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Preface and Overview

Chapter 1: Preface and Overview

Mathematical modeling forms the backbone of scientific and engineering disciplines, enabling researchers to understand and solve complex real-world problems. These models provide a simplified representation of intricate systems, facilitating analysis and the development of practical solutions. However, balancing model simplicity and accuracy has always been challenging. While overly simplified models may lack the necessary precision, highly complex models often lead to equally complex problem-solving processes.

In recent years, advancements in mathematical modeling have made it possible to address these challenges through interval analysis. Interval analysis is a powerful tool that considers uncertainties within mathematical models, providing a more realistic and accurate representation of real-world systems. By expressing variables as intervals rather than single values, it becomes possible to account for uncertainties and analyze how they affect the overall system behavior.

This book explores the application of interval analysis in solving problems with interval uncertainties. It seeks to bridge the gap between overly simplified and overly complex models by providing a robust and practical approach to addressing uncertainties. By employing interval analysis techniques, researchers and engineers can obtain more reliable results and gain deeper insights into the behavior of complex systems.

The book begins with an introduction to mathematical modeling and the challenges of simplifying and understanding complex systems. It highlights the compromises made to balance model simplicity and accuracy. The subsequent chapters delve into the fundamentals of interval analysis, presenting topics such as the algebra of interval sets, interval representations, interval functions, and techniques for solving linear systems with interval parameters.

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Building upon this foundation, the book explores stability and controllability analysis based on interval analysis. It discusses techniques for testing stability, including the Routh–Hurwitz stability test and interval stability tests using linear matrix inequalities. Moreover, the book addresses controllability and observability concepts, shedding light on the essential properties of dynamic systems.

The book also covers the application of interval analysis in optimal control problems. It presents indirect and direct methods for solving optimal control problems and examines how these methods can be used to analyze and solve problems affected by interval uncertainties. Quadratic optimal control problems with interval uncertainties are also discussed, along with practical simulations to demonstrate the implementation of these approaches. What follows is a brief explanation of the chapters of this book.

Chapter 2: Introduction

This chapter delves into the fundamental concepts underlying mathematical modeling and its importance in analyzing real-world issues. It discusses how assumptions are often employed to simplify mathematical models, providing an overview of the compromises between model simplicity and precision. Additionally, it explores recent developments that have improved the accuracy and efficiency of these models, leading to more robust solutions for applied problems. However, it also highlights the potential limitations that arise as a result.

Chapter 3: Literature Review

Building upon the foundation laid in Chapter 2, this chapter focuses on reviewing the existing literature surrounding control systems. It emphasizes adjusting and controlling internal state variables to enhance system performance. The chapter also provides insights into classical control systems and their aim to improve specific functional characteristics such as transient behavior, settling time, and overshoot. Furthermore, it explores advanced studies wherein the optimal behavior of a system is achieved through the minimization or maximization of performance indices.

Chapter 4: Introduction to Interval Analysis and Solving the Problems with Interval Uncertainties

Chapter 4 introduces interval analysis and its role in handling problems affected by interval uncertainties. The chapter covers various topics, including the algebra of interval sets, interval representations, and interval functions; solving linear systems with interval parameters, interval derivatives, and integrals; and determining minimal intervals. Moreover, it explores advanced techniques such as the centered inclusion method, the Interval Runge–Kutta Method (IRKM) for interval differential equations, and interval uncertainty analysis based on orthogonal functions.

Chapter 5: Stability and Controllability Based on Interval Analysis

This chapter focuses on stability and controllability analysis using interval analysis. It explains how interval stability and controllability provide valuable insights into system behavior. The chapter investigates stability tests, such as the Routh–Hurwitz stability test and the interval Routh–Hurwitz stability test (Kharitonov Method). It also delves into interval stability based on linear matrix inequalities (LMIs). Furthermore, the concepts of controllability and observability are explored, shedding light on the essential properties of dynamic systems.

Chapter 6: Optimal Control of the Systems with Interval Uncertainties

In Chapter 6, the book focuses on optimal control of systems affected by interval uncertainties. It presents both indirect and direct methods for solving optimal control problems. The chapter examines how these methods can be employed to analyze and solve problems through techniques such as Euler–Lagrange equations, the interval Runge–Kutta method, the Chebyshev inclusion method, and the piecewise interval Chebyshev method (PICM). Quadratic optimal control problems with interval uncertainties and the interval quadratic regulator are also addressed based on indirect methods. Application-oriented simulations are included to illustrate the practical implementation of these approaches.

Chapter 7: Conclusions

The final chapter of the book serves as a summary and conclusion. It highlights the key findings, contributions, and limitations discussed in the preceding chapters. Furthermore, it emphasizes the importance of interval analysis in handling problems affected by uncertainties. The chapter concludes by suggesting potential areas of future research and the potential applications of the methods presented throughout the book.

In conclusion, this book comprehensively explores interval analysis and its application in solving problems affected by interval uncertainties. By striking a balance between model simplicity and accuracy, interval analysis offers robust solutions for addressing real-world complexities. This book will be valuable for researchers, scientists, and engineers seeking effective problem-solving techniques in diverse fields.