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Computational intelligence techniques for multicriteria decision aiding: An overview

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1.1 Introduction

Real world decision-making problems are usually too complex and ill-structured to be considered through the examination of a single criterion, attribute or point of view that will lead to an 'optimal' decision. In fact, such a single-dimensional approach is merely an oversimplification of the actual nature of the problem at hand, and it can lead to unrealistic decisions. A more appealing approach would be the simultaneous consideration of all pertinent factors that are related to the problem. However, through this approach some very essential issues/questions emerge: how can several and often conflicting factors be aggregated into a single evaluation model? Is this evaluation model unique and/or 'optimal'? In addressing such issues, one has to bear in mind that each decision-maker (DM) has his/her own preferences, experiences, and decision-making policy.

The field of multicriteria decision aid (MCDA) is devoted to the study of problems that fit the above context. Among others, MCDA focuses on the development and implementation of decision support tools and methodologies to confront complex decision problems involving multiple criteria, goals or objectives of conflicting nature. It has to

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be emphasized through, that MCDA techniques and methodologies are not just some mathematical models aggregating criteria that enable one to make optimal decisions in an automatic manner. Instead, MCDA has a strong decision support focus. In this context the DM has an active role in the decision-modeling process, which is implemented interactively and iteratively until a satisfactory recommendation is obtained that fits the preferences and policy of a particular DM or a group of DMs.

Even though MCDA has developed as a major and well-distinguished field of operations research, its interaction with other disciplines has also received much attention. This is understood if one considers the wide range of issues related to the decision process, which the MCDA paradigm addresses. These involve among others the phases of problem structuring, preference modeling, the construction and characterization of different forms of criteria aggregation models, as well as the design of interactive solution and decision aid procedures and systems. The diverse nature of these topics often calls for an interdisciplinary approach.

A significant part of the research on the connections of MCDA with other disciplines has focused on intelligent systems. Over the past decades enormous progress has been made in the field of artificial intelligence, in areas such as expert systems, knowledge-based systems, case-based reasoning, fuzzy logic, and data mining. This chapter focuses on computational intelligence, which has emerged as a distinct sub-field of artificial intelligence involved with the study of adaptive mechanisms to enable intelligent behavior in complex and changing environments (Engelbrecht 2002). Typical computational intelligence paradigms include machine learning algorithms, evolutionary computation and nature-inspired computational methodologies, as well as fuzzy systems. We provide an overview of the main contributions of popular computational intelligence approaches in MCDA, covering areas such as multiobjective optimization, preference modeling, and model building through preference disaggregation.

The rest of the chapter is organized as follows: Section 1.2 presents an introduction to the MCDA paradigm, its main concepts and methodological streams. Section 1.3 is devoted to the overview of the connections between MCDA and computational intelligence, focusing on three main fields of computational intelligence, namely statistical learning/data mining, fuzzy set theory, and metaheuristics. Finally, Section 1.4 concludes the chapter and discusses some future research directions.

1.2 The MCDA paradigm

1.2.1 Modeling process

The major goal of MCDA is to provide a set of criteria aggregation methodologies that enable the development of decision support models considering the DM's preferential system and judgment policy. Achieving this goal requires the implementation of complex processes. Most commonly, these processes do not lead to optimal solutions/decisions, but to satisfactory ones that are in accordance with the DM's policy. Roy (1985) introduced a general framework that covers all aspects of the MCDA modeling philosophy (Figure 1.1).

The first level of the process, involves the specification of a set A of feasible alternative solutions for the decision problem at hand. This set can be continuous or discrete. In the

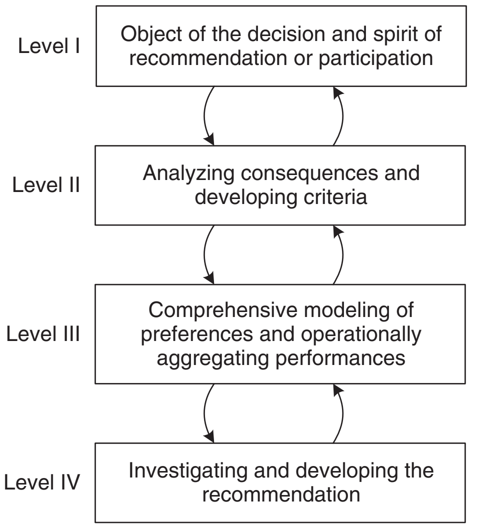

Figure 1.1 The MCDA modeling process.

former case, it is specified through a set of constraints. In the case where A is discrete, it is assumed that the DM can list the alternatives which will be subject to evaluation within the given decision-making framework. The form that the output of the analysis should have is also defined at the first phase of the process. This involves the selection of an appropriate decision 'problematic', which may involve: (a) the choice of the best alternative or a set of good alternatives; (b) the ranking of the alternatives from the best to the worst ones; (c) the classification of the alternatives into predefined categories; and (d) the description of the alternatives and their characteristics.

The second stage involves the identification of all factors related to the decision. MCDA assumes that these factors have the form of criteria. A criterion is a real function f measuring the performance of the alternatives on each of their individual characteristics. The set of selected criteria $\{f_1, \ldots, f_n\}$ must form a consistent family of criteria. A consistent family of criteria is characterized by the following properties (Bouyssou 1990):

- Monotonicity: If alternative $\mathbf{x}$ is preferred over alternative $\mathbf{y}$, the same should also hold for any alternative $\mathbf{z}$ such that $f_k(\mathbf{z}) \geq f_k(\mathbf{x})$ for all k.
- Completeness: If $f_k(\mathbf{x}) = f_k(\mathbf{y})$ for all criteria, then the DM should be indifferent between alternatives $\mathbf{x}$ and $\mathbf{y}$.
- Nonredundancy: The set of criteria satisfies the nonredundancy property if the elimination of any criterion results to the violation of monotonicity and/or completeness.

Once a consistent family of criteria has been specified, the next step is to proceed with the specification of the criteria aggregation model that meets the requirements of the problem. Finally, the last stage involves all the necessary supportive actions needed for the successful implementation of the results of the analysis and the justification of the model's recommendations.

1.2.2 Methodological approaches

MCDA provides a wide range of methodologies for addressing decision-making problems of different types. The differences between these methodologies involve the form of the models, the model development process, and their scope of application. On the basis of these characteristics, Pardalos *et al*. (1995) suggested the following four main streams in MCDA research:

- Multiobjective mathematical programming.
- Multiattribute utility/value theory.
- Outranking relations.
- Preference disaggregation analysis.

The following subsections provide a brief overview of these methodological streams.

1.2.2.1 Multiobjective mathematical programming

Multiobjective mathematical programming (MMP) extends the well-known single objective mathematical programming framework to problems involving multiple objectives. Formally, a MMP problem has the following form:

$$\begin{array}{ll} \max & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x})\} \\ \text{subject to:} & \mathbf{x} \in \mathcal{A} \end{array} \tag{1.1}$$

where $\mathbf{x}$ is the vector of the decision variables, $f_1, f_2, \dots, f_n$ are the objective functions (in maximization form), and $\mathcal{A}$ is the set of feasible solutions defined through multiple constraints.

In a MMP context, the objectives are assumed to be in conflict, which implies that it is not impossible to find a solution that maximizes all the objectives simultaneously. In that regard, efficient solutions (Pareto optimal or nondominated solutions) are of interest. A solution $\mathbf{x}^*$ is referred to as efficient if there is no other solution $\mathbf{x}$ that dominates $\mathbf{x}^*$, i.e., $f_k(\mathbf{x}) \geq f_k(\mathbf{x}^*)$ for all k and $f_j(\mathbf{x}) > f_j(\mathbf{x}^*)$ for at least one objective j. An overview of the MMP theory and different techniques for finding Pareto optimal solutions can be found in the books of Steuer (1985), Miettinen (1998), Ehrgott and Gandibleux (2002), and Ehrgott (2005).

An alternative approach to model multiobjective optimization problems is through goal programming formulations. In the context of goal programming a function of the deviations from some pre-specified goals is optimized. The goals are set by the DM and may represent ideal points on the objectives, some benchmark or reference points, or a set of satisfactory target levels on the objectives that should be met as closely as possible. The general form of a goal programming formulation is the following:

$$\begin{array}{lll} \min & \mathcal{F}(d_k^+, d_k^-; \mathbf{w}) & \\ \text{subject to:} & \mathbf{x} \in \mathcal{A} & \\ & f_k(\mathbf{x}) + d_k^+ - d_k^- = s_k, & k = 1, \dots, n \\ & d_k^+, d_k^- \geq 0, & k = 1, \dots, n \end{array} \tag{1.2}$$

where s_k is the target level (goal) set for objective k, d_k^+ and d_k^- are the deviations from the target, and $\mathcal{F}$ is a function of the deviations, which is parameterized by a vector $\mathbf{w}$ of weighting coefficients. These coefficients may either represent the trade-offs between the deviations corresponding to different objectives or indicate a lexicographic ordering of the deviations' significance (pre-emptive goal programming). An overview of the theory and applications of goal programming can be found in Aouni and Kettani (2001), Jones and Tamiz (2002), as well as in the book of Jones and Tamiz (2010).

1.2.2.2 Multiattribute utility/value theory

Multiattribute utility/value theory (MAUT/MAVT) extends the traditional utility theory to the multidimensional case.[1] MAVT has been one of the cornerstones of the development of MCDA and its practical applications. The objective of MAVT is to model and represent the DM's preferential system into a value function $V(\mathbf{x})$, where $\mathbf{x}$ is the vector with the data available over a set of n evaluation criteria. The value function is defined on the criteria space, such that:

$$\begin{aligned} V(\mathbf{x}) > V(\mathbf{y}) &\Rightarrow \mathbf{x} \succ \mathbf{y} \\ V(\mathbf{x}) = V(\mathbf{y}) &\Rightarrow \mathbf{x} \sim \mathbf{y} \end{aligned} \tag{1.3}$$

where $\succ$ denotes preference and $\sim$ denotes indifference. The most commonly used form of value function is the additive one:

$$V(\mathbf{x}) = \sum_{k=1}^{n} w_k v_k(x_k) \tag{1.4}$$

where $w_k \geq 0$ is the trade-off constant for criterion k (usually the trade-off constants are assumed to sum up to one) and $v_k(x_k)$ is the corresponding marginal value function, which defines the partial value (performance score) of the alternatives on criterion k, in a predefined scale (e.g., in [0, 1]). If the marginal value function is assumed to be linear the additive model reduces to a simple weighted average of the criteria. Keeney and Raiffa (1993) present in detail the theoretical principles of MAVT under both certainty and uncertainty, and discuss the independence conditions that characterize different types of value models (e.g., additive, multiplicative, multi-linear).

1.2.2.3 Outranking techniques

The foundations of the outranking relation theory (ORT) have been set by Bernard Roy during the late 1960s through the development of the ELECTRE family of methods (ELimination Et Choix Traduisant la REalité; Roy 1968). Since then, ORT has been widely used by MCDA researchers, mainly in Europe. All ORT techniques operate in two major stages. The first stage involves the development of an outranking relation, whereas the second stage involves the exploitation of the outranking relation in order to perform the evaluation of the alternatives for choice, ranking, and classification purposes.

An outranking relation can be defined as a binary relation used to estimate the strength of the preference for an alternative $\mathbf{x}$ over an alternative $\mathbf{y}$. In comparison with MAVT, outranking techniques have two special features:

[1] The term 'utility' is used in the context of decision making under uncertainty, whereas the term 'value' is preferred for decisions in a certain environment. Henceforth, the term 'value' will be used throughout the chapter.

- An outranking relation is not necessarily transitive: in MAVT models the evaluation results are transitive. On the other hand, models developed on the basis of outranking relations allow intransitivities.
- An outranking relation is not complete: the main preference relations used in a MAVT modeling framework involve preference and indifference as defined in (1.3). In addition to these two relations, outranking methods also consider the incomparability relation, which arises when comparing alternatives with very special characteristics and diverse performance on the criteria.

The most popular methods implementing the outranking relations framework are the ELECTRE methods (Roy 1991), as well as the PROMETHEE methods (Brans and Mareschal 2005), with different variants for addressing choice, ranking and classification problems.

1.2.2.4 Preference disaggregation analysis

The development of the MCDA model can be performed through direct or indirect procedures. The former are based on structured communication sessions between the analyst and the DM, during which the analyst elicits specific information about the DM's preferences (e.g., weights, trade-offs, goals, etc.). The success of this approach is heavily based on the willingness of the DM to participate actively in the process, as well as the ability of the analyst to guide the interactive process in order to address the DM's cognitive limitations. This kind of approach is widely used in situations involving decisions of strategic character.

However, depending on the selected criteria aggregation model, a considerable amount of information may be needed by the DM. In 'repetitive' decisions, where time limitations exist, the above direct approach may not be applicable. Disaggregation methods (Jacquet-Lagrèze and Siskos 2001) are very helpful in this context. Disaggregation methods use regression-like techniques to infer a decision model from a set of decision examples on some reference alternatives, so that the model is as consistent as possible with the actual evaluation of the alternatives by the DM. This model inference approach provides a starting basis for the decision-aiding process. If the obtained model's parameters are in accordance with the actual preferential system of the DM, then the model can be directly applied to new decision instances. On the other hand, if the model is consistent with the sample decisions, but its parameters are inconsistent with the DM's preferential system (which may happen if, for example, the decision examples are inadequate), then the DM has a starting basis upon which he/she can provide recommendations to the analyst about the calibration of the model in the form of constraints about the parameters of the model. Thus, starting with a model that is consistent with a set of reference examples, an interactive model calibration process is invoked.

Jacquet-Lagrèze and Siskos (1982) introduced the paradigm of preference disaggregation in the context of decision aiding through the development of the UTA method (UTilité Additive), which enables the development of evaluation models in the form of an additive value function for ranking purposes. A comprehensive review of this methodological approach of MCDA can be found in Jacquet-Lagrèze and Siskos (2001) and Siskos *et al*. (2005). Recent research has focused on extensions covering:

- other types of decision models, including among others outranking models (Doumpos and Zopounidis 2002b 2004 Mousseau *et al*. 2001), and rule-based models (Greco *et al*. 2001);
- other decision problematics (e.g., classification, Doumpos and Zopounidis 2002a);
- new modeling forms in the context of robustness decision-making (Dias *et al*. 2002),(Greco *et al*. 2008b).

1.3 Computational intelligence in MCDA

Computational intelligence has evolved rapidly over the past couple of decades and it is now considered as a distinct sub-field that emerged within the area of artificial intelligence. Duch (2007) discusses the unique features of computational intelligence as opposed to the artificial intelligence paradigm, analyzes the multiple aspects of computational intelligence and introduces a definition of the field as 'the science of solving non-algorithmizable problems using computers or specialized hardware.' Craenen and Eiben (2003) view artificial intelligence and computational intelligence as two complementary fields of 'machine intelligence.' In their view, artificial intelligence is mostly concerned with knowledge-based approaches whereas computational intelligence is a different stream involved with non-knowledge-based principles.

In the following subsections, we focus on three major computational intelligence paradigms, namely statistical learning/data mining, fuzzy sets, and metaheuristics, which all have been extremely popular among researchers and practitioners involved with the area of computational intelligence. We analyze the contributions of the paradigms within the context of decision-making problems by overviewing their connections with MCDA.

1.3.1 Statistical learning and data mining

Hand *et al*. (2001) define data mining as 'the analysis of (often large) observational data sets to find unsuspected relationships and to summarize the data in novel ways that are both understandable and useful to the data owner.' Statistical learning plays an important role in the data mining process, by describing the theory that underlies the identification of such relationships and providing the necessary algorithmic procedures.

Modern statistical learning and data mining adopt an *algorithmic modeling culture* as described by Breiman (2001), in which the focus is shifted from data models to the characteristics and predictive performance of learning algorithms. This approach is very different from the MCDA paradigm (a discussion of the similarities and differences in the context of the preference disaggregation approach of MCDA can be found in Doumpos and Zopounidis 2011b as well as in the work of Waegeman *et al*. 2009). Nevertheless, the algorithmic developments in statistical learning and data mining, such as the focus on the analysis of large scale data sets, as well as the wide range of different types of generalized modeling forms employed in these fields, provide new capabilities in the context of MCDA.

1.3.1.1 Artificial neural networks

Artificial neural networks (ANNs) can be considered as directed acyclic graphs with nodes (neurons) organized into layers. The most popular feed-forward architecture consists of a layer of input nodes, a layer of output nodes, and a series of intermediate processing layers. The input nodes correspond to the information that is available for every input vector, whereas the output nodes provide the recommendations of the network. The nodes in the intermediate (hidden) layers are parallel processing units that define the input–output relationship. Every neuron at a given layer receives as input the weighted average of the outputs of the neurons at the preceding layer and maps it to an output signal through a predefined transformation function.

Depending on the topology of the network and the selection of the neurons' transformation functions, a neural network can model real functions of arbitrary complexity. This flexibility has made ANNs a very popular modeling approach in addressing complex real-world problems in engineering and management. This characteristic has important implications for MCDA, mainly with respect to modeling general preference structures.

Within this context, ANNs have been successfully used for learning generalized MCDA models from decision examples in a preference disaggregation setting. Wang and Malakooti (1992), and Malakooti and Zhou (1994) used feedforward ANN models to learn an arbitrary value function for ranking a set of alternatives, as well as to learn a relational multicriteria model based on pairwise comparisons (binary relations) among the alternatives. Generalized network decision models have a function free form, which is less restricted by the assumptions imposed in MAVT (Keeney and Raiffa 1993). Experimental simulation results showed that ANN models performed very well in representing various forms of decision models, outperforming other popular model development techniques based on linear programming formulations. Wang *et al.* (1994) applied a similar ANN model to a job shop production system problem.

In a different framework compared with the aforementioned studies, Stam *et al.* (1996) used ANNs within the context of the analytic hierarchy process (AHP; Saaty 2006). AHP is based on a hierarchical structuring of the decision problem, with the overall goal on the top of the hierarchy and the alternatives at the bottom. With this hierarchical structure, the DM is asked to perform pairwise comparisons of the elements at each level of the hierarchy with respect to the elements of the preceding (higher) level. Stam *et al.* investigated two different ANN structures for accurately approximating the preferences ratings of the alternatives, within the context of imprecise preference judgments by the DM. They showed that a modified Hopfield network has very close connections to the mechanics of the AHP, but found that this network formulation cannot provide good results in estimating the mapping from a positive reciprocal pairwise comparison matrix to its preference rating vector. On the other hand, a feed-forward ANN model was found to provide very good approximations of the preference ratings in the presence of impreciseness. This ANN model was actually superior to the standard principal eigenvector method.

Similar ANN-based methodologies have also be used to address dynamic MCDA problems (where the DM's preferences change over time; Malakooti and Zhou 1991), to learn fuzzy preferences (Wang 1994a,b; Wang and Archer 1994) and outranking relations

(Hu 2009), to provide support in group decision-making problems (Wang and Archer 1994), as well as in multicriteria clustering (Malakooti and Raman 2000).

ANNs have also been employed for preference representation and learning in multiobjective optimization. Within this context, Sun *et al*. (1996) proposed a feed-forward ANN model, which is trained to represent the DM's preference structure. The trained ANN model serves as a value function, which is maximized in order to identify the efficient solution that best fits the DM's preferences. Sun *et al*. (2000) used a similar feed-forward ANN approach to facilitate the interactive solution process in multiobjective optimization problems. Other ANN architectures have also been used as multiobjective optimizers (Gholamian *et al*. 2006; McMullen 2001) and hybrid evaluation systems (Raju *et al*. 2006; Sheu 2008).

A comprehensive overview of the contributions of ANNs in MCDA in provided by Hanne in Chapter 5.

1.3.1.2 Rule-based models

Rule-based and decision tree models are very popular within the machine learning research community. The symbolic nature of such models makes them easy to understand, which is important in the context of decision aiding. During the last decade significant research has been devoted to the use of such approaches as preference modeling tools in MCDA.

In particular, a significant part of the research related to the use of rule-based models in MCDA has focused on rough set theory (Pawlak 1982; Pawlak and Słowiński 1994), which provides a complete and well-axiomatized methodology for constructing decision rule preference models from decision examples. Rough sets have been initially introduced as a methodology to describe dependencies between attributes, to evaluate the significance of attributes and to deal with inconsistent data in the context of machine learning. However, significant research has been conducted on the use of the rough set approach as a methodology for preference modeling in multicriteria decision problems (Greco *et al*. 1999, 2001). The decision rule models developed through the rough set approach for MCDA problems are built on the basis of the dominance relation. Each 'if... then...' decision rule is composed of a condition part specifying a partial profile on a subset of criteria to which an alternative is compared using the dominance relation, and a conclusion part suggesting a decision recommendation.

Decision rule preference models have been initially developed in the context of multicriteria classification problems. In this case the recommendations in the conclusion part of each rule involve the assignment of the alternatives either in a specific class or a set of classes. Extensions to ranking and choice decision problems have been developed by Greco *et al*. (2001) and Fortemps *et al*. (2008), whereas Greco *et al*. (2008a) presented a dominance-based rough set approach for multiobjective optimization.

The decision rule preference model has also been considered in terms of conjoint measurement (Greco *et al*. 2004) and Bayesian decision theory (Greco *et al*. 2007). Greco *et al*. (2004) showed that there is an equivalence of simple cancellation property, a general discriminant function and a specific outranking relation, on the one hand, and the decision rule model on the other hand. They also showed that the decision rule model resulting from the dominance-based rough set approach has an advantage over the usual functional and relational models because it permits the handling of inconsistent decision instances. Inconsistency decision instances often appear due to the instability of preferences, the incomplete determination of criteria, and the hesitation of the DM.

In Chapter 6, Szeląg *et al*. provide a comprehensive presentation of rule-based decision models, focusing on MCDA ranking problems.

1.3.1.3 Kernel methods

Kernel methods are widely used for pattern classification, regression analysis, and density estimation. Kernel methods map the problem data to a high dimensional space (feature space), thus enabling the development of complex nonlinear decision and prediction models, using linear estimation methods (Schölkopf and Smola 2002). The data mapping process is implicitly defined through the introduction of (positive definite) kernel functions. Support vector machines (SVMs) are the most widely used class of kernel methods. Recently, they have also been used within the context of preference learning for approximating arbitrary utility/value functions and preference aggregation.

Herbrich *et al*. (2000) illustrated the use of kernel approaches, within the context of SVM formulations, for representing value/ranking functions of the generalized form $V(\mathbf{x}) = \mathbf{w}\phi(\mathbf{x})$, where ϕ is a possibly infinite-dimensional and in general unknown feature mapping. The authors derived bounds on the generalizing performance of the estimated ranking models, based on the margin separating objects in consecutive ranks.

Waegeman *et al*. (2009) extended this approach to relational models. In this case, the preference model of the form $f(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{w}\phi(\mathbf{x}_i, \mathbf{x}_j)$ is developed to represent the preference of alternative i compared with alternative j. This framework is general enough to accommodate special modeling forms. For instance, it includes value models as a special case, and similar techniques can also be used to kernelize Choquet integrals. As an example, Waegeman *et al*. illustrated the potential of this framework in the case of valued concordance relations, which are used in the ELECTRE methods.

Except for the development of generalized decision models, kernel methods have also been employed for robust model inference purposes. For instance, Evgeniou *et al*. (2005) showed how the regularization principle (which is at the core of the theory of kernel methods) is related to the robust fitting of linear and polynomial value function models in ordinal regression problems. Doumpos and Zopounidis (2007) employed the same regularization principle for developing new improved linear programming formulations for fitting additive value functions in ranking and classification problems. The development of additive value function was also addressed by Dembczynski *et al*. (2006) who presented a methodology integrating the dominance-based rough set approach and SVMs.

SVMs have also be used in the context of multiobjective optimization (Aytug and Sayin 2009; Yun *et al*. 2009) in order to approximate the set of Pareto optimal solutions in complex nonlinear problems. Multiobjective and goal programming formulations has also been used for training SVM models (Nakayama and Yun 2006; Nakayama *et al*. 2005). Finally, hybrid systems based on SVMs have been proposed. For instance, Jiao *et al*. (2009) combined SVMs with the UTADIS disaggregation method (Doumpos and Zopounidis 2002a) for the development of accurate multi-group classification models.

1.3.2 Fuzzy modeling

Decision making is often based on fuzzy, ambiguous, and vague judgments. Verbal expressions such as ‘almost,’ ‘usually,’ ‘often,’ etc., are simple yet typical examples of the ambiguity and vagueness often encountered in the decision-making process. The fuzzy set theory first introduced by Zadeh (1965), provides the necessary modeling tools for such situations. The concept of a fuzzy set is at the center of this approach. In the traditional set theory, a set is considered as a collection of well defined and distinct objects, which implies that sets have clearly defined (crisp) boundaries. Therefore, a statement of the form 'object x belongs to set A' is either true or false. On other hand, a fuzzy set has no crisp boundaries, and every object is associated with a degree of membership with respect to a fuzzy set.

Since its introduction, fuzzy set theory has been an extremely active research field with numerous practical applications in engineering and management. Its uses in the context of decision aiding have also attracted much interest.

1.3.2.1 Fuzzy multiobjective optimization

The traditional multiobjective programming framework assumes that all the parameters of the problem are well-defined. However, imprecision, vagueness, and uncertainty can make the specification of goals, targets, objectives, and constraints troublesome and unclear. Bellman and Zadeh (1970) were the first to explore optimization models in the context of fuzzy set theory. Zimmermann (1976, 1978) further investigated this idea both in the case of single-objective problems as well as in the context of multiobjective optimization.

Fuzzy multiobjective programming formulations have a similar form to conventional multiobjective programming problems (i.e., the optimization of several objective functions over some constraints). The major distinction between these two approaches is that while in deterministic multiobjective programming all objective functions and constraints are specified in a crisp way, in fuzzy multiobjective programming they are specified using the fuzzy set theory through the introduction of membership functions. Fuzzy coefficients for the decision variables in the objective function and the constraints can also be introduced.

A major advantage of fuzzy multiobjective programming techniques over conventional mathematical programming with multiple objectives, is that it provides a framework to address optimization problems within a less strict context regarding the sense of the imposed constraints, as well as the degree of satisfaction of the DM from the compromise solutions that are obtained (i.e., introduction of fuzzy objectives).

The FLIP method (Słowiński 1990) for multiobjective linear programming problems is a typical example of the integration of the fuzzy set theory with multiobjective optimization techniques. FLIP considers uncertainty through the definition of all problem parameters (objective function coefficients, variables' coefficients in the constraints, right-hand side coefficients) as fuzzy numbers, each one associated with a possibility distribution. The recent book by Sakawa *et al*. (2011) presents a comprehensive overview of the theory of fuzzy multiobjective programming including stochastic problems, whereas Roubens and Teghem (1991) present a survey of fuzzy multiobjective programming and stochastic multiobjective optimization and perform a comparative investigation of the two areas.

A detailed presentation of the principles and techniques for fuzzy multiobjective optimization is presented by Sakawa in Chapter 10.

1.3.2.2 Fuzzy preference modeling

Preference modeling is a major research topic in MCDA. The modeling of a DM's preferences can be viewed within the context of MAVT models as well as in the context of outranking relations (Fodor and Roubens 1994; Roubens 1997). The concept of outranking relation is closely connected with the philosophy of fuzzy sets. For instance, in ELECTRE methods the outranking relation $\mathbf{x}_i \; S \; \mathbf{x}_j$ is constructed to evaluate whether alternative i is at least as good as alternative j. Similarly, in the PROMETHEE methods a preference relation is constructed to measure the preference for alternative i over alternative j. In both sets of methods the outranking/preference relations are not treated in a crisp setting. Instead, the relations are quantified by proper measures (e.g., credibility index in ELECTRE and preference index in PROMETHEE) representing the strength of the outranking/preference of one alternative over another. For instance, the credibility index $\sigma(\mathbf{x}_i, \mathbf{x}_j)$ used in ELECTRE methods represents the validity of the affirmation 'alternative i outranks alternative j.' Thus, it is a form of membership function. Perny and Roy (1992) provided a comprehensive discussion on the use of fuzzy outranking relations in preference modeling together with an analysis of the characteristics and properties of such relations.

Despite the above fundamental connection between commonly used MCDA outranking techniques and fuzzy theory, it should be noted that traditional outranking methods consider crisp data. However, many extensions for handling fuzzy data in outranking methods have been proposed. For instance, Czyzak and Słowiński (1996) considered the evaluations of the alternatives on the criteria as fuzzy numbers in order to construct an outranking relation. Common aggregation operators (e.g., maximum and minimum) are employed to aggregate these fuzzy numbers in order to perform the necessary concordance and discordance tests similarly to the traditional outranking relations approach. Roubens (1996) presented several procedures for aggregating fuzzy criteria in an outranking context for choice and ranking problem, whereas a more recent overview of this research direction is given by Bufardi *et al*. (2008). Fuzzy relations can also be used to handle the fuzziness that characterize the DM's preferences. For instance, Siskos (1982) proposed a methodology using disaggregation techniques to build a fuzzy outranking relation on the basis of the information represented in multiple additive value functions which are compatible with the DM's preferences, thus modeling the DM's fuzzy preferential system.

Fuzzy preference modeling approaches have also been developed in the context of MAVT. Grabisch (1995; 1996) introduced an approach to manage uncertainty in the MAVT framework through the consideration of the concept of fuzzy integrals initially introduced by Sugeno (1974). In the proposed approach fuzzy integrals are used instead of the additive and multiplicative aggregation operators that are commonly used in MAVT in order to aggregate all attributes into a single evaluation index (value function). The major advantageous feature of employing fuzzy integrals within the MAVT context is their ability to consider the interactions among the evaluation criteria including redundancy and synergy. On the other hand, the major drawback of such an approach that is a consequence of its increased complexity over simple aggregation procedures (e.g., weighted average), involves the increased number of parameters that should be defined, either directly by the DM, or employing heuristics and optimization techniques. The use of the Choquet integral as an aggregation function has also attracted much interest among MCDA researchers. Marichal and Roubens (2000) first introduced a methodology implementing this approach in a preference disaggregation context. Some work on this topic can be found in the papers of Angilella *et al*. (2004; 2010) and Kojadinovic (2004; 2007), while a review of this area is given in the paper by Grabisch *et al*. (2008). Other applications of the fuzzy set theory to MAVT are discussed in the book by Lootsma (1997).

A final class of decision models developed within the context of fuzzy set theory that attracted much interest in the context of MCDA is based on the ordered weighted averaging (OWA) approach first introduced by Yager (1988). An OWA aggregation model, is a particular case of the Choquet integral, which is similar to a simple weighted average model. However, instead of weighting the criteria, an OWA model assigns weights to the relative position of one criterion value with respect to the other values (Torra 2010). In this way, OWA models allow for different compensation levels to be modeled. For instance, assigning high weights to low performances lead to a noncompensatory mode, whereas compensation can be allowed if higher weight is given to good performance levels. In the context of decision making under uncertainty, the OWA aggregation scheme is a generalization of the Hurwicz rule. Yager (1993) and Xu and Da (2003) provide overviews of different OWA models, whereas Yager (2004) extends this framework to consider different criteria priorities in an MCDA context.

1.3.3 Metaheuristics

Metaheuristics have been one of the most active and rapidly evolving fields in computational intelligence and operations research. Their success and development is due to the highly complex nature of many decision problems. As a consequence the corresponding mathematical models are nonlinear, nonconvex, and/or combinatorial in nature, thus making it very difficult to solve them through traditional optimization algorithms. Metaheuristics and evolutionary techniques have been very successful in dealing with computationally intensive optimization problems, as they make few or no assumptions about the problem and can search very large solution spaces very efficiently. In the context of MCDA, such methods have been primarily used for multiobjective optimization. Their use for fitting complex decision models in a preference disaggregation setting has also attracted some interest.

1.3.3.1 Evolutionary methods and metaheuristics in multiobjective optimization

Traditional multiobjective optimization techniques seek to find an efficient solution that best fits the preferences of a DM. The solution process is performed iteratively so that the DM's preferences are progressively specified and refined until the most satisfactory solution is identified. During this process a series of optimization problems needs to be solved, which may not be easy in the case of combinatorial or highly complex nonlinear and nonconvex problems. Furthermore, in such procedures the DM is often not provided with a full idea of the whole set of Pareto optimal solutions. Metaheuristics are well-suited in this context as they are applicable in all types of computationally intensive multiobjective optimization problems and enable the approximation of complex Pareto sets in a single run of an algorithmic procedure.

Different classes of algorithms can be identified in this research direction. Approaches based on genetic algorithms (GAs) are probably the most popular. GAs are computational procedures that mimic the process of natural evolution for solving complex optimization problems. They implement stochastic search schemes to evolve an initial population (set) of solutions through selection, mutation, and crossover operators until a good solution is reached. The first GA-based approach for multiobjective optimization problem was proposed by Schaffer (1985). During the 1990s and the 2000s many other algorithms implementing a similar GA approach have been proposed. A comprehensive presentation of this approach can be found in the book by Deb (2001), whereas Konak *et al.* (2006) presented a tutorial and review of the field.

The differential evolution (DE) algorithm has also attracted much interest for multiobjective optimization. DE has been introduced by Storn and Price (1997) as a powerful alternative to GAs, which is well-suited to continuous optimization problems. Similarly to a GA, DE also employs evolution operators to evolve a generation of solutions, but it is based on greedy search strategies, which ensure that solutions are strictly improved in every iteration of the algorithm. Abbass and Sarker (2002) presented one of the first implementations of the DE scheme in multiobjective optimization. Some recent extensions have been presented by Gong *et al.* (2009), Krink and Paterlini (2011), and Wang and Cai (2012), whereas Mezura-Montes *et al.* (2008) present a review of DE-based multiobjective optimization algorithms.

A third class of computational intelligence techniques for solving multiobjective optimization problems involves other metaheuristic algorithms, such as simulated annealing, tabu search, ant colony optimization, and particle swarm optimization, which have been proved very successful in solving complex optimization problems of a combinatorial nature. The use of such algorithms in multiobjective optimization can be found in Landa Silva *et al.* (2004), Molina *et al.* (2007), Bandyopadhyay *et al.* (2008), Doerner *et al.* (2008), and Elhossini *et al.* (2010). Jones *et al.* (2002) present an overview of the field, whereas Ehrgott and Gandibleux (2008) focus on recent approaches, where metaheuristics are combined with exact methods.

In Chapter 8, Jaimes and Coello Coello present in detail different interactive methods for multiobjective optimization.

1.3.3.2 Preference disaggregation with evolutionary techniques

Inferring simple decision-making models (e.g., additive or linear value functions) from decision examples poses little computational problems. Most existing preference disaggregation techniques use linear programming for this purpose (Jacquet-Lagrèze and Siskos 2001; Zopounidis and Doumpos 2002). However, more complex models cannot be constructed with exact methods. Metaheuristics are well-suited in this context and have attracted the interest of MCDA researchers over the past few years.

Most of the research on this area has focused on outranking models. Goletsis *et al.* (2004) used a GA for the development of an outranking model based on the philosophy of the ELECTRE methods in a medical classification problem. Belacel *et al.* (2007) used the reduced variable neighborhood search metaheuristic to infer the parameters of the PROAFTN outranking method from a set of reference examples. Focusing on the same outranking method Al-Obeidat *et al.* (2011) used a particle swarm optimization algorithm. Fernandez *et al.* (2009) developed a model based on a fuzzy indifference relation for classification purposes. In order to infer the parameters of the model from a set of reference examples they used the NSGA-II multiobjective evolutionary algorithm (Deb *et al.* 2002) considering four measures related to the inconsistencies and the correct recommendations of the decision model. A similar approach was also presented by Fernandez and Navarro (2011). Doumpos *et al.* (2009) presented a methodology based on the differential evolution algorithm for estimating all the parameters of an ELECTRE TRI model from assignment examples in classification problems under both the optimistic and the pessimistic assignment rules (Roy and Bouyssou 1993). Doumpos and Zopounidis (2011a) applied this methodology to a large data set for the development of credit rating models and demonstrated how the special features of ELECTRE TRI can provide useful insights into the relative importance of the credit rating criteria and the characteristics of the alternatives. Eppe *et al*. (2011) employed the NSGA-II algorithm for inferring the parameters of PROMETHEE II models from decision instances. The authors suggested a bi-objective approach according to which the model is developed so that the number of inconsistencies compared with the DM's evaluation of the reference alternatives is minimized and the robustness of the model's parameters estimates is maximized. In contrast to all the aforementioned studies, which focused on outranking models, Doumpos (2012) considered the construction of a nonmonotone additive value function, assuming that the marginal value functions are quasi-convex. The differential evolution algorithm was used to infer the additive function from reference examples in a classification setting.

1.4 Conclusions

In a dynamic environment characterized by increasing complexity and considerable uncertainties, the interdisciplinary character of decision analysis and decision aiding is strengthened. Complex and ill-structured decision problems in engineering and management cannot be handled in a strictly defined methodological context. Instead, integrated approaches often need to be implemented, combining concepts and techniques from various research fields. In this context, the relationship between artificial intelligence and MCDA has attracted much interest among decision scientists.

This chapter presented an overview of this area, focusing on the computational intelligence paradigm. Computational intelligence has been one of the most active areas in artificial intelligence research, with numerous applications engineering and management systems. The overview focused on the contributions of computational intelligence methodologies in decision support, covering important issues such as the introduction of new preference modeling techniques, advanced algorithmic solution procedures for complex problems, as well as new techniques for constructing decision models. The advances in each of these areas provides new capabilities for extending the research and practice of the MCDA paradigm, thus enabling its use in new ill-structured decision domains, characterized by uncertainty, vagueness, and imprecision, complex preference and data structures, and high data dimensionality.

The active research conducted in this area is expected to continue to grow at a rapid pace. Future research could cover various issues. For instance, the integration with other artificial intelligence paradigms, including knowledge management, representation, and engineering, natural language processing, intelligent agents (see Chapter 11), evidential reasoning (see Chapter 7) and Bayesian inference, is an interesting research area providing a wide range of new potentials for enhancing decision aiding. The implementation of the state-of-the-art research results into intelligent decision support systems taking advantage of the rapid advances in data management and web-based technologies is also important for disseminating new results among researchers and improving their applicability in practice (see Chapter 2). Finally, given the wide arsenal of existing approaches, their comprehensive comparative empirical evaluation is necessary in order to identify their strengths, weaknesses, and limitations, under different decision aiding settings.

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