + \theta \epsilon_{t-1}^i + \beta \log \sigma_{t-1}^i, \tag{1.21}$$

where ϵ_t^i follows a distribution with an expectation equal to 0 and variance equal to 1. A negative correlation between returns and volatility is obtained when θ is negative: for such cases, a negative return increases the logarithm of the volatility by $\alpha + \theta$. There are different competing time series models to measure this phenomenon such as Ding *et al.*'s (1993) Asymmetric Power ARCH model or Glosten *et al.*'s (1993) GJR GARCH model. A large part of the literature still focuses on the EGARCH model, as it generates a leverage effect while maintaining the positivity of the conditional variance at all times. Grouping the empirical evidence obtained in the literature by commodity sector, we get the following insights from these empirical works: in the case of precious metals, consistent evidence shows that these commodities exhibit a positive leverage effect. Such findings have been uncovered in Batten and Lucey (2007), Tully and Lucey (2007), Batten *et al.* (2008) and Baur (2012) by using various samples and modeling approaches. Hammoudeh and Yuan (2008) and McKenzie *et al.* (2001) present more mixed evidence when it comes to industrial metals: Hammoudeh and Yuan (2008) find equity-like leverage effects in the case of copper, as well as insignificant leverage effects for gold and silver. McKenzie *et al.* (2001) estimated such time series models to the metals traded on the London Metal Exchange, and found no empirical evidence that there is some leverage effect for such commodities. The case of energy is maybe the case for which the highest number of estimations has been performed. Morana (2001) finds that the leverage effect helps the forecasting of oil prices. Hammoudeh *et al.* (2003), Lee and Zyren (2007), Chang *et al.* (2010), Singh *et al.* (2011) and Du *et al.* (2011) unanimously diagnose equity-like leverage effects. In the case of agricultural products, Zheng *et al.* (2008) find that returns on many food products are positively correlated to their volatility: the scarcity argument is here getting the upper hand. Finally, Brooks and Prokopczuk (2011) also find various signs for leverage effects, by using a continuous time model: they find a positive correlation between returns and volatility in the case of gold, silver and soybean. This correlation is negative in the case of the S&P 500, crude oil, gasoline and wheat. However, this correlation is statistically different from zero only in the cases of crude oil, gold, silver, soybeans and in the case of the S&P 500. Interestingly, when comparing the absolute value of these correlations, the highest value obtained in the case of commodities is only half of the S&P 500.

This question of leverage effects in commodities matters for several reasons, the first of which being risk management and the computation of Value-at-Risk (VaR). Forecasting the volatility of commodities is essential for such a purpose – and leverage effects are proved to make a difference. Then, the estimation of volatility is also very useful for the computation of hedge ratios, as illustrated in Kroner and Sultan (1993), Lien and Tse (2000) and Chen *et al.* (2001). Hedging a position requires correctly estimating the ratio of the hedging instrument's volatility to the asset to be hedged. The presence of leverage effects can have a marked impact here. Finally, Giamouridis and Tamvakis (2001) raise an interesting point: the addition to an equity portfolio – with a negative return to volatility spillover – of assets with positive leverage effect can increase the diversification of the final portfolio. Thus, having assets with opposite leverage effects can be a potential source of risk reduction.

1.3.1.3 The Modeling Approach

The previously quoted empirical evidence has been obtained by using various empirical approaches. Despite this fact, most of the conclusions listed above are consistent across academic contributions. We now turn our attention towards the estimation of the leverage effect in commodities. When doing so, the econometrician is faced with an essential issue: a negative (respectively positive) leverage effect creates negative (positive) skewness in financial return samples. In turn, it has been assumed that the presence of positive or negative skewness is a sign of the leverage effect. This is clearly an abusive shortcut: negative skewness can be generated by large negative jumps, triggering or not a rise in volatility. The leverage effect is often referred to as 'conditional skewness', given that it creates skewness from the dynamics of σ_t^i. Conversely, the portion of skewness that is not explained by the leverage effect is referred to as 'unconditional skewness', given that it is generated by a data-generating process whose parameters are not time dependent. By using the notation from Equation (1.21), ϵ_t^i captures this part of skewness that remains unexplained by the relay from past returns to the dynamics of volatility. For our empirical applications, we need to decide on a suitable distribution for ϵ_t^i. There are several candidates for such a purpose. Giot and Laurent (2003) have used a skewed Student distribution. Hung *et al.* (2008) make use of the heavy-tail distribution presented in Politis (2004). Ane and Labidi (2001) use a mixture of Gaussian distributions. A mixture of Gaussian distributions has been widely used and applied to other financial assets such as stocks and currencies in Kon (1984), Akgiray and Booth (1987), Tucker and Pond (1998), Lekkos (1999) and Alexander and Lazar (2006). An application to option pricing can be found in Monfort and Pegoraro (2006): beyond the empirical performance obtained when it comes to pricing options, they discuss the merit of the mixture of Gaussian distributions, emphasizing its connection to a non-parametric estimation of the conditional distribution of returns. A suitable competitor to the mixture of Gaussian distribution could be the Generalized Hyperbolic distribution (GH hereafter). The GH distribution was introduced by Barndorff-Nielsen and Blaesild (1981), and applied to finance in Eberlein and Prause (2002) and Chorro *et al.* (2010). These two distributions are interesting candidates as they encompass both fat tails and asymmetry of the conditional distribution of financial returns.

The difficulty that we are faced with now is a numerical one. When considering a model such that:

$$r_t^i = \mu^i + \sigma_t^i \epsilon_t^i, \tag{1.22}$$

where ϵ_t^i follows one of the previously mentioned distributions, the total skewness and kurtosis of the sample now has two sources of explanations σ_t^i and ϵ_t^i that appear in the equation in a multiplicative way. The usual estimation scheme here is to use the Quasi Maximum Likelihood (QML) approach. This amounts to first assuming that ϵ_t^i follows a centered Gaussian distribution to estimate the parameters driving the volatility process, and estimating the parameters characterizing ϵ_t^i in a second step by holding the estimated volatility parameters fixed. By using such an estimation approach, Chorro *et al.* (2010) have shown that the estimated parameters suffer from a strong bias, leading then to misleading conclusions concerning leverage effects in financial markets. Here, we will use the recursive estimation approach presented in Chorro *et al.* (2010), making it possible to disentangle both sources of skewness and kurtosis.[22] One

[22] See Chorro *et al.* (2010) for additional details regarding the recursive estimation methodology employed, and for additional references regarding this approach. Similar contributions are available in Song *et al.* (2005) and Fan *et al.* (2007). The intuition behind this estimation method is to perform multiple likelihood maximization by fixing either the volatility's or the conditional distribution's

key conclusion from their simulations and estimation strategy was that the GH and mixture of two Gaussian distributions delivered fairly comparable results in terms of their ability to capture the tails of the conditional distribution of financial returns. Given the numerical simplicity of the mixture of Gaussian distribution, we focus on this conditional distribution.

When ϵ_t^i follows a mixture of Gaussian distributions MN($\phi, \mu_1, \sigma_1, \mu_2, \sigma_2$), its density $f(.)$ is given by the following equation:

$$f\left(\epsilon_t^i\right) = \phi g\left(\epsilon_t^i; \mu_1, \sigma_1\right) + (1-\phi) g\left(\epsilon_t^i; \mu_2, \sigma_2\right), \tag{1.23}$$

where $g(.; \mu, \sigma)$ is the density of Gaussian distribution with expectation μ and standard deviation σ; ϕ is a parameter taking its value between 0 and 1, mixing the two Gaussian distributions. As noted previously, despite the simplicity of this distribution, Monfort and Pegoraro (2006) showed its ability to replicate a wide range of empirical distributions, which is exactly what we need to disentangle these two sources of skewness and kurtosis. What is more, the mixture of Gaussian distributions has an appealing economic intuition: for example, with two Gaussian distributions, markets are implicitly viewed as being made of two types of different agents or two types of potential market configurations – bullish and bearish ones. The mixing parameter does the arbitrage between either the market power or the frequency of occurrence of each market episode. Interestingly, the unconditional distribution of a Markov switching with two regimes is a mixture of Gaussian distributions as well.

1.3.1.4 Empirical Findings

Now, we use again a dataset of commodities and more standard assets made of daily close-to-close returns from 1995 to 2012. We use the recursive estimation of Chorro *et al.* (2010), applied to the EGARCH model with a mixture of normal distributions (EGARCH-MN) defined by Equations (1.21) to (1.23). Estimated parameters are presented in Table 1.12, along with the standard deviations associated with each of these estimated parameters. Several key conclusions can be drawn from this table:

- First of all, the sign of the θ parameter that drives the leverage effect in the EGARCH model is globally consistent with the findings in the previous literature. It is found to be positive in the case of agricultural products and precious metals – both when considered individually and as an asset class through the GSCI indices. For example, in the case of gold it is found to be equal to 0.054, and in the case of the GSCI precious metals we obtain 0.0262. In the case of coffee it is equal to 0.0877, and for the GSCI agricultural sub-index we find 0.0425. It is consistently found to be negative in the case of energy and industrial metals. Again, this is consistent when considering the individual commodities and the two GSCI indices. In the case of Brent, we find $\theta = -0.029$ and in the case of GSCI Energy sub-index $\theta = -0.0353$. In the case of copper, this parameter is estimated to be equal to -0.027 and in the case of GSCI Industrial Metals -0.0524. In this respect, energy and industrial metals have a tendency to be characterized by equity-like leverage effects: the S&P 500 has a θ parameter estimated to be equal to -0.1637.
- Second, for most of the individual commodities, the leverage parameter is not found to be statistically different from zero. There are only four cases for which the leverage parameter

parameters each time, until this sequential optimization stops improving the value of the log-likelihood function. Monte Carlo results in Chorro *et al.* (2010) show that the convergence rate for such estimation is lower, but the estimation bias is strongly reduced and the efficiency of the estimates much higher than that of both the maximum or quasi maximum likelihood methods.

Table 1.12 EGARCH-MN parameters estimated

	Volatility Parameters				Conditional Distribution Parameters				
	ω	θ	α	β	ϕ	μ_1	σ_1	μ_2	σ_2
Gold	−0.2626	0.054	0.1588	0.9729	0.2678	−0.0125	0.7053	0.0257	0.3129
	(0.0076)	(0.0262)	(0.0095)	(0.0015)	(0.0539)	(0.0202)	(0.0402)	(0.0074)	(0.0169)
Silver	−0.2063	0.0387	0.1771	0.9775	0.207	−0.1178	0.7501	0.0449	0.3214
	(0.0065)	(0.0232)	(0.0101)	(0.0016)	(0.0321)	(0.0342)	(0.0362)	(0.0071)	(0.0105)
Platinum	−0.3781	0.0267	0.2301	0.9581	0.2587	−0.0302	0.6974	0.0286	0.3273
	(0.0115)	(0.0237)	(0.0149)	(0.0025)	(0.0394)	(0.0207)	(0.0325)	(0.0074)	(0.0112)
Aluminum	−0.2432	−0.0081	0.1538	0.9744	0.8918	0.0144	0.4029	−0.1167	0.841
	(0.0094)	(0.0053)	(0.0108)	(0.0019)	(0.0294)	(0.007)	(0.01)	(0.0635)	(0.0612)
Copper	−0.2617	−0.027	0.1841	0.9706	0.1902	−0.0684	0.7304	0.0228	0.351
	(0.0081)	(0.0205)	(0.0116)	(0.0019)	(0.0414)	(0.0411)	(0.0426)	(0.0083)	(0.0122)
Nickel	−0.2633	0.0135	0.1472	0.9658	0.2143	−0.0008	0.7605	0.0052	0.3602
	(0.0082)	(0.0329)	(0.0125)	(0.0022)	(0.0425)	(3e-04)	(0.0429)	(0.0066)	(0.0129)
Zinc	−0.1116	0.0194	0.086	0.9876	0.2282	−0.0296	0.7009	0.0125	0.3366
	(0.0031)	(0.014)	(0.0047)	(8e-04)	(0.0498)	(0.04)	(0.0407)	(0.0087)	(0.014)
Lead	−0.1792	0.0084	0.1508	0.9803	0.253	−0.061	0.6687	0.0292	0.3365
	(0.0049)	(0.007)	(0.0078)	(0.0012)	(0.0443)	(0.0304)	(0.0321)	(0.0085)	(0.0116)
WTI	−0.2178	−0.0139	0.1209	0.9719	0.1268	−0.1277	0.7751	0.0296	0.4102
	(0.0098)	(0.0324)	(0.0135)	(0.0025)	(0.0465)	(0.0607)	(0.0655)	(0.0094)	(0.0132)
Brent	−0.3402	−0.029	0.1765	0.956	0.3032	−0.0426	0.657	0.0351	0.3632
	(0.0126)	(0.0412)	(0.0173)	(0.0032)	(0.0846)	(0.0298)	(0.041)	(0.0107)	(0.0212)
Gasoil	−0.2423	−0.0264	0.1403	0.9703	0.2339	−0.03	0.6804	0.026	0.3997
	(0.0112)	(0.0242)	(0.0141)	(0.0027)	(0.1271)	(0.0482)	(0.0717)	(0.0141)	(0.0284)
Natural Gas	−0.2366	0.0204	0.1594	0.9664	0.8507	−0.009	0.3813	0.0822	0.8278
	(0.0075)	(0.0219)	(0.0141)	(0.0025)	(0.0358)	(0.0087)	(0.012)	(0.0444)	(0.0556)
Heating Oil	−0.1906	−0.0192	0.1136	0.9762	0.0597	−0.1763	0.9197	0.0227	0.44
	(0.0123)	(0.0526)	(0.0149)	(0.003)	(0.0257)	(0.1089)	(0.0991)	(0.0085)	(0.01)
Corn	−0.3231	0.0119	0.2099	0.9624	0.1204	0.1453	0.9	−0.0088	0.3749
	(0.0117)	(0.0183)	(0.0158)	(0.0027)	(0.0244)	(0.0518)	(0.0565)	(0.0061)	(0.0095)

Wheat	−0.0714	0.0418	0.0488	0.9917	0.8575	−0.0241	0.398	0.1807	0.8009
	(0.0044)	(0.0168)	(0.0055)	(0.001)	(0.032)	(0.0089)	(0.0099)	(0.0534)	(0.049)
Coffee	−0.3771	0.0877	0.1492	0.9464	0.6869	−0.0013	0.3256	0.0176	0.7047
	(0.0118)	(0.0361)	(0.0186)	(0.0032)	(0.0526)	(0.0008)	(0.0161)	(0.029)	(0.035)
Sugar	−0.156	−0.0102	0.1093	0.9813	0.9042	0.0174	0.3989	−0.1491	0.9647
	(0.0061)	(0.0289)	(0.0088)	(0.0015)	(0.0216)	(0.0086)	(0.0091)	(0.0698)	(0.0673)
Cocoa	−0.1543	0.0138	0.1051	0.9822	0.1735	0.0045	0.8133	0.004	0.3861
	(0.0084)	(0.0028)	(0.0108)	(0.0019)	(0.0364)	(0.0032)	(0.0487)	(0.0069)	(0.0115)
Cotton	−0.1724	0.0188	0.1195	0.9801	0.0601	0.0842	1.1023	−0.0042	0.4053
	(0.0063)	(0.0155)	(0.0087)	(0.0015)	(0.0189)	(0.0728)	(0.1094)	(0.0053)	(0.0097)
Soybean	−0.3608	−0.0008	0.1926	0.9578	0.2874	−0.0388	0.6946	0.0274	0.3405
	(0.0131)	(0.0005)	(0.0164)	(0.0028)	(0.041)	(0.0274)	(0.0299)	(0.0093)	(0.0115)
Rice	−1.1153	−0.0121	0.3449	0.8516	0.923	−0.0098	0.3906	0.2071	1.0572
	(0.0258)	(0.0104)	(0.0364)	(0.0063)	(0.0166)	(0.008)	(0.0083)	(0.0788)	(0.0796)
GSCI Agri.	−0.283	0.0425	0.1651	0.9699	0.2146	−0.0099	0.6836	−0.0006	0.3879
	(0.0108)	(0.0188)	(0.0121)	(0.0021)	(0.0668)	(0.0172)	(0.0493)	(0.0011)	(0.0146)
GSCI Energy	−0.3127	−0.0353	0.1609	0.9605	0.2758	−0.0525	0.6488	0.0337	0.3953
	(0.013)	(0.0195)	(0.0169)	(0.0031)	(0.1402)	(0.0401)	(0.0589)	(0.0125)	(0.0301)
GSCI Ind. Metals	−0.3268	−0.0524	0.2037	0.9642	0.1284	−0.1093	0.7515	0.0233	0.3933
	(0.0111)	(0.0239)	(0.0137)	(0.0023)	(0.05)	(0.0662)	(0.0633)	(0.0084)	(0.0142)
GSCI Prec. Metals	−0.1963	0.0262	0.1237	0.9797	0.1226	−0.1081	0.8599	0.0342	0.3615
	(0.0062)	(0.0267)	(0.0077)	(0.0012)	(0.0292)	(0.0504)	(0.0606)	(0.0073)	(0.0108)
S&P 500	−0.2538	−0.1637	0.12	0.9706	0.4759	−0.0536	0.5345	0.0665	0.2758
	(0.0098)	(0.039)	(0.0123)	(0.002)	(0.088)	(0.0227)	(0.0237)	(0.0114)	(0.0254)
10Y US	−0.1642	−0.0361	0.2176	0.9869	0.2131	0.0122	0.6543	−0.0196	0.3645
	(0.0079)	(0.0284)	(0.0104)	(0.0017)	(0.07)	(0.024)	(0.0477)	(0.0078)	(0.0157)
US Dollar	−0.0982	−0.01	0.0759	0.9923	0.5026	0.0203	0.3249	−0.0284	0.6114
	(0.0107)	(0.0107)	(0.0083)	(0.0015)	(0.1339)	(0.0142)	(0.0424)	(0.0189)	(0.0386)

is found to be significant: gold (0.054), wheat (0.0418), coffee (0.0877) and cocoa (0.0138). The rest of these parameters do not pass the significance test. On the contrary, each GSCI index but the precious metals one is found to have a statistically significant leverage effect of the previously mentioned sign. Hence, as an asset class – once individual patterns have been averaged out – commodity markets do exhibit leverage effects. The case of precious metals may be explained by the fact that precious metals are increasingly used as industrial metals, thus mixing two kinds of patterns.

– Finally, as presented in Figure 1.17, the magnitude of leverage effects in the commodity markets is less pronounced than those in the US equity market. Coffee set apart, the absolute maximum value for this parameter is around 0.05, when in the case of S&P 500 it is close to -0.15. Another way of analyzing these results and getting some sense from these estimation figures is to compute Engle and Ng's (1993) news impact curve. This curve represents how volatility responds to the magnitude and sign of past returns. Suppose that volatility is at its long-term level $\bar{\sigma}^i$. By following Equation (1.20), the increase in volatility given a past positive or negative shock in returns can be computed the following way:

$$\log \sigma_{t+1}^2 = \omega + \alpha \left|\epsilon_{t-1}^i\right| + \theta \epsilon_{t-1}^i + \beta \log \bar{\sigma}_{t-1}^i. \tag{1.24}$$

This allows us to chart the potential variations of volatility depending on past realized returns. Such news impact curves are presented in Figure 1.15 by comparing the news impact curves of the GSCI commodity sectors to the one associated to the S&P 500, the 10-year rates and the dollar trade-weighted index. When negative (positive) leverage effects are observed on a market, this curve is asymmetric and skewed to the left (right). In the case of commodities – and consistent with our previous estimation results – the asymmetry of the news impact curve is far less pronounced than those associated with the S&P 500. As in the case of copper, the bulk of the asymmetry of returns on these commodities seems to come from the conditional distribution; that is, from ϵ_t^i.

The case of copper is very illustrative of the difficulties in disentangling leverage from unconditional skewness: Hammoudeh and Yuan (2008) and Brooks and Prokopczuk (2011) find evidence of statistically significant leverage effects for this commodity. When using a more robust estimation approach that makes it possible to discriminate between both sources of skewness, we conclude that most of this skewness comes from the unconditional skewness, as the leverage effect is not found to be statistically significant. For example, the bulk of the skewness in copper seems to come from the mixture of Gaussian distributions. The first Gaussian distribution has a negative expectation (-6.84%) and a volatility that is higher than the volatility of the second Gaussian distribution. The expectation of the second Gaussian distribution is positive, and of a lower absolute value than that of the first distribution. Hence, what has been diagnosed so far to be return to volatility spillovers in copper – thus generating skewness in market returns – is in fact an historical 19.02% chance that copper exhibits strong negative returns with higher volatility. This asymmetry in the conditional distribution is the primary contributor to the negative skewness found in the returns of copper.

Figure 1.16 presents the log-densities of ϵ_t^i estimated by using the EGARCH-MN model in the case of the four GSCI indices, compared with the results obtained in the case of S&P 500, the US 10-year rate and the trade-weighted US Dollar. Clearly, the main difference between all of these commodity indices and these more standard financial assets lies in the tails of their conditional distribution. In the case of commodities, tails are always decreasing at a slower rate

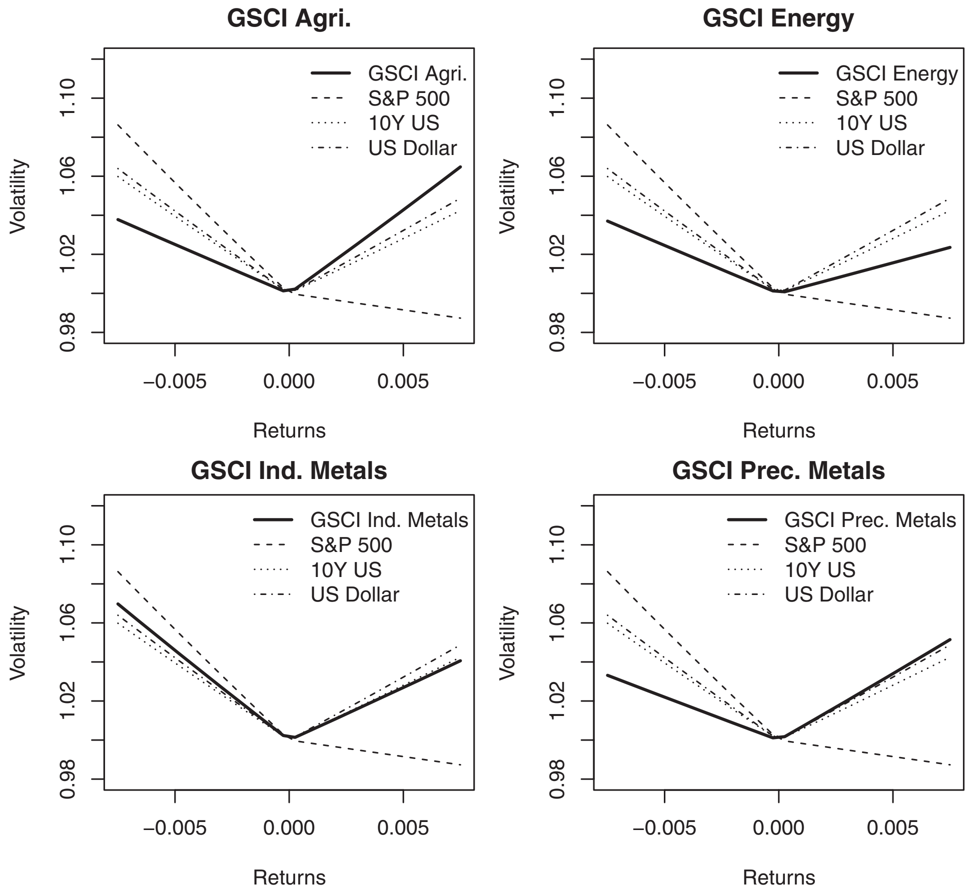

Figure 1.15 Volatility signature plots for commodity sectors

than in the S&P 500 case, for example: extreme returns have a higher frequency of occurrence in the case of commodities than in US equity.

Another way to consider this tail activity is to investigate the frequency and size of the jumps affecting the returns on commodities. In the next section, we investigate such an issue.

1.3.2 Twenty Years of Jumps in Commodity Markets

Given the great importance of tail events in commodities, investigating and measuring jump activity in such returns is an essential step toward understanding the dynamics of these markets. We have previously captured the tail behavior through the mixture of Gaussian distributions, by using a distribution flexible enough to capture tail patterns without having to understand what is at stake behind these extreme events. Now, we propose to measure more precisely what is happening in these tails: tail events in financial economics are usually modeled by jumps, i.e. extreme events – some market observers consider them as discontinuous events – that appear rarely in financial markets. Rarely here means that the number of jumps per year is lower than the number of trading days in a year. However, the scale of these extreme returns is such

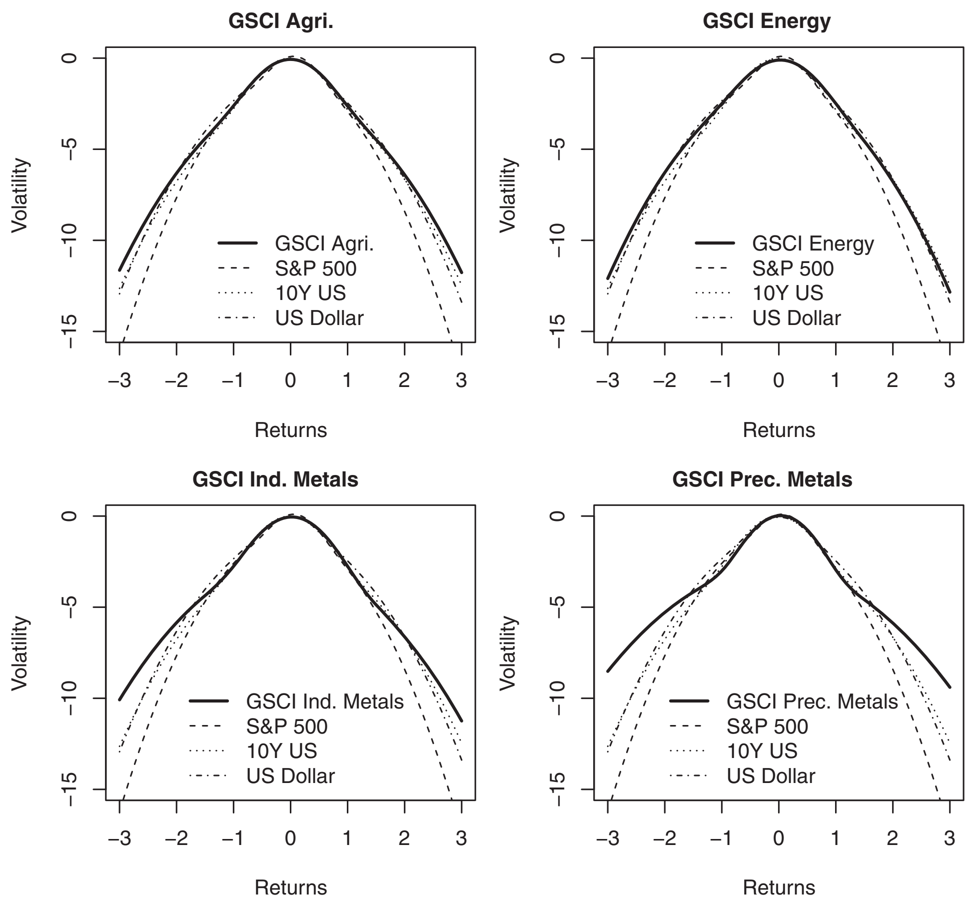

Figure 1.16 Logarithm of the conditional distributions across commodity sectors

that even though they occur rarely, they can change the direction of markets. The challenge (as in the case of volatility) is that these jumps are only a sort of concept that we use to put reality into equations. Indeed, jumps are not directly observable and their estimation is even more complex than volatility. As we will explain later, a full measurement of the jump activity in any market can only be done by computing three quantities that are increasingly difficult to estimate: the date for each jump, the sign of each jump and the absolute magnitude of each jump.

1.3.2.1 Jumps in Commodities

Jumps have helped financial economists to take decisive steps towards understanding the pricing mechanism implicit in financial markets. A wide literature has investigated the use of jump processes in equity markets, with a large part of it focusing on option pricing. Merton (1976) is often considered as one of the seminal contributions to this field: he proposed to add jumps to the pure Gaussian Black and Scholes (1973) model. By building on previous

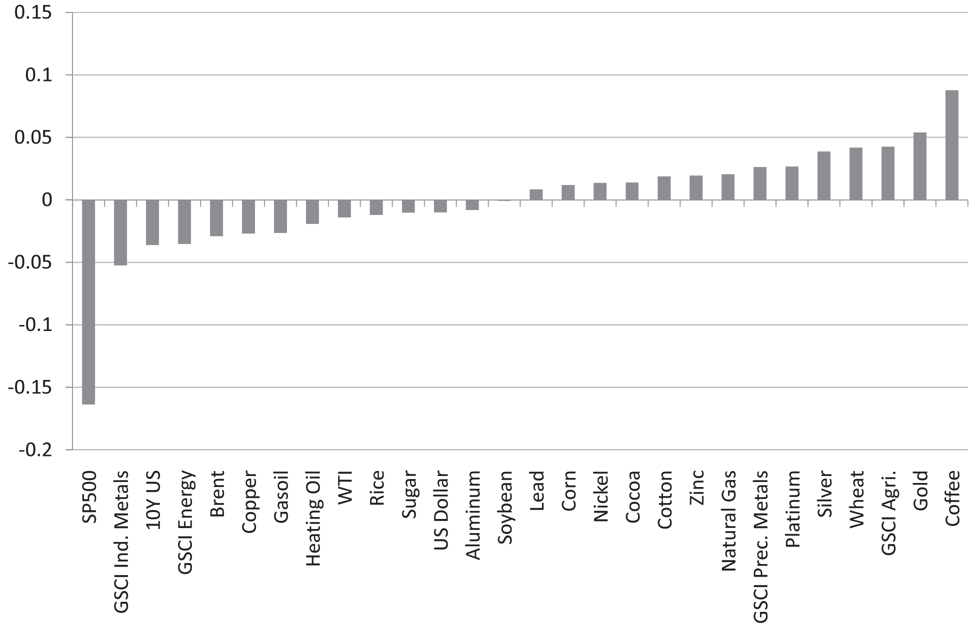

Figure 1.17 Estimated θ across commodities

notation,[23] r_t is again the close-to-close return obtained from investing in asset i on date t−1. A discrete time version of Merton's (1976) model could be written as follows:

$$r_t = \mu + \sigma\epsilon_t + \sum_{j=0}^{N_t} x_{j,t}, \tag{1.25}$$

where μ is the returns' expectation before jumps, σ is the volatility of the Gaussian part of the data-generating process of returns, N_t is the number of jumps at time t and $x_{j,t}$ is the scale of the jth jump on date t. Various distributions can be considered here for ϵ_t, N_t and for $x_{j,t}$. However, ϵ_t is now usually assumed to follow a Gaussian distribution: tail events are captured by $\sum_{j=0}^{N_t} x_{j,t}$ while more normal days of trading are modeled through $\mu + \sigma\epsilon_t$. In a continuous time setting, $\mu + \sigma\epsilon_t$ is referred to as the continuous part of the process, while the jump part is the discontinuous one. Given that our focus here is set in discrete time, none of these components are continuous, but the spirit remains the same. Ball and Torous (1985) have presented early empirical results underlying the interest of adding jumps when it comes to option pricing. This jump-based framework has been extended to stochastic volatility in Bates (1996) and Backshi *et al.* (1997), based on the work of Heston (1993). Empirical estimates for the parameters of such models have been produced in Andersen *et al.* (2012), Chernov *et al.* (2003), Eraker *et al.* (2003) and Eraker (2004). In comparison to the dynamic research activity in equity markets, commodities have been subject to less attention. Early models were proposed in Brennan and Schwartz (1985), Gibson and Schwartz (1990), Schwartz (1997) and Schwartz and Smith (2000). These contributions focused on the Gaussian part of the

[23] We dropped the i indexation insofar as it simplifies the notation.

returns-generating process. Given that the previous contributions focused on the modeling of the term structure of futures, jumps have been set apart since the contribution of Hilliard and Reis (1998) showing that futures do not incorporate jump-related premia. In a more recent study, Deng (2005) details the merits of adding jumps for non-storable commodities such as electricity. Finally, Aravindhakshan and Brorsen (2011) and Brooks and Prokopczuk (2011) present the empirical interest of adding jumps to model the dynamics of various commodities. Brooks and Prokopczuk (2011) provide estimates to the jump parameters in Equation (1.25), by assuming that N_t follows a Poisson distribution driven by a jump intensity parameter λ, and that jump sizes $x_{j,t}$ are following a Gaussian distribution with expectation μ_x and variance σ_x^2. On average, they find that the jump intensity[24] for gold, silver, crude oil, gasoline, soybean and wheat is much more important than in the case of the S&P 500: for example, in the case of crude oil, they find that around 6.2 jumps occurred on average per year over the 1985–2010 period whereas in the S&P 500 case, only 1.8 jumps[25] were to be expected. They find significant differences as well between commodities, as, for example, silver prices are expected to jump about 22 times per year. On average, they also find that the expected size of a jump in the case of commodities is smaller than the expected size of a jump in the US equity case: silver has an expected jump equal to -46% whereas in the case of S&P 500 this parameter is estimated to be equal to -258%. Aravindhakshan and Brorsen (2011) have obtained similar figures in the case of wheat and the Commodities Research Bureau (CRB) index, disentangling positive from negative jumps. Ielpo and Sévi (2011) present empirical estimates of such quantities by using an intra-day dataset for crude oil prices: their estimates are model-free, in the sense that jumps are extracted directly from 5-minute returns by using the Huang and Tauchen (2005) method[26] that does not rely on a given specification of $\sum_{j=0}^{N_t} x_{j,t}$. While Brooks and Prokopczuk (2011) obtain around 6 jumps per year for the WTI, Ielpo and Sévi (2011) find around 37 jumps. The expected size of these jumps is, however, much smaller than those of Brooks and Prokopczuk (2011). Two elements can help explain these differences: the first one being the difficulty of disentangling the effects of volatility dynamics from the effects of jumps. Jumps create additional skewness and kurtosis in total returns, as does stochastic volatility. As presented in the previous section, the fact that these two components can have a similar impact on the final return makes their estimation complex. Second, it appears that daily data and intra-day datasets provide different figures when it comes to estimates of volatility parameters or jump components. The reason for this is that the definition of jumps from a daily perspective is somewhat different when considered from an intra-day point of view: intra-day datasets can show two jumps in one day that average each other out as they are of comparable size but of opposite sign. This type of jump will not be diagnosed as a jump in a daily dataset, as they will not appear in such datasets. Obtaining these daily and intra-day datasets is still an empirical challenge so far, as illustrated in Ielpo and Sévi (2012).

1.3.2.2 Estimating Jumps from Daily Returns

Even though such model-free intra-day datasets are, by essence, probably the best way to estimate and characterize the jump activity in any financial market, such intra-day datasets are

[24] The jump intensity measures the probability of observing at least one jump over a given period of time. With annual figures, if $\lambda = 10$, this means that the expected number of jumps over a year should be 10.

[25] A similar result is obtained in Eraker (2004).

[26] We do not provide much technical explanation regarding the extraction of jumps from intra-day datasets. The interested reader can find additional information in Huang and Tauchen (2005) and Ielpo and Sévi (2011).

costly and require an extensive cleaning process. Conversely, daily prices for futures or spot prices are much easier to obtain, and can offer interesting insights into the jump activity in commodity markets. Laurent *et al.* (2011) have proposed a test based on daily data making it possible to estimate the number of days over which a jump may have happened. The test is based on the standardization of returns: returns are scaled through the estimation of their expectation and volatility in a robust way. Their method is based on an improvement of the method presented in Franses and Ghijsels (1999). We briefly review the methodology before moving to the analysis of empirical results applied to our sample of commodities.

Returns are assumed to be accurately described by a combination of an ARMA-GARCH component r_t and an additive jump component $a_t I_t$, where $I_t = 1$ when day t includes jumps of a total size a_t:

$$r_t^* = r_t + a_t I_t \tag{1.26}$$

$$\phi(L)(r_t - \mu) = \theta(L)\sigma_t \epsilon_t \text{ where } \epsilon_t \overset{i.i.d.}{\sim} N(0,1) \tag{1.27}$$

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 \sigma_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{1.28}$$

where $\phi(L)$ and $\theta(L)$ are polynomials of the lag operator with unit roots outside the unit circle. The jump detection rule is based on the fact that if ϵ_t begins Gaussian, ex-jump scaled returns should also be conditionally Gaussian. Rewriting the previous equations to have an explicit conditional drift μ_t and volatility σ_t, the model can be stated as follows:

$$r_t = \mu + \sigma_t \epsilon_t \tag{1.29}$$

$$\mu_t = \mu + \sum_{i=1}^{\infty} \lambda_i \sigma_{t-i} \epsilon_{t-i}. \tag{1.30}$$

When scaling the ex-jump return r_t as follows:

$$\tilde{J}_t = \frac{r_t - \mu_t}{\sigma_t}, \tag{1.31}$$

$\tilde{J}_t$ should evolve between two values that can be used as thresholds under the null hypothesis that no jump occurred on date t. Following Lee and Mykland (2008), Laurent *et al.* (2011) assume that $|\tilde{J}_t|$ follows a Gumbel distribution.[27] On the computations of the threshold at a given risk level, see Laurent *et al.* (2011), Equation (2.10). The key part of their approach is to disentangle r_t from $a_t I_t$. To do so, they use a robust estimation method inspired by Muler and Yohai (2008) and Muler *et al.* (2009).[28] Through Monte Carlo experiments, they show that their estimation methodology proved well enough behaved to conduct a jump analysis on the YENUSD currency, producing a list of dates for which jumps have occurred.

We intend to reproduce this analysis in the context of commodity markets.

1.3.2.3 Jumps in Commodity Markets

Figure 1.18 presents the time series of returns for the S&P 500, the 10-year rate and the US Dollar that will be used as a benchmark for analyzing the results obtained with commodities.

[27] Following the extreme value theory (EVT), the maximum of n i.i.d. realizations of the absolute value of a standard normal variable is asymptotically Gumbel distributed.

[28] For additional details on the methodology, see Laurent *et al.* (2011), pages 7–12.

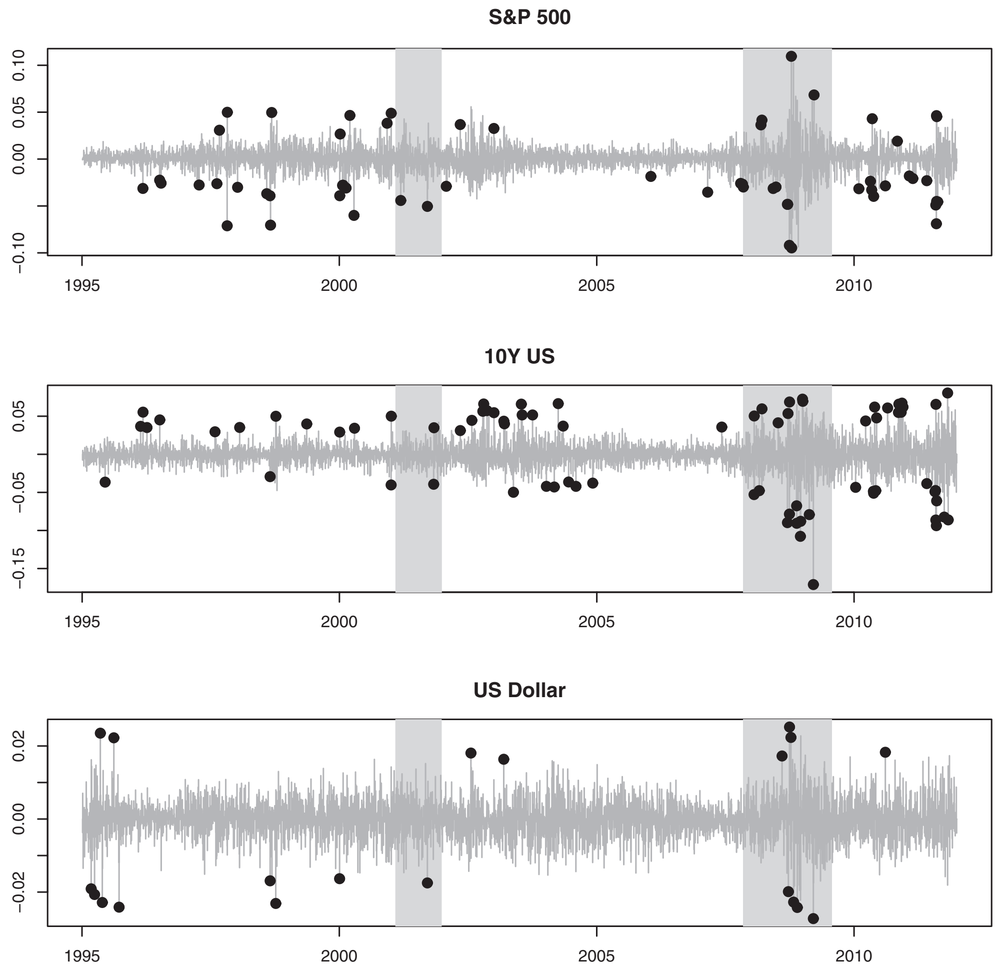

Figure 1.18 Estimated jumps for the S&P 500, the US Dollar and the US 10Y rate. Jumps are marked by black dots

Figure 1.19 presents similar graphics in the case of the GSCI indices. Table 1.13 presents various statistics obtained by applying Laurent *et al.*'s (2011) approach. We compute the total number of jumps obtained over the 1995–2012 period, as well as the average return observed over such days depending whether the return was positive or negative. In addition, we compute the average return of the 5 to 20 days following a jump. These statistics will help us understand how jumps are actually affecting the dynamics of returns on commodities. Three main conclusions can be drawn from our estimations:

– First of all, for most of the commodities, the total number of detected jumps is higher than those of the S&P 500, the US 10-year rate or the US Dollar. Platinum and lead seem to have the highest number of jumps across commodities, with respectively 109 and 101 jumps over the period, which is around 6 jumps a year. Hence, with this jump robust estimation method, the jump activity diagnosed in commodities is slightly below what was detected in Brooks

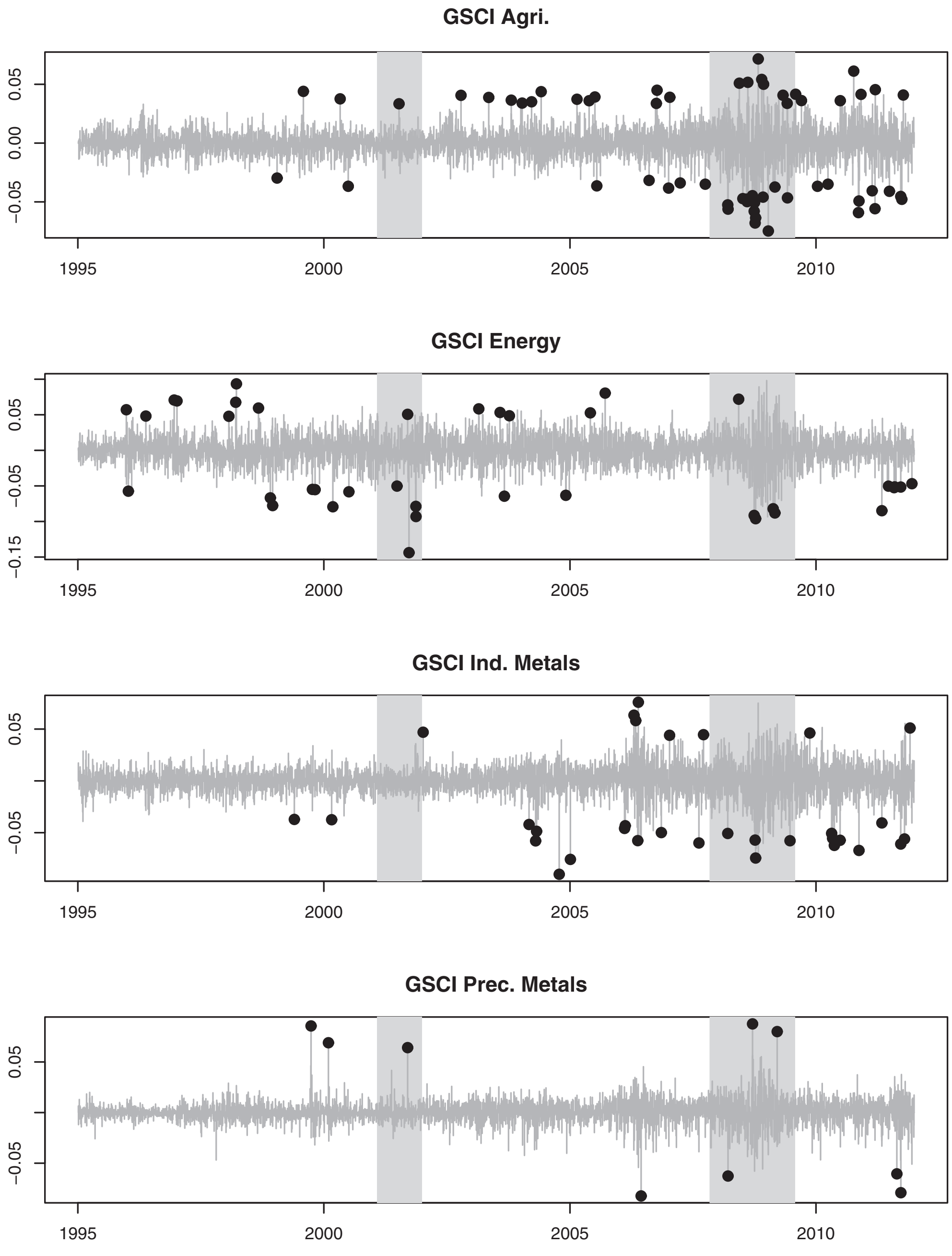

Figure 1.19 Estimated jumps for the GSCI indices. Jumps are marked by black dots

Table 1.13 Summary statistics from Laurent *et al.*'s (2011) jump estimation method

		Positive jumps					Negative jumps				
	Total Nb. Jumps	Nb. Pos. Jumps	Avg. Return	Next 5D	Next 10D	Next 20D	Nb. Neg. Jumps	Avg. Return	Next 5D	Next 10D	Next 20D
Gold	21	8	6.75	2.67	0.41	−0.17	13	−4.94	−0.54	0.26	1.29
Silver	91	31	6.02	0.73	1.13	0.05	60	−7.39	−0.42	−0.09	−0.98
Platinum	109	53	4.44	0.08	−0.32	−0.68	56	−5.07	−0.3	−1.26	−0.99
Aluminum	57	21	4.44	0.55	−0.56	−0.5	36	−5.03	0.04	−0.9	−1.5
Copper	86	30	5.7	−0.28	−0.55	0.78	56	−5.4	−0.1	−1.09	−1.67
Nickel	78	34	7.82	0.16	−2.9	−4.56	44	−8.12	−0.39	−0.63	0.84
Zinc	83	28	5.39	−1.02	−1.65	−2.95	55	−5.95	0.26	0.86	−0.15
Lead	101	39	6.71	0.74	1.28	0.9	62	−6.71	−0.29	0.4	−0.66
WTI	46	17	8.16	−3.05	−4.43	−6.41	29	−8.24	1.93	2.58	1.62
Brent	55	24	8.11	−3.4	−6.7	−6.92	31	−7.99	1.97	1.41	1.29
Gasoil	44	23	6.87	−3.62	−7.61	−8.12	21	−7.46	0.42	−0.59	−1.71
Natural Gas	52	30	13.72	−2.72	−4.61	−4.75	22	−12.42	2.81	−2.65	−5.4
Heating Oil	49	20	7.37	−1.01	−1.8	−2.08	29	−8.31	−0.02	0.51	0.33
Corn	80	56	5.38	−0.04	−0.42	−0.4	24	−6.93	0.1	1.69	2.61
Wheat	80	54	6.73	−0.43	−1.32	−1.58	26	−7.17	−1.28	0.68	−2.09
Coffee	88	45	8.25	−1.44	−1.55	−2.68	43	−8.67	−0.92	−0.12	−0.91
Sugar	55	17	8.45	1.38	0.34	1.11	38	−8.35	−0.32	0.15	1.53
Cocoa	81	37	6.55	0.31	−0.36	0.26	44	−6.83	−0.99	−1.01	−0.98
Cotton	84	47	5.7	0.36	0.58	−0.68	37	−6.59	−1.1	−1.44	−3.13
Soybean	72	36	4.46	0.06	0.73	1.19	36	−5.72	1.26	2.06	2.6
Rice	82	58	5.69	−0.3	0.38	0.8	24	−6.17	−0.33	0.42	−0.6
GSCI Agri.	59	29	4.24	0.66	0.12	0.61	30	−4.65	−0.62	−0.64	−2.1
GSCI Energy	38	15	6.2	−1.05	−3.24	−5.3	23	−7.12	0.72	0.11	−0.09
GSCI Ind. Metals	32	8	5.38	0.52	0.3	−2.46	24	−5.57	0.82	−0.27	−0.49
GSCI Prec. Metals	9	5	7.73	1.92	0.09	−5.65	4	−7.12	1.74	1.32	3.13
S&P 500	58	17	4.53	0.07	−1.51	−1.32	41	−3.9	0.99	0.13	−0.39
10Y US	76	44	5.13	−0.01	0.23	1.51	32	−6.27	−0.1	−1.73	−2.18
US Dollar	20	8	2.04	0.67	1.28	2.01	12	−2.12	0.33	0.88	0.61

and Prokopczuk (2011): they found an average of 7.5 jumps for the commodities considered. Note that we also find a large discrepancy between commodities: while we obtain 21 jumps for gold, silver has 91 jumps over the period. Similarly, wheat is characterized by 80 jumps whereas sugar has only 55 jumps. However, a common pattern appears from our estimates: the commodities displaying the highest number of jumps are agricultural commodities. Then energy and industrial metals also display an important number of jumps. The lowest number of jumps is obtained for precious metals: the GSCI sector index has only 9 jumps detected. This again is consistent with the fact that precious metals are fit for various uses. From gold to platinum we also observe discrepancies in terms of the number of jumps detected.

– Second, when comparing the number of positive and negative jumps, we cast light on a key difference between commodities and standard assets. Whereas the number of negative jumps in the case of S&P 500 is much higher (around two thirds) than the number of positive jumps, commodities exhibit a completely different story. Agricultural products have a tendency to have more positive than negative jumps – in line with Giamouridis and Tamvakis's (2001) hypothesis that there should be more positive than negative extreme returns. For the rest of commodities, it ranges from a pretty balanced situation, as in the case of platinum for which the number of positive (negative) days with jumps is equal to 53 (56). In the case of copper, we have 56 negative jumps vs. 30 positive jumps. This casts additional light on the origins of the diversification effect observed when adding commodities to a diversified portfolio. It also provides hints regarding which commodity offers the strongest relation to the worldwide business cycle: industrial metals and energy display a jump structure which is close to the S&P 500. This point will be further discussed in Part II of the book.
– Coming back to the origin of trends in commodity markets, a strong and negative/positive shock could create a long-lasting direction in markets through what is now called the 'jump to volatility channel' uncovered in a couple of recent contributions by Ait-Sahalia, Cacho-Diaz and Laeven (2011), Carr and Wu (2011) and Fulop, Li and Yu (2011). A negative (positive) jump in asset returns is passed on to market volatility when entering a bear (bull) market period. While this type of relationship exists on equity markets (and can be used to build investment strategies), there is currently no previous evidence of such patterns on commodity markets. To investigate this question, we present in Table 1.13 the average return following a positive or a negative jump. When a negative jump creates a prolonged negative trend on a given market, we obtain negative returns over the next 5 to 20 days. Conversely, when a negative (respectively positive) jump is followed by a mean reversion behavior, the upcoming returns following such a jump should be of an opposite sign. For example, in the case of soybean, a positive jump is usually followed by positive returns on soybeans in the following days. In the case of natural gas, when a negative jump occurs, a negative trend is observed over the next 20 trading days. Brent constitutes a fairly good example of a market which exhibits mean reversion after a shock: after either a positive or a negative shock, a return of the opposite sign is usually observed during the next few days. When considering the GSCI indices, we obtain results that are very consistent with the individual analyses: agricultural markets are typically affected by trends created by a positive or a negative shock. A positive shock to the energy or the industrial metals sector is usually followed by some mean reversion, while a negative shock triggers a pattern that is very similar to the equity case: after some mean-reversion, the average performance over 20 days is also negative. Finally, precious metals are typically affected by mean reverting

Table 1.14 Returns (in %) realized on common jump days on commodity indices, S&P 500, US Dollar and US 10Y rate

	GSCI Agri.	GSCI Energy	GSCI Ind. Metals	GSCI Prec. Metals	S&P 500	10Y US	US Dollar
08-Mar-96					−3.1	5.5	
05-Jul-96					−2.2	4.5	
27-Aug-98					−3.9	−2.9	
07-Oct-98						5	−2.3
03-Jan-00						2.9	−1.6
03-Jan-01					4.9	5	
14-Sep-01		5.1		6.4			
17-Sep-01					−5		−1.7
08-May-02					3.7	3.1	
15-Oct-02	4.1					5.7	
02-Jan-03					3.3	5.5	
13-Mar-03						4.3	1.6
17-Mar-08	−5.3		−5.1				
18-Mar-08					4.2	6	
19-Mar-08	−5.6			−6.3			
06-Jun-08		7.2			−3.1		
08-Aug-08	−5						1.7
15-Sep-08					−4.8	−9	
17-Sep-08				8.8	−4.8		
29-Sep-08	−5.8	−9.2			−9.2	−7.9	
30-Sep-08						6.9	2.5
06-Oct-08	−6.8		−5.7				
10-Oct-08	−6.4	−9.6	−7.4				2.2
29-Oct-08	7.2						−2.3
24-Nov-08	5.4						−2.4
17-Feb-09		−8.2				−7.9	
02-Mar-09	−3.7	−8.8					
18-Mar-09						−17.1	−2.7
12-Jan-10	−3.7					−4.3	
27-Apr-10			−5.1		−2.4		
20-May-10					−4	−5.1	
11-Aug-10					−2.9		1.8
12-Nov-10	−5.9					5.5	
16-Nov-10	−4.9		−6.7				
01-Dec-10	4.1					5.5	
22-Feb-11	−4.1				−2.1		
05-May-11		−8.5	−4.1				
01-Jun-11					−2.3	−3.8	
04-Aug-11		−5.2			−4.9	−8.6	
08-Aug-11		−5.1			−6.9	−9.4	
10-Aug-11					−4.5	−6.1	
22-Sep-11	−4.5	−5.2	−6.1				

patterns: a positive (negative) jump in precious metals should trigger a negative (positive) trend over the next 20 days.

– Finally, one last interesting question when it comes to jumps is assessing at what frequency jumps appear on several markets simultaneously. This question is also raised by Brooks and Prokopczuk (2011): they find a higher than expected probability of co-jumps between commodities belonging to different sectors, such as silver and soybean. Following our estimates, we find at least 40 days in our sample for which we observe a jump in at least two of the GSCI indices and the standard assets previously listed. Several conclusions are drawn from Table 1.14. First, standard assets and commodities exhibit a tendency to jump together. Second, this tendency seems to be more salient during crisis periods: most of the co-jumps found occur either during the 2001 or the 2008–2011 financial crisis. These jumps can be triggered by Central Bank actions, such as the Fed lowering its target rate following 9/11, triggering a positive jump in precious metals and a negative jump in the S&P 500. Negative jumps are then observed in the agricultural, energy and industrial metals' prices at the end of September 2001. Such a pattern is also observed in September 2008, following the collapse of Lehman Brothers. Finally, market crashes similar to the episode of August 2011 can be followed by a negative jump in the price of energy products.

Hence, several common patterns within commodities have been uncovered, and between commodities and more standard asset classes. The next chapter will document the relationship between these markets, both from a static and from a dynamic perspective.

REFERENCES

Aingworth, DD., Das, SR., Motwani, R., 2006. A Simple Approach for Pricing Equity Options With Markov Switching State Variables. *Quantitative Finance* 6(2):95–105

Ait-Sahalia, Y., Cacho-Diaz, J., Laeven, RJA. 2011. Modelling Financial Contagion Using Mutually Exciting Jump Processes. *NBER Working Paper* #15850, National Bureau of Economic Research, Boston, Massachusetts, USA

Akgiray, V., Booth, GG., 1987. Compound Distribution Models of Stock Returns: An Empirical Comparison. *The Journal of Financial Research* 10:269–280

Al-Anaswah, N., Wilfling, B., 2011. Identification of Speculative Bubbles using Statespace Models with Markov-switching. *Journal of Banking and Finance* 35:1073–1086

Alexander, C., Lazar, E. 2006. Normal Mixture Garch (1,1): Applications to Exchange Rate Modelling. *Journal of Applied Econometrics* 21(3):307–336

Alizadeh, A., Nomikos, N., Pouliasis, P. 2008. A Markov Regime Switching Approach for Hedging Energy Commodities. *Journal of Banking and Finance* 32:1970–1983

Amihud, Y., Mendelson, H. 1987. Trading Mechanisms and Stock Returns: An Empirical Investigation. *Journal of Finance* 42(3):533–553

Amisano, G., Giacomini, R. 2007. Comparing Density Forecasts via Weighted Likelihood Ratio Tests. *Journal of Business and Economic Statistics* 25:177–190

Andersen, TG., Dobrev, D., Schaumburg, E., 2012. Jump-robust Volatility Estimation Using Nearest Neighbor Truncation. *Journal of Econometrics* 169(1):75–93

Ane, T., Labidi, C. 2001. Revisiting the Finite Mixture of Gaussian Distributions with Applications to Futures Markets. *Journal of Futures Markets* 21(4):347–376

Ang, A., Bekaert, G. 2002. International Asset Allocation with Regime Shifts. *Review of Financial Studies* 15:1137–1187

Ang, A., Chen, J. 2002. Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics* 63:443–494

Ang, A., Timmermann, A. 2011. Regime Changes and Financial Markets. *NBER Working Paper* #17182, National Bureau of Economic Research, Boston, Massachusetts, USA

Anson, MJP. 2002. *Handbook of Alternative Assets*, Wiley Finance, Chichester, UK

Aravindhakshan, SC., Brorsen, BW. 2011. Identifying Jumps and Systematic Risk in Futures. *Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis*, Forecasting and Market Risk Management, USA

Ardeni, PG., Wright, B. 1992. The Prebisch–Singer Hypothesis: A Reappraisal Independent of Stationarity Hypotheses. *Economic Journal* 102:803–812

Asness, C. 1994. Variables That Explain Stock Returns. *Ph.D. Dissertation* University of Chicago, USA

Asness, C., Moskowitz, TJ., Pedersen, LH. 2010. Value and Momentum Everywhere. *AQR Capital*, University of Chicago and National Bureau of Economic Research, USA

Bakshi, G., Cao, C., Chen, Z. 1997. Empirical Performance of Alternative Option Pricing Models. *The Journal of Finance* 52(5):2003–2049

Ball, CA., Torous, WN. 1985. On Jumps in Common Stock Prices and Their Impact on Call Option Pricing. *The Journal of Finance* 40(1):155–173

Barndorff-Nielsen, OE., Blaesild, P. 1981. Hyperbolic Distributions and Ramifications: Contributions to Theory and Applications. In: Taillie, C., Patil, GP., Baldessari, BA. (Eds), *Statistical Distributions in Scientific Work*, Vol. 4., pages 19–44, Reidel, Dordrecht, The Netherlands

Barndorff-Nielsen, O., Shephard, N. 2002. Econometric Analysis of Realized Volatility and its Use in Estimating Stochastic Volatility Models. *Journal of the Royal Statistical Society, Series B* 64:253–280

Barndorff-Nielsen, O., Shephard, N. 2004. Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics* 2:1–37

Bates, DS. 1996. Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies* 9(1):69–107

Batten, JA., Ciner, C., Lucey, BM. 2008. The Macroeconomic Determinants of Volatility in Precious Metals Markets. *Resources Policy* 35(2):65–71

Batten, JA., Lucey, BM. 2007. Volatility in the Gold Futures Market. *Applied Economics Letters* 17(2):187–190

Baur, DG., 2012. The Asymmetric Volatility of Gold. *Journal of Alternative Investments*, 14(4):26–38

Bernard, JT., Khalaf, L., Kichian, M., McMahon, M. 2008. Forecasting Commodity Prices: GARCH, Jumps, and Mean Reversion. *Journal of Forecasting* 27(4):279–291

Bertholon, H., Monfort, A., Pegoraro, F. 2007. Pricing and Inference with Mixtures of Conditionally Normal Processes. *Banque de France Working Paper Series* #188, Paris, France

Bessembinder, H. 1992. Systematic Risk, Hedging Pressure and Risk Premiums in Futures Markets. *Review of Financial Studies* 5(4):637–667

Black, F. 1976. Studies of Stock Price Volatility Changes. *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistics*, USA

Black, F., Scholes, M. 1973. The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy* 81(3):637–654

Bleaney, M., Greenaway, D. 1993. Long-run Trends in the Relative Price of Primary Commodities and in the Terms of Trade in Developing Countries. *Oxford Economic Papers* 45:349–363

Bodie, Z., Rosansky, VI. 1980. Risk and Return in Commodity Futures. *Financial Analysts Journal* 36(3):27–39

Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31:307–327

Box, GEP., Jenkins, GM., Reinsel, GC. 2008. *Time Series Analysis: Forecasting and Control*, Wiley Series in Probability and Statistics, Chichester, UK

Brennan, MJ. 1991. The Price of Convenience and the Valuation of Commodity Contingent Claims. In: Land, D., Oeksendal, B. (Eds), *Stochastic Models and Options Values*, Elsevier

Brennan, M., Schwartz, E. 1985. Evaluating Natural Resource Investments. *Journal of Business* 58:135–157

Brooks, C., Prokopczuk, M. 2011. The Dynamics of Commodity Prices. *ICMA Centre Discussion Papers in Finance* #DP2011–09, USA

Campbell, JY., Hentschel, L. 1992. No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns. *Journal of Financial Economics* 31:281–318

Carr, P., Wu, X. 2011. Leverage Effect, Volatility Feedback, and Self-Exciting Market Disruptions: Disentangling the Multi-Dimensional Variations in S&P500 Index Options. *Bloomberg Portfolio Research Paper* #2009–03-Frontiers, New York, USA

Casassus, J., Collin-Dufresne, P. 2005. Stochastic Convenience Yield Implied from Commodity Futures and Interest Rates. *Journal of Finance* 60:2283–2331

Cashin, P., McDermott, JC. 2002. The Long-Run Behavior of Commodity Prices: Small Trends and Big Variability. *IMF Staff Papers* 49(2):175–199

Chang, CL., McAleer, M., Tansuchat, R. 2010. Analyzing and Forecasting Volatility Spillovers, Asymmetries and Hedging in Major Oil Markets. *Energy Economics* 32(6):1445–1455

Chauvet, M., Potter, S. 2000. Coincident and Leading Indicators of the Stock Market. *Journal of Empirical Finance* 7:87–111

Chen, SS., Lee, CF., Shrestha, K., 2001. On a Mean-Generalized Semivariance Approach to Determining the Hedge Ratio. *Journal of Futures Markets* 21:581–598

Chernov, M., Gallant, AR., Ghysels, E., Tauchen, G. 2003. Alternative Models for Stock Price Dynamics. *Journal of Econometrics* 116(1):225–257

Cheung, YW., Ng, LK. 1992. Stock Price Dynamics and Firm Size: An Empirical Investigation. *Journal of Finance* 47:1985–1997

Chevallier, J., Ding, W., Ielpo, F. 2012. Implementing a Simple Rule for Dynamic Stop-Loss Strategies. *Journal of Investing* Winter:1–4

Chorro, C., Guegan, D., Ielpo, F. 2010. Likelihood-Related Estimation Methods and Non-Gaussian GARCH Processes. Universite Paris 1 Pantheon-Sorbonne *HAL-SHS Working Papers* #00523371, Lyon, France

Chorro, C., Guegan, D., Ielpo, F. 2012. Option Pricing for GARCH-type Models with Generalized Hyperbolic Innovations. *Quantitative Finance* 12(7):1–50

Christie, AA. 1982. The Stochastic Behavior of Common Stock Variances – Value, Leverage and Interest Rate Effects. *Journal of Financial Economics* 3:145–166

Clare, A., Seaton, J., Smith, PN., Thomas, S. 2012. Breaking into the Blackbox: Trend Following, Stop Losses, and the Frequency of Trading: the case of the S&P500. *Discussion Papers*, Department of Economics, University of York, UK

Cochrane, J., Piazzesi, M. 2005. Bond Risk Premia. *American Economic Review* 95(1):138–160

Connor, G., Korajczyk, RA. 1993. A Test for the Number of Factors in an Approximate Factor Model. *Journal of Finance* 48(4):1263–1291

Cootner, PH. 1960. Returns to Speculators: Telser Versus Keynes. *The Journal of Political Economy* 68(4):396–404

Cuddington, J. 1992. Long-Run Trends in 26 Primary Commodity Prices. *Journal of Development Economics* 39:207–227

Cuddington, J., Urzua, C. 1989. Trends and Cycles in the Net Barter Terms of Trade: A New Approach. *Economic Journal* 99:426–442

Damodaran, A. 1993. A Simple Measure of Price Adjustment Coefficients. *Journal of Finance* 48(1):387–400

Deaton, A., Laroque, G. 1992. On the Behaviour of Commodity Prices. *Review of Economic Studies* 59:1–23

Deaves, R., Krinsky, I. 1995. Do Futures Prices For Commodities Embody Risk Premiums? *Journal of Futures Markets* 15(6):637–648

Dempster, MAH., Medova, E., Tang, K. 2010. Long and Short Term Jumps in Commodity Futures Prices. *SSRN Working Paper* #1107966, Social Science Research Network, USA

Deng, SJ. 1999. Stochastic Models of Energy Commodity Prices and Their Applications: Mean-reversion with Jumps and Spikes. *Proceedings of the Fourth Annual POWER Conference*, Berkeley, USA

Deng, SJ. 2005. Valuation of Investment and Opportunity-To-Invest in Power Generation Assets with Spikes in Electricity Price. *Managerial Finance* 31(6):95–115

Diebold, FX., Yilmaz, K. 2012. Better to Give than to Receive: Forecast-Based Measurement of Volatility Spillovers. *International Journal of Forecasting* 28(1):57–66

Ding, Z., Granger, CWJ., Engle, R., 1993. A Long Memory Property of Stock Market Returns and a New Model. *Journal of Empirical Finance* 1:83–106

Dionne, G., Gauthier, G., Hammami, K., Maurice, M., Simonato, JG. 2011. A Reduced Form Model of Default Spreads with Markov-Switching Macroeconomic Factors. *Journal of Banking and Finance* 35:1984–2000

Du, X., Yu, CL., Hayes, DJ., 2011. Speculation and Volatility Spillover in the Crude Oil and Agricultural Commodity Markets: A Bayesian analysis. *Energy Economics* 33(3):497–503

Duffie, D., Gray, S., Hoang, P. 1995. Volatility in Energy Prices. In: Pai, L., Field, P. (Eds) *Managing Energy Price Risk*, pages 273–290, Risk Publications, UK

Eberlein, E., Prause, K. 2002. The Generalized Hyperbolic Model: Financial Derivatives and Risk Measures. In: Geman, H., Madan, D., Pliska, S., Vorst, T. (Eds), *Mathematical Finance-Bachelier Congress 2000*, pages 245–267. Springer Verlag, Berlin, Germany

Engle, R. 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* 50(4):987–1007

Engle, R., Ng, S. 1993. Measuring and Testing the Impact of News on Volatility. *Journal of Finance* 48(5):1749–1778

Eraker, B. 2004. Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. *The Journal of Finance* 59(3):1367–1404

Eraker, B., Johannes, M., Polson, N. 2003. The Impact of Jumps in Volatility and Returns. *The Journal of Finance* 58(3):1269–1300

Erb, CB., Campbell, RH. 2006. The Strategic and Tactical Value of Commodity Futures. *Financial Analysts Journal* 62(2):69–97

Fan, Y., Pastorello, S., Renault, E., 2007. Maximisation by Parts in Extremum Estimation. *Working Paper*, University of North Carolina, USA

Ferraro, D., Rogoff, K., Rossi, B. 2011. Can Oil Prices Forecast Exchange Rates? *Working Paper* #11–05, Duke University, Department of Economics, USA

Franses, P., Ghijsels, H. 1999. Additive Outliers, GARCH and Forecasting Volatility. *International Journal of Forecasting* 15:1–9

Fuertes, AM., Miffre, J., Rallis, G. 2008. Tactical Allocation in Commodity Futures Markets: Combining Momentum and Term Structure Signals. *EDHEC Business School Working Paper*, Lille, France

Fulop, A., Li, J., Yu, J. 2011. Bayesian Learning of Self-Exciting Return Dynamics. *SMU Economics and Statistics Working Paper Series* #03–2012, Singapore Management University, Singapore

Gatumel, M., Ielpo. F., 2011. The Number of Regimes Across Asset Returns: Identification and Economic Value. Universite Paris 1 Pantheon-Sorbonne *HAL-SHS Working Papers* #00658540, Lyon, France

Geman, H., Nguyen, VN. 2005. Soybean Inventory and Forward Curve Dynamics. *Management Science* 51:1076–1091

Ghoshray, A. 2011a. Underlying Trends and International Price Transmission of Agricultural Commodities. *Asian Development Bank Economics Working Paper Series* #257, Mandaluyong City, Philippines

Ghoshray, A. 2011b. A Reexamination of Trends in Primary Commodity Prices. *Journal of Development Economics* 95(2):242–251

Giamouridis, DG., Tamvakis, MN. 2001. The Relation Between Return and Volatility in the Commodity Markets. *Journal of Alternative Investments* 4(1):54–62

Gibson, R., Schwartz, E. 1990. Stochastic Convenience Yield and the Pricing of Oil Contingent Claims. *Journal of Finance* 45:959–976

Giot, P., Laurent, S. 2003. Market Risk in Commodity Markets: a VaR Approach. *Energy Economics* 25(5):435–457

Glosten, LR., Jagannathan, R., Runkle, DE. 1993. On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance* 48(5):1779–1801

Gorton, GB., Rouwenhorst, KG. 2005a. Facts and Fantasies about Commodity Futures. *Financial Analysts Journal* 62(2):47–68

Gorton, GB., Rouwenhorst, KG. 2005b. A Note on Erb and Harvey (2005). *Yale School of Management Working Paper* #2595, Yale School of Management, Yale, USA

Gorton, GB., Hayashi, F., Rouwenhorst, KG. 2012. The Fundamentals of Commodity Futures Returns. *Yale ICF Working Paper* #07–08, Yale ICF, Yale, USA

Gourieroux, C., Jasiak, J. 2001. *Financial Econometrics: Problems, Models, and Methods*. Princeton University Press, Princeton, USA

Grilli, E., Yang, M. 1988. Primary Commodity Prices, Manufactured Goods Prices, and the Terms of Trade of Developing Countries: What the Long Run Shows. *World Bank Economic Review* 2:1–47

Haigh, M., Holt, M. 2002. Crack Spread Hedging: Accounting for Time-Varying Volatility Spillovers in the Energy Futures Markets. *Journal of Applied Econometrics* 17(3):269–289

Hamilton, JD., 1989. A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica* 57(2):357–384

Hammoudeh, S., Yuan, Y. 2008. Metal Volatility in Presence of Oil and Interest Rate Shocks. *Energy Economics* 30(2):606–620

Hammoudeh, S., Li, H., Jeon, BG. 2003. Causality and Volatility Spillovers among Petroleum Prices of WTI, Gasoline and Heating Oil in Different Locations. *The North American Journal of Economics and Finance* 14(1):89–114

Hansen, B.E. 1992. The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov-Switching Model of GNP. *Journal of Applied Econometrics* 7:S61-S82

Helg, R. 1991. A Note on the Stationarity of the Primary Commodities Relative Price Index. *Economics Letters* 36:55–60

Henry, Ò., 2009. Regime Switching in the Relationship between Equity Returns and Short-Term Interest Rates in the UK. *Journal of Banking and Finance* 33:405–414

Heston, SL. 1993. A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies* 6(2):327–343

Hilliard, J., Reis, J. 1998. Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot. *Journal of Financial and Quantitative Analysis* 33:61–86

Hillier, D., Draper, P., Faff, R. 2006. Do Precious Metals Shine? An Investment Perspective. *Financial Analysts Journal* 62(2): 98–106

Huang, X., Tauchen, G. 2005. The Relative Price Contribution of Jumps to Total Price Variance. *Journal of Financial Econometrics* 3:456–499

Hung, JC., Lee, MC., Liu, HC. 2008. Estimation of Value-at-Risk for Energy Commodities via Fat-Tailed GARCH Models. *Energy Economics* 30:1173–1191

Ielpo, F., Sévi, B. 2011. Do Jumps Help in Forecasting the Density of Returns? *SSRN Working Paper*, Social Science Research Network, USA

Ielpo, F., Sévi, B. 2012. Empirical Bias in Intraday Volatility Measures. *Finance Research Letters* 9(4):231–237

Ilmanen, A. 2011. *Expected Returns: An Investor's Guide to Harvesting Market Rewards*. Wiley Finance, Chichester, UK

Irwin, SH., Zulauf, CR., Gerlow, ME., Tinker, JN. 1997. A Performance Comparison of a Technical Trading System with ARIMA models for Soybean Complex Prices. *Advances in Investment Analysis and Portfolio Management* 4:193–203

Jegadeesh, N., Titman, S. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48:65–91

Kaldor, N. 1939. Speculation and Economic Theory. *Review of Economic Studies* 7:1–27

Kat, HM., Oomen, RCA. 2007a. What Every Investor Should Know About Commodities, Part I: Univariate Return Analysis. *Journal of Investment Management* 5(1):1–25

Kat, HM., Oomen, RCA. 2007b. What Every Investor Should Know About Commodities, Part II: Multivariate Return Analysis. *Journal of Investment Management* 5(3):1–25

Kessler, S., Scherer, B. 2009. Varying Risk Premia in International Bond Markets. *Journal of Banking and Finance* 33(8):1361–1375

Keynes, JM. 1930. *A Treatise on Money*. MacMillan, London, UK

Kolb, RW. 1992. Is Normal Backwardation Normal? *Journal of Futures Markets* 12(1):75–90

Kon, SJ. 1984. Models of Stock Returns: A Comparison. *Journal of Finance* 39(1):147–165

Koutmos, G. 1998. Asymmetries in the Conditional Mean and the Conditional Variance: Evidence From Nine Stock Markets. *Journal of Economics and Business* 50(3):277–290

Kroner, KF., Sultan, J. 1993. Time-Varying Distributions and Dynamic Hedging with Foreign Currency Futures. *Journal of Financial and Quantitative Analysis* 28(4):535–551

Laurent, S., Lecourt, C., Palm, FC. 2011. Testing for Jumps in GARCH models: a Robust Approach. *Working Paper*, Maastricht University, Maastricht, Belgium

Lee, SS., Mykland, PA. 2008. Jumps in Financial Markets: a New Nonparametric Test and Jump Dynamics. *Review of Financial Studies* 21(6):2535–2563

Lee, T., Zyren, J. 2007. Volatility Relationship between Crude Oil and Petroleum Products. *Atlantic Economic Journal* 35(1):97–112

Lekkos, I. 1999. Distributional Properties of Spot and Forward Interest Rates: USD, DEM, GBP and JPY. *The Journal of Fixed Income* 8(4):35–54

Leon, J., Soto, R., 1997. Structural Breaks and Long Run Trends in Commodity Prices. *Journal of International Development* 9:347–366

Lien, D., Tse, YK., 2000. Hedging Downside Risk with Futures Contracts. *Applied Financial Economics* 10:163–170

Liu, P., Tang, K. 2011. The Stochastic Behavior of Commodity Prices with Heteroskedasticity in the Convenience Yield. *Journal of Empirical Finance* 18:211–224

Longin, F., Solnik, B. 2001. Extreme Correlation of International Equity Markets. *Journal of Finance* 56:649–676

Lukac, LP., Brorsen, BW., Irwin, SH. 1988. A Test of Futures Market Disequilibrium using Twelve Different Trading Systems. *Applied Economics* 20(5):623–639

Maheu, J., McCurdy, TH. 2000. Identifying Bull and Bear Markets in Stock Returns. *Journal of Business & Economic Statistics* 18(1):100–112

Maheu, J., McCurdy, TH., Song, Y. 2012. Components of Bull and Bear Markets: Bull Corrections and Bear Rallies. *Journal of Business and Economic Statistics* 30(3):391–403

Manoliu, M., Tompaidis, S. 2002. Energy Futures Prices: Term Structure Models with Kalman Filter Estimation. *Applied Mathematical Finance* 9:21–43

Marshall, BR., Cahan, RH., Cahan, JM. 2008. Can Commodity Futures be Profitably Traded with Quantitative Market Timing Strategies? *Journal of Banking and Finance* 32:1810–1819

McKenzie, M., Mitchell, H., Brooks, R., Faff, R. 2001. Power ARCH Modeling of Commodity Futures Data on the London Metal Exchange. *European Journal of Finance* 7:22–38

Merton, R. 1976. Option Pricing when Underlying Stock Returns are Discontinuous. *Journal of Financial Economics* 3:125–144

Miffre, J., Rallis, G. 2007. Momentum Strategies in Commodity Futures Markets. *Journal of Banking and Finance* 31(6):1863–1886

Monfort, A., Pegoraro, F. 2006. Option Pricing with Mixture of Gaussian Distributions. *Working Paper of the Banque de France*, Paris, France

Morana, C. 2001. A Semiparametric Approach to Short-Term Oil Price Forecasting. *Energy Economics* 23(3):325–338

Moskowitz, TJ., Ooi, YH., Pedersen, LH. 2012. Time Series Momentum. *Journal of Financial Economics* 104(2):228–250

Muler, N., Pena, D., Yohai, V. 2009. Robust Estimation for ARMA Models. *Annals of Statistics* 37:816–840

Muler, N., Yohai, V. 2008. Robust Estimates for GARCH Models. *Journal of Statistical Planning and Inference* 138:2918–2940

Nelson, DB. 1991. Conditional Heteroskedasticity in Asset Returns. *Econometrica* 59:347–370

Newbold, P., Pfaffenzeller, S., Rayner, A. 2005. How Well are Long-Run Commodity Price Series Characterized by Trend Components? *Journal of International Development* 17(4):479–494

Newey, WK., West, KD. 1987. A Simple, Positive Semidefinite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55:703–708

Ng, V., Pirrong, S. 1996. Price Dynamics in Refined Petroleum Spot and Futures Markets. *Journal of Empirical Finance* 2:359–388

Pindyck, R. 2001. The Dynamics of Commodity Spot and Futures Markets: A Primer. *The Energy Journal* 22(3):1–30

Pindyck, R. 2004. Volatility and Commodity Price Dynamics. *Journal of Futures Markets* 24:1029–1047

Politis, ND. 2004. A Heavy Tailed Distribution for ARCH Residuals with Application to Volatility Prediction. *Annals of Economics and Finance* 5:283–298

Prebisch, R. 1950. *The Economic Development of Latin America and its Principal Problems*. United Nations, New York, USA

Psaradakis, Z., Spagnolo, N. 2003. On the Determination of the Number of Regimes in Markov-Switching Autoregressive Models. *Journal of Time Series Analysis* 24(2):237–252

Sadorsky, P. 2006. Modeling and Forecasting Petroleum Futures Volatility. *Energy Economics* 28(4):467–488

Sapsford, D. 1985. The Statistical Debate on the Net Barter Terms of Trade Between Primary Commodities and Manufactures: A Comment and Some Additional Evidence. *Economic Journal* 95:781–788

Schwartz, E. 1997. The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *Journal of Finance* 52:923–973

Schwartz, E., Smith, J. 2000. Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science* 46:893–911

Schwert, GW. 1988. Tests for Unit Roots: A Monte Carlo Investigation. *NBER Technical Working Papers* #0073, National Bureau of Economic Research, Boston, Massachusetts, USA

Schwert, GW. 1989a. Why does Stock Market Volatility Change Over Time? *Journal of Finance* 44:1115–1153

Schwert, GW. 1989b. Business Cycles, Financial Crises and Stock Volatility. *Carnegie–Rochester Conference Series on Public Policy* 31:83–125

Sekkel, R. 2011. International Evidence on Bond Risk Premia. *Journal of Banking and Finance* 35(1):174–181

Serletis, A. 1994. A Cointegration Analysis of Petroleum Futures Prices. *Energy Economics* 16:93–97

Singer, H. 1950. The Distribution of Gains Between Investing and Borrowing Countries. *American Economic Review: Papers and Proceedings* 11:473–485

Singh, A., Karali, B., Ramirez, OA. 2011. High Price Volatility And Spillover Effects In Energy Markets. *Proceedings of the 2011 Annual Meeting*, Agricultural and Applied Economics Association, Pittsburgh, Pennsylvania, USA

Smith, A., Naik, PA., Tsai, C. 2006. Markov-Switching Model Selection Using Kullback–Leibler Divergence. *Journal of Econometrics* 134(2):553–577

Song, PX., Fan, Y., Kalfeisch, D. 2005. Maximization by Parts in Likelihood Inference. *Journal of the American Statistical Association* 100:1145–1158

Sorensen, C. 2002. Modeling Seasonality in Agricultural Commodity Futures. *Journal of Futures Markets* 22(5):393–426

Szakmary, A., Shen, Q., Sharma, S. 2010. Trend Following Trading Strategies in Commodity Futures: a Reexamination. *Journal of Banking and Finance* 34: 409–426

Trolle, A., Schwartz, E. 2009. Unspanned Stochastic Volatility and the Pricing of Commodity Derivatives. *Review of Financial Studies* 22:4423–4461

Tucker, AL., Pond, L. 1998. The Probability Distribution of Foreign Exchange Price Changes: Tests of Candidate Processes. *Review of Economics and Statistics* 11:638–647

Tully, E., Lucey, BM. 2007. A Power GARCH Examination of the Gold Market. *Research in International Business and Finance* 21(2):316–325

Wang, C., Yu, M. 2004. Trading Activity and Price Reversals in Futures Markets. *Journal of Banking and Finance* 28:1337–1361

Wang, T., Wu, J., Yang, J. 2008. Realized Volatility and Correlation in Energy Futures Markets. *Journal of Futures Markets* 28:993–1011

Wickham, P., Reinhart, C. 1994. Commodity Prices – Cyclical Weakness or Secular Decline? *IMF Working Papers* #94/7, International Monetary Fund, Washington DC, USA

Zheng, Y., Kinnucan, H., Thompson, H. 2008. News and Volatility of Food Prices. *Applied Economics* 40(13):1629–1635