IN THIS CHAPTER

» Identifying angles and their names

» Understanding trig speak

» Finding trig applications in the basics

Chapter 1

Taking On Trig Technicalities

How did Columbus find his way across the Atlantic Ocean? How did the Egyptians build the pyramids? How did early astronomers measure the distance to the moon? No, Columbus didn't follow a yellow brick road. No, the Egyptians didn't have LEGO instructions. And, no, there isn't a tape measure long enough to get to the moon. The common answer to all these questions is trigonometry.

Trigonometry is the study of angles and triangles and the wonderful things about them and that you can do with them. For centuries, humans have been able to measure distances that they can't reach because of the power of this mathematical subject.

Taking Trig for a Ride: What Trig Is

"What's your angle?" That question isn't a come-on such as, "What's your astrological sign?" In trigonometry, you can measure angles in both degrees and radians. You can position the angles into triangles and circles and make them do special things. Actually, angles drive trigonometry. Sure, you have to consider algebra and other math to make it all work. But you can't have trigonometry without angles. Put an angle measure into a trig function, and out pops a special, unique number. What do you do with that number? Read on, because that's what trig is all about.

Sizing up the basic figures

Segments, rays, and lines are some of the basic forms found in geometry, and they're just as important in trigonometry. As I explain in the following sections, you use those segments, rays, and lines to form angles and triangles and other geometric and trig forms.

Drawing segments, rays, and lines

A *segment* is a straight figure drawn between two endpoints. You usually name it by its endpoints, which you indicate by capital letters. Sometimes, a single letter names a segment; this single letter is positioned at about the middle of the segment. For example, in a triangle, a lowercase letter may refer to a segment opposite the angle labeled with the corresponding uppercase letter.

A *ray* is another straight figure that has an endpoint on one end, and then it just keeps going forever in some specified direction. You name rays by their endpoint first and then by any other point that lies on the ray. You indicate that the other end goes on forever by using an arrow point.

A *line* is a straight figure that goes forever and ever in either direction. You only need two points to determine a particular line — and only one line can go through both of those points. You can name a line by any two points that lie on it.

Figure 1-1 shows a segment, ray, and line and the different ways you can name them using points.

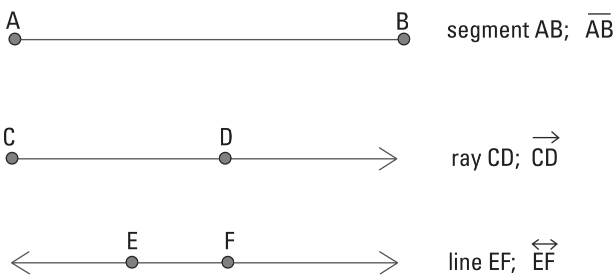


FIGURE 1-1: Segment AB, ray CD, and line EF.

Intersecting lines

When two lines intersect — if they do intersect — they can only do so at one point. They can't double back and cross one another again. And some curious things happen when two lines intersect. The angles that form between those two lines are related to one another. Any two angles that are next to one another and share a side are called *adjacent angles.* In Figure 1-2, you see several sets of intersecting lines and marked angles. The top two figures indicate two pairs of adjacent angles. Can you spot the other two pairs? The angles that are opposite one another when two lines intersect also have a special name. Mathematicians call these angles *vertical angles.* They don't have a side in common. The two middle pairs in Figure 1-2 are vertical angles. Vertical angles are always equal in measure.

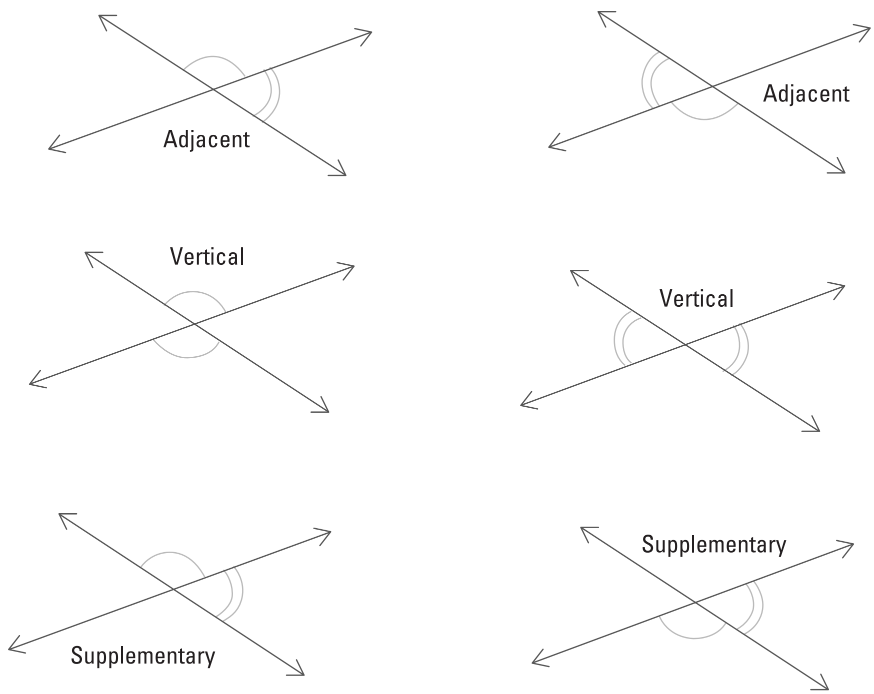


FIGURE 1-2: Intersecting lines form adjacent, vertical, and supplementary angles.

Why are these different angles so special? They're different because of how they interact with one another. The adjacent angles here are called *supplementary angles.* The sides that they don't share form a straight line, which has a measure of 180 degrees. The bottom two figures show supplementary angles. Note that these are also adjacent.

Identifying angles and their names

When two lines, segments, or rays touch or cross one another, they form an angle or angles. In the case of two intersecting lines, the result is four different angles. When two segments intersect, they can form one, two, or four angles; the same goes for two rays.

These examples are just some of the ways that you can form angles. Geometry, for example, describes an angle as being created when two rays have a common endpoint. In practical terms, you can form an angle in many ways, from many figures. The business with the two rays means that you can extend the two sides of an angle out farther to help with measurements, calculations, and practical problems.

Describing the parts of an angle is pretty standard. The place where the lines, segments, or rays cross is called the *vertex* of the angle. From the vertex, two sides extend.

Naming angles by size

You can name or categorize angles based on their size or measurement in degrees and radians. For more on radian measures, go to Chapter 4. Figure 1-3 shows examples of each of the following angles.

- **Acute:** An angle with a positive measure less than 90 degrees (less than $\frac{\pi}{2}$ radians).
- **Obtuse:** An angle measuring more than 90 degrees but less than 180 degrees (between $\frac{\pi}{2}$ and π radians).
- **Right:** An angle measuring exactly 90 degrees (or $\frac{\pi}{2}$ radians).
- **Straight:** An angle measuring exactly 180 degrees (a straight line or π radians).
- **Oblique:** An angle measuring more than 180 degrees (more than π radians).

Naming angles by letters

How do you name an angle? Why does it even need a name? In most cases, you want to be able to distinguish a particular angle from all the others in a picture. When you look at a photo in a newspaper, you want to know the names of the different people and be able to point them out. With angles, you should feel the same way.

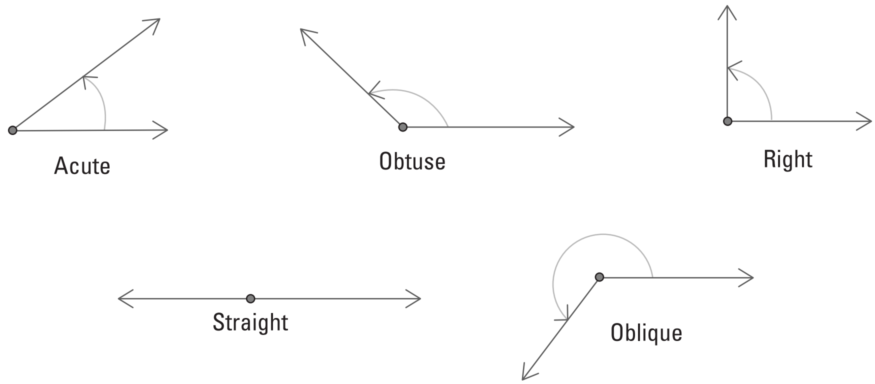


FIGURE 1-3: Types of angles — acute, obtuse, right, straight, and oblique.

You can name an angle in one of three different ways.

- **By its vertex alone:** Often, you name an angle by its vertex alone because such a label is efficient, neat, and easy to read. In Figure 1-4, you can call the angle *A*. You only use this type of name if there aren't any angles adjacent at the vertex *A*. It has to stand alone.
- **By a point on one side, followed by the vertex, and then a point on the other side:** For example, you can call the angle in Figure 1-4 angle *BAC* or angle *CAB*. This naming method is helpful if someone may be confused as to which angle you're referring to in a picture. ***Remember:*** Make sure you always name the vertex in the middle.
- **By a letter or number written inside the angle:** Usually, that letter is Greek; in Figure 1-4, however, the angle has the letter *w*. Often, you use a number for simplicity if you're not into Greek letters or if you're going to compare different angles later.

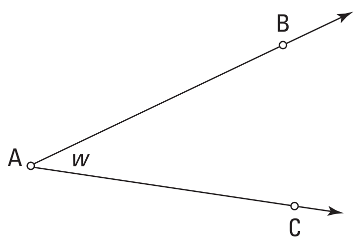


FIGURE 1-4: Naming an angle.

Taking on triangles and their angles

All on their own, angles are certainly very exciting. But put them into a triangle, and you've got icing on the cake. Triangles are one of the most frequently studied geometric figures. When angles are part of a triangle, they have many characteristics.

Angles in triangles

A triangle always has three angles. The angles in a triangle have measures that always add up to 180 degrees — no more, no less. A triangle named ABC (often written ΔABC) has angles A, B, and C, and you can name the sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, depending on which two angles the side is between. The angles themselves can be acute, obtuse, or right. If the triangle has either an obtuse or right angle, then the other two angles have to be acute.

Naming triangles by their shape

Triangles can have special names based on their angles and sides. They can also have more than one name — a triangle can be both acute and isosceles, for example. Here are their descriptions, and check out Figure 1-5 for the pictures.

- **Acute triangle:** A triangle where all three angles are acute.
- **Right triangle:** A triangle with a right angle (the other two angles must be acute).

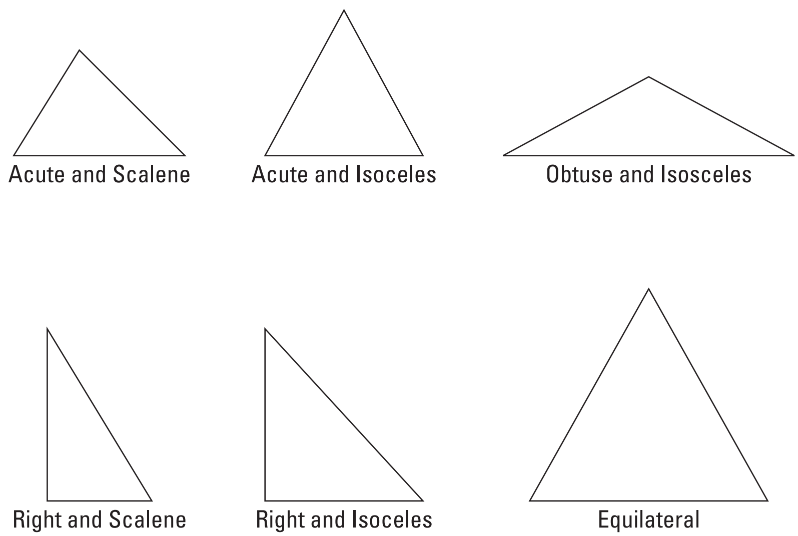


FIGURE 1-5: Triangles can have more than one name, based on their characteristics.

- **Obtuse triangle:** A triangle with an obtuse angle (the other two angles must be acute).
- **Isosceles triangle:** A triangle with two equal sides; the angles opposite those sides are equal, too. The equal angles have to be acute.
- **Equilateral triangle:** A triangle where all three side lengths are equal, and all the angles measure 60 degrees.
- **Scalene triangle:** A triangle with no angles or sides of the same measure.

Going outside the triangle

Another angle that comes up frequently in trigonometry is an exterior angle. When one side of a triangle is extended on both sides of its vertex, an exterior angle is formed. An angle and its exterior angle share the vertex, and the measure of the exterior angle is quickly deduced (as shown in Figure 1-6)! Here are some characteristics of an exterior angle:

- An exterior angle to a triangle is always supplementary to the angle it's adjacent to in the triangle. This means that the sum of the angle and its exterior angle is always 180 degrees.
- An exterior angle to a triangle has a measure that is equal to the sum of the two non-adjacent angles in the triangle.

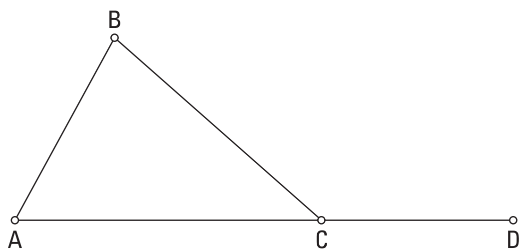


FIGURE 1-6: Angle *BCD* is adjacent to angle *BCA*.

Making a circle work from every angle

A *circle* is a geometric figure that needs only two parts to identify it and classify it: its *center* (or middle) and its *radius* (the distance from the center to any point on the circle). Technically, the center isn't a part of the circle; it's just a sort of anchor or reference point. The circle consists only of all those points that are the same distance from the center.

Radius, diameter, circumference, and area

After you've chosen a point to be the center of a circle and you know how far that point is from all the points that lie on the circle, you can draw a fairly decent picture. With the measure of the radius, you can tell a lot about the circle: its *diameter* (the distance from one side to the other, passing through the center), its *circumference* (how far around it is), and its *area* (how many square inches, feet, yards, meters — what have you — fit into it). Figure 1-7 shows these features. A *chord* of a circle is a segment that joins any two points on the circle. A chord can be a *diameter.*

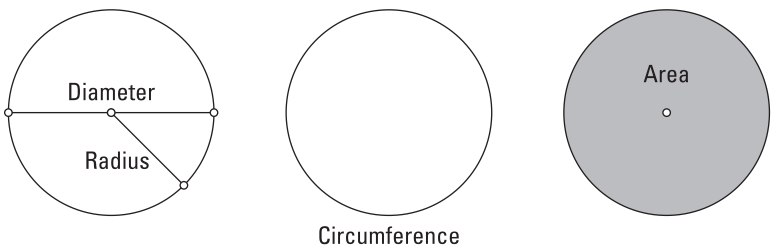


FIGURE 1-7: The different features of a circle.

Ancient mathematicians figured out that the circumference of a circle is always a little more than three times the diameter of the circle. Since then, they narrowed that "little more than three times" to a value called *pi* (pronounced "pie"), designated by the Greek letter π. The decimal value of π isn't exact; in fact, it goes on forever and ever. However, most of the time, people refer to it as being approximately 3.14 or $\frac{22}{7}$, whichever form works best in specific computations.

The formula for figuring out the circumference of a circle is tied to π and the diameter:

TRIG RULES

Circumference of a circle $C = \pi d = 2\pi r$

The *d* represents the measure of the diameter, and *r* represents the measure of the radius. The diameter is always twice the radius, so either form of the equation works.

Similarly, the formula for the area of a circle is tied to π and the radius:

TRIG RULES

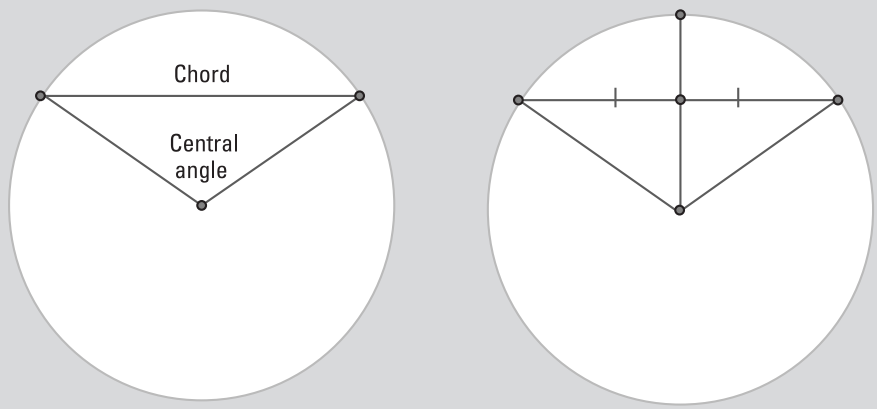
Area of a circle $A = \pi r^2$

This formula reads, "Area equals pi are squared." (And here I thought that *pies are round.*)

Example: Find the radius, circumference, and area of a circle if its diameter is equal to 10 feet in length.

DON'T GIVE ME THAT *JIVA*

The ancient Greek mathematician Ptolemy was born some time at the end of the first century. Ptolemy based his version of trigonometry on the relationships between the chords of circles and the corresponding central angles of those chords. Ptolemy came up with a theorem involving four-sided figures that you can construct with the chords. (See the section, "Finding Trig Applications in the Basics," later in this chapter.) In the meantime, mathematicians in India decided to use the measure of *half* a chord and *half* the angle to try to figure out these relationships. Drawing a radius from the center of a circle through the middle of a chord (halving it) forms a right angle, which is important in the definitions of the trig functions. These half-measures were the beginning of the sine function in trigonometry. In fact, the word *sine* actually comes from the Hindu word *jiva.*



If the diameter (d) is equal to 10, you write this value as $d=10$. The radius is half the diameter, so the radius is 5 feet, or $r=5$. You can find the circumference by using the formula $C=\pi d=\pi\cdot 10\approx 3.14\cdot 10\approx 31.4$. So, the circumference is about $31\frac{1}{2}$ feet around. You find the area by using the formula $A=\pi r^2=\pi\cdot 5^2=\pi\cdot 25\approx 3.14\cdot 25\approx 78.5$, so the area is about $78\frac{1}{2}$ square feet.

Chord versus tangent

You show the diameter and radius of a circle by drawing segments from a point on the circle either to or through the center of the circle. But two other straight figures have a place on a circle. One of these figures is called a chord, and the other is a tangent.

- A *chord* of a circle is a segment that you draw from one point on the circle to another point on the circle (see Figure 1-8). A chord always stays inside the circle. The largest chord possible is the diameter — you can't get any longer than that segment.
- A *tangent* to a circle is a line, ray, or segment that touches the outside of the circle at exactly one point, as shown in Figure 1-8. It never crosses into the circle. A tangent can't be a chord, because a chord touches a circle in two points, crossing through the inside of the circle. Any radius drawn to a tangent is perpendicular to that tangent.

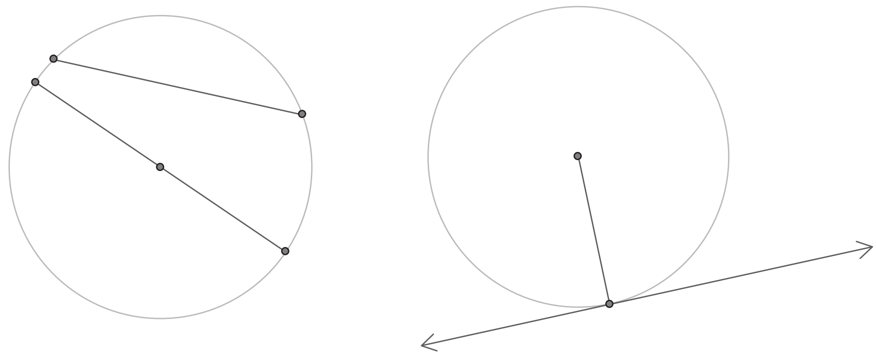


FIGURE 1-8: Chords and a tangent of a circle.

Looking at angles in a circle

There are several ways of drawing an angle in a circle, and each has a special way of computing the size of that angle. The four different types of angles are central, inscribed, interior, and exterior. In Figure 1-9, you see examples of these different types of angles.

Central angle

A *central angle* has its vertex at the center of the circle, and the sides of the angle are two radii of the circle. The measure of the central angle is the same as the measure of the arc that the two sides cut out of the circle.

Inscribed angle

An *inscribed angle* has its vertex on the circle, and the sides of the angle are two chords of the circle. The measure of the inscribed angle is half that of the arc that the two sides cut out of the circle.

Example: Find the measure of the arc formed by the two rays of an inscribed angle if the measure of the angle is 60 degrees (see Figure 1-9).

If the measure of the inscribed angle is 60 degrees, then the arc is twice that measure, or 120 degrees.

Interior angle

An *interior angle* has its vertex at the intersection of two lines that intersect inside a circle or of two chords of the circle. The sides of the angle lie on the intersecting lines. The measure of an interior angle is the average of the measures of the two arcs that are cut out of the circle by those intersecting lines.

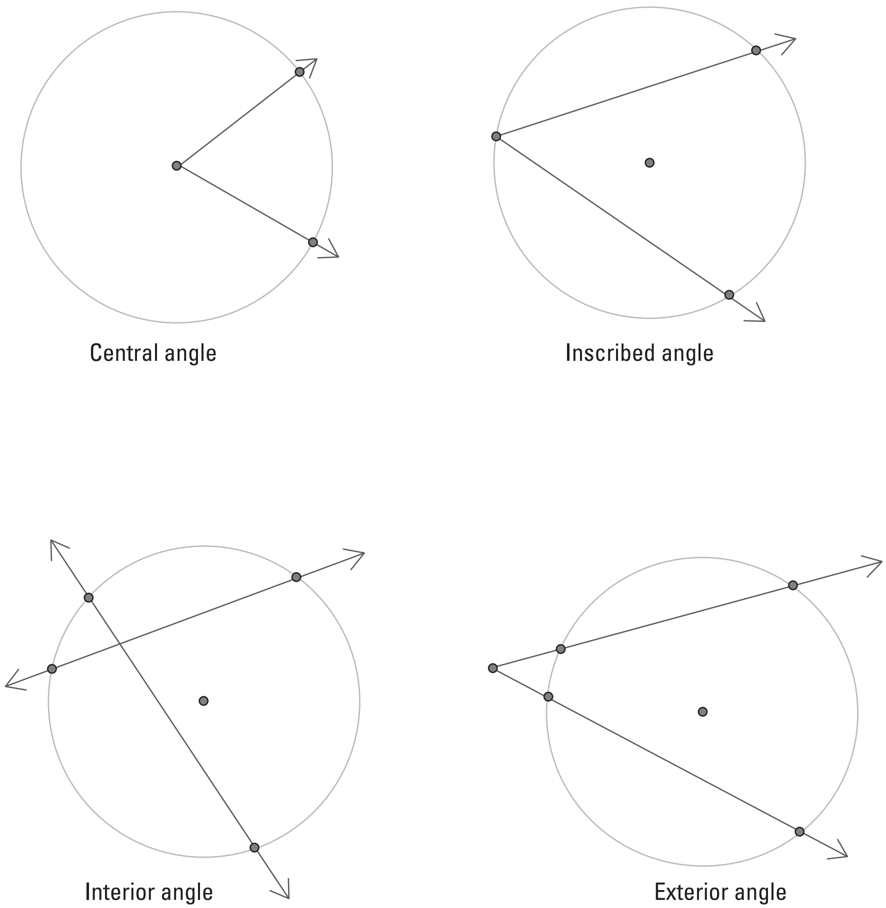


FIGURE 1-9: Measuring angles in a circle.

Exterior angle

An *exterior angle* has its vertex where two rays share an endpoint outside a circle. The sides of the angle are those two rays. The measure of an exterior angle is found by dividing the difference between the measures of the intercepted arcs by two.

Example: Find the measure of angle *EXT*, given that the exterior angle cuts off arcs of 20 degrees and 108 degrees (see Figure 1-10).

Find the difference between the measures of the two intercepted arcs and divide by 2:

$$\frac{108-20}{2}=\frac{88}{2}=44$$

The measure of angle *EXT* is 44 degrees.

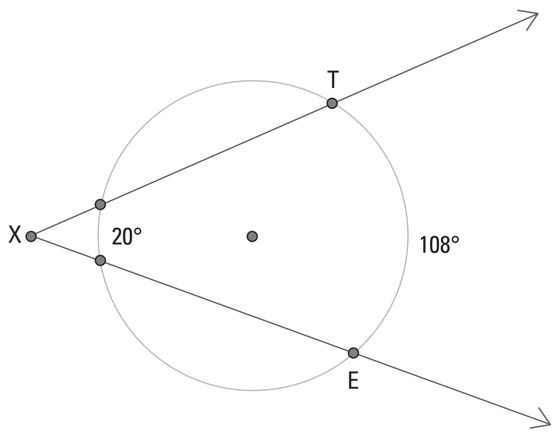


FIGURE 1-10: Calculating the measure of an exterior angle.

Sectioning sectors

A *sector* of a circle is a section of the circle between two *radii* (plural for radius). You can consider this part like a piece of pie cut from a circular pie plate (see Figure 1-11).

You can find the area of a sector of a circle if you know the angle between the two radii. A circle has a total of 360 degrees all the way around the center, so if that central angle determining a sector has an angle measure of 60 degrees, then the sector takes up $\frac{60}{360}$, or $\frac{1}{6}$, of the degrees all the way around. In that case, the sector has $\frac{1}{6}$ the area of the whole circle.

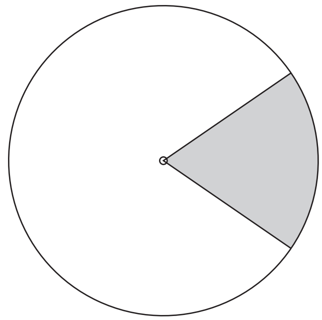


FIGURE 1-11: A sector of a circle.

Example: Find the area of a sector of a circle if the angle between the two radii forming the sector is 80 degrees and the diameter of the circle is 9 inches.

1. **Find the area of the circle.**

   The area of the whole circle is $A = \pi r^2 = \pi \cdot (4.5)^2 \approx 3.14(20.25) \approx 63.585$, or about $63\frac{1}{2}$ square inches.

2. **Find the portion of the circle that the sector represents.**

   The sector takes up only 80 degrees of the circle. Divide 80 by 360 to get $\frac{80}{360} = \frac{2}{9} \approx 0.222$.

3. **Calculate the area of the sector.**

   Multiply the fraction or decimal from Step 2 by the total area to get the area of the sector: $0.222(63.585) \approx 14.116$. The whole circle has an area of almost 64 square inches, and the sector has an area of just over 14 square inches.

Understanding Trig Speak

Any math or science topic has its own unique vocabulary. Some very nice everyday words have new and special meanings when used in the context of that subject. Trigonometry is no exception.

Making the words fit the triangle

Every triangle has six measurable parts: three sides and three angles. If you measure the sides and then pair up those measurements (taking two at a time), you have three different pairings. Do division problems with the pairings — changing the order in each pair — and you have six different answers. These six different answers represent the six trig functions. For example, if your triangle has sides measuring 3, 4, and 5, then the six divisions are $\frac{3}{4}$, $\frac{4}{3}$, $\frac{3}{5}$, $\frac{5}{3}$, $\frac{4}{5}$, and $\frac{5}{4}$.

In Chapter 6, you find out how all these fractions work in the world of trig functions by using the different sides of a right triangle. And then, in Chapter 7, you take a whole different approach as you discover how to define the trig functions with a circle.

The six trig functions are named *sine, cosine, tangent, cotangent, secant,* and *cosecant.* Many people confuse the spoken word *sine* with *sign* — you can't really tell the difference when you hear it unless you're careful with the context. You can "go off on a tangent" in some personal dealings, but that phrase has a whole different meaning in trig. Cosigning a loan isn't what trig has in mind, either. The other three ratios are special to trig speak — you can't confuse them with anything else.

Interpreting trig abbreviations

REMEMBER

Even though the word *sine* isn't all that long, you have a three-letter abbreviation for this trig function and all the others. Mathematicians find using abbreviations easier, and those versions fit better on calculator keys. The functions and their abbreviations are

sine: sin	**cosine:** cos
tangent: tan	**cotangent:** cot
secant: sec	**cosecant:** csc

As you can see, the first three letters in the full name make up the abbreviations, except for cosecant's.

Noting notation

Angles are the main focus in trigonometry, and you can work with them even if you don't know their measure. Many angles and their angle measures have general rules that apply to them. You can name angles by one letter, three letters, or a number, but to do trig problems and computations, mathematicians commonly refer to the angle names and their measures with Greek letters.

The most commonly used letters for angle measures are α (alpha), β (beta), γ (gamma), and θ (theta). Also, many equations use the variable x to represent an angle measure.

TECHNICAL STUFF

Algebra has conventional notation involving superscripts, such as the 2 in x^2. In trigonometry, superscripts have most of the same rules and characteristics as in other mathematics. But trig superscripts often look very different. Table 1-1 presents a listing of some of the ways that trig uses superscripts.

TABLE 1-1 **How You Use Superscripts in Trig**

How to Write in Trig Notation	Alternate Notation	What the Superscript Means
$\sin^2\theta$	$(\sin\theta)^2$	Square the sine of the angle theta.
$(\sin\theta)^{-1}$	$\frac{1}{\sin\theta}$	Find the reciprocal of the sine of theta.
$\sin^{-1}\theta$	$\arcsin\theta$	Find the angle theta given its sine.

The first entry in Table 1-1 shows how you can save having to write parentheses every time you want to raise a trig function to a power. This notation is neat and efficient, but it can be confusing if you don't know the "code." The second entry shows you how to write the reciprocal of a trig function. It means you should take the value of the function and divide it into the number 1. The last entry in Table 1-1 shows how you write the *inverse sine* function. Using the -1 superscript between sine and the angle means that you're talking about inverse sine (or *arcsin*), not the reciprocal of the function. In Chapter 14, I cover the inverse trig functions in great detail, making this business about the notation for an inverse trig function even more clear.

Functioning with angles

The functions in algebra use many operations and symbols that are different from the common add, subtract, multiply, and divide signs in arithmetic. For example, take a look at the square-root operation, $\sqrt{25}=5$. Putting 25 under the *radical* (square-root symbol) produces an answer of 5. Other operations in algebra, such as absolute value, factorial, and step-function, are used in trigonometry, too. But the world of trig expands the horizon, introducing even more exciting processes.

When working with trig functions, you have a whole new set of values to learn or find. For instance, putting 25 into the sine function looks like this: sin 25. The answer that pops out is either 0.423 or -0.132, depending on whether you're using degrees or radians (for more on those two important trig concepts, head on over to Chapters 3 and 4). You can't usually determine or memorize all the values that you get by putting angle measures into trig functions. Fortunately, you can use scientific calculators or computers or even cell phones to study trigonometry.

In general, when you apply a trig function to an angle measure, you get some real number (if that angle is in its domain). Some angles and trig functions have nice values, but most don't. Table 1-2 shows the trig functions for a 30-degree angle.

TABLE 1-2 The Trig Functions for a 30-Degree Angle

Trig Function	Exact Value	Value Rounded to Three Decimal Places
sin 30°	$\frac{1}{2}$	0.500
cos 30°	$\frac{\sqrt{3}}{2}$	0.866
tan 30°	$\frac{\sqrt{3}}{3}$	0.577
cot 30°	$\sqrt{3}$	1.732
sec 30°	$\frac{2\sqrt{3}}{3}$	1.155
csc 30°	2	2.000

Some characteristics that the entries in Table 1-2 confirm are that the sine and cosine functions always have values that are between and include –1 and 1. Also, the secant and cosecant functions always have values that are equal to or greater than 1 or equal to or less than –1. (I discuss these properties in more detail in Chapter 6.) And you'll find a complete listing of the values of the most commonly used functions in the Appendix.

How about trying this out? Do you have your cell phone handy? Or, just go to a calculator and type in **sin (45)**. Did you get 0.7071 (or maybe a few more digits)? That's the sine of 45 degrees! You'll better understand how this wonderful relationship works in later chapters.

Making triangles less radical

A *radical* is a mathematical symbol that means, "Find the number that multiplies itself by itself one or more times to give you the number under the radical." You can see why you use a symbol such as $\sqrt{\ }$ rather than all those words. Radicals represent values of functions that are used a lot in trigonometry. In Chapter 6, I define the trig functions by using a right triangle. To solve for the lengths of a right triangle's sides by using the Pythagorean Theorem, you have to compute some square roots, which use radicals. Some basic answers to radical expressions are $\sqrt{16}=4$, $\sqrt{121}=11$, $\sqrt[3]{8}=2$, and $\sqrt[4]{81}=3$.

These examples are all *perfect squares, perfect cubes,* or *perfect fourth roots,* which means that the answer is a number that ends — the decimal doesn't go on forever. The following section discusses a way to simplify radicals that aren't perfect roots.

Simplifying radical forms

Simplifying a radical form means to rewrite it with a smaller number under the radical — if possible. You can simplify this form only if the number under the radical has a perfect square or perfect cube (or perfect whatever factor) that you can factor out.

Example: Simplify $\sqrt{80}$.

The number 80 isn't a perfect square, but one of its factors, 16, is. You can write the number 80 as the product of 16 and 5, write the two radicals separately, and then evaluate each radical. The resulting product is the simplified form:

$$\sqrt{80}=\sqrt{16\cdot 5}=\sqrt{16}\sqrt{5}=4\sqrt{5}$$

Example: Simplify $\sqrt[3]{250}$.

The number 250 isn't a perfect cube, but one of its factors, 125, is. Write 250 as the product of 125 and 2; separate, evaluate, and write the simplified product: $\sqrt[3]{250}=\sqrt[3]{125\cdot 2}=\sqrt[3]{125}\sqrt[3]{2}=5\sqrt[3]{2}$.

Approximating answers

As wonderful as a simplified radical is, and as useful as it is when you're doing further computations in math, sometimes you just need to know approximately how much the value's worth.

Approximating an answer means to shorten the actual value in terms of the number of decimal places. You may find approximating especially useful when the decimal value of a number goes on forever without ending or repeating. When you approximate an answer, you *round* it to a certain number of decimal places, letting the rest of the decimal values drop off. Before doing that, though, you need to consider how big a value you're dropping off. If the numbers that you're dropping off start with a 5 or greater, then bump up the last digit that you leave on by 1. If what you're dropping off begins with a 4 or less, then just leave the last remaining digit alone.

THEY CALLED THIS SIMPLER?

Some ancient mathematicians didn't like to write fractions unless they had a numerator of 1. They only liked the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, and so on. So what did they do when they needed to write the fraction $\frac{5}{6}$? They wrote $\frac{1}{2}+\frac{1}{3}$ instead (because $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$). What a pain to have to write $\frac{1}{2}+\frac{1}{4}+\frac{1}{10}$ rather than $\frac{17}{20}$. Or maybe you prefer this approach, too?

Example: Round the number 3.141592654 to four decimal places, three decimal places, and two decimal places.

- **Four decimal places.** Look at the first five decimal places: 3.141<u>5</u>9. The 5 in the fourth place either stays as is or gets rounded up to the next digit. Because the next digit is 9, and 9 is greater than 5, you round the 5 up to a 6. Rounded to four places, 3.141592654 rounds to 3.1416.
- **Three decimal places.** Look at the first four decimal places: 3.14<u>1</u>5. The 1 in the third place either stays as is or gets rounded up to the next digit. Because the next digit is 5, and anything 5 or greater gives a "bump up," you round the 1 to a 2. Rounded to three places, 3.141592654 rounds to 3.142.
- **Two decimal places.** Look at the first three decimal places: 3.1<u>4</u>1. The 4 in the second place either stays as is or gets rounded up to the next digit. Because the next digit is 1, and 1 is smaller than 5, you leave the 4 as is. Rounded to two places, 3.141592654 rounds to 3.14.

Use this technique when approximating radical values. Using a calculator, the decimal value of $\sqrt{80}$ is about 8.94427191. Depending on what you're using this value for, you may want to round it to two, three, four, or more decimal places. Rounded to three decimal places, this number is 8.944.

Equating and Identifying

Trigonometry has the answers to so many questions in engineering, navigation, and medicine. The ancient astronomers, engineers, farmers, and sailors didn't have the current systems of symbolic algebra and trigonometry to solve their problems, but they did well and set the scene for later mathematical developments. People today benefit big-time by having ways to solve equations in trigonometry that are quick and efficient; trig now includes special techniques and identities that you can play with — all thanks to the mathematicians of old who created the systems that we use today.

The methods that you use for solving equations in algebra take a completely different turn from usual solutions when you use trig *identities* (in short, equivalences that you can substitute into equations in order to simplify them). To make matters easier (or, some say, to *complicate* them), the different trig functions can be written many different ways. They almost have split personalities. When you're solving trig equations and trig identities, you're sort of like a detective working your way through to substitute, simplify, and solve. What answers should you expect when solving the equations? Why, angles, of course!

Take, for example, one trigonometric equation: $\sin\theta + \cos^2\theta = 1$.

The point of the problem is to figure out what angle or angles should replace the θ to make the equation true. In this case, if θ is 0 degrees, 90 degrees, or 180 degrees, the equation is true.

If you replace θ with 0 degrees in the equation, you get

$$\begin{aligned} \sin 0° + (\cos 0°)^2 &= 1 \\ 0 + (1)^2 &= 1 \\ 1 &= 1 \end{aligned}$$

If you replace θ with 90 degrees in the equation, you get

$$\begin{aligned} \sin 90° + (\cos 90°)^2 &= 1 \\ 1 + (0)^2 &= 1 \\ 1 &= 1 \end{aligned}$$

Something similar happens with 180 degrees and all the other angle measures that work in this equation (there are an infinite number of solutions). But remember that not just any angle will work here. I carefully chose the angles that are *solutions,* which are the angles that make the equation true. In order to solve trig equations like this one, you have to use inverse trig functions, trig identities, and various algebraic techniques. You can find all the details on how to use these processes in Chapters 10 through 15. And when you've got those parts figured out, dive into Chapter 16, where the equation-solving comes in.

In this particular case, you need to use an *identity* to solve the equation for all its answers. You replace the $\cos^2\theta$ with $1 - \sin^2\theta$ so that all the terms have a sine in them — or just a number. You actually have several other choices for changing the identity of $\cos^2\theta$. I chose $1 - \sin^2\theta$, but some other choices include $\frac{1}{\sec^2\theta}$ and $\frac{1 + \cos 2\theta}{2}$. You can discover how to actually solve equations like this one in Chapter 16.

This example just shows you that the identity of the trig functions can change an expression significantly — but according to some very strict rules.

Finding Trig Applications in the Basics

The basics of trigonometry helped the ancients with everyday problems, and they're here now to give you some insights and help with occasional challenges.

Measuring fencing

You have a circular corral that you've divided into four equal portions so you can keep your cow, pony, pig, and rooster separate from one another, as shown in Figure 1-12.

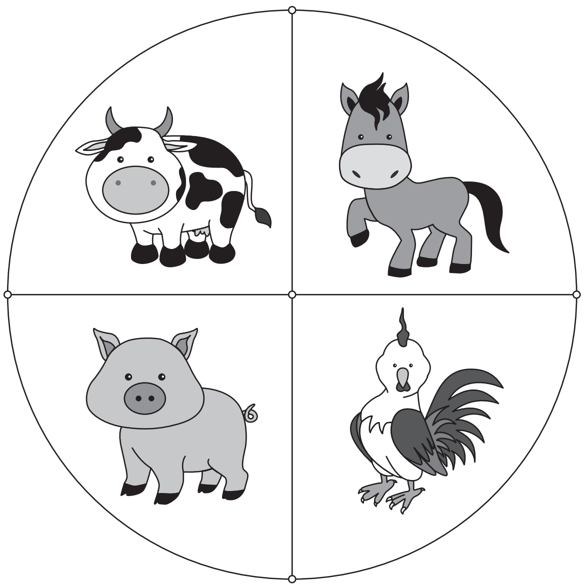


FIGURE 1-12: Dividing the circle into four equal parts.

You need some fencing to keep the animals in and apart. You know that the corral has a radius of exactly 40 feet. What is the total number of feet needed to create the corral?

Because the radius of the corral is 40 feet, and you need four of these lengths for the crisscross portion of the fencing, you need a total of 160 feet. To find the circular portion, you need the circumference of the circle. You can use either the radius or diameter in the formula $C = \pi d = 2\pi r$ to find that circumference: $C = 2\pi r = 2\pi(40) = 80\pi \approx 251.33$ feet. Add this to the crisscross portion and you have $251.33 + 160 = 411.33$ total feet of fencing.

Ptolemy's Theorem

Earlier in this chapter, I made reference to the Greek mathematician Ptolemy and his work with circles and chords. Ptolemy's Theorem says that, "When you have a quadrilateral (four-sided figure) circumscribed by a circle, the product of the measures of the two diagonals in that figure is equal to the sum of the products of the opposite sides of the figure." Take a look at Figure 1-13, where *AD* and *BC* are the diagonals, and *AB* and *CD* are one pair of opposite sides and *AC* and *BD* are the other pair.

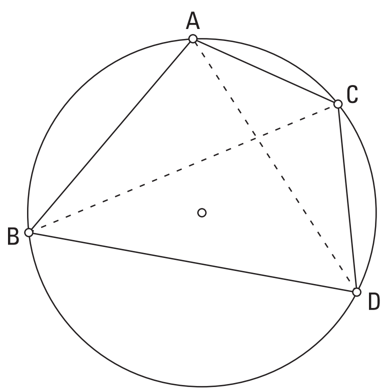


FIGURE 1-13: The product of the diagonals equals the sum of the products of the opposite sides.

You are able to measure each segment in the figure except *BD*. Given the other measures, determine that missing measure.

Here are the measures you know: *AC* = 5, *AB* = 8, *CD* = 6, *AD* = 10, and *BC* = 10.5.

So you know the product of the diagonals: $10 \times 10.5 = 105$.

The product of the diagonals is equal to the sum of the products of the opposite sides, so: $105 = (AB)(CD) + (AD)(BC) = (8)(6) + (10)(BC) = 48 + (10)(BC)$.

Solving the equation for BC:

$$\begin{aligned}105 &= 48 + (10)(BC)\\ 57 &= 10(BC)\\ \frac{57}{10} &= BC\end{aligned}$$

So BC is 5.7 units in length.

Dealing with radicals

One of my favorite special right triangles is the triangle with acute angles measuring 30 degrees and 60 degrees. And a great feature of this special triangle is that there's a wonderful relationship between the lengths of the sides of the triangle. In Figure 1-14, you see that the hypotenuse of the right triangle is always twice as long as the shortest side, and the middle-length side is always $\sqrt{3}$ or about 1.7 times as long as the shortest side.

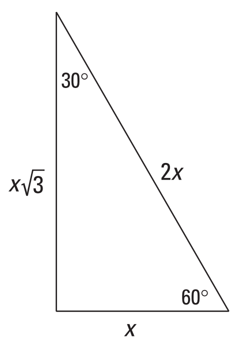


FIGURE 1-14: The special 30-60-90 degree right triangle.

Example: You know that the hypotenuse of one of these triangles measures 20 inches. How long are the other two sides? (Give the radical answer correct to 3 decimal places.)

The shortest side is half the hypotenuse, so it measures 10 inches.

The middle-length side (opposite the 60-degree angle) is 10 times $\sqrt{3}$. Using your calculator, you have $10\sqrt{3} \approx 10(1.732050808) = 17.32050808$. Rounded to three places, that's 17.321 inches.

Skateboarding

You're going to build a skateboard ramp that has an angle of 130 degrees as you approach and an angle of 130 degrees as you descend. This requires an isosceles triangle, as shown in Figure 1-15. What is the measure of the top angle, where you switch from going up to going down?

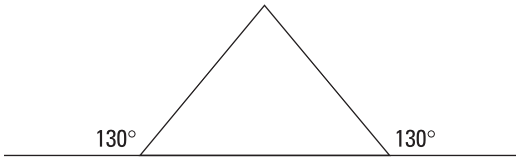


FIGURE 1-15: Skateboarding over a triangle.

The 130-degree angle in the approach is an exterior angle of the triangle. Subtract 130 from 180, and you find that the interior angle in the triangle is 50 degrees.

The other exterior angle is also 130 degrees, so you have another angle in the triangle of 50 degrees. Add those two interior angles together, and you have 100 degrees, leaving 80 degrees for the top angle measure. Seems a bit steep to me!!!!