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Electric Circuit Notations and Elementary Concepts

In this introductory chapter, we start by defining the electric circuit notations and elementary circuit concepts, including symbolisms used in representation of time-domain and AC phasor variables, such as voltage and currents, used throughout Chapters 1–12 of this textbook. These include multivariable arrays, the so-called vectors in matrix linear-algebra parlance, as well as concepts of time-domain instantaneous power and complex power in AC phasor formulation of electric circuits.

A basic and fundamental complex-algebra concept to all that will follow is the well-known Euler formulation that yields the following identities:

$$e^{j(\omega t+\phi)} = \cos(\omega t + \phi) + j\sin(\omega t + \phi) \tag{1.1}$$

$$\cos(\omega t + \phi) = \mathrm{Re}[e^{j(\omega t+\phi)}] \tag{1.2}$$

and

$$\sin(\omega t + \phi) = \mathrm{Im}[e^{j(\omega t+\phi)}] \tag{1.3}$$

With these identities, Eqs. (1.1)–(1.3), we are ready to discuss frequency-domain root mean square (RMS) phasor representation of time-domain AC voltages and currents.

1.1 Frequency-Domain RMS Phasor Representation of Time-Domain AC Voltages and Currents

Here, lower-case variables (symbols) such as v, i, e, and p stand for time-domain voltages, currents, electromotive forces (emfs), and real powers, respectively. Also here, upper-case variables (symbols) such as, V, I, E, and P stand for RMS magnitudes (absolute values) of voltage phasors, current phasors, emf phasors, and average real power in phasor complex power computations, respectively.

Electric Machinery and Drives: An Electromagnetics Perspective, First Edition.
Nabeel A. O. Demerdash, JiangBiao He, and Hao Chen.

Companion website: www.wiley.com/go/demerdash/electricmachineryanddrives

Meanwhile, variables (symbols) such as $\overline{V}$, $\overline{I}$, $\overline{E}$, and $\overline{S}$ stand for RMS phasor complex form voltages, currents, emfs, and complex power (real and reactive) in phasor complex power computations, respectively.

Accordingly, in steady-state AC circuit analysis, one can write the following:

$$v(t) = \sqrt{2}V\cos(\omega t + \phi) \tag{1.4}$$

where V is the RMS AC voltage magnitude (absolute value), $\omega = (2\pi f)$ is the angular frequency in electrical radians per second, f is the AC frequency in Hertz, t is the time in seconds, and ϕ is the phase angle of the voltage signal.

Hence, based on the Euler identities given earlier, Eqs. (1.1)–(1.3), one can rewrite $v(t)$ as follows:

$$v(t) = \sqrt{2}V\left\{\mathrm{Re}\left[e^{j(\omega t+\phi)}\right]\right\} \tag{1.5}$$

or

$$v(t) = \sqrt{2}V\mathrm{Re}\left[e^{j\omega t}\cdot e^{j\phi}\right] \tag{1.6}$$

In AC phasor form, the term ($e^{j\omega t}$) is common to all voltage, current, and other signals. Hence, all the information that is needed is in terms (V) and ($e^{j\phi}$). Hence, in RMS phasor notation, the voltage, $\overline{V}$, can be written as follows in exponential phasor form:

$$\overline{V} = V(e^{j\phi}) \tag{1.7}$$

or in Cartesian coordinate polar and rectangular complex forms as follows:

$$\overline{V} = (V\angle\phi) = (V\cos\phi) + j(V\sin\phi) \tag{1.8}$$

Similarly, for an instantaneous steady-state time-domain, current, $i(t)$, given by the following equation:

$$i(t) = \sqrt{2}I\cos(\omega t + \psi) \tag{1.9}$$

The corresponding Euler formulation gives

$$i(t) = \sqrt{2}I\left\{\mathrm{Re}\left[e^{j(\omega t+\psi)}\right]\right\} \tag{1.10}$$

and for the corresponding RMS phasor notation, the current, $\overline{I}$, can be written as follows:

$$\overline{I} = I(e^{j\psi}) \tag{1.11}$$

or, in Cartesian coordinate polar and rectangular complex forms, the current, $\overline{I}$, can be written as follows:

$$\overline{I} = (I\angle\psi) = (I\cos\psi) + j(I\sin\psi) \tag{1.12}$$

In this text, $\overline{V}^*$ is the conjugate of the phasor $\overline{V}$ and $\overline{I}^*$ is the conjugate of the phasor $\overline{I}$. Therefore,

$$\overline{V}^* = \text{Conj}(\overline{V}) = (V\angle - \phi) = (V\cos\phi) - j(V\sin\phi) \tag{1.13}$$

and

$$\overline{I}^* = \text{Conj}(\overline{I}) = (I\angle - \psi) = (I\cos\psi) - j(I\sin\psi) \tag{1.14}$$

1.2 Time-Domain and RMS Frequency-Domain Power Concepts Using Consumer System Formulation and Notations

Consider the two-terminal "system or device" shown in Figure 1.1. The current i (or $\overline{I}$) is taken to be flowing in a positive orientation when flowing into the terminal designated with a positive voltage polarity, v (or $\overline{V}$). In such a case, the instantaneous input power, $p(t)$, is given by the following:

$$p(t) = vi = v(t)i(t) \tag{1.15}$$

where a positive $p(t)$ means that real power (watts) is being consumed and a negative $p(t)$ means that real power (watts) is being generated.

Meanwhile, in phasor frequency-domain, the complex power, $\overline{S}$, is given by the following:

$$\overline{S} = \overline{V}\cdot\overline{I}^* = P + jQ \tag{1.16}$$

where P is the real power in watts and Q is the reactive power in vars, and once again, P is positive means that watts is being consumed (as in a motoring mode) and P is negative means that watts is being generated (as in a generating mode), and Q is the reactive power in vars, in which Q is positive means that vars is being consumed (as in inductive loads) and Q is negative means that vars is being generated (as in capacitive loads). The reader is urged to examine the complex powers, $\overline{S}_1$, $\overline{S}_2$, $\overline{S}_3$, and $\overline{S}_4$, associated with the current phasors, $\overline{I}_1$, $\overline{I}_2$, $\overline{I}_3$, and $\overline{I}_4$, respectively, relative to the terminal voltage phasor, $\overline{V} = V\angle 0°$, associated with the two-terminal system (or device), shown in Figure 1.1.

In the Consumer System Notation:

$$p(t) = vi = v(t)i(t)$$

Positive $p(t)$ means watts is consumed (load/motoring).
Negative $p(t)$ means watts is generated (generator/source).

Now, for a multiterminal (or port), n, system (or device), the voltages can be written in matrix/array (vector) form in the time-domain, $\underline{V}$, or phasor frequency-domain, $\underline{\overline{V}}$, respectively, as follows:

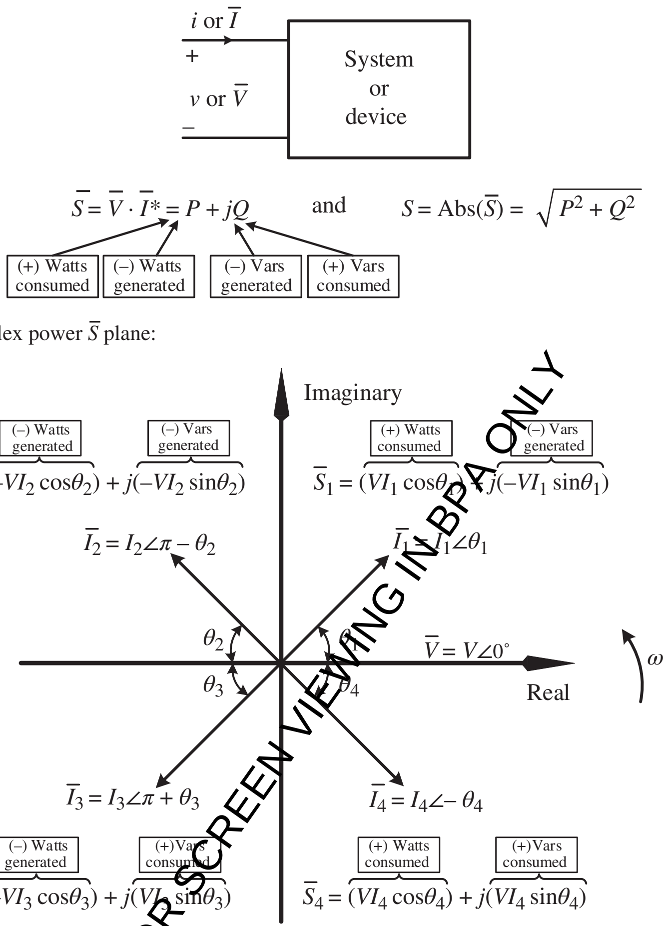


Figure 1.1 A summary graphical and formulation representation of the consumer system in the time-domain and frequency-domain phasor power computation.

$$\underline{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} \tag{1.17}$$

and

$$\underline{\overline{V}} = \begin{bmatrix} \overline{V}_1 \\ \overline{V}_2 \\ \overline{V}_3 \\ \vdots \\ \overline{V}_n \end{bmatrix} \tag{1.18}$$

Similarly, the currents can be written in matrix/array (vector) form in the time-domain, $\underline{I}$, or phasor frequency-domain, $\underline{\overline{I}}$, respectively, as follows:

$$\underline{I} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_n \end{bmatrix} \tag{1.19}$$

and

$$\underline{\overline{I}} = \begin{bmatrix} \overline{I}_1 \\ \overline{I}_2 \\ \overline{I}_3 \\ \vdots \\ \overline{I}_n \end{bmatrix} \tag{1.20}$$

Therefore, the instantaneous time-domain power, $p(t)$, can be written as follows in an n-polyphase device:

$$p(t) = \underline{V}^t \cdot \underline{I} = v_1 i_1 + v_2 i_2 + \cdots + v_n i_n \tag{1.21}$$

where $\underline{V}^t$ is the transpose of $\underline{V}$.

Meanwhile, the frequency-domain phasor computation of the complex power, $\overline{S}$, can be written as follows:

$$\overline{S} = \underline{\overline{V}}^t \cdot (\underline{\overline{I}}^*) \tag{1.22}$$

That is,

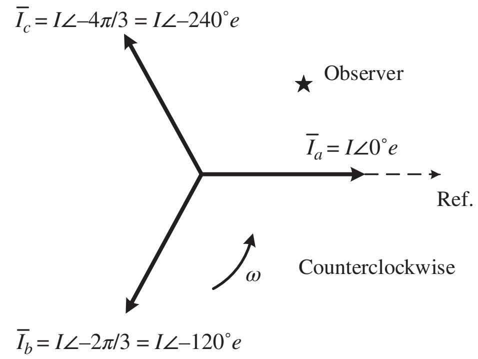
$$\overline{S} = \overline{V}_1 \cdot \overline{I}_1^* + \overline{V}_2 \cdot \overline{I}_2^* + \cdots + \overline{V}_n \cdot \overline{I}_n^* \tag{1.23}$$

where $\underline{\overline{V}}^t$ is the transpose of $\underline{\overline{V}}$.

1.3 Elementary Concepts of Complex Real and Reactive Power in Balanced Three-Phase Circuits and Devices Using Consumer System Notations

A balanced three-phase set of current phasors, $\overline{I}_a$, $\overline{I}_b$, and $\overline{I}_c$, is shown in Figure 1.2. We will always assume counterclockwise rotation for such phasor diagrams throughout this textbook, unless it is explicitly stated otherwise. Accordingly, for an "observer" located at the star-point in this diagram, the observer will see an *a, b, c, a, b, c,* … sequence, that is, a positive (+) sequence as designated in this figure. Meanwhile, if the rotation of this phasor diagram is reversed to a clockwise orientation, the sequence would become an *a, c, b, a, c, b,* … one, that is, a negative (−) sequence.

If one would interchange the locations of the $\overline{I}_b$ and $\overline{I}_c$ phasors, as shown in Figure 1.3, and still preserve the counterclockwise rotation and the location of the



The sequence is $a, b, c, a, b, c, \ldots$
(+) Positive sequence
If the rotation is reversed to a clockwise one, the sequence would become:
$a, c, b, a, c, b, \ldots$
(–) Negative sequence

Figure 1.2 A balanced three-phase set of current phasors, $\bar{I}_a$, $\bar{I}_b$, and $\bar{I}_c$, and the concepts of positive *a*, *b*, *c* and negative *a*, *c*, *b* sequencing.

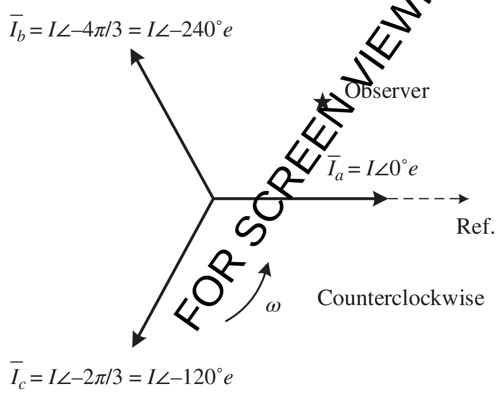


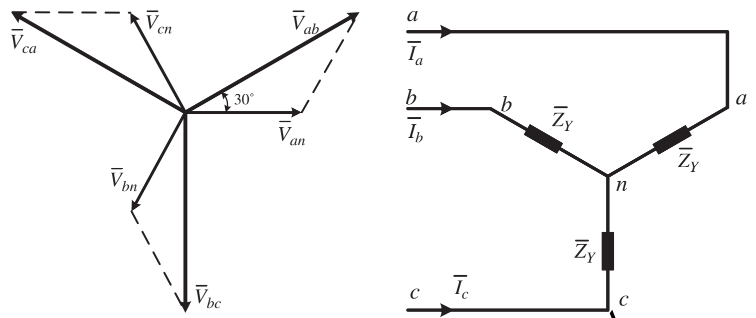
Figure 1.3 Another approach to negative *a*, *c*, *b* sequencing in balanced three-phase set of currents, $\bar{I}_a$, $\bar{I}_c$, and $\bar{I}_b$.

The sequence is $a, c, b, a, c, b, \ldots$
(–) Negative sequence

star-point "observer," the result will be an *a, c, b, a, c, b,* … sequence. That is, one will be seeing a negative (−) sequence as designated in Figure 1.3.

Meanwhile, we discuss complex power, $\bar{S}$, in the context of a balanced three-phase *Y*-connected impedance load with an isolated neutral, *n*. See Figure 1.4 in which the impedance per-phase is $\bar{Z}_Y$, and the phase-currents, $\bar{I}_a$, $\bar{I}_b$, and $\bar{I}_c$, which are also equal to the *a*, *b*, *c* line-currents as depicted in this figure.

Y-Connected load:



$\overline{V}_{an} = V\angle 0 = V\angle 0^\circ$

$\overline{V}_{bn} = V\angle -2\pi/3 = V\angle -120^\circ$

$\overline{V}_{cn} = V\angle -4\pi/3 = V\angle -240^\circ$

$\overline{Z}_Y = Z_Y\angle\theta$

$\overline{I}_a = \overline{V}_{an}/\overline{Z}_Y = V\angle 0^\circ/\overline{Z}_Y = V/Z_Y = \angle -\theta$

$\overline{I}_b = \overline{V}_{bn}/\overline{Z}_Y = V\angle -120^\circ/\overline{Z}_Y = V/Z_Y = \angle -\theta -120^\circ$

$\overline{I}_c = \overline{V}_{cn}/\overline{Z}_Y = V\angle -240^\circ/\overline{Z}_Y = V/Z_Y = \angle -\theta -240^\circ$

Figure 1.4 A balanced Y-connected impedance load and its associate line-to-neutral and line-to-line voltages and line currents, $\overline{I}_a$, $\overline{I}_b$, and $\overline{I}_c$.

Also, given in Figure 1.4 are the formulations for the phase (line-to-neutral) voltages, $\overline{V}_{an}$, $\overline{V}_{bn}$, and $\overline{V}_{cn}$, as well as phase-currents. The phasor diagram of the voltages depicts the graphical/schematic phasor representation of the line-to-line voltages, $\overline{V}_{ab}$, $\overline{V}_{bc}$, and $\overline{V}_{ca}$. Here, one can write the complex power, $\overline{S}$, as follows:

$$\overline{S} = \overline{V}_{an}\cdot\overline{I}_a^{\,*} + \overline{V}_{bn}\cdot\overline{I}_b^{\,*} + \overline{V}_{cn}\cdot\overline{I}_c^{\,*} = P + jQ \tag{1.24}$$

where P is the real power in watts and Q is the reactive power in vars. Again, here, one can write the following:

$$S = \text{Abs}(\overline{S}) = \sqrt{P^2 + Q^2} \tag{1.25}$$

and in the case of a balanced three-phase load, the power factor, PF, is

$$PF = \cos(\theta) = P/S \tag{1.26}$$

where the angle, θ, is the power factor angle given by

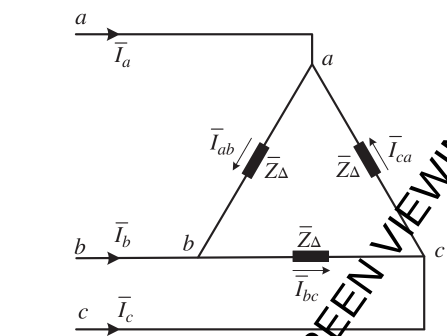
$$\theta = \cos^{-1}(P/S) = \cos^{-1}(PF) \tag{1.27}$$

which is the angle of the impedance per-phase, $\overline{Z}_Y$, where $\overline{Z}_Y = Z_Y\angle\theta$; see Figure 1.4. Furthermore, in this balanced three-phase Y-connected impedance load, one can write the following for $\overline{S}$:

$$\overline{S} = P + jQ = 3\overline{V}_{an}\overline{I}_a^* = 3\overline{V}_{bn}\overline{I}_b^* = 3\overline{V}_{cn}\overline{I}_c^* \tag{1.28}$$

The schematic of a balanced Δ-connected impedance load is shown in Figure 1.5, in which an impedance, $\overline{Z}_\Delta$, interconnects lines a to b, lines b to c, and lines c to a, respectively. The formulations for the line-to-line voltages, $\overline{V}_{ab}$, $\overline{V}_{bc}$, and $\overline{V}_{ca}$, are depicted in Figure 1.5, including the delta (Δ) branch currents, $\overline{I}_{ab}$, $\overline{I}_{bc}$, and $\overline{I}_{ca}$. Also, depicted in Figure 1.5 are the line currents, $\overline{I}_a$, $\overline{I}_b$, and $\overline{I}_c$.

Comparing the line currents, $\overline{I}_a$, $\overline{I}_b$, and $\overline{I}_c$, for the Y-connected balanced load of Figure 1.4 and the Δ-connected balanced load of Figure 1.5, one can conclude



$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn}$$
$$\overline{V}_{ab} = \sqrt{3}V\angle 30^\circ$$
$$\overline{V}_{bc} = \overline{V}_{bn} - \overline{V}_{cn}$$
$$\overline{V}_{bc} = \sqrt{3}V\angle -90^\circ$$
$$\overline{V}_{ca} = \overline{V}_{cn} - \overline{V}_{an}$$
$$\overline{V}_{ca} = \sqrt{3}V\angle -210^\circ = \sqrt{3}V\angle 150^\circ$$

$$\overline{I}_{ab} = \overline{V}_{ab}/\overline{Z}_\Delta = \sqrt{3}V\angle 30^\circ/\overline{Z}_\Delta$$
$$\overline{I}_{bc} = \overline{V}_{bc}/\overline{Z}_\Delta = \sqrt{3}V\angle -90^\circ/\overline{Z}_\Delta$$
$$\overline{I}_{ca} = \overline{V}_{ca}/\overline{Z}_\Delta = \sqrt{3}V\angle 150^\circ/\overline{Z}_\Delta = \sqrt{3}V\angle -210^\circ/\overline{Z}_\Delta$$

$$\overline{I}_a = \overline{I}_{ab} - \overline{I}_{ca} = \sqrt{3}V\angle 0^\circ/\overline{Z}_\Delta$$
$$\overline{I}_b = \overline{I}_{bc} - \overline{I}_{ab} = \sqrt{3}V\angle -120^\circ/\overline{Z}_\Delta$$
$$\overline{I}_c = \overline{I}_{ca} - \overline{I}_{bc} = \sqrt{3}V\angle -240^\circ/\overline{Z}_\Delta$$

Figure 1.5 Balanced Δ-connected impedance load and its line-to-line voltages and line currents, $\overline{I}_a$, $\overline{I}_b$, and $\overline{I}_c$.

that for the two loads to draw the same line, a, b, and c, currents, one must be able to write the following for each line current, $\overline{I}_a$, $\overline{I}_b$, and $\overline{I}_c$:

$$\overline{I}_a = (V\angle 0^\circ/\overline{Z}_Y) = (3V\angle 0^\circ/\overline{Z}_\Delta) \tag{1.29}$$

$$\overline{I}_b = (V\angle -120^\circ/\overline{Z}_Y) = (3V\angle -120^\circ/\overline{Z}_\Delta) \tag{1.30}$$

and

$$\overline{I}_c = (V\angle -240^\circ/\overline{Z}_Y) = (3V\angle -240^\circ/\overline{Z}_\Delta) \tag{1.31}$$

Hence, for the two Y-connected and Δ-connected loads to be equivalent at their a, b, and c terminals, we must have

$$\overline{Z}_Y = (\overline{Z}_\Delta/3) \tag{1.32}$$

$$\overline{Z}_\Delta = (3\overline{Z}_Y) \tag{1.33}$$

In summary, Eqs. (1.32) and (1.33) constitute the Δ-to-Y impedance transformation and the Y-to-Δ impedance transformation for balanced three-phase impedance loads, respectively. The complex power, $\overline{S}$, drawn from the source is identical for both impedance loads in Eqs. (1.32) and (1.33).