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Disorder or Uncertainty?

This book is not a novel, and I think it is acceptable to give away the plot at the very outset. Entropy is a thermal property of matter, and when real (as opposed to idealised) macroscopic changes take place in the world, the total amount of entropy goes up. This is the celebrated second law of thermodynamics, so celebrated, in fact, that saying 'the second law' alone is often enough to convey which field it relates to. It is due to the efforts of Ludwig Boltzmann (1844–1906) and Josiah Willard Gibbs (1839–1903) that we now connect thermodynamic entropy with statistical ideas; with the uncertainty that prevails in the microscopic state of the world if we have only limited information about it. The growth of entropy when constraints on a system are removed, to initiate change, is a consequence of an increase in this uncertainty: the number of possibilities for the microscopic state goes up, and so does the entropy.

It is often said that the rise in entropy is related to the natural tendency for disorder to increase, and while this can sometimes help to develop intuition, it can be misleading. The atoms of a crystalline solid held within a thermally insulated box have evidently chosen to arrange themselves as a regular lattice. They might instead have arranged themselves as a liquid with the same total energy, but at a lower temperature since some of the kinetic energy would need to be converted into potential energy in order to melt the solid. But they did not. Nature sometimes has a preference for spatially ordered instead of disordered systems: if we set up the system in the molten state, the material would spontaneously freeze.

A better interpretation is that the spatially ordered arrangement of atoms in the solid has a larger number of underlying microstates than the cooler, but spatially disordered fluid. The disorder in atomic velocities is larger at the higher temperature (and even here I would rather say the *uncertainty* in velocities is larger) and this gives a greater overall uncertainty surrounding the actual microstate of the system, when in equilibrium, if the atoms are arranged as a solid. The selection rule imposed by Nature for the choice of macrostate is to maximise the uncertainty.

An uncertain situation might convey the idea of disorder or untidiness, but we need to take care when we build analogies between entropy and untidy situations. My desk is very disordered, but this does not mean that it has more entropy than it would have if I were

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to tidy it. A disordered desk and a tidy desk are just two particular arrangements of the system. But if I defined the term 'untidy' to encompass a certain set of arrangements of items on my desk, while another, much smaller, set of arrangements is classed as 'tidy', then I could start to make statistical statements about the likely condition (tidy/untidy) of my desk in the future, as long as I had a model of how the arrangement of items changed from day to day, as a result of my usual activities. I could define 'tidy' such that the fraction of desk area showing through the jumble is greater than 75%, say. Then a tidy desktop (few configurations, lots of desk showing) would most likely develop into an untidy desktop (many configurations, less desk showing) as the days (or even minutes!) passed. An untidy desk would probably remain untidy, though its evolution into a tidy desk is not beyond all expectation.

But this is as far as ideas concerning the loss of order and gain of untidiness should be taken. A key point is that we could start the process with everything scattered randomly over the desk. This is not a tidy or an ordered initial condition. It is, on the other hand, a *definite* initial condition, with no uncertainty attached to it. If entropy is uncertainty, then a definite initial state has the same (zero) amount of entropy whether it is tidy or untidy, ordered or disordered. It is the certainty in configuration that is lost if we fail to follow the details of the desktop dynamics as time progresses, not the tidiness or the order. The rise in this uncertainty is equivalent to the increase in entropy.

As an extension to this reasoning, the initial condition might be that the system is in one of a certain number of configurations, perhaps similar to one another, but perhaps completely different: an arbitrary collection of my favourite desktop arrangements. Such a slightly indefinite initial state would evolve into a more indefinite state: a low but nonzero entropy situation evolves into one with a higher entropy. This is a more sophisticated description of the evolution of a complex system than a picture of order turning into disorder. This is the meaning of the second law.

Really, discussions of desks or even rooms becoming untidy should include shutting the door to the room (and maybe putting up an entropy hazard warning sign!). We leave the occupant to rearrange things according to his or her wishes. The configuration of the room changes with time and, from the other side of the door, we do not know exactly how it proceeds. All we can do is occasionally ask the occupant for some information that does not specify the exact arrangement, but instead is more generic, such as how much desk is showing. Our knowledge about the state of affairs inside the room is steadily impaired, and eventually goes to a minimum, based on what we can discover remotely.

This is how we interrogate a macroscopic system, allowing us to close in on the meaning of thermodynamic entropy. The macroscopic equilibrium state of a gas is described by a measurable density and temperature, but this is insufficient to specify its exact microscopic state, which would be a list of the locations and velocities of all the atoms, at least from a classical physics perspective. This is an occasion when admitting 'I do not know what is going on' is extremely profound. Thermodynamic entropy is a measure of this uncertainty: it is proportional to the logarithm of the number of microscopic configurations compatible with the available measurements or information. We can categorise those configurations into different classes, such as 'gas concentrated in a corner' or 'gas spread out uniformly in the container', and then estimate the likelihood that the system might be found in each class, as long as probabilities for each microscopic configuration are provided. We choose these probabilities on the basis of what we might know about the dynamics or by sophisticated 'best judgement'.

For an isolated system in equilibrium, equal probabilities for all configurations are often assumed, which is perhaps an oversimplification, but it implies that the system is most likely to be found in the macroscopic class that possesses the greatest number of configurations. If the system were disturbed by the release of some constraint (say a change in confining volume), it would eventually find a new equilibrium, and again take the class with the most microscopic states. In equilibrium, the macroscopic state with the greatest uncertainty is chosen. In this way, an arrow of macroscopic change (or of time, loosely) emerges and it is characterised by entropy increase.

It is sometimes said that the universe is falling to bits, or that everything is going wrong, but this a profoundly pessimistic view of the events that we attempt to describe with the second law. The statement that disorder is always on the increase carries the same gloomy view about the future. But does the interpretation that uncertainty is increasing offer anything more positive?

The evolution of the universe is a consequence of the rules of interaction between the component particles and fields, many of which we have determined in detail in the laboratory. These rules recognise no such thing as pessimism or decline. The universe is simply following a dynamical trajectory. But one of the core features of the dynamics is that transfers take place between participants in a way that seems to favour the sharing out of energy or space between them. The attributes of the universe are being mixed up in a manner that is hard to follow and our failure to grasp and retain the detail of all this is what is meant by the growth of uncertainty. However, we could interpret this failure as a reflection of the richness of the dynamics of the world and all its possibilities. We could perhaps view the second law more positively as a statement about the extraordinary complexity and promise that the universe can offer as it evolves.

The growth of entropy is our rationalisation of this complexity. We can explain the direction of macroscopic change, including events taking place in a test tube as well as processes occurring in the wider cosmos, on the basis of a simply stated and implemented rule of Nature. We can do this without having to delve too deeply into the microscopic laws: it seems that in certain important ways they all have a similar effect. The second law is a reflection of an underlying imperative to mix, share and explore, such that certain macroscopic events happen frequently, because they are nearly inevitable under such circumstances, while others occur more rarely.

So if we wish to ascribe a motivation to the workings of the universe, instead of arguing that the natural direction of change is towards disorder and destruction, we might regard the dynamics as essentially egalitarian and, as an indirect consequence, potentially benevolent. Particles of a gas with more than their fair share of energy naturally tend to pass some to their slower neighbours. Energy will flow, but this does not mean that the exceptional cannot arise. The toolbox of physical processes available to the world is so well stocked that the flow can be partly intercepted and put to use in building and maintaining complex structures. Nature will find opportunities to feed off energy flows in extraordinary ways: mixing and sharing seem to have the capacity to build as well as to dissipate, at least until the mixing is completed. These are themes that are worth developing.