

# Chapter 1

## Time Value of Money

**W**hich is worth more: one dollar today, or one dollar tomorrow?

### The Standard Theory

The standard theory of the time value of money states that a dollar in your hand today is worth more than the same dollar in your hand in the future. This is true even if there is no inflation.

Here's one example. Would you rather have \$100 today, or \$100,000,000 in a million years? Even if you were absolutely certain that the \$100,000,000 would be "yours" in a million years, it would be worth nothing to you, because you would not expect to be around to enjoy it. Further, even if your great, great, great, etc., grandchildren could be assured of receiving it, it would probably mean little to you.

But what about more realistic waiting periods? For example, suppose you have the choice between \$100 now, and \$100 in a year from now. The standard theory says that the rational person will

always choose the \$100 now, because you could do everything with \$100 now that you could do with it in a year from now, and you could invest it to earn interest between now and then.

This standard theory is right, but it has a few important assumptions buried in it. The most important of these assumptions is that you can save money at no cost and at no risk between now and the future. In thinking about the standard theory, or any economic theory, it is important to remember the assumption of “all other things equal” (from the Latin, *ceteris paribus*).

### *Assumption: You Can Save Money at No Cost*

The assumption that you can save money at no cost means that if you have a certain amount of money today, you can save that money for any period at no cost. No cost means that you don't have to pay any storage charges, or insurance charges, or handling charges, or taxes, or charges of any kind.

In the last hundred years or so, citizens in developed economies have gotten used to the idea that they can save money, in a bank for example, and not have to pay for the service. We will not be surprised if financial historians of the future look back and see this as a sort of financial magical thinking, which has indirectly had catastrophic costs by making the entire banking system unstable.

In general, if you want to store any valuable commodity, you would have to do it yourself – perhaps in your home, in a rented vault, or perhaps even burying it in the ground. You would not expect that you can store it securely for free.

In principle, money is no different. But the way banking has developed over the past several hundred years, bankers, with the willing cooperation of their depositors, have purported to “save” money for free for their depositors. Of course, nothing is free. The bankers have been willing to offer this service for free so that they could get their hands on the money. Do they “save” it for you? Absolutely not. They lend it out. This lending puts your money at risk, and generates income for the bank, provided that the bank doesn't suffer too many loan losses. If the bank suffers too many

loan losses, they may not even be able to pay you back the money that you deposited.<sup>1</sup>

Despite the inherent instability of this practice, it has become so entrenched in the modern banking system that it seems like a permanent feature. We don't think history will prove it so, but for the time being (and perhaps for the foreseeable future), it is.

### *Assumption: You Can Save Money at No Risk*

The assumption that you can save money at no risk of not getting it back (in other words, that you can save money with the guarantee that you'll receive 100% of it back) is easy to understand in principle. It is a bit surprising that so many people believe it to be literally true. There is no truly riskless proposition in the material world. It may be the case that some things carry extremely low risk, but in our world, that risk is never actually zero.

Nevertheless, most people in the developed world behave as though they can put money in the bank, and have no risk of not getting it back in the future.

As long as you can put money in the bank and expect to receive it back in the future, and the bank doesn't charge you for the service, a dollar now is worth more than a dollar in the future, because you can turn it into a dollar in the future by storing it. Anything you can do with a dollar in a year, you could do with a dollar now because you could just wait. But you can do things with the dollar now, such as spend it now, that you cannot do with a dollar in a year. So the dollar now is worth at least as much as the dollar in the future.<sup>2</sup>

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<sup>1</sup> In the current system, deposits may be "insured" by government guarantees. These are not true insurance, because it is not optional, it is not truly risk-based, and it is ultimately backed by taxpayer funds. Furthermore, it is very likely the case that deposit insurance results in bankers taking greater risks with depositors' funds than they would without it, and thereby make the entire system less stable. The financial crisis of 2008 was a case in point, and ended up costing taxpayers at least half a trillion dollars. (See Deborah Lucas, 2019, "Measuring the Costs of Bailouts," *Annual Review of Financial Economics* 11: 85–100.) For a discussion of how deposit insurance causes instability, see Asli Demirgüç-Kunt and Enrica Detragiache, 2002, "Does Deposit Insurance Increase Banking System Stability? An Empirical Investigation," *Journal of Monetary Economics* 49: 1373–1406.

<sup>2</sup> The pure time value of money analysis does not depend on the assumption of there being no risk of not getting your money back, but the assumption simplifies the analysis.

## Time Value of Money and Compound Growth

Economists, financial professionals, and talking heads often conflate the time value of money with the phenomenon of compound growth. You don't need to worry about the finer points of theory as discussed earlier, but you do need to understand the math of exponential or compound growth.

Perhaps the easiest way to understand compound growth is to think in terms of "interest on interest." Suppose you have \$100 and you can earn 10% per year. (For the purpose of illustration, we assume that the 10% earnings are risk-free, but remember the real world is never risk-free.) Table 1.1 shows how your money would grow if you get paid interest at the end of every year, and reinvest the interest.

**Table 1.1** Compound Growth Assuming Interest Payments Are Reinvested

Year	Beginning Principal	Interest	Ending Principal
1	100.00	10.00	110.00
2	110.00	11.00	121.00
3	121.00	12.10	133.10
4	133.10	13.31	146.41
5	146.41	14.64	161.05
6	161.05	16.11	177.16
7	177.16	17.72	194.87
8	194.87	19.49	214.36
9	214.36	21.44	235.79
10	235.79	23.58	259.37
11	259.37	25.94	285.31
12	285.31	28.53	313.84
13	313.84	31.38	345.23
14	345.23	34.52	379.75
15	379.75	37.97	417.72
16	417.72	41.77	459.50
17	459.50	45.95	505.45
18	505.45	50.54	555.99
19	555.99	55.60	611.59
20	611.59	61.16	672.75

In the first year, you earn \$10 of interest, which is 10% of your \$100. But in the second year, you earn \$11, because you earned your 10% on your original \$100, but you also earned 10% on your \$10 of interest. That “interest on interest” earned you \$1 in year two. You earned a total of \$11, which could be thought of as \$10 on your original \$100, plus \$1 on the interest you earned the first year.

Each year, you still earn the \$10 on your original \$100, but the “interest on interest” gets bigger every year. By the ninth year, you are earning more “interest on interest” than on your original \$100!

This compound growth is the phenomenon that people get so excited about. Almost everyone who has ever amassed significant wealth legitimately (i.e. other than by stealing it) has done it, at least in part, by putting compound interest, or exponential growth (another term for the same phenomenon), to work. Just in case you’re not convinced, here is a bit of *argumentum ad verecundiam* (just a fancy sounding way of saying *argument from authority*).

“Compounding is the magic of investing.”

—*Jim Rogers*

“The effects of compounding even moderate returns over many years are compelling, if not downright mind boggling.”

—*Seth Klarman*

“Understanding both the power of compound interest and the difficulty of getting it is the heart and soul of understanding a lot of things.”

—*Charlie Munger*

“The sooner you start, the more compounding can do for you. If, beginning at the age of twenty, you sock away just \$100 a month in stocks, and your portfolio compounds at 10%, which is what stocks have provided historically, you will be a millionaire when you retire at sixty-five.”

—*Ralph Wanger*<sup>3</sup>

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<sup>3</sup> All these quotations were sourced from <http://www.valuwalk.com/2016/10/compounding-quotes/>. We have not independently verified them.

## Comparing Values Across Time

The ability to compare values across time, a process called *present value analysis*, is useful to understand the process of building wealth.

Suppose I told you that a gallon of regular gas is \$3.50 at the Exxon station and \$3.65 at the Chevron station down the block. Assuming both stations are equally convenient, safe, busy, etc., you would have no difficulty telling me that the Exxon station was a better deal.

But what if I told you I was going to buy an item you're selling, and I'll pay you either \$65 for it now, or \$68 in a year. Now how easy it is for you to say which is a better deal? As we explained in the previous section, you know that a dollar today is worth more than a dollar in a year. But is \$65 today worth more than \$68 in a year?

For most people, thinking about money across time does not come naturally. You have to work at it, just as you do for most skills. It's not hard, but you do have to practice.

It's great to have the tools of present value in your toolbox, but it's even better if you know how to use them. You learn by practice. You might keep your eye out for retail offers such as per month pricing versus subscription discounts for longer terms, for investment advertisements, and for claims made by politicians, and apply the tools of present value analysis where you can.

### *Real-World Compounding Versus "Pure" Theory of Time Value of Money*

In the basic models discussed in this chapter, we always assumed a known and constant rate of return. However, in the real world, there is always risk, and rates of return are rarely, if ever, stable over time. Even so, these models can give you a rough idea of what something is worth now compared to some time in the future. As long as you don't believe that your models represent reality, you will be well served by being as comfortable as possible with making present

value calculations, future value calculations, and in general using the concepts of compounding in a wide variety of situations.

In order to compare the value of \$65 today to \$68 in a year, you may use the following formula:

$$FV = PV(1 + r)^t \quad (1)$$

In this formula, *FV* represents the *future value* of your investment in the future, while *PV* represents the *present value*, or value today, of your investment. The length of time, in years, is represented by *t* while the rate of return is represented by *r*. (This formula is explained in the Appendix to this chapter.)

For the purposes of the above example, \$65 is the *present value*, \$68 is the *future value*, and *t* is one year.

Whether \$65 now is worth more than \$68 in a year, depends on your own personal *discount rate*. A rate of 4.6% per year makes \$65 now equal to \$68 in a year. There are several ways to think about this, but the most useful one for investing is that if the market rate of interest (e.g. the rate on one-year loans of the same risk category as the loan in question) is 4.6%, then the market value of \$65 now is the same as the market value of \$68 in a year.

## Appendix

### *The Math of Compounding*

In the example in this chapter, interest was compounded each year. It is possible to compound more often, for example monthly, weekly, or daily. It turns out that you can even compound continuously and still get a meaningful answer. If you can use a calculator, you can make all the time value of money calculations you will need. It is often easier, however, to use a computer spreadsheet. You can also do the calculations using natural logs and the exponential function, but in my experience the need or usefulness of those in the real world has been limited.

There is one equation that you really should know your way around backwards and forwards, because that equation is at the heart of the miracle of compounding. Here it is:

$$FV = PV(1 + r)^t \quad (1)$$

Where  $FV$  is the Future Value (the value you'll have in the future),  $PV$  is the Present Value (the value you have now),  $r$  is the growth rate (or interest rate) per period, and  $t$  is the number of periods. Often,  $r$  is expressed as an annual interest rate, and  $t$  is measured in years. So, for example, if the  $PV$  is \$100,  $r$  is 10%, and  $t$  is 7, the future value will be \$194.87.

Here is the calculation:

$$FV = 100 \times (1 + 0.1)^7 = 100 \times 1.1^7 = 100 \times 1.9487 = 194.87 \quad (2)$$

You might have noticed that you could have looked this up in Table 1, because that table used \$100 as the beginning value, 10% annual interest, and compounded annually.

### More Frequent Compounding

What if we used the same 10% annual interest rate, but compounded twice a year? The formula is equation (1), but now  $r$  is 5% because

interest is paid twice, so we divide 10% by 2 and  $t$  is 14, because there are now twice as many periods. So the calculation is:

$$FV = 100 \times (1 + 0.05)^{14} = 100 \times 1.05^{14} = 100 \times 1.9799 = 197.99 \quad (3)$$

We can generalize the compounding formula to

$$FV = PV \left(1 + \frac{r}{n}\right)^{nt} \quad (4)$$

where  $r$  is the annual interest rate,  $t$  is the number of years, and  $n$  is the number of times we're compounding during the year.

As we make  $n$  larger and larger, we are compounding over shorter and shorter periods of time. We can use calculus to show that as  $n$  goes to infinity, the limit of the function becomes:

$$FV = PVe^{rt} \quad (5)$$

where  $e$  is the base of the natural log function, approximately 2.71828.

You don't have to understand where equation (4) or (5) comes from to use it. For most purposes, either one is fine.

### *A Little Algebra*

Equations (1), (2), and (3) are good if we know the value now, the interest rate, and the number of years. But what if we know how much we're going to get in the future, when we're going to get it, and the interest rate, and we want to calculate what that is worth today? We want the present value, and we can get it by just rearranging one of our equations. Let's take equation (1) and solve it for  $PV$ . We get:

$$PV = \frac{FV}{(1 + r)^t} \quad (6)$$

Rearranging equation (5) gives us

$$PV = \frac{FV}{e^{rt}} = FVe^{-rt} \quad (7)$$

Given any three of  $PV$ ,  $FV$ ,  $r$  and  $t$ , it is possible to solve for the fourth one. Above we gave equations solving for  $FV$  and  $PV$ . As an

exercise, you can try solving for  $r$ , and then for  $t$ . Following are the answers.

$$r = \sqrt[t]{\frac{FV}{PV}} - 1 = \left(\frac{FV}{PV}\right)^{\left(\frac{1}{t}\right)} - 1 \quad (8)$$

For example, 55 years ago a man bought a building for \$200,000. Now the building is worth \$10 million. His annual rate of return is:

$$.0737 = \left(\frac{10,000,000}{200,000}\right)^{\left(\frac{1}{55}\right)} - 1 \quad (9)$$

The man earned a 7.37% compound annual rate of return.

Fortunately, calculators and computers make it simple to perform these calculations.

Now we will solve for  $t$ . The equation is:

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}, \quad (10)$$

where  $\ln$  is the natural logarithm function.

Again, it is easy to perform these calculations with a calculator or computer. For example, suppose you put \$1,000 into an investment that you expect will earn 5% each year indefinitely. How long would it take to grow to \$10,000?

$$t = \frac{\ln\left(\frac{10,000}{1000}\right)}{\ln(1+.05)} = 47.19 \quad (11)$$

## Exercises

Here are 40 sample problems, consisting of 10 each where you are to solve for  $PV$ ,  $FV$ ,  $r$ , and  $t$ . The answers are on the following page.

**Table 1.2** Chapter 1 Exercises

Exercise #	$FV$	$r$	$t$	$PV$
1	795		14	76.51
2	734	0.068		210.30
3	17	0.094		7.57
4	246	0.122	7	
5		0.018	14	303.03
6	752		19	59.34
7		0.023	15	396.02
8	875		2	685.25
9		0.168	15	6.91
10	972	0.09		206.06
11	723	0.147	19	
12	56	0.055		47.69
13	542	0.175	10	
14		0.082	4	227.64
15	412	0.05		338.95
16	669	0.086	15	
17		0.076	19	98.71
18	325	0.178	16	
19	438	0.009		372.76
20	417		9	171.16
21	848	0.049	20	
22	490		18	282.84
23	704		18	329.96
24		0.012	5	77.25
25		0.159	0	321.00
26	505	0.084	18	
27	592	0.042		319.38
28	754		13	95.79
29	422		20	25.78
30	933	0.04		498.14
31	667	0.127	19	
32	91		10	16.95
33	166	0.179	2	
34	875	0.046		426.09
35	293	0.017		278.55
36		0.033	13	9.84
37		0.167	1	419.88
38	530		12	61.26
39	391	0.07	5	
40		0.117	9	237.17

## Answers

**Table 1.2** Chapter 1 Answers

Exercise #	<i>FV</i>	<i>r</i>	<i>t</i>	<i>PV</i>
1	795	0.182	14	76.5107
2	734	0.068	19	210.3014
3	17	0.094	9	7.5734
4	246	0.122	7	109.8968
5	389	0.018	14	303.0266
6	752	0.143	19	59.3389
7	557	0.023	15	396.0233
8	875	0.13	2	685.2533
9	71	0.168	15	6.9122
10	972	0.09	18	206.0579
11	723	0.147	19	53.3867
12	56	0.055	3	47.6904
13	542	0.175	10	108.0490
14	312	0.082	4	227.6384
15	412	0.05	4	338.9534
16	669	0.086	15	194.0791
17	397	0.076	19	98.7091
18	325	0.178	16	23.6352
19	438	0.009	18	372.7633
20	417	0.104	9	171.1648
21	848	0.049	20	325.7512
22	490	0.031	18	282.8395
23	704	0.043	18	329.9553
24	82	0.012	5	77.2523
25	321	0.159	0	321.0000
26	505	0.084	18	118.2400
27	592	0.042	15	319.3787
28	754	0.172	13	95.7883
29	422	0.15	20	25.7843
30	933	0.04	16	498.1363
31	667	0.127	19	68.7973
32	91	0.183	10	16.9509
33	166	0.179	2	119.4209
34	875	0.046	16	426.0898
35	293	0.017	3	278.5510
36	15	0.033	13	9.8353
37	490	0.167	1	419.8800
38	530	0.197	12	61.2556
39	391	0.07	5	278.7776
40	642	0.117	9	237.1682