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Introduction

1.1 Composite Structures

Composite materials are used extensively in engineering and industrial applications; from traditional structural strengthening to advanced aerospace and defence systems, and from nanoscale carbon nanotube (CNT) applications to large scale turbines and power plants and so on. Their highly flexible design allows prescribed tailoring of material properties, fitted to the engineering requirements. These include a wide variety of properties across various length scales, including nano and micro-mechanical structural needs, thermo-mechanical specifications and even electro-magneto-mechanical characteristics. In addition, multilayer and orthotropic functionally graded materials (FGMs) have been increasingly used in advanced material systems in high-tech industries to withstand hostile operating conditions, where conventional homogeneous composites may fail.

Composite materials are created by the combination of two or more materials to form a new material with enhanced properties compared to those of the individual constituents. By this definition, reinforced concrete, as a mixture of stone, sand, cement and steel, wood comprised of cellulose and lignin, and bone consisting of collagen and apatite can be regarded as special types of composites. The conventional forms of composites, however, are made of two main ingredients: fibres and matrix. Fibres are required to have a number of specifications, such as high elasticity modulus and ultimate strength, and must retain their geometrical and mechanical properties during fabrication and handling. The matrix constituent must be chemically and thermally compatible with the fibres over a long period of time, and is meant to bind together the fibres, protect their surfaces, and transfer stresses to the fibres efficiently.

High specific strength, excellent fatigue durability, significant corrosion, chemical and environmental resistances, especially important in food and chemical processing plants, cooling towers, offshore platforms and so on, designable mechanical properties, electromagnetic transparency or electrical insulation, together with relatively fast deployment and low maintenance have made composites attractive materials for almost all engineering applications.

Despite excellent characteristics, composites suffer from a number of shortcomings, such as brittleness, high thermal and residual stresses, poor interfacial bonding strength and low toughness, which may facilitate the process of unstable cracking under different conditions, such as imperfection in material strength, fatigue, yielding and production faults. These failures

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can cause extensive damage accompanied by substantial reduction in stiffness and load bearing capacity, decreased ductility and the possibility of abrupt collapse mechanisms. The problem becomes even more important in intensive concentrated loading conditions, such as moving and dynamic loadings, high velocity impact and explosion.

Moreover, composite materials are utilized in thin forms which are susceptible to various types of defects. Cracking, the most likely type of defect in these structures, can be initiated and propagated under different production imperfections and service circumstances, such as initial weakness in material strength, fatigue and yielding. Therefore, the study of the crack stability and load bearing capacity of these types of structures, which directly affect the safety and economics of many important industries, has become an important topic of research for the computational mechanics community.

1.2 Failures of Composites

Layered, orthotropic, sometimes inhomogeneous and multi-material characteristics of composites allow the possibility for occurrence of various failure modes under different loading conditions. In general, however, the failure modes of composite plies can be categorized into four classes: fibre failure, ply delamination, matrix cracking and fibre/matrix deboning. These failure modes, or any combination of them, reduce and may ultimately eliminate the composite action altogether.

1.2.1 Matrix Cracking

The matrix material is the lowest strength component in a composite action to withstand a specific loading. The brittle nature of matrix cracking is the main source of failure in composites and may initiate other modes of failure, such as delamination and debonding.

1.2.2 Delamination

Delamination, also called interlaminar debonding or interface cracking, is among the most commonly encountered failure modes in composite laminates and may become a major source of concern in the performance and safety of composites by reducing the ductility, stiffness and strength of the composite specimen and even cause sudden brittle fracture mechanisms.

Delaminations can be initiated or extended from high stress concentrations that originate from mechanical effects, such as manufacturing, transportation and service effects, such as temperature, moisture, matrix shrinkage, or from general loading conditions, especially sudden concentrated loadings such as impact and explosion.

These effects may become more severe around curved sections, sudden changes of cross sections, and free edges. One important aspect of delamination failure is that substantial internal damage may exist in the interface adjacent plies without any apparent external destruction.

1.2.3 Fibre/Matrix Debonding

A perfect bonding between the fibre and matrix is necessary to ensure the composite action. Any debonding, or even local sliding, may substantially affect the overall strength of the

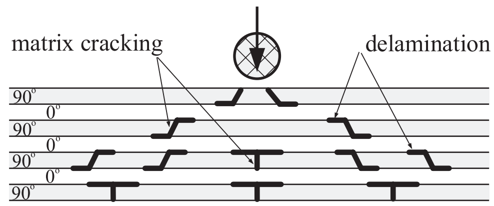

Figure 1.1 Main modes of cracking in composites.

composite specimen. It is generally accepted that the composite material should be designed and manufactured in such a way that fibre/matrix debonding never occurs before matrix cracking and delamination.

1.2.4 Fibre Breakage

Fibre breakage is probably the last mode of failure of a composite specimen prior to its collapse. Once the fibres are broken, the load bearing capacity of the specimen suddenly drops to almost zero.

1.2.5 Macro Models of Cracking in Composites

Homogeneous composites are primarily assumed to behave in an orthotropic linear elastic state. Equivalent homogeneous orthotropic material properties are determined based on the assumption of an equivalent smeared fibre/matrix mixture. This is certainly the case for most numerical solutions at the macroscopic level. As a result, fracture is assumed to occur only in an in-plane cracking mode (matrix/fibre cracking) or in an interlaminar cracking state (delamination), as depicted in Figure 1.1.

1.3 Crack Analysis

In this section, a brief review of the main available theoretical approaches for analysis of crack stability and propagation is presented. There are different classifications in crack analysis. For example, from the geometrical point of view, a crack may be represented as an internal discontinuity or external boundary (discrete crack model), characterizing a strong displacement discontinuity (Figure 1.2a), or its equivalent continuum mechanical effects (in terms of stiffness and strength reduction) can be considered within the numerical model in a distributed fashion without explicitly defining its geometry (smeared crack model) (see Figure 1.2b).

1.3.1 Local and Non-Local Formulations

Early attempts to simulate crack problems by numerical methods adopted a simple rule to check the stress state at any sampling point against a material strength criterion. The constitutive

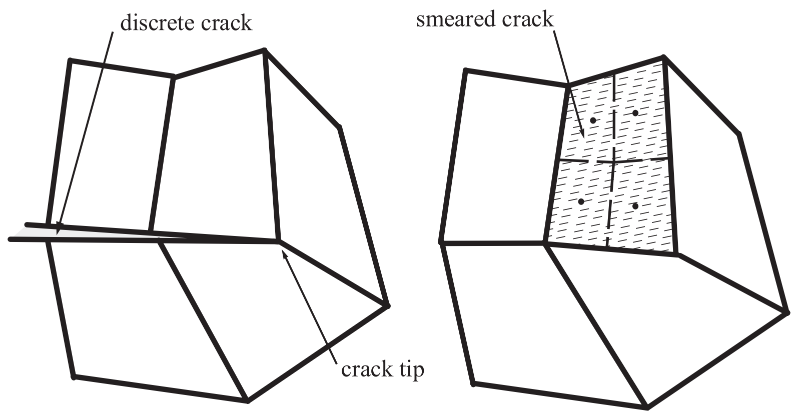

Figure 1.2 Discrete and smeared crack models in a typical finite element mesh.

behaviour of the point was only affected by its own local stress–strain state (point 1 in Figure 1.3). Soon it was realized that cracking could not be regarded solely as a local point-wise stress-based criterion, and such a local approach for fracture analysis may become size or mesh dependent and unreliable.

The remedy was the introduction of non-local formulations based on characteristic length scales (Bazant and Planas, 1997), defined for the material constitutive law as an intrinsic material property, or for a numerical model based on the geometrical requirements. To clarify the basic idea, consider a very simplified case (point 2 in Figure 1.3), where the fracture behaviour of this point is determined from a non-local criterion expressed in terms of the state

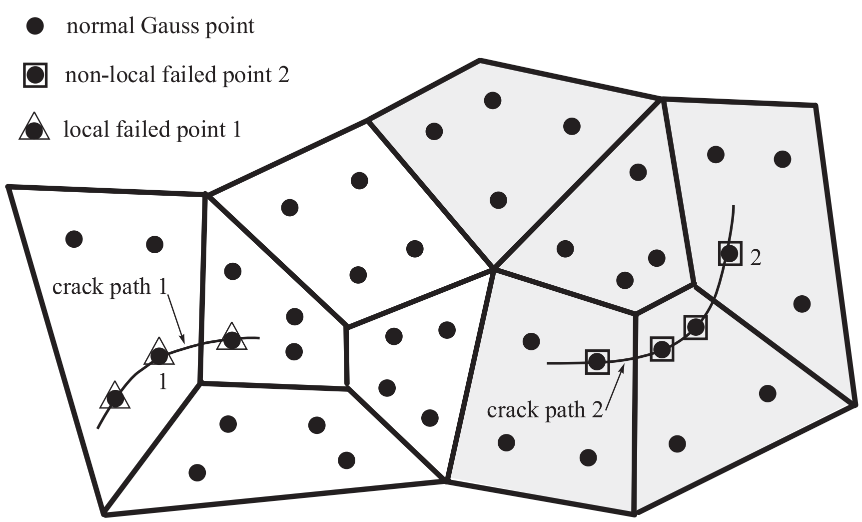

Figure 1.3 Local and non-local evaluation of the cracking state.

variables (including the length scale) at that point and a number of surrounding points in its support domain.

1.3.2 Theoretical Methods for Failure Analysis

Three fundamental approaches are available for discussion of the effects of defects and failures: continuum-based plasticity and damage mechanics and the crack-based approach of fracture mechanics. All three approaches can be implemented within different numerical methods. These methods, however, are applied to fundamentally different classes of failure problems. While the theory of plasticity and damage mechanics are basically designed for problems where the displacement field, and usually the strain field, remain continuous everywhere, fracture mechanics is essentially formulated to deal with strong discontinuities (cracks) where both the displacement and strain fields are discontinuous across a crack surface (Mohammadi, 2003a, 2008).

In practice, however, damage mechanics and the theory of plasticity have been modified and adapted for failure/fracture analysis of structures with strong discontinuities and fracture mechanics is sometimes used for weak discontinuity problems. It is, therefore, difficult to distinguish between the practical engineering applications exclusively associated with each class.

1.3.2.1 Plasticity

Plasticity theory is well developed to deal with plastic deformations and is based on various local-failure criteria, written in terms of local (point-wise) state variables, such as stress tensor and elastic and plastic strain components. Most of the plasticity based crack analyses are based on softening plasticity models of smeared cracking, which may become mesh or size dependent, if higher order formulations (such as the Cosserate or gradient theories) are not adopted. Plasticity models are capable of predicting the initiation of crack as well as predicting its growth, and can be readily implemented in different numerical techniques.

1.3.2.2 Fracture Mechanics

In contrast, the theory of linear elastic fracture mechanics (LEFM) is based on the existence of an initial crack or flaw and adopts the laws of thermodynamics to formulate an energy-based criterion for analysis of the existing crack. Such an approach ensures the size-independence of the solution. Clearly, the method is based on an explicit discrete definition of crack in the form of internal or external boundaries. Another important aspect of LEFM is its capability in deriving the singular stress field, predicted by the analytical solution at a crack tip. These two specifications have substantially complicated the numerical techniques designed for fracture mechanics analysis.

In addition to original linear elastic formulation, fracture mechanics has been extended to limited nonlinear behaviour and plasticity around the crack tip, forming the theory of elastoplastic fracture mechanics (EPFM).

1.3.2.3 Damage Mechanics

Damage mechanics has been increasingly adopted to analyze failure in various engineering application involving concrete, rock, metals, composites, and so on. Damage mechanics is a non-local approach (similar to fracture mechanics) but with a formulation apparently similar to the softening theory of plasticity. In damage mechanics, both the strength and stiffness of a material point are decreased if it experiences some level of damage. This is in contrast to the classical theory of plasticity, where the stiffness remains unchanged and only the strength is updated according to the hardening/softening behaviour.

Thermodynamics principles are adopted to derive the necessary formulation based on the micro-cracking state of the material and one of the fundamental assumptions of equivalent strain or equivalent strain energy principles to relate the equivalent undamaged model with the real damaged one. Such an equivalent undamaged model holds the continuity of the model intact.

1.4 Analytical Solutions for Composites

1.4.1 Continuum Models

Conventional continuum lamination models for analysis of composites are based on a composite element, which considers the fibre/matrix mixture as an equivalent homogeneous orthotropic continuum laminate, with perfect bond between the constituents in each single lamina, no strain discontinuity across the interface, and a regular arrangement of fibres. Equilibrium equations, compatibility conditions and the linear elastic Hook's law determine the elasticity constants and govern the stress–strain constitutive law or its generalized form. The classical lamination theory formulates the multilayered laminate based on variations of fibre orientation, stacking sequence and ply-level material properties. More advanced models assume a viscoelasticity model for the matrix. A number of analytical models have also been developed to account for a number of failure modes in each ply, but they are, unfortunately, limited to very simplified geometries, and specific orientations, stacking and loading conditions.

1.4.2 Fracture Mechanics of Composites

Linear elastic fracture mechanics (LEFM) is based on the existence of a crack or a flaw and determines the state of its stability and possible propagation. Its non-local nature guarantees the size or mesh (in case of a finite element analysis) independency of the solution. Definitions of non-local concepts such as the stress intensity factor, energy release rate and energy-based criteria allow the classical fracture mechanics to be extended to nonlinear problems. Most of the research in this field, however, can be classified into four categories: static cracking in a single orthotropic material, dynamic orthotropic cracking, orthotropic bimaterial interface cracks and fracture in orthotropic FGMs. All categories include topics on definition and evaluation of stress intensity factors, associated J and interaction integrals, deriving the asymptotic solutions, crack propagation criteria, and so on.

The fracture mechanics of composite structures has been studied by many researchers. Beginning with the pioneering work by Muskelishvili (1953), several others such as Sih, Paris and Irwin (1965), Bogy (1972), Bowie and Freese (1972), Barnett and Asaro (1972), Kuo and Bogy (1974), Tupholme (1974), Atluri, Kobayashi and Nakagaki (1975a), Forschi and Barret (1976), Boone, Wawrzynek and Ingraffea (1987), Viola, Piva and Radi (1989) and, more recently, Lim, Choi and Sankar (2001), Carloni and Nobile (2002), Carloni, Piva and Viola (2003) and Nobile and Carloni (2005) have proposed solutions for various anisotropic static and quasi-static crack problems.

Simultaneously, several researchers have contributed to finding the elastodynamic fields around a propagating crack within an anisotropic medium, including Achenbach and Bazant (1975), Arcisz and Sih (1984), Piva and Viola (1988), Viola, Piva and Radi (1989), Shindo and Hiroaki (1990), De and Patra (1992), Gentilini, Piva and Viola (2004), Kasmalkar (1996) and Chen and Erdogan (1996). Lee, Hawong and Choi (1996) derived the dynamic stress and displacement components around the crack tip of a steady state propagating crack in an orthotropic material. The same subject was then followed by Gu and Asaro (1997), Rubio-Gonzales and Mason (1998), Broberg (1999), Lim, Choi and Sankar (2001), Federici *et al*. (2001), Nobile and Carloni (2005), Piva, Viola and Tornabene (2005) Sethi *et al*. (2011), and Abd-Alla *et al*. (2011), among others.

The research has not been limited to single layer orthotropic homogeneous composites. Analytical solutions for delamination in multilayer composites have also been investigated comprehensively. The first attempt was probably by Williams (1959) who discovered the oscillatory near-tip behaviour for a traction-free interface crack between two dissimilar isotropic elastic materials, followed by several others such as Erdogan (1963), Rice and Sih (1965), Malysev and Salganik (1965), England (1965), Comninou (1977), Comninou and Schmuser (1979), Sun and Jih (1987), Hutchinson, Mear and Rice (1987) and Rice (1988), among others. The study of interface cracks between two anisotropic materials was performed by Gotoh (1967), Clements (1971) and Willis (1971), followed by Wang and Choi (1983a, 1983b), Ting (1986), Tewary, Wagoner and Hirth (1989), Wu (1990), Gao, Abbudi and Barnett (1992) and Hwu (1993a, 1993b), Bassani and Qu (1989), Sun and Manoharan (1989), Suo (1990), Yang, Sou and Shih (1991), Hwu (1993b), Qian and Sun (1998), Lee (2000) and Hemanth *et al*. (2005).

Fracture mechanics of FGMs has similarly been an active topic for analytical research. For instance, Yamanouchi *et al*. (1990), Holt *et al*. (1993), Ilschner and Cherradi (1995), Nadeau and Ferrari (1999), Takahashi *et al*. (1993), Pipes and Pagano (1970, 1974), Pagano (1974), Kurihara, Sasaki and Kawarada (1990), Niino and Maeda (1990), Sampath *et al*. (1995), Kaysser and Ilschner (1995), Erdogan (1995) and Lee and Erdogan (1995) have studied various aspects of FGM properties. Despite material inhomogeneity, Sih and Chen (1980), Eischen (1983) and Delale and Erdogan (1983) have shown that the asymptotic crack-tip stress and displacement fields for certain classes of FGMs follow the general form of homogeneous materials and Ozturk and Erdogan (1997) and Konda and Erdogan (1994) analytically solved for crack-tip fields in inhomogeneous orthotropic infinite FGM problems. Evaluation of the J integral for determining the mixed-mode stress intensity factors in general FGM problems was studied by Gu and Asaro (1997), Gu, Dao and Asaro (1999), Anlas, Santare and Lambros (2000) and, in particular, Kim and Paulino (2002a, 2002b, 2000c, 2003a, 2003b, 2005) who examined and developed three independent formulations: non-equilibrium, incompatibility and constant-constitutive-tensor, for the J integral.

1.5 Numerical Techniques

Due to the limitations and inflexible nature of analytical methods in handling arbitrary complex geometries and boundary conditions and general crack propagations, several numerical techniques have been developed for solving composite fracture mechanics problems.

The finite element method has been widely used for fracture analysis of structures for many years and is probably the first choice of analysis for general engineering problems, including fracture, unless a better solution is proposed. Despite outstanding advantages, alternative methods are also available, including the adaptive finite/discrete element method (DEM), the boundary element method (BEM), a variety of meshless methods, the extended finite element method (XFEM), the extended isogeometric analysis (XIGA) and, more recently, advanced multiscale techniques. In the following, a brief review of a number of studies on fracture analysis of composites for each class of numerical methods is presented.

1.5.1 Boundary Element Method

Cruse (1988), Aliabadi and Sollero (1998) and García-Sánchez, Zhang and Sáez (2008) developed boundary element solutions for quasi-static crack propagation and dynamic analysis of cracks in orthotropic media. In the boundary element method, a number of elements are used to discretize the boundary of the problem domain, and the domain itself is analytically represented in the governing equations. The boundary element method, regardless of all the benefits, cannot be readily extended to nonlinear systems and is not suited to general crack propagation problems (Figure 1.4c).

1.5.2 Finite Element Method

Most of the performed numerical analyses of structures prior to the end of the twentieth century were related to the finite element method. The finite element method can be easily adapted to complex geometries and general boundary conditions and is well developed into almost every possible engineering application, including nonlinear, inhomogeneous, anisotropic, multilayer, large deformation, fracture and dynamic problems. Several general purpose finite element softwares have been developed, verified and calibrated over the years and are now available to almost anyone who asks (and pays) for them. Furthermore, concepts of FEM are now offered by all engineering departments in the form of postgraduate and even undergraduate courses.

Introduction and fast development of the finite element method drastically changed the extent of application of LEFM from classical idealized models to complex practical engineering problems. After earlier application of FEM to fracture analysis of composites (for instance by Swenson and Ingraffea, 1988) a large number of studies adopted FEM to simply obtain the displacement, strain and stress fields required for numerical evaluation of fracture mechanics parameters such as the stress intensity factors, the energy release rate or the J integral to assess the stability of crack (Rabinovitch and Frosting, 2001; Pesic and Pilakoutas, 2003; Rabinovitch, 2004; Colombi, 2006; Lu *et al*., 2006; Yang, Peng and Kwan, 2006; Bruno, Carpino and Greco, 2007; Greco, Lonetti and Blasi, 2007).

Later, various techniques were developed within the FEM framework to allow reproduction of a singular stress field at a crack tip and to facilitate simulation of arbitrary crack propagations.

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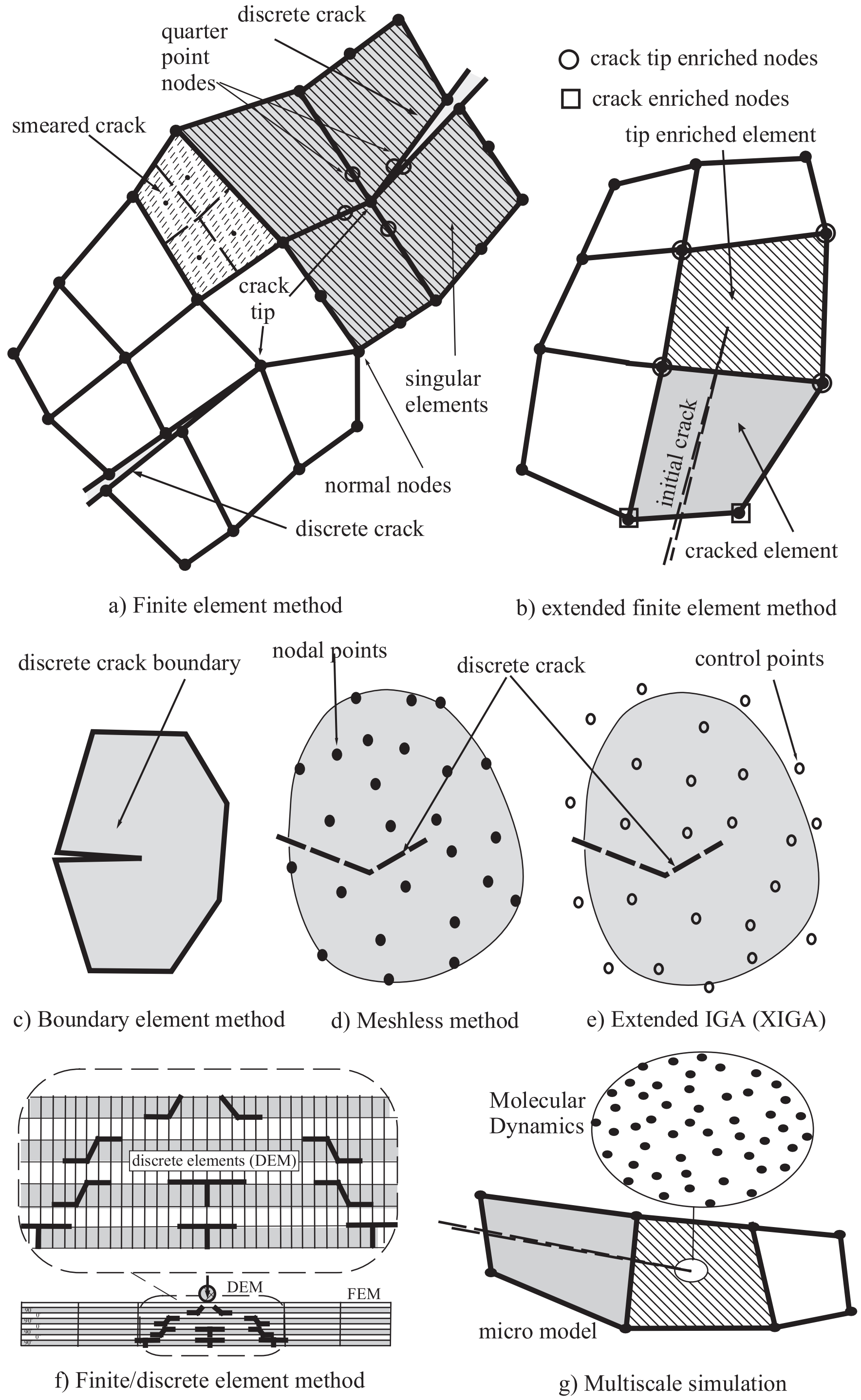

Figure 1.4 Various numerical methods for crack analysis.

Singular finite elements, developed to accurately represent crack-tip singular fields (Owen and Fawkes, 1983), provide the major advantage of simple construction of the model by simply moving the nearby midside nodes to the quarter points with no other changes in the finite element formulation being required (Figure 1.4a). Prior to the development of XFEM, singular elements were the most popular approach for fracture analysis of structures. Singular elements, however, have to be used in a finite element mesh, where crack faces have to match element boundaries. This largely limits their application to general crack propagation problems, unless combined with at least a local adaptive finite element scheme.

1.5.3 Adaptive Finite/Discrete Element Method

The adaptive finite element method, combined with the concepts of contact mechanics of the discrete element method (DEM), has been adopted in several studies for simulation of progressive crack propagation in composites under quasi-static and dynamic/impact loadings. They include a variety of crack models, such as smeared crack, discrete inter-element crack models, cracked interface elements and the discrete contact element, which may use general contact mechanics algorithms to simulate progressive delamination and fracture problems (Mohammadi, Owen and Peric, 1997; Sprenger, Gruttmann and Wagner, 2000; Wu, Yuan and Niu, 2002; Mohammadi and Forouzan-sepehr, 2003; Wong and Vecchio, 2003; Wu and Yin, 2003; Mohammadi, 2003b; Lu *et al*., 2005; Wang, 2006; Teng, Yuan and Chen, 2006; Mohammadi and Mousavi, 2007; Moosavi and Mohammadi, 2007; Mohammadi, 2008; Rabinovitch, 2008).

In a composite progressive fracture analysis, part of the composite model, which is potentially susceptible to damage, is represented by discrete elements and the rest of the specimen is modelled with coarser finite elements to reduce the analysis time (Figure 1.4f). Each discrete element can be discretized by a finite element mesh; finer for the plies closer to the damaged region and coarser elsewhere. A contact methodology controls the debonding mechanisms and all post delamination behaviour, such as sliding (Mohammadi, Owen and Peric, 1997; Mohammadi, 2008). On occurrence of a crack or after a crack propagation step based on a simple comparison of the computed stress state with the adhesive strength (Parker, 1981; O'Brien, 1985; Rowlands, 1985; Roberts, 1989; Taljsten, 1997), adaptive schemes are used to locally remesh the finite element model to ensure matching of element edges and crack faces. Nonlinear material properties and geometric nonlinearities can be considered in the basic FE formulation. The method, however, is numerically expensive and the nodal alignments may cause numerical difficulties and mesh dependency to some extent in propagation problems.

1.5.4 Meshless Methods

Meshless methods include a wide variety of numerical methods with different approximation techniques, diverse solution schemes, variable levels of accuracy and dissimilar applications. Ironically, in a general view, they have nothing in common but being different from the finite element method. The main idea of meshless methods is to avoid a predefined fixed connectivity between the nodal points which are used to define the geometry and to set necessary degrees of freedom to discretize the governing equation. As a result of such a connection-free style of nodal discretization, any existing cracks or crack propagation paths can be efficiently embedded geometrically within the numerical model (Figure 1.4d).

Meshless methods have been used extensively for analysis of various engineering problems (Belytschko *et al*., 2002). They include the frequently used classes of the element-free Galerkin method (EFG) (Belytschko, Lu and Gu, 1994; Belytschko, Organ and Krongauz, 1995; Ghorashi, Valizadeh and Mohammadi, 2011), the meshless local Petrov–Galerkin (MLPG) (Atluri and Shen, 2002), smoothed particle hydrodynamics (SPH) (Belytschko *et al*., 2000; Madani and Mohammadi, 2011; Ostad and Mohammadi, 2012), the finite point method (FPM) (Onate *et al*., 1995; Bitaraf and Mohammadi, 2010), isogeometric analysis (IGA) (Hughes, Cottrell and Bazilevs, 2005) and its extended version (XIGA) (Ghorashi, Valizadeh and Mohammadi, 2012), and other approaches including the reproducing kernel particle method (RKPM) (Liu *et al*., 1996), HP-clouds (Duarte and Oden, 1995), the equilibrium on line method (ELM) (Sadeghirad and Mohammadi, 2007) and the smoothed finite element method (Liu and Trung, 2010), among others. It is noted that some of the mentioned methods are in fact a combination of finite element and meshless concepts. By this definition, the extended finite element method (XFEM) may somehow be included.

Despite higher accuracy and flexible adaptive schemes, the majority of meshless methods are yet to be user friendly because of weak versatility to deal with arbitrary boundary conditions and geometries, complicated theoretical bases, high numerical expense and the need for sensitivity analysis, calibration and difficult stabilization schemes in many of them.

1.5.5 Extended Finite Element Method

Since the introduction of the extended finite element method (XFEM) for fracture analysis by Belytschko and Black (1999), Moës, Dolbow and Belytschko (1999) and Dolbow (1999), based on the mathematical foundation of the partition of unity finite element method (PUFEM), proposed earlier by Melenk and Babuska (1996) and Duarte and Oden (1996), XFEM methodology has been rapidly extended to a vastly wide range of applications that somehow include a local discontinuity or singularity within the solution.

The natural extension of FEM into XFEM allows new capabilities while preserving the finite element original advantages. The two main superiorities of XFEM are its capability in reproducing the singular stress state at a crack tip, and allowing several cracks or arbitrary crack propagation paths to be simulated on an independent unaltered mesh (Figure 1.4b). In fact, while the presence of the crack is not geometrically modelled and the mesh does not need to conform to the virtual crack path, the exact analytical solutions for singular stress and discontinuous displacement fields around the crack (tip) are reproduced by inclusion of a special set of enriched shape functions that are extracted from the asymptotic analytical solutions.

Apart from earlier works that were directed towards the development of the extended finite element method for linear elastic fracture mechanics (LEFM), simulation of failure and localization has been the target of several studies including, Jirásek and Zimmermann (2001a, 2001b), Sukumar *et al*. (2003), Dumstorff and Meschke (2003), Patzak and Jirásek (2003), Ventura, Moran and Belytschko (2005), Samaniego and Belytschko (2005), Areias and Belytschko (2005a, 2005b, 2006) and Song, Areias and Belytschko (2006). In addition, various contact problems have been simulated by XFEM. For example, Dolbow, Moes and Belytschko (2000c, 2001), Shamloo, Azami and Khoei (2005), Belytschko, Daniel and Ventura (2002), Khoei and Nikbakht (2006), Khoei, Shamloo and Azami (2006) and Khoei *et al*. (2006) have used simple Heaviside enrichments to deal with contact discontinuities in different problems, while Ebrahimi, Mohammadi and Kani (2012) have recently proposed a numerical approach within the partition of unity finite element method to determine the order of singularity at a sliding contact corner.

Moreover, several dynamic fracture problems have been studied by XFEM. Among them, Peerlings *et al*. (2002), Belytschko *et al*. (2003), Oliver *et al*. (2003), Ventura, Budyn and Belytschko (2003), Chessa and Belytschko (2004, 2006), Belytschko and Chen (2004), Zi *et al*. (2005), Rethore, Gravouil and Combescure (2005a), Rethore *et al*. (2005), Menouillard *et al*. (2006), Gregoire *et al*. (2007), Nistor, Pantale and Caperaa (2008), Prabel, Marie and Combescure (2008), Combescure *et al*. (2008), Gregoire, Maigre and Combescure (2008), Motamedi (2008), Kabiri (2009), Gravouil, Elguedj and Maigre (2009a, 2009b) and Rezaei (2010) have discussed various aspects of general dynamic fracture problems.

In the past decade, development of XFEM has substantially contributed to new studies of fracture analysis of various types of composite materials. Dolbow and Nadeau (2002) employed XFEM to simulate fracture behaviour of micro-structured materials with a focus on functionally graded materials. Then, Dolbow and Gosz (2002) described a new interaction energy integral method for the computation of mixed mode stress intensity factors in functionally graded materials. In a related contribution, Remmers, Wells and de Borst (2003) presented a new formulation for delamination in thin-layered composite structures. A study of bimaterial interface cracks was performed by Sukumar *et al*. (2004) by developing new bimaterial enrichment functions. Nagashima, Omoto and Tani (2003) and Nagashima and Suemasu (2004, 2006) described the application of XFEM to stress analyses of structures containing interface cracks between dissimilar materials and concluded the need for orthotropic enrichment functions to represent the asymptotic solution for a crack in an orthotropic material. Hettich and Ramm (2006) simulated the delamination crack as a jump in the displacement field without using any crack-tip enrichment. Also, a number of XFEM simulations have focused on thermo-mechanical analysis of orthotropic FGMs (Dag, Yildirim and Sarikaya, 2007), 3D isotropic FGMs (Ayhan, 2009; Zhang *et al*., 2011; Moghaddam, Ghajar and Alfano, 2011) and frequency analysis of cracked isotropic FGMs (Natarajan *et al*., 2011). Recently, Bayesteh and Mohammadi (2012) have used XFEM with orthotropic crack-tip enrichment functions to analyze several FGM crack stability and propagation problems.

Development of independent orthotropic crack-tip enrichment functions was reported in a series of papers by Asadpoure, Mohammadi and Vafai (2006, 2007), Asadpoure and Mohammadi (2007) and Mohammadi and Asadpoure (2006). Later, Motamedi (2008) and Motamedi and Mohammadi (2010a, 2010b, 2012) studied the dynamic crack stability and propagation in composites based on static and dynamic orthotropic enrichment functions and Esna Ashari (2009) and Esna Ashari and Mohammadi (2009, 2010b, 2011a, 2012) have further extended the method for orthotropic bimaterial interfaces.

1.5.6 Extended Isogeometric Analysis

Since the introduction of isogeometric analysis (IGA) by Hughes, Cottrell and Bazilevs (2005) a new and fast growing chapter has been opened in unifying computer aided design (CAD) and numerical solutions by using the non-uniform rational B-splines (NURBS) functions (Figure 1.4e). The method, in fact, can be categorized with other meshless methods, but it is briefly introduced due to its fast growing state and excellent potentials.

IGA has been successfully adopted in several engineering problems, including structural dynamics (Cottrell, Hughes and Bazilevs, 2009; Hassani, Moghaddam and Tavakkoli, 2009), Navier–Stokes flow (Nielsen *et al*., 2011), fluid–solid interaction (Bazilevs *et al*., 2009), shells

(Benson *et al*., 2010a; Benson *et al*., 2011; Kiendl *et al*., 2010; Kiendl *et al*., 2009), damage mechanics (Verhoosel *et al*., 2010a), cohesive zone simulations (Verhoosel *et al*., 2010b), heat transfer (Anders, Weinberg and Reichart, 2012), large deformation (Benson *et al*., 2011), electromagnetic (Buffa, Sangalli and Vazquez, 2010), strain localization (Elguedj, Rethore and Buteri, 2011), contact mechanics (Lu, 2011; Temizer, Wriggers and Hughes, 2011), topology optimization (Hassani, Khanzadi and Tavakkoli, 2012) and crack propagation (Verhoosel *et al*., 2010b, Benson *et al*., 2010b; De Luycker *et al*., 2011; Haasemann *et al*., 2011; Ghorashi, Valizadeh and Mohammadi, 2012).

The first attempt at enhancing IGA for crack problems was reported by Verhoosel *et al*. (2010b) and followed by Benson *et al*. (2010c), De Luycker *et al*. (2011) and Haasemann *et al*. (2011). Recently, a full combination of XFEM and IGA methodologies has been developed for general mixed mode crack propagation problems by the introduction of extended isogeometric analysis (XIGA) by Ghorashi, Valizadeh and Mohammadi (2012). XIGA uses the superior concepts of XFEM to extrinsically enrich the versatile IGA control points with Heaviside and crack-tip enrichment functions.

1.5.7 Multiscale Analysis

The traditional borders between simulation of mechanics, physics and even biology problems have been removed by recent computational advances in the form of multiscale simulations. Most problems in science involve many scales in time and space. An example is the overall stress state in a cracked solid which can well be described by macroscopic continuum equations, but requires details on a microscale at the tip of the crack. In a multiscale method, a part of the model which requires a more accurate theoretical basis or numerical approximation, due to lack of theoretical bases or existing inconsistencies of many conventional models, is simulated by a finer modelling scale, which can better represent details of the material behaviour and the interacting effects of material constituents (Figure 1.4g).

A common difficulty with the simulation of these problems and many others in physics, chemistry, engineering and biology is that an attempt to represent all fine scales will lead to an enormous computational model with unacceptably long computation times and huge memory requirements, even for state of the art modern supercomputers. On the other hand, if the discretization at a coarse level ignores the fine scale information then the solution may not be physically meaningful. Therefore, the challenging task of incorporating the influence of the fine scales into the coarse model must logically be determined.

Several multiscale techniques have been developed for different applications including the bridging scale method, the bridging domain method, homogenization techniques, the quasi continuum method, the heterogeneous multiscale method, and so on. These methods can be incorporated into multiscale FEM, meshless or XFEM methodologies to analyze various multiscale problems at macro/micro, micro/nano, macro/nano, macro/micro/nano and bio/nano interface levels (Mohammadi, 2012).

1.6 Scope of the Book

This text is dedicated to discussing various aspects of application of the extended finite element method for fracture analysis of composites on the macroscopic scale. Nevertheless, many subjects can be similarly used for fracture analysis of other materials, even in microscopic scales.

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The book is designed as a textbook, which provides all the necessary theoretical bases before discussing the numerical issues. This preliminary chapter briefly introduced the subject of fracture in composite structures, and summarily reviewed the existing classes of analytical and numerical techniques. In each case, a short description and a number of reference works were presented. The aim was to provide a general overview of the wide extent of applications without going into detail.

Chapter 2 provides a review of classical isotropic fracture mechanics, which quickly examines the basic concepts and fundamental formulations but does not provide proofs or in depth detail. It is, in fact, a slightly modified and corrected edition of a similar chapter in Mohammadi (2008). The chapter begins with an introduction to the basics of the theory of elasticity, and is followed by discussions on classical problems of LEFM. Asymptotic solutions for displacement and stress fields in different fracture modes are presented and basic concepts of stress intensity factors, energy release rate and the J contour integral and its modifications, such as the equivalent domain integral and the interaction integral, are explained. The methodology of extracting mixed mode stress intensity factors is addressed and various mixed mode fracture criteria are explained. The chapter briefly reviews issues related to the basic finite element models of fracture mechanics and describes the basic formulation of popular singular finite elements. The chapter ends by extending some of the basic ideas of LEFM to elastoplastic fracture mechanics (EPFM).

Chapter 3, which is a redesigned and completed edition of the XFEM chapter in Mohammadi (2008), begins with the concepts of the partition of unity and the extended finite element method. It is followed by a detailed XFEM formulation for analysis of cracks in isotropic materials. This section constitutes the basic relations for modelling displacement discontinuity across a crack and the stress singularity at the crack tip. The chapter continues with three sections on available options for strong discontinuity enrichment functions, weak discontinuity enrichments for material interfaces, and a collection of several crack-tip enrichments for various engineering applications. A review of the level set method for tracking moving boundaries is provided before the concluding section which includes simulations and discussion of a wide range of problems to assess the accuracy, performance and efficiency of XFEM in dealing with various discontinuity and singularity problems, which include classical in-plane mixed mode fracture mechanics problems, cracking in plates and shells, simulation of shear band creation and propagation, self-similar fault rupture, sliding contact, hydraulic fracture, and dislocation dynamics.

Static orthotropic fracture analysis is the comprehensive subject of Chapter 4. It begins with a review of anisotropic and orthotropic elasticity, followed by a complete discussion on the available analytical solutions for near crack-tip fields in orthotropic materials. These fields are then used to develop the orthotropic enrichment functions for the XFEM formulation. A section is then devoted to orthotropic mixed mode fracture mechanics, which discusses orthotropic mixed mode criteria, the J integral, crack propagation criteria for orthotropic media and other related issues. Finally, a number of numerical simulations are provided to illustrate the validity, robustness and efficiency of the proposed approach for evaluation of mixed mode stress intensity factors in homogeneous orthotropic materials.

Chapter 5 is devoted to dynamic fracture analysis of composites. After a comprehensive literature review, analytical solutions for near crack-tip in dynamic states are provided for isotropic and orthotropic media. Then, a section on dynamic stress intensity factors discusses the dynamic fracture criteria for different cases of stationary and moving cracks. It also includes details of the dynamic J and interaction integrals and explains the existing techniques for determining the dynamic stress intensity factors. The next section provides the dynamic XFEM formulation and discusses various options for the dynamic crack-tip enrichment functions. A separate section is dedicated to time integration techniques and reviews existing developments in the fields of the time and time–space extended finite element methods (TXFEM and STXFEM, respectively). The chapter concludes with numerical simulations and comprehensive discussions of several dynamic fracture problems, including stationary and propagating cracks in orthotropic media. The results, usually in the form of time histories, complicate the process of comparisons with available reference data.

Orthotropic functionally graded materials are comprehensively discussed in Chapter 6. Again, after a literature review, analytical asymptotic solutions are presented for a crack in an inhomogeneous orthotropic medium. Then, fracture mechanics concepts, such as the toughness, stress intensity factors and crack propagation criteria, are explained. A comprehensive discussion is dedicated to various options for deriving the interaction integral for an inhomogeneous medium. It is followed by details of inhomogeneous XFEM formulation which also includes necessary enrichment functions and the numerical requirement for a transition domain. Several numerical examples from simple tensile FGM plates to crack propagation in an orthotropic FGM bending beam are presented and discussed.

Delamination or interlaminar crack analysis by XFEM is discussed in Chapter 7 with a comprehensive review of the existing techniques, followed by the concepts of fracture mechanics for bimaterial interface cracks. This includes the analytical solutions for the displacement and stress fields near an interlaminar crack tip in an orthotropic bimaterial problem. The simplified case of isotropic bimaterial is also discussed. Then, the interaction integral and the method of computing stress intensity factors are explained. The chapter also includes a section on delamination propagation criteria, before presenting the details of bimaterial XFM. Several numerical issues are addressed and the necessary strong and weak discontinuity enrichment functions and bimaterial crack-tip enrichment functions are explained. The chapter ends with a broad section on numerical simulations, which include conventional composite bimaterial applications and a number of FRP-strengthening problems from the retrofitting industry.

The final chapter presents a number of on-going research topics based on new engineering applications for orthotropic materials or new numerical tools for more efficiently tackling the cracking in composites. It begins by introducing the orthotropic version of the extended isogeometric analysis (XIGA) for fracture analysis of composites. This section briefly reviews the basic concepts of NURBS and IGA methodology and explains the enrichment techniques within an extended IGA framework (XIGA). Numerical issues are addressed and a number of simple simulations, both isotropic and orthotropic, are presented and discussed. The next section is dedicated to the newly developed idea of plane strain anisotropic dislocation dynamics. Related XFEM formulation and necessary enrichment functions for the self-stress state of edge dislocations are explained. A number of numerical simulations are presented without any available reference results. The chapter concludes with two brief sections on orthotropic biomaterial applications of XFEM and the piezoelectric problems.