

IN THIS CHAPTER

- » Refreshing your memory on numbers and variables
- » Accepting the importance of graphing
- » Preparing for pre-calculus by understanding the vocabulary

Chapter 1

Preparing for Pre-Calculus

Pre-calculus is the bridge (drawbridge, suspension bridge, covered bridge) between Algebra II and calculus. In its scope, you review concepts you've seen before in math, but then you quickly build on them. You see some brand-new ideas, and even those build on the material you've seen before; the main difference is that the problems now get even more interesting and challenging (for example, going from linear systems to nonlinear systems). You keep on building until the end of the bridge span, which doubles as the beginning of calculus. Have no fear! What you find here will help you cross the bridge (toll free).

Because you've probably already taken Algebra I, Algebra II, and geometry, it's assumed throughout this book that you already know how to do certain things. Just to make sure, though, I address some particular items in this chapter in a little more detail before moving on to the material that is pre-calculus.

If there is any topic in this chapter that you're not familiar with, don't remember how to do, or don't feel comfortable doing, I suggest that you pick up another *For Dummies* math book and start there. If you need to do this, don't feel like a failure in math. Even pros have to look up things from time to time. Use these books like you use encyclopedias or the Internet — if you don't know the material, just look it up and get going from there.

Recapping Pre-Calculus: An Overview

Don't you just love movie previews and trailers? Some people show up early to movies just to see what's coming out in the future. Well, consider this section a trailer that you see a couple months before the *Pre-Calculus For Dummies* movie comes out! The following list presents some items you've learned before in math, and some examples of where pre-calculus will take you next.

» **Algebra I and II:** Dealing with real numbers and solving equations and inequalities.

Pre-calculus: Expressing inequalities in a new way called *interval notation*.

You may have seen solutions to inequalities in set notation, such as $\{x | x > 4\}$. This is read in inequality notation as $x > 4$. In pre-calculus, you often express this solution as an interval: $(4, \infty)$. (For more, see Chapter 2.)

» **Geometry:** Solving right triangles, whose sides are all positive.

Pre-calculus: Solving non-right triangles, whose sides aren't always represented by positive numbers.

You've learned that a length can never be negative. Well, in pre-calculus you sometimes use negative numbers for the lengths of the sides of triangles. This is to show where these triangles lie in the coordinate plane (they can be in any of the four quadrants).

» **Geometry/trigonometry:** Using the Pythagorean Theorem to find the lengths of a triangle's sides.

Pre-calculus: Organizing some frequently used angles and their trig function values into one nice, neat package known as the *unit circle* (see Unit 3).

In this book, you discover a handy shortcut to finding the sides of triangles — a shortcut that is even handier for finding the trig values for the angles in those triangles.

» **Algebra I and II:** Graphing equations on a coordinate plane.

Pre-calculus: Graphing in a brand-new way with the polar coordinate system (see Chapter 16).

Say goodbye to the good old days of graphing on the Cartesian coordinate plane. You have a new way to graph, and it involves goin' round in circles. I'm not trying to make you dizzy; actually, polar coordinates can make you some pretty pictures.

» **Algebra II:** Dealing with imaginary numbers.

Pre-calculus: Adding, subtracting, multiplying, and dividing complex numbers gets boring when the complex numbers are in rectangular form $(a + bi)$. In pre-calculus, you become familiar with something new called *polar form* and use that to find solutions to equations you didn't even know existed.

Checking in on Number Basics and Processes

When entering pre-calculus, you should be comfy with sets of numbers (natural, integer, rational, and so on). By this point in your math career, you should also know how to perform operations with numbers. You can find a quick review of these concepts in this section. Also, certain

properties hold true for all sets of numbers, and it's helpful to know them by name. I review them in this section, too.

Understanding the multitude of number types: Terms to know

Mathematicians love to name things simply because they can; it makes them feel special. In this spirit, mathematicians attach names to many sets of numbers to set them apart and cement their places in math students' heads for all time.



TIP

» **The set of natural or counting numbers: $\{1, 2, 3 \dots\}$.** Notice that the set of natural numbers doesn't include 0.

» **The set of whole numbers: $\{0, 1, 2, 3 \dots\}$.** The set of whole numbers consists of all the natural numbers plus the number 0.

» **The set of integers: $\{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$.** The set of integers includes all positive and negative natural numbers and 0.

Dealing with integers is like dealing with money: Think of positives as having it and negatives as owing it. This becomes important when operating on numbers (see the next section).

» **The set of rational numbers:** The numbers that can be expressed as a fraction where the numerator and the denominator are both integers. The word *rational* comes from the idea of a ratio (fraction or division) of two integers.

Examples of rational numbers include (but in no way are limited to) $\frac{1}{5}$, $-\frac{7}{2}$, and 0.23. A rational number is any number in the form $\frac{p}{q}$ where p and q are integers, but q is never 0. If you look at any rational number in decimal form, you notice that the decimal either stops or repeats.

» **The set of irrational numbers:** All numbers that can't be expressed as fractions. Examples of irrational numbers include $\sqrt{2}$, $\sqrt[3]{4}$, and π .

The decimal value of an irrational number never ends and never repeats.

» **The set of all real numbers:** All the sets of numbers previously discussed. For an example of a real number, think of a number $\dots$ any number. Whatever it is, it's real. Any number from the previous bullets works as an example. The numbers that aren't real numbers are imaginary.

Like telemarketers and pop-up ads on the web, real numbers are everywhere; you can't get away from them — not even in pre-calculus. Why? Because they include all numbers except the following.

- **A fraction with a zero as the denominator:** Such numbers don't exist and are called *undefined*.
- **The square root of a negative number:** These numbers are part of *complex numbers*; the negative root is the *imaginary* part (see Chapter 15). And this extends to any even root of a negative number.
- **Infinity:** Infinity is a concept, not an actual number. It describes a behavior.

» **The set of imaginary numbers: square roots of negative numbers.** Imaginary numbers have an imaginary unit, like i , $4i$, and $-2i$. Imaginary numbers used to be considered to be made-up numbers, but mathematicians soon realized that these numbers pop up in the real world. They are still called imaginary because they're square roots of negative numbers, but they are a part of the language of mathematics. The imaginary unit is defined as $i = \sqrt{-1}$. (For more on these numbers, head to Chapter 15.)

» **The set of complex numbers: the sum or difference of a real number and an imaginary number.** Complex numbers include these examples: $3 + 2i$, $2 - \sqrt{2}i$, and $4 - \frac{2}{3}i$. However, they also cover all the previous lists, including the real numbers (3 is the same thing as $3 + 0i$) and imaginary numbers ($2i$ is the same thing as $0 + 2i$).



REMEMBER

The set of complex numbers is the most complete set of numbers in the math vocabulary because it includes real numbers (any number you can possibly think of), imaginary numbers (i), and any combination of the two.



YOUR
TURN

In Questions 1 to 4, determine the different ways you can refer to each number. Your choices are: natural, whole, integer, rational, irrational, and real. Select all that apply.

1 -14	2 $\sqrt{15}$
3 $\frac{49}{9}$	4 $\sqrt{-16}$

Looking at the fundamental operations you can perform on numbers

From positives and negatives to fractions, decimals, and square roots, you should know how to perform all the basic operations on all real numbers. These operations include adding, subtracting, multiplying, dividing, taking powers of, and taking roots of numbers. The *order of operations* is the way in which you perform these operations.



TIP

The mnemonic device used most frequently to remember the order is PEMDAS, which stands for

1. **P**arentheses (and other grouping devices such as brackets and division lines)
2. **E**xponents (and roots, which can be written as exponents)
3. **M**ultiplication and **D**ivision (done in order from left to right)
4. **A**ddition and **S**ubtraction (done in order from left to right)



REMEMBER

One type of operation that is often overlooked or forgotten about is absolute value. *Absolute value* gives you the distance from zero on the number line. Absolute value should be included with the parentheses step because you have to consider what's inside the absolute-value bars first (because the bars are a grouping device). Don't forget that absolute value is always positive or zero. Hey, even if you're walking backward, you're still walking!



EXAMPLE

Q. Simplify the expression using the order of operations: $\frac{-3(x+4)^2 + 3x}{\sqrt{27}}$.

A. $\frac{-\sqrt{3}(x^2 + 7x + 16)}{3}$. First, simplify the numerator. Do this by squaring the binomial, multiplying through by -3 , and then combining like terms.

$$\frac{-3(x+4)^2 + 3x}{\sqrt{27}} = \frac{-3(x^2 + 8x + 16) + 3x}{\sqrt{27}} = \frac{-3x^2 - 24x - 48 + 3x}{\sqrt{27}} = \frac{-3x^2 - 21x - 48}{\sqrt{27}}$$

Next, factor the -3 from the terms in the numerator and simplify the denominator.

$$\frac{-3x^2 - 21x - 48}{\sqrt{27}} = \frac{-3(x^2 + 7x + 16)}{\sqrt{9 \cdot 3}} = \frac{-3(x^2 + 7x + 16)}{\sqrt{9}\sqrt{3}} = \frac{-3(x^2 + 7x + 16)}{3\sqrt{3}}$$

Divide the numerator and denominator by 3. And, finally, rationalize the denominator.

$$\frac{-3(x^2 + 7x + 16)}{3\sqrt{3}} = \frac{-(x^2 + 7x + 16)}{\sqrt{3}} = \frac{-(x^2 + 7x + 16)}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}(x^2 + 7x + 16)}{3}$$

YOUR
TURN

In Questions 5 and 6, simplify each expression using the *order of operations*.

5 $3x(x-1)^2 + x$

6 $\frac{-6 \pm \sqrt{(-6)^2 - 4(-2)(8)}}{2(-2)}$

Knowing the properties of numbers:

Truths to remember

Remembering the properties of numbers is important because you use them consistently in pre-calculus. You may not often use these properties by name in pre-calculus, but you do need to know when to use them. The following list presents properties of numbers.

- » **Reflexive property:** $a = a$. For example, $10 = 10$.
- » **Symmetric property:** If $a = b$, then $b = a$. For example, if $5 + 3 = 8$, then $8 = 5 + 3$.
- » **Transitive property:** If $a = b$ and $b = c$, then $a = c$. For example, if $5 + 3 = 8$ and $8 = 4 \cdot 2$, then $5 + 3 = 4 \cdot 2$.
- » **Commutative property of addition:** $a + b = b + a$. For example, $2 + 3 = 3 + 2$.
- » **Commutative property of multiplication:** $a \cdot b = b \cdot a$. For example, $2 \cdot 3 = 3 \cdot 2$.
- » **Associative property of addition:** $(a + b) + c = a + (b + c)$. For example, $(2 + 3) + 4 = 2 + (3 + 4)$.
- » **Associative property of multiplication:** $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. For example, $(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$.
- » **Additive identity:** $a + 0 = a$. For example, $-3 + 0 = -3$.
- » **Multiplicative identity:** $a \cdot 1 = a$. For example, $4 \cdot 1 = 4$.
- » **Additive inverse property:** $a + (-a) = 0$. For example, $2 + (-2) = 0$.
- » **Multiplicative inverse property:** $a \cdot \frac{1}{a} = 1$. For example, $2 \cdot \frac{1}{2} = 1$. (Remember: $a \neq 0$.)
- » **Distributive property:** $a(b + c) = a \cdot b + a \cdot c$. For example, $10(2 + 3) = 10 \cdot 2 + 10 \cdot 3 = 20 + 30 = 50$.
- » **Multiplicative property of zero:** $a \cdot 0 = 0$. For example, $5 \cdot 0 = 0$.
- » **Zero-product property:** If $a \cdot b = 0$, then $a = 0$ or $b = 0$. For example, if $x(x + 2) = 0$, then $x = 0$ or $x + 2 = 0$.



REMEMBER

If you're trying to perform an operation that isn't on the previous list, then the operation probably isn't correct. After all, algebra has been around since 1600 B.C., and if a property exists, someone has probably already discovered it. For example, it may look inviting to say that $10(2 + 3) = 10 \cdot 2 + 3 = 23$, but that's incorrect. The correct process and answer is $10(2 + 3) = 10 \cdot 2 + 10 \cdot 3 = 50$. Knowing what you can't do is just as important as knowing what you can do.



EXAMPLE

Q. Use the associative and commutative properties to simplify the expression:
 $(5x + 3) + (4y - 5) + (6x + 11)$.

A. $11x + 4y + 9$. Rewrite with the variable terms grouped followed by the numbers:
 $5x + 6x + 4y + 3 - 5 + 11$. Now combine the like terms: $(5x + 6x) + 4y + (3 - 5 + 11) = 11x + 4y + 9$.



EXAMPLE

Q. Use the distributive property to simplify the expression: $-3(5x^2 - 1) - x(2x + 3) - 2$.

YOUR
TURN

A. $-17x^2 - 3x + 1$. First, distribute the -3 and $-x$ over their respective parentheses; then combine like terms: $-3(5x^2 - 1) - x(2x + 3) - 2 = -15x^2 + 3 - 2x^2 - 3x - 2 = -17x^2 - 3x + 1$.

In Questions 7 and 8, simplify each using the distributive, associative, and commutative properties.

7

$$16a^2 + 4ab + 5b^2 - 2a^2 + 3ab - 9b$$

8

$$3(x - y) + 4(3x - 2y) + 9y$$

Looking at Visual Statements: When Math Follows Form with Function

Graphs are great visual tools. They're used to display what's going on in math problems, in companies, and in scientific experiments. For instance, graphs can be used to show how something (like the price of real estate) changes over time. Surveys can be taken to get facts or opinions, the results of which can be displayed in a graph. Open up the newspaper on any given day and you can find a graph in there somewhere.

Hopefully, the preceding paragraph answers the question of why you need to understand how to construct graphs. Even though in real life you don't walk around with graph paper and a pencil to make the decisions you face, graphing is vital in math and in other walks of life. Regardless of the absence of graph paper, graphs are indeed everywhere.

For example, when scientists go out and collect data or measure things, they often arrange the data as x and y values. Typically, scientists are looking for some kind of general relationship between these two values to support their hypotheses. These values can then be graphed on a coordinate plane to show trends in data. For example, a good scientist may show with a graph that the more you read this book, the more you understand pre-calculus! (Another scientist may show that people with longer arms have bigger feet. Boring!)

Using basic terms and concepts

Graphing equations is a huge part of pre-calculus, and eventually calculus, so it's good to review the basics of graphing before getting into the more complicated and unfamiliar graphs that you will see later in the book.

Although some of the graphs in pre-calculus will look very familiar, some will be new — and possibly intimidating. This book will get you more familiar with these graphs so that you will be more comfortable with them. However, the information in this chapter is mostly information that you remember from Algebra II. You did pay attention then, right?



TECHNICAL
STUFF

» Each point on the coordinate plane on which you construct graphs — that is, a plane made up of the horizontal (x -) axis and the vertical (y -) axis, creating four quadrants — is called a coordinate pair (x,y) , also often referred to as a *Cartesian coordinate pair*.

The name *Cartesian coordinates* comes from the French mathematician and philosopher who invented all this graphing stuff, René Descartes. Descartes worked to merge algebra and Euclidean geometry (flat geometry), and his work was influential in the development of analytic geometry, calculus, and cartography.

» A *relation* is a set (which can be empty, but in this book I only consider nonempty sets) of ordered pairs that can be graphed on a coordinate plane. Each relation is kind of like a computer that expresses x as input and y as output. You know you're dealing with a relation when the set is given in those curly brackets (like these: $\{ \}$) and has one or more points inside. For example, $R = \{(2,-1), (3,0), (-4,5)\}$ is a relation with three ordered pairs. Think of each point as (input, output) just like from a computer.

» The *domain* of a relation is the set of all the input values, usually listed from least to greatest. For example, the domain of set R is $\{-4, 2, 3\}$. The *range* is the set of all the output values, also often listed from least to greatest. For example, the range of R is $\{-1, 0, 5\}$. If any value in the domain or range is repeated, you don't have to list it twice. Usually, the domain is the x -variable and the range is y .



REMEMBER

If different variables appear, such as m and n , input (domain) and output (range) usually go alphabetically, unless you're told otherwise. In this case, m would be your input/domain and n would be your output/range. But when written as a point, a relation is always (input, output).

Graphing linear equalities and inequalities

When you first figured out how to graph a line on the coordinate plane, you learned to pick domain values (x) and plug them into the equation to solve for the range values (y). Then, you went through the process multiple times, expressed each pair as a coordinate point, and connected the dots to make a line. Some mathematicians call this the ol' *plug-and-chug method*.

After a bit of that tedious work, somebody said to you, "Hold on! You can use a shortcut." That shortcut involves an equation called *slope-intercept form*, and it's expressed as $y = mx + b$. The variable m stands for the slope of the line (see the next section, "Gathering information from graphs"), and b stands for the y -intercept (or where the line crosses the y -axis). You can change equations that aren't written in slope-intercept form to that form by solving for y . For example, graphing $2x - 3y = 12$ requires you to subtract $2x$ from both sides first to get $-3y = -2x + 12$. Then you divide every term by -3 to get $y = \frac{2}{3}x - 4$.

In the first quadrant, this graph starts at -4 on the y -axis; to find the next point, you move up 2 and right 3 (using the slope). Slope is often expressed as a fraction because it's rise over run — in this case $\frac{2}{3}$.



Q. Write the linear equation $4x - 5y - 100 = 0$ in slope-intercept form.

EXAMPLE

A. $y = \frac{4}{5}x - 20$. First, subtract $4x$ and add 100 to each side.

$$4x - 5y - 100 = 0 \rightarrow -5y = -4x + 100$$

Then, divide each term by -5 : $\frac{-5y}{-5} = \frac{-4x}{-5} + \frac{100}{-5} \rightarrow y = \frac{4}{5}x - 20$. The line has a slope of $\frac{4}{5}$ and a y -intercept at $(0, -20)$.

Inequalities are used for comparisons, which are a big part of pre-calculus. They show a relationship between two expressions (greater-than, less-than, greater-than-or-equal-to, and less-than-or-equal-to). Graphing inequalities starts exactly the same as graphing equalities, but at the end of the graphing process (you still put the equation in slope-intercept form and graph), you have two decisions to make:

» Is the line *dashed*, indicating $y <$ or $y >$, or is the line *solid*, indicating $y \leq$ or $y \geq$?

» Do you shade under the line for $y <$ or $y \leq$, or do you shade above the line for $y >$ or $y \geq$? Simple inequalities (like $x < 3$) express all possible answers. For inequalities, you show all possible answers by shading the side of the line that works in the original equation.

For example, when graphing $y < 2x - 5$, you follow these steps (refer to Figure 1-1):

1. Graph the line $y = 2x - 5$ by starting off at -5 on the y -axis, marking that point, and then moving up 2 and right 1 to find a second point.
2. When connecting the dots, you produce a straight dashed line through the points.
3. Shade on the bottom half of the graph to show all possible points in the solution.
4. Check your choice on the shading by checking a point. You have $(5, 0)$ in your shaded area, so inserting it into $y < 2x - 5$, you have $0 < 2(5) - 5$, which is a true statement. It checks!

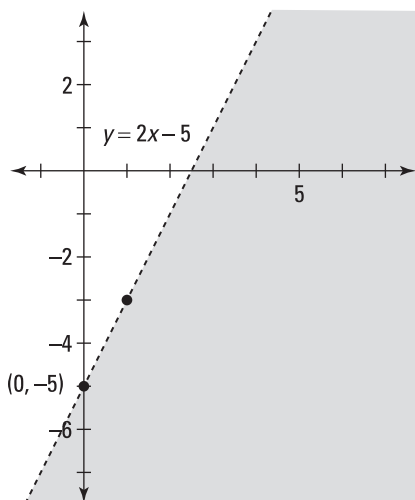
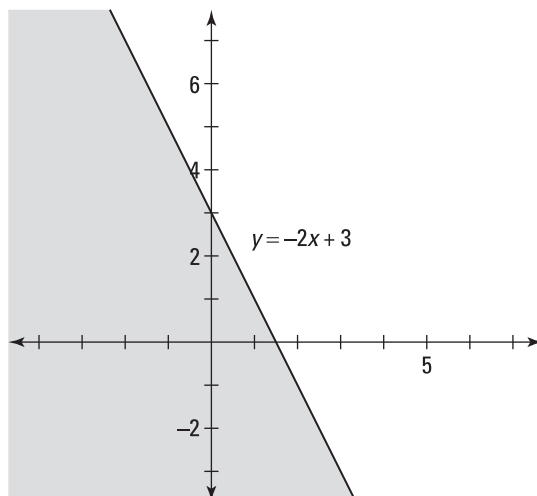


FIGURE 1-1:
The graph of
 $y < 2x - 5$.



EXAMPLE

Q. Write the inequality represented by the graph.



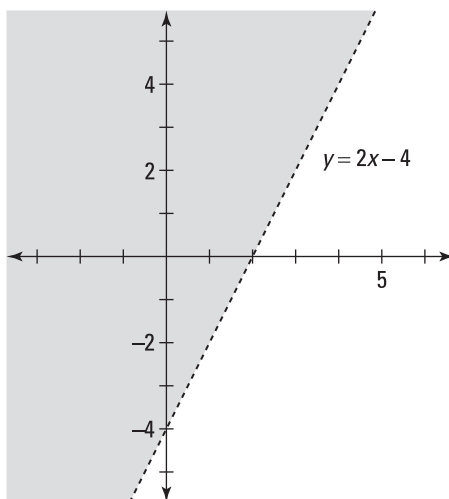
A. $y \leq -2x + 3$. The graph of the shaded area shows all the points below the graph of $y = -2x + 3$. Write the inequality $y \leq -2x + 3$, using less-than-or-equal to because the line is solid. Select a point in the shaded area such as $(0,0)$ and check to be sure the inequality is correct. Substituting, you have $0 \leq 0 + 3$, which is a true statement.



YOUR
TURN

9 Write the equation $-11 - 3y = 7x$ in slope-intercept form.

10 Write the inequality represented by the graph.



Gathering information from graphs

After getting you used to coordinate points and graphing equations of lines on the coordinate plane, typical math books and teachers begin to ask you questions about the points and lines that you've been graphing. The three main things you'll be asked to find are the distance between two points, the midpoint of the segment connecting two points, and the exact slope of a line that passes through two points.

Calculating distance



TECHNICAL
STUFF

Knowing how to calculate distance by using the information from a graph comes in handy in a big way, so allow this quick review of a few things first. *Distance* is how far apart two objects, or two points, are. To find the distance, d , between the two points (x_1, y_1) and (x_2, y_2) on a coordinate plane, for example, use the following formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

You can use this equation to find the length of the segment between two points on a coordinate plane whenever the need arises. For example, to find the distance between A(-6,4) and B(2,1), first identify the parts: $x_1 = -6$ and $y_1 = 4$; $x_2 = 2$ and $y_2 = 1$. Plug these values into the distance formula: $d = \sqrt{(2 - (-6))^2 + (1 - 4)^2}$. This problem simplifies to $\sqrt{73}$.

Finding the midpoint



TECHNICAL
STUFF

Finding the midpoint of a segment pops up in pre-calculus topics like conics (see Chapter 17). To find the midpoint of the segment connecting two points, you just average their x values and y values and express the answer as an ordered pair: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.

You can use this formula to find the center of various graphs on a coordinate plane, but for now you're just finding the midpoint. You find the midpoint of the segment connecting the two points A(-6,4) and B(2,1) by using this formula. You have $M = \left(\frac{-6 + 2}{2}, \frac{4 + 1}{2} \right)$ or $\left(-2, \frac{5}{2} \right)$.



TECHNICAL
STUFF

Figuring a line's slope

When you graph a linear equation, slope plays a role. The slope of a line tells how steep the line is on the coordinate plane. When you're given two points (x_1, y_1) and (x_2, y_2) and are asked to find the slope of the line between them, you use the following formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

If you use the points $A(-6, 4)$ and $B(2, 1)$ and plug the values into the formula, the slope is $-\frac{3}{8}$.

Positive slopes always move up when going to the right or move down going to the left on the plane. Negative slopes move either down when going right or up when going left. (Note that if you moved the slope down and left, it would be negative divided by negative, which has a positive result.) Horizontal lines have zero slope, and vertical lines have undefined slope.



TIP

If you ever get the different types of slopes confused, remember the skier on the ski-slope:

- » When they're going uphill, they're doing a lot of work (+ slope).
- » When they're going downhill, the hill is doing the work for them (– slope).
- » When they're standing still on flat ground, they're not doing any work at all (0 slope).
- » When they hit a wall (the vertical line), they're done for and they can't ski anymore (undefined slope)!



EXAMPLE

Q. Find the distance between the points $(11, -2)$ and $(-1, 4)$.

A. $6\sqrt{5}$. Using the distance formula, $d = \sqrt{(-1 - 11)^2 + (4 - (-2))^2} = \sqrt{(-12)^2 + 6^2} = \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$.

Q. Find the midpoint of the two points $(0, -3)$ and $(-1, 2)$.

A. $\left(-\frac{1}{2}, -\frac{1}{2}\right)$. Using the midpoint formula, $M = \left(\frac{0 + (-1)}{2}, \frac{-3 + 2}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$.

Q. Find the slope of the line through the two points $(4, 6)$ and $(-8, 5)$.

A. $\frac{1}{12}$. Using the slope formula, $m = \frac{5 - 6}{-8 - 4} = \frac{-1}{-12} = \frac{1}{12}$.



YOUR
TURN

11 Find the distance between the points and the midpoint of the two points.

12 Find the slope of the line that goes through the points.

Getting Yourself a Graphing Calculator

It's highly recommended that you purchase a graphing calculator for pre-calculus work. Since the invention of the graphing calculator, the emphasis and time spent on calculations in the classroom and when doing homework have changed because the grind-it-out computation isn't necessary. Many like doing most of the work with the calculator, but others prefer not to use one. A graphing calculator does so many things for you, and, even if you don't use it for every little item, you can always use one to check your work on the big problems.

Many different types of graphing calculators are available, and their inner workings are all different. To figure out which one to purchase, ask for advice from someone who has already taken a pre-calculus class, and then look around on the Internet for the best deal.



REMEMBER

Many of the more theoretical concepts in this book, and in pre-calculus in general, are lost when you use your graphing calculator. All you're told is, "Plug in the numbers and get the answer." Sure, you get your answer, but do you really know what the calculator did to get that answer? Nope. For this reason, this book goes back and forth between using the calculator and doing complicated problems longhand. But whether you're allowed to use the graphing calculator or not, be smart with its use. If you plan on moving on to calculus after this course, you need to know the theory and concepts behind each topic.

The material found here can't even begin to teach you how to use your unique graphing calculator, but the good *For Dummies* folks at Wiley supply you with entire books on the use of them, depending on the type you own. However, I can give you some general advice on their use. Here's a list of hints that should help you use your graphing calculator:

» **Always double-check that the mode (degrees versus radians) in your calculator is set according to the problem you're working on.** Look for a button somewhere on the calculator that says *mode*. Depending on the brand of calculator, this button allows you to change things like degrees or radians, or $f(x)$ or $r(\theta)$. For example, if you're working in degrees, you must make sure that your calculator knows that before you use it to solve a problem. The same goes for working in radians. Some calculators have more than ten different modes to choose from. Be careful!

» **Make sure you can solve for y before you try to construct a graph.** You can graph anything in your graphing calculator as long as you can solve for y to write it as a function. The calculators are set up to accept only equations that have been solved for y .

Equations that you have to solve for x often aren't true functions and aren't studied in pre-calculus — except conic sections, and students generally aren't allowed to use graphing calculators for this material because it's entirely based on graphing (see Chapter 17).

» **Be aware of all the shortcut menus available to you and use as many of the calculator's functions as you can.** Typically, under your calculator's graphing menu you can find shortcuts to other mathematical concepts (like changing a decimal to a fraction, finding roots of numbers, or entering matrices and then performing operations with them). Each brand of graphing calculator is unique, so read the manual. Shortcuts give you great ways to check your answers!



REMEMBER

» **Type in an expression exactly the way it looks, and the calculator will do the work and simplify the expression.** All graphing calculators do order of operations for you, so you don't even have to worry about the order. Just be aware that some built-in math shortcuts automatically start with grouping parentheses.

For example, most calculators start a square root off as $\sqrt{}$ so all information you type after that is automatically inside the square root sign until you close the parentheses. For instance, $\sqrt{(4+5)}$ and $\sqrt{(4)} + 5$ represent two different calculations and, therefore, two different values (3 and 7, respectively). Some smart calculators even solve the equation for you. In the near future, you probably won't even have to take a pre-calculus class; the calculator will take it for you!

Okay, after working through this chapter, you're ready to take flight into pre-calculus. Good luck to you and enjoy the ride!

Practice Questions Answers and Explanations

- 1 **Integer, rational, real.** The number -14 is a negative integer. It is rational because it can be written as a fraction, such as $\frac{-14}{1}$. And it is a real number.
- 2 **Irrational, real.** The number 15 is not a perfect square, so its decimal value continues on forever without repeating.
- 3 **Rational, real.** The fraction $\frac{49}{9}$ can be written as the repeating decimal $5.\bar{4}$.
- 4 **None of those given.** This is an imaginary number, which can be written as $4i$.
- 5 **$3x^3 - 6x^2 + 4x$.** First, square the binomial, then distribute the $3x$. And, finally, combine like terms. $3x(x-1)^2 + x = 3x(x^2 - 2x + 1) + x = 3x^3 - 6x^2 + 3x + x = 3x^3 - 6x^2 + 4x$
- 6 **1,4.** Perform the square and multiplication under the radical, and multiply the factors in the denominator. Then add the terms under the radical and take the root.

$$\frac{-6 \pm \sqrt{(-6)^2 - 4(-2)(8)}}{2(-2)} = \frac{-6 \pm \sqrt{36 + 64}}{-4} = \frac{-6 \pm \sqrt{100}}{-4} = \frac{-6 \pm 10}{-4}$$

Now write the two answers, one by adding 10 and the other by subtracting 10:

$$\begin{aligned} \frac{-6 \pm 10}{-4} &= \\ \frac{-6 + 10}{-4} \text{ or } \frac{-6 - 10}{-4} &= \\ \frac{4}{-4} \text{ or } \frac{-16}{-4} &= \\ -1 \text{ or } 4 \end{aligned}$$

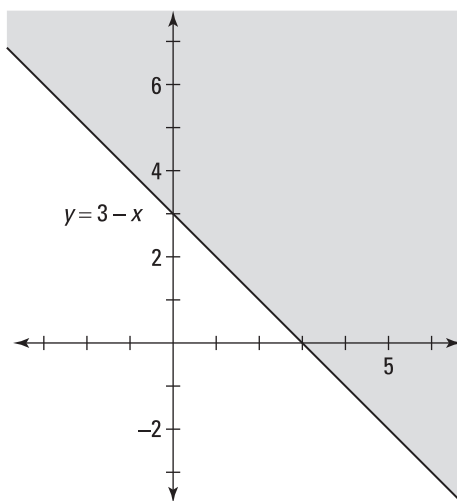
- 7 **$14a^2 + 7ab + 5b^2 - 9b$.** Rearrange the terms: $16a^2 + 4ab + 5b^2 - 2a^2 + 3ab - 9b = 16a^2 - 2a^2 + 4ab + 3ab + 5b^2 - 9b = 14a^2 + 7ab + 5b^2 - 9b$.
- 8 **$15x - 2y$.** First, distribute the 3 and 4. Then rearrange the terms and combine like terms. $3(x-y) + 4(3x-2y) + 9y = 3x - 3y + 12x - 8y + 9y = 3x + 12x - 3y - 8y + 9y = 15x - 2y$
- 9 **$y = -\frac{7}{3}x - \frac{11}{3}$.** Add 11 to each side, and then divide each side by -3 . $-11 - 3y = 7x \rightarrow -3y = 7x + 11 \rightarrow \frac{-3y}{-3} = \frac{7x}{-3} + \frac{11}{-3} \rightarrow y = -\frac{7}{3}x - \frac{11}{3}$
- 10 **$y > 2x - 4$.** The line is dashed, so the $>$ symbol is used. The shaded side is above the line. To check, use the point $(0,0)$ and you have $0 > 0 - 4$, which is a true statement.
- 11 **$4\sqrt{5}$ and $(-2, 1)$.** Using the distance formula, $d = \sqrt{(0 - (-4))^2 + (-3 - 5)^2} = \sqrt{16 + 64} = \sqrt{80} = \sqrt{16} \sqrt{5} = 4\sqrt{5}$. Using the midpoint formula, $M = \left(\frac{-4 + 0}{2}, \frac{5 + (-3)}{2} \right) = (-2, 1)$.
- 12 **-2 .** Using the slope formula, $m = \frac{-3 - 5}{0 - (-4)} = \frac{-8}{4} = -2$.

If you're ready to test your skills a bit more, take the following chapter quiz that incorporates all the chapter topics.

Whaddya Know? Chapter 1 Quiz

Quiz time! Complete each problem to test your knowledge on the various topics covered in this chapter. You can then find the solutions and explanations in the next section.

- 1 Which of the following numbers are *integers*? $\left\{-3, 0, 8.795, \frac{19}{5}, \sqrt{16}, \pi, \sqrt{10}\right\}$
- 2 Find the midpoint of the points $(3, -2)$ and $(-5, 5)$.
- 3 Simplify: $\frac{-5 - \sqrt{(-5)^2 - 4(-3)(-2)}}{(-3)^2}$
- 4 Simplify: $5x(3x - 4)^2 + 2(x + 1)^2 - 3$
- 5 Write an inequality that describes the graph shown here.



- 6 Which of the following numbers are *rational numbers*? $\left\{-3, 0, 8.795, \frac{19}{5}, \sqrt{16}, \pi, \sqrt{10}\right\}$
- 7 Find the distance between the points $(-8, 4)$ and $(1, 6)$.
- 8 Simplify: $\frac{x^2 - 5x - (9 - 5x)}{7 + x - 4}$
- 9 Find the slope of the line through the points $(-4, 0)$ and $(6, -2)$.
- 10 Simplify: $6 \cdot 100 \left(1 + \frac{0.04}{4}\right)^3$

Answers to Chapter 1 Quiz

- 1 $-3, 0, \sqrt{16}$. These are integers. They can be written as a positive or negative whole number or 0. The numbers $8.795, \frac{19}{5}, \pi, \sqrt{10}$ are not integers.

2 $\left(-1, \frac{3}{2}\right)$. Using the midpoint formula, $M = \left(\frac{3+(-5)}{2}, \frac{-2+5}{2}\right) = \left(-1, \frac{3}{2}\right)$.

- 3 $-\frac{2}{3}$. Working inside the radical, first square the number and multiply the three factors. Square the denominator. Simplify under the radical. Then simplify and reduce the fraction.

$$\frac{-5 - \sqrt{(-5)^2 - 4(-3)(-2)}}{(-3)^2} = \frac{-5 - \sqrt{25 - 24}}{9} = \frac{-5 - \sqrt{1}}{9} = \frac{-6}{9} = -\frac{2}{3}$$

- 4 $45x^3 - 118x^2 + 84x - 1$. First, square both binomials and distribute the factors, multiplying them. Then combine like terms.

$$\begin{aligned} 5x(3x-4)^2 + 2(x+1)^2 - 3 &= 5x(9x^2 - 24x + 16) + 2(x^2 + 2x + 1) - 3 \\ &= 45x^3 - 120x^2 + 80x + 2x^2 + 4x + 2 - 3 \\ &= 45x^3 - 118x^2 + 84x - 1 \end{aligned}$$

- 5 $y \geq 3 - x$. The shaded area is above the line. The line is solid, so $\geq$ is used to include the points on the line. To check the accuracy of the shading, select a point in the inequality. Using the point $(2, 2)$, you have $2 \geq 3 - 2$, which is true.

- 6 $-3, 0, 8.795, \frac{19}{5}, \sqrt{16}$. These are all rational because each can be written as a fraction with an integer in the numerator and denominator. The numbers π and $\sqrt{10}$ are not rational; their decimal values go on forever without ever repeating.

7 $\sqrt{85}$. Using the distance formula, $d = \sqrt{(1-(-8))^2 + (6-4)^2} = \sqrt{9^2 + 2^2} = \sqrt{81+4} = \sqrt{85}$.

- 8 $x - 3$. Distribute the negative sign over the two terms in the parentheses. Combine like terms and simplify the denominator. Factor the numerator, and then reduce the fraction.

$$\frac{x^2 - 5x - (9 - 5x)}{7 + x - 4} = \frac{x^2 - 5x - 9 + 5x}{7 + x - 4} = \frac{x^2 - 9}{x + 3} = \frac{(x-3)(\cancel{x+3})}{\cancel{x+3}} = x - 3$$

9 $-\frac{1}{5}$. Using the slope formula, $m = \frac{-2-0}{6-(-4)} = \frac{-2}{10} = -\frac{1}{5}$.

- 10 **618.1806**. First, simplify the expression in the parentheses. Next, raise the result to the power, and then multiply by the product in front.

$$6 \cdot 100 \left(1 + \frac{0.04}{4}\right)^3 = 6 \cdot 100(1.01)^3 = 6 \cdot 100(1.030301) = 618.1806$$

