Section I

Introduction

Superconductivity is, as will be developed during the text, a phase state that is *independent of the magnetothermal history*, which is very different to the condition of a *perfect conductor*. As will be developed, the perfect conductor and the superconductor would have different end states depending on the *history* of the cooling and application of a magnetic field.

However, we start the text with a short recap of resistivity and conduction in metals leading up to an understanding *scattering rate*, which will feature in our development of superconductivity. This gives us a starting point in our discussion as we move from conductor to *super*conductor.

The Chapter 2 in this section sets out the key stages in the historical development of superconductivity. The history of the subject is the route taken by the majority of texts, although this in itself does not make it the *correct* way to go, and seems an appropriate way to introduce the theory and experimental evidence. It is also the view of many that an overview of the historical development of a subject gives the reader a context in which to place their learning.

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Resistivity and Conduction in Metals

1.1 INTRODUCTION

In this chapter, the ideas of resistivity and conduction will be revised. Both resistivity and conduction are important ideas in the development of *superconductivity*, and hence, it is important to have these ideas securely understood with a symbolic representation that will be used throughout the whole text.

1.2 RESISTIVITY

The resistivity of a material is a constant, albeit temperature dependent, for that material in the same way that density is. Resistivity is defined as

$$\rho = \frac{\varepsilon}{J}. \tag{1.1}$$

Given that ε is measured in V m^{-1} and J in A m^{-2}, we can, with reference to Figure 1.1, write

$$p = \frac{\frac{V}{l}}{\frac{I}{A}} = R\frac{A}{l}. \tag{1.2}$$

Discovering Superconductivity: An Investigative Approach, First Edition. Gren Ireson.

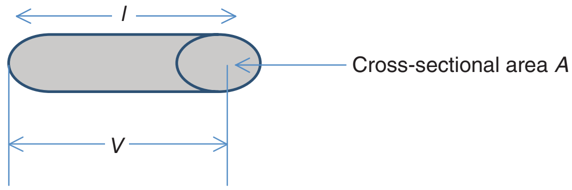

Figure 1.1 Resistivity in terms of resistance, length and cross-sectional area.

From Equation 1.1 for resistivity, we can define *conductivity* as the reciprocal of resistivity giving the conductivity, σ, as

$$\sigma = \frac{J}{\varepsilon}. \tag{1.3}$$

The value of resistivity, or conductivity, for a range of materials, 273 K, is given in Table 1.1.

In a *superconductor*, as we will explore later, the resistivity drops to, within the tolerance of measurement, zero implying that the conductivity tends to infinity.

Investigation

Using conducting putty, see Figure 1.2, design and carry out an investigation to verify or refute Equation 1.2.

Figure 1.2 Using conducting putty.

Table 1.1 Resistivity and conductivity of some common metals or alloys.

Material	Resistivity ($\times 10^{-8}$ Ω m)	Conductivity ($\times 10^{8}$ S m^{-1})
Copper	1.54	0.65
Gold	2.05	0.49
Tin	11.50	0.09
Brass	6.30	0.16
Constantan	49.00	0.02
Nichrome	107.30	0.01
Manganin	41.50	0.02

Source: Adapted from Kaye and Laby (1995).

1.3 CONDUCTION IN METALS

A simple model of conduction in metals can be developed by considering each atom giving up one, or more, outer electrons. The free electrons can flow through the lattice but collide with the stationary, positive, ions. If the metal is in an electric field then the random motion of the electrons will 'drift' in a direction dictated by the direction of the field, the drift velocity, v_d, is typically of the order of 10^{-4} m s^{-1}.

Considering the section of conductor of length l m, as shown in Figure 1.3, if the conductor contains n electrons per unit volume then the charge in the section is

$$Q = neAl, \tag{1.4}$$

where e is the charge on the electron, 1.6×10^{-19} C.

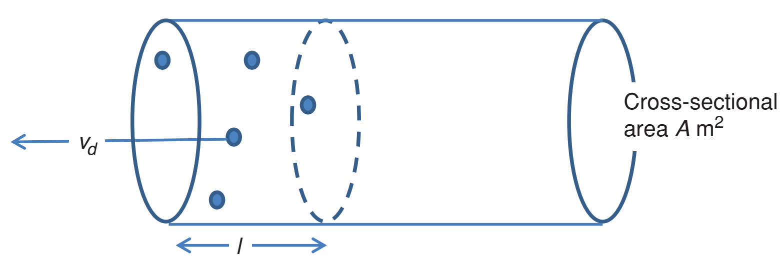

Figure 1.3 A macroscopic view of drift velocity.

If the drift velocity of the electrons is v_d then the time taken for this volume of charge to pass a fixed point is

$$t = \frac{v_d}{l}. \tag{1.5}$$

Given that current I is given by

$$I = \frac{\partial Q}{\partial t}. \tag{1.6}$$

Then for a constant current, we can write

$$I = neAv_d. \tag{1.7}$$

Or for any charge carrier a generalised expression for the current is

$$I = \frac{\partial Q}{\partial t} = n|Q|Av_d, \tag{1.8}$$

which, in turn, allows for current density J, see Equations 1.1 and 1.3, to be written with drift velocity as a vector:

$$\vec{J} = nQ\vec{v}_d \quad \text{and} \quad \vec{\varepsilon} = \rho\vec{J}. \tag{1.9}$$

Table 1.1 gave values of resistivity at 273 K; the resistivity of metallic conductors tends to increase with temperature. As the temperature increases so does the amplitude of vibration of the ions in the lattice and this impedes the drift of the conduction electrons. Impurities in the conductor may also impede the drift of the electrons. The variation in resistivity is nonlinear, see Equation 2.1, but over a small temperature range, taking a reference temperature, T_0, of 273 K with the resistivity, ρ_0, measured at this temperature we can write

$$\rho_T = \rho_0[1 + \alpha(T - 273)], \tag{1.10}$$

where T is the absolute temperature and α is the temperature coefficient of resistivity.

The α values for some common metals are given in Table 1.2.

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Table 1.2 Alpha coefficients of common conductors.

Conductor	α coefficient (K^{-1})
Brass	0.00200
Constantan	0.00001
Copper	0.00393
Manganin	0.00000
Nichrome	0.00040

Source: Adapted from Young and Freedman (2000).

Investigations

1. Using a copper conductor design and carry out an investigation to verify or refute the α coefficient value for copper given above.
2. Given that, for copper, n is of the order of 8.5×10^{28} m^{-3} design and carry out an investigation to verify or refute that v_d is of the order of 10^{-4} m s^{-1}.

1.4 REVISITING OHM'S LAW

In this short exploration of Ohm's law, we will use the *free-electron model*, that is one in which we can assume that the conduction electrons are free to move through the volume of metal rather like molecules of a gas in a closed container and in which the conduction electrons only collide with the metal lattice and not one another.

Using the gas molecule analogy should give us a distribution of speed given by the *Maxwell–Boltzmann distribution*, which gives the probability of finding a molecule at a given speed and proportional to $\sqrt{T}$ where T is the absolute temperature.

You may recall from work on thermal properties of material, or statistical mechanics, that the function governing the distribution of molecular speeds, f_v, is

$$f_v = 4\pi \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2kT}}, \tag{1.11}$$

which gives solutions for the most probable, average and root-mean-square speeds all of which are proportional to $\sqrt{T}$.

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Investigation

A note on Maxwell–Boltzmann

Starting with Equation 1.11, it is possible to calculate the most probable, average and root-mean-square speeds.

This investigation will take you through the steps required.

If the translational kinetic energy of a molecule is written as E_K show that Equation 1.11 can be written in the form

$$f_v = \frac{8\pi}{m}\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} E_K e^{\frac{-E_K}{kT}}. \qquad (1.12)$$

The most probable speed is when the curve described by Equation 1.11 has its maximum value, show that the most probable speed, v_{most}, is given by

$$v_{\text{most}} = \sqrt{\frac{2kT}{m}}. \qquad (1.13)$$

The hint here is to take $\frac{\mathrm{d}f_v}{\mathrm{d}v} = 0$.

The average speed, v_{av}, can be found from $v_{\text{av}} = \int_0^\infty v f_v \mathrm{d}v$, carry out the integration and show that v_{av} is given by

$$v_{\text{av}} = \sqrt{\frac{8kT}{\pi m}} \qquad (1.14)$$

The hint here is to take Equation 1.11, consider substituting v^2 and then integrate by parts.

The root-mean-square speed, v_{rms}, can be found from $v_{\text{rms}} = \sqrt{\int_0^\infty v^2 f_v \mathrm{d}v}$, carry out the integration and show that v_{rms} is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}. \qquad (1.15)$$

The dependence of f_v on the absolute temperature gives the familiar family of curves (Figure 1.4).

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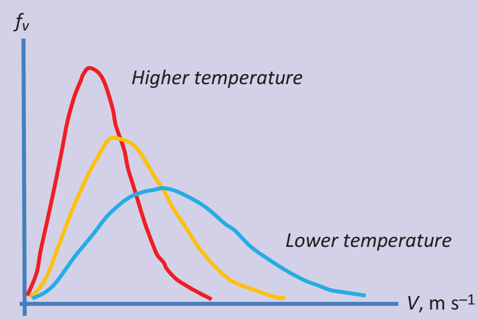

Figure 1.4 Distribution of molecular speeds at various temperatures.

The root-mean-square speed, when applied to an ideal gas model, can be used to give a distribution of speeds as a ratio of $\frac{v}{v_{\text{rms}}}$, as shown in Table 1.3.

The above distribution and results of the investigation allow us to express the root-mean-square speed and average speed in terms of the most probable speed as

$$v_{\text{rms}} = \sqrt{\frac{3}{2}} v_{\text{most}}, \tag{1.16}$$

$$v_{\text{av}} = \frac{2}{\sqrt{\pi}} v_{\text{most}}. \tag{1.17}$$

Table 1.3 Distribution of molecular speeds.

v/v_{rms}	Proportion less than v/v_{rms}
0.2	0.01
0.4	0.08
0.6	0.22
0.8	0.41
1.0	0.61
1.2	0.77
1.4	0.88
1.6	0.95
1.8	0.98
2.0	0.99

Source: Adapted from Young and Freedman (2000).

However, this distribution, based on classical physics, does not hold for the conduction electrons. The conduction electrons are better described as moving with an effective speed v_{ef}. This speed is several orders of magnitude greater than the drift velocity in Equation 1.7.

If an electric field is applied to the metal the conduction electrons acquire a *drift velocity*, v_d, in the opposite direction to the applied field. The path of the conduction electrons, therefore, becomes a combination of the two. However, whilst the magnitude of v is effectively the same for all conduction electrons the motion is randomly distributed and collisions with the metal lattice gives a *velocity* which sums to zero. Thus, the *drift velocity* can be considered to be independent of this contribution.

Since the force on a charged particle in an electric field is $F = q\mathbf{E}$ and for any motion $F = ma$ we can write, for the electron

$$a = \frac{e\mathbf{E}}{m_e}. \tag{1.18}$$

This acceleration can be applied after each collision with the metal lattice. If the mean time between the collisions is $\boldsymbol{\tau}$ then the drift velocity can be expressed as $v_d = a\tau$, which when combined with Equation 1.16 gives

$$v_d = \frac{e\mathbf{E}_\tau}{m_e}. \tag{1.19}$$

Combining this with Equation 1.9 and using the charge on the electron, e, we can write

$$\vec{E} = \left(\frac{m_e}{e^2 n\tau}\right)\vec{J}. \tag{1.20}$$

When an analysis of the conduction electrons in a metal is undertaken using a *Fermi gas* model the conductivity is given by $\sigma = ne^2\tau/m_e$.

Since conductivity is the reciprocal of resistivity, using this result and Equation 1.9, we can write

$$\rho = \frac{m_e}{ne^2}\tau^{-1}, \tag{1.21}$$

where τ^{-1} is the scattering rate.

Scattering rate will become important in our discussion of electrons as charge carriers in a superconductor.

REFERENCES

Kaye, G.W.C. and Laby, T.H. (1995) *Tables of Physical and Chemical Constants*, 16th edn, Longman, London.

Young, H.D. and Freedman, R.A. (2000) *Sears and Zemansky's University Physics*, 10th edn, Addison-Wesley, San Francisco.