CHAPTER 1

INTRODUCTION

CHAPTER GOALS

- Introduce the concept of electric and magnetic fields
- Describe the electromagnetic spectrum
- Provide guidelines for handling numbers and dimensions
- Explain how electromagnetics is fundamental to wireless communications
- Review time harmonic signals and basic wave propagation
- Describe the use of phasors for representing time-harmonic signals

We are immersed in electromagnetic fields. They are everywhere, being generated naturally (e.g., solar radiation and lightning), and by us (e.g., radio stations, cell phones, and power lines). The modern office, kitchen, and automobile are all stuffed full of devices that rely on electricity, and magnetic fields are in action anywhere an electric motor is running. Wireless communications have electromagnetics at its very core: Voice and data information is transmitted and received via antennas and high-frequency electronics, components requiring knowledge of electromagnetics to design and understand. The study of electromagnetics is necessary for understanding even simple electronic components such as resistors, capacitors, and inductors.

Human beings have been aware of magnetic materials for as long as there has been recorded history. The Greek Thales of Miletus recorded evidence of both static electricity and magnetic attraction around 600 B.C. But it was not until the latter half of the 18th century, and the 19th century in particular, that progress was made in recognizing and understanding electromagnetic phenomena. The timeline in Figure 1.1 shows some of the key advances[1.1].

[1.1] The 20th century began with a flood of engineering applications for electromagnetics, let by the prolific engineers Thomas Alva Edison and Nikola Tesla.

Applied Electromagnetics: Early Transmission Lines Approach, Second Edition. Stuart M. Wentworth.

Companion website: www.wiley.com/go/wentworthapplied2e

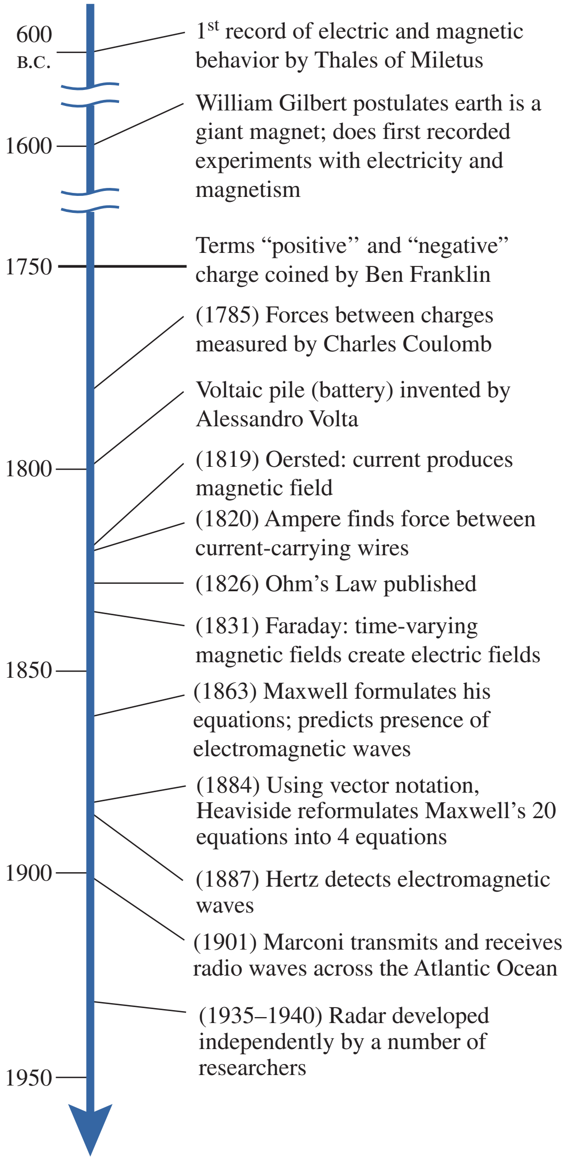

FIGURE 1.1 The key historical events in electromagnetics.

The real era of understanding began after Alessandro Volta invented the voltaic cell, allowing research to be conducted with controlled currents. From there, Oersted's discovery that electric currents create magnetic fields, and Faraday's discovery that magnetic fields changing with time create electric fields, culminated in James Clerk Maxwell's unification of electricity and magnetism in the concise four equations known as Maxwell's Equations. Development and understanding of these four equations is a primary goal of this text.

Before getting started on the first fundamental chapter of the text on electrostatics, several topics are addressed. First, it is instructive to briefly summarize some key concepts of electromagnetic fields, which we will do in the following section. This is followed by a brief overview of the electromagnetic spectrum, showing the basic relationships between frequency and wavelength and identifying specific frequency bands of interest in wireless communications. This will also describe how electromagnetics is pivotal to the application of wireless communications. Then, some suggestions are presented dealing with units and numeric precision in solving problems. The chapter concludes with a review of wave fundamentals and phasors.

1.1 ELECTROMAGNETIC FIELDS

Electromagnetic fields consist of an intimate linkage between electric and magnetic fields. This section introduces these fields and this linkage, describing key concepts behind electromagnetic theory without the hindrance of mathematical detail.

1.1.1 Electric Fields

Matter contains atoms, and in the classical physics view, atoms contain electrons, protons, and neutrons. Among the fundamental forces in nature is the electromagnetic force between charged objects. As indicated in Figure 1.2, there is a repulsive force between a pair of like charges (such as two protons), and an attractive force between a pair of unlike charges (an electron and a proton). The attraction (or repulsion) between the charges is a function of the magnitude of the charges and their separation distance.

If the force exerted on an electron is measured as it is moved from point to point in the vicinity of a nucleus, the spatial distribution of the force can be mapped. Such a mapping is called a *field*. The field concept is extremely useful in dealing with situations where bodies interact from a distance.

Moving a negative charge around the positive charge indicated in Figure 1.3 can map a *vector field*. A vector contains both magnitude and

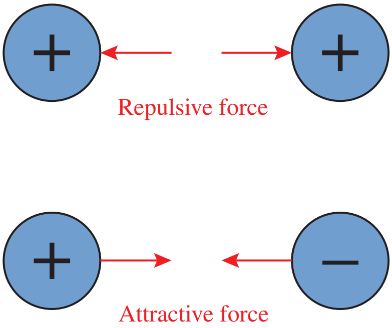

FIGURE 1.2 Forces acting between charges.

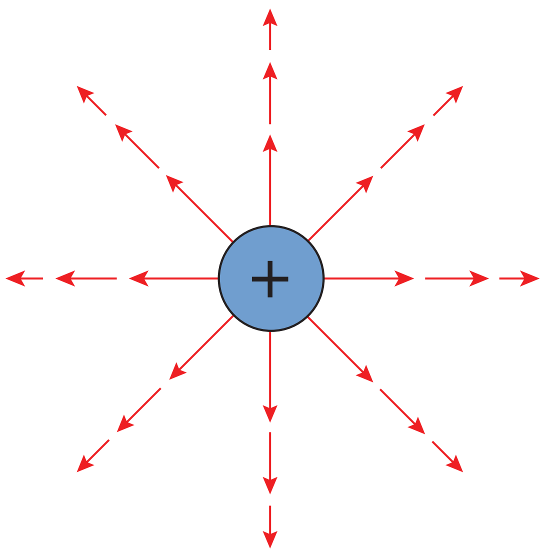

FIGURE 1.3 Force vectors acting on a negative charge (not shown) at specific points around a positively charged nucleus.

direction information. So, every point in the vector field indicates the magnitude of the force acting on the negative charge as well as the direction of this force. For instance, the magnitude decreases as the square of the distance from the nucleus, but the force vector always points away from the nucleus. This is analogous to the mapping of a gravity field; the values always point to the center of gravity. In contrast, consider a topographical map. This is mapping of scalar quantities, where each point in the field represents a single value—in this case the magnitude of land elevation as a function of position.

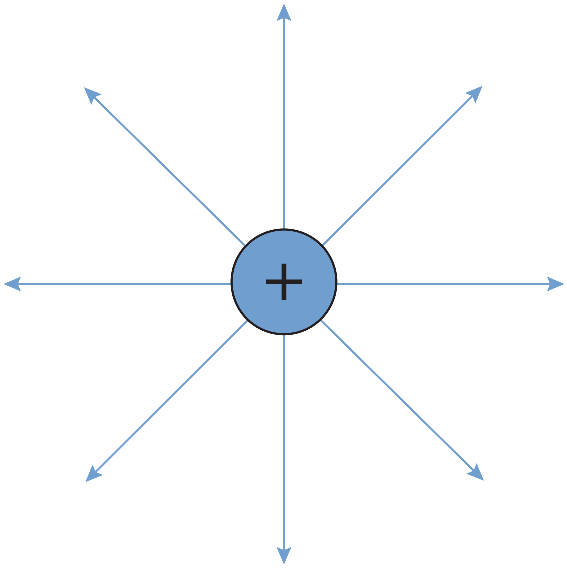

FIGURE 1.4 Electric field emanating from a positive charge.

The amount of force exerted by the nucleus on the electron is proportional to the product of both their charges. If the force is divided by the electron's charge, the result is a field value that can be used to find the force exerted by the nucleus on any arbitrary amount of charge. This is termed the *electric field* of the nucleus (Figure 1.4). Thus, a charge or a collection of charges can be considered the source of an electric field. Lines of the electric field begin on positive charges and end on negative charges. Once the field is known, the force can be calculated on any charged object that is placed within the field.

One electric field device is the capacitor, consisting of a pair of conductive plates separated by a thin layer of non-conducting material like air, plastic, or ceramic (Figure 1.5[a]). Positive charge is stored on one plate, while negative charge is stored on the other. The degree to which a capacitor can store charge is the *capacitance*. Increasing the amount of charge on the plates results in an increase in energy stored by the capacitor. Because the plates have opposite charges, an electric field is established between them and this field becomes stronger with increasing charge (Figure 1.5[b]). The stored energy of a capacitor is related to the intensity of the electric field.

The material between the plates is called a *dielectric*. The material property known as *permittivity* influences the amount of electric field between the plates for a given amount of charge. The capacitance is directly proportional to the dielectric permittivity, ε (epsilon). The permittivity of most materials can be

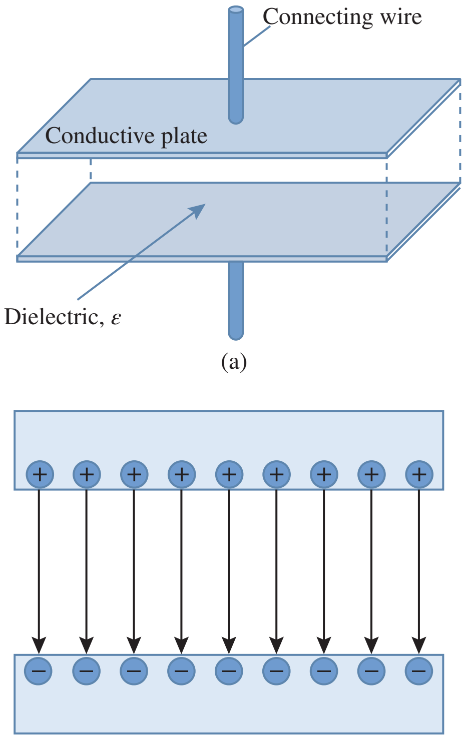

FIGURE 1.5 (a) Simplified view of a parallel plate capacitor. (b) Cross section view shows charge on the plates and resulting electric field lines.

represented as the product of the permittivity of free space, ε_0, and the relative permittivity, ε_r, of the dielectric, or

$$\varepsilon = \varepsilon_r \varepsilon_0 \tag{1.1}$$

A dielectric material's relative permittivity is commonly referred to as its dielectric constant. The free space permittivity is given by

$$\varepsilon_0 = 8.854\ \frac{\text{pF}}{\text{m}} \tag{1.2}$$

where pF is a picofarad and $1\ \text{pF} = 10^{-12}\ \text{F}$. Values of ε_r for some common materials are given in Appendix E.

1.1.2 Magnetic Fields

Whereas charged particles establish an electric field, moving charges generate a magnetic field. The moving charges can be a current or can be the internal motions of electrons contained within magnetic materials. The concept of magnetic fields is similar to that of electric fields, with a key difference. While electric field lines start on a positive charge and end on a negative charge, magnetic field lines have no start and end points. Rather, they form continuous loops, encompassing the source of a magnetic field. Figure 1.6 indicates the closed loop magnetic field lines for a bar magnet.

Analogous to electric fields and capacitors, it is possible to store energy in the magnetic field of inductors. A typical inductor is a coil or length of conductive wire such that current in the wire establishes a magnetic field. Figure 1.7 shows a wire wound inductor, where the field is enhanced by winding the wire around magnetic material characterized by material permeability, μ (mu). The permeability is the product of free space permeability, μ_0, and the

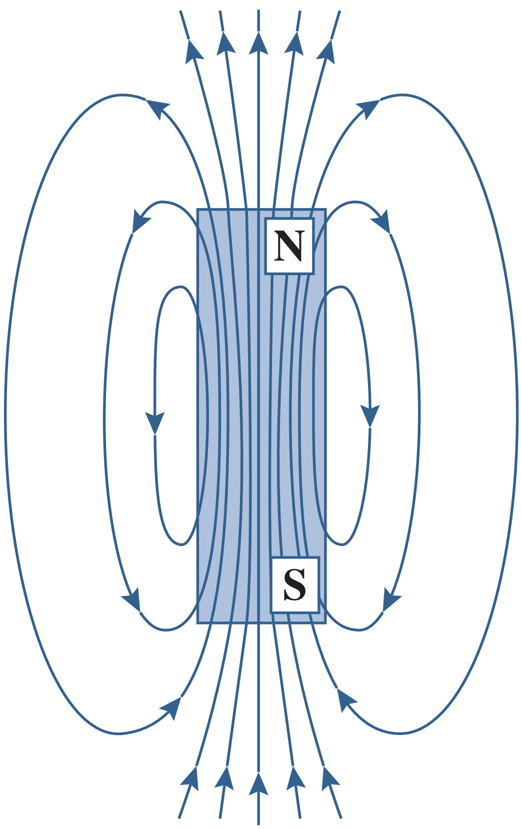

FIGURE 1.6 Bar magnet (with north (N) and south (S) ends shown) generates magnetic field lines that form closed loops.

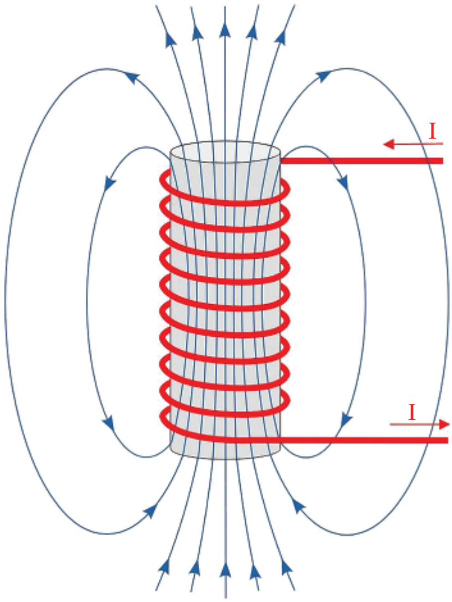

FIGURE 1.7 Magnetic field lines in a wire-wound inductor. The wire wraps a core of permeability μ.

relative permeability of the material, μ_r. So,

$$\mu = \mu_r \mu_0 \tag{1.3}$$

The free space permeability is given by

$$\mu_0 = 4\pi \times 10^{-7} \, \frac{\mathrm{H}}{\mathrm{m}} \tag{1.4}$$

Values of μ_r for some common materials are given in Appendix E.

1.1.3 Field Linkage

In 1801, Andre Marie Ampère discovered that electric current produces magnetic fields. In 1831, Michael Faraday showed that time-varying magnetic fields produce an electric field. James Clerk Maxwell then postulated that a time-varying electric field would produce a magnetic field. This insight led to Maxwell's equations. Experiments confirmed his theory, and the important link between electricity and magnetism began to be fully appreciated. Since a time-varying magnetic field (produced by running a sinusoidal current source through a wire) would generate a time-varying electric field, and since this electric field would in turn generate a magnetic field, and so on, then Maxwell reasoned that an electromagnetic wave would move or propagate away from the

TABLE 1.1 Maxwell's Equations in Differential Form

$$\nabla \cdot \mathbf{D} = Q_v$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}$$

time-varying source. Since the fields are intimately linked, their combination is termed an electromagnetic field. Furthermore, Maxwell theorized that light itself is an electromagnetic wave and that all such waves travel at the speed of light. Although initially met with great skepticism, Maxwell's theory was experimentally verified in 1888 in a series of brilliant experiments by Heinrich Hertz. The relationship between electric and magnetic fields developed by Maxwell in 1863 was elegantly reformulated by Heaviside in 1884 to a set of four concise equations, known as Maxwell's equations (Table 1.1).

Energy is carried by electromagnetic waves. The simplest treatment of waves considers propagation in space free of any material (i.e., *free space*, or a vacuum), but wave propagation also occurs in various media. Waves may be constrained, or guided, by specific structures called waveguides. A transmission line is a special type of waveguide that most often consists of a pair of conductors separated by a dielectric material.

The next chapter begins with the study of electromagnetics by observing waves traveling on transmission lines. Beginning with transmission lines is beneficial because wave phenomena (i.e., attenuation, reflection at boundaries, and standing waves) are presented based on relatively simple circuit theory rather than on more complicated electromagnetic theory. Understanding more general electromagnetic wave propagation will require an understanding of Maxwell's equations, which in turn requires a solid foundation in electric and magnetic fields. Also, these equations contain vectors and vector algebra, and employ a variety of coordinate systems to view and understand the fields. Thus, the equations of Table 1.1 are gradually developed in Chapters 3–5.

1.2 ELECTROMAGNETIC SPECTRUM

Maxwell predicted that light consists of electric and magnetic fields oscillating in tandem. Such an *electromagnetic wave* propagates in a vacuum with velocity

$c = 2.998 \times 10^8$ m/s. Over a very broad frequency range, a continuous[1.2] spectrum of electromagnetic radiation is possible. The spectrum shown in Figure 1.8 ranges from 0.1 Hz up to 10^{23} Hz, where a hertz (Hz)[1.3] is equal to one cycle per second. In vacuum, the frequency f and wavelength λ are related by the speed of light,

$$c = \lambda f \tag{1.5}$$

So, the spectrum can also be indicated in terms of wavelength.

Below 300 GHz, it is customary to refer to electromagnetic waves in terms of frequencies. For instance, the microwave bands employed in radar and communications applications are in terms of frequency, as shown in the Figure 1.8. Above 300 GHz, or at wavelengths below 1 mm, waves are more likely to be referred to in terms of wavelength. Thus, the spectrum of visible light is listed by wavelength in the Figure 1.8.

For wireless communications, the amount of information that can be communicated directly scales with frequency. Therefore, engineers are challenged to continue to develop circuitry, transmitters, and receivers that can handle the ever-increasing frequencies. Furthermore, it may be recalled from physics that the energy U of a photon is proportional to frequency as

$$U = hf \tag{1.6}$$

where h is Planck's constant ($h = 6.63 \times 10^{-34}$ J $\cdot$ s). The energy from frequencies above that of visible light (i.e. X-rays) is capable of breaking covalent bonds in DNA molecules, and therefore, poses a cancer risk. Cell phones operate at frequencies about 5 orders of magnitude lower than visible light and are only capable of adding an insignificant amount of localized heat to the body.

At frequencies associated with ultraviolet and visible light, the signal is severely *attenuated*[1.4] by material media and clouds. Fiber-optic and line-of-site communication schemes are utilized at such frequencies. Wireless communication is undertaken at certain microwave frequencies, between 1 and 100 GHz, where there are windows of relatively low signal attenuation in the atmosphere. Some of the important low attenuation windows are <18 GHz, 26–40 GHz, and 94 GHz.

Efficient transmission of signals requires an antenna of dimension roughly on the order of the signal wavelength. Some AM radio stations require antennas as long as 100 m, and wireless transmission at lower frequencies becomes challenging.

[1.2] The spectrum is continuous, that is, down to the level of a discrete quantum.

[1.3] The hertz is named in honor of German physicist Heinrich R. Hertz (1857–1894).

[1.4] If the amplitude of the wave decreases in the direction of propagation, it is said to *attenuate*.

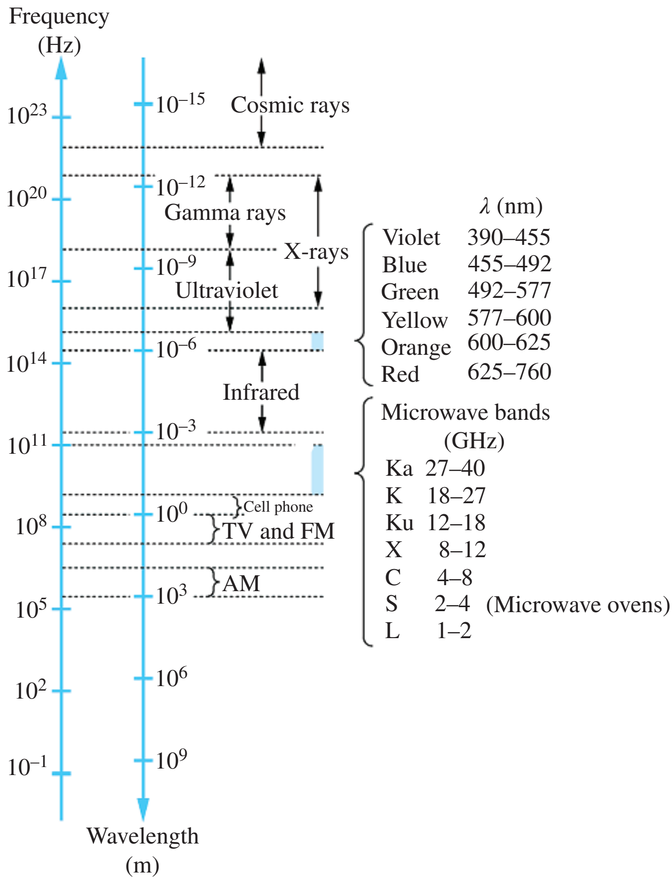

FIGURE 1.8 The electromagnetic spectrum.

1.3 NUMERIC CONSIDERATIONS

For many problems in this text, students must calculate numerical solutions. Rarely is this number, by itself, correct unless the appropriate units are included. For instance, suppose 12 V is dropped across a resistor when 0.2 A passes through it. The resistance is not 60, rather it is 60 Ω.

When reporting large or small numbers, scientific notation gives way to engineering notation, whereby appropriate prefixes are used for multiples of 10^3 (or submultiples of 10^{-3})[1.5]. Common prefix multipliers are listed

[1.5] A notable exception is the ubiquitous use of the centimeter, or a hundredth of a meter.

TABLE 1.2 Engineering Notation Prefixes

Prefix	Pronunciation	Multiplier
T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
m	milli	10^{-3}
µ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}

in Table 1.2. Why use engineering notation instead of scientific notation? Consider an 18 GHz frequency. I can say "eighteen times 10 to the 9 hertz" or I can say "eighteen gigahertz." An efficient engineer prefers brevity, and students must get in the habit of speaking in engineering language before assuming their careers.

Occasionally, students report their answers to a calculation by including all of the digits shown on their calculator. For instance, the resistance calculated at the start of this section might be expressed as "60.127 Ω". This implies more precision than is warranted. Generally, three significant digits (here, "60.1 Ω") are sufficient.

Solving problems may involve calculation with multiple quantities and a variety of units. Students should make sure their answers, including the units, follow from the calculation. Conversion factors (also called "unity ratios")[1.6] are used to convert a quantity in terms of one unit into the terms of another. A good approach is to use a horizontal line to separate numerator and denominator quantities, and vertical lines to separate quantities to be multiplied together. This *dimensional analysis* approach is a convenient way to include the conversion factors and prevents common mistakes like dividing by a conversion factor when it should be multiplied or improperly accounting for number prefixes.

EXAMPLE 1.1

To demonstrate the dimensional analysis approach to a simple problem, find the voltage dropped across a 1.10 kΩ resistor when 10.6 mA passes through it. How much power is dissipated?

[1.6] See Table D.2.

This is a straightforward Ohm's Law problem, where $V = IR$. So,

$$V = \frac{10.6\text{ mA}}{} \left| \frac{1.1\text{ k}\Omega}{} \right| \left| \frac{\text{A}}{1000\text{ mA}} \right| \left| \frac{1000\ \Omega}{1\text{ k}\Omega} \right| \left| \frac{\text{V}}{\Omega \cdot \text{A}} \right|$$

Some of the units in the numerator and in the denominator (like mA) will cancel, leaving

$$V = 11.7\text{ V}$$

Dissipated power in the resistor is $P = VI$. So,

$$P = \frac{11.7\text{ V}}{1} \left| \frac{10.6\text{ mA}}{1} \right| \left| \frac{\text{A}}{1000\text{ mA}} \right| \left| \frac{\text{W}}{\text{V} \cdot \text{A}} \right|$$

$$= \frac{124 \times 10^{-3}\text{ W}}{1} \left| \frac{10^3\text{ mW}}{\text{W}} \right| = 124\text{ mW}$$

Drill 1.1 If 1.08 V is measured across a resistor that has 7.43 mA passing through it, (a) calculate R. (b) How much power is dissipated in the resistor? (*answers:* (a) $R = 145\ \Omega$, (b) $P = 8.02$ mW)

Drill 1.2 Suppose a 9 V battery is connected to a resistor to dissipate 100 mW for a small heating application. (a) How much current must pass through R? (b) What value of R is needed? (*answers*: (a) $I = 11.1$ mA, (b) $R = 810\ \Omega$)

EXAMPLE 1.2

Find the energy associated with a photon of blue light at 475 nm.

From (1.6) the energy of a photon is $U = hf$ where $h = 6.63 \times 10^{-34}$ J · s. Since speed of light $c = \lambda f$,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8\text{ m}}{s} \left| \frac{1}{475\text{ nm}} \right| \left| \frac{10^9\text{ nm}}{\text{m}} \right| \left| \frac{\text{Hz} \cdot \text{s}}{1} \right|$$

$$= \frac{631 \times 10^{12}\text{ Hz}}{1} \left| \frac{\text{THz}}{10^{12}\text{ Hz}} \right| = 631\text{ THz}$$

Now the energy is found as

$$U = hf = (6.63 \times 10^{-34}\text{ J} \cdot \text{s})(631 \times 10^{12}\text{Hz})\left(\frac{1}{\text{Hz} \cdot \text{s}}\right) = 4.18 \times 10^{-19}\text{J}$$

Drill 1.3 Find the energy associated with a photon at 100 GHz. (*answer:* 66.3×10^{-24} J)

Drill 1.4 Popular for high-power applications, the Nd:YAG laser has a typical wavelength of 632.8 nm. What is its operating frequency? (*answer:* 474 THz)

1.4 WIRELESS COMMUNICATIONS

Cellular telephones are truly sophisticated feats of engineering. In addition to basic phone service, modern cell phone options include Internet access, text messaging, e-mail, music playing, camera, gaming, personal digital assistant (PDA), and global positioning system (GPS) navigation.

There is a limited number of frequency channels available to handle cellular communications; far fewer than would seem necessary to handle the millions of transactions that take place daily. The way a cellular system can handle all of these transactions is to break up a town or city into multiple small sections, or cells, each serviced by its own cell tower. These cells are typically arranged in a hexagonal grid, as shown in Figure 1.9. Each cell, since it has six neighbors, can use one-seventh of the available frequency channels. Since the transmit and receive power is not strong enough to communicate with a tower two cells away, non-adjacent cells can use the same frequency channels.

Communication between the cell phone and the tower is represented in Figure 1.10. The towers are tied into the conventional phone grid. The signal being transmitted by the cell phone is at a different frequency than the received signal frequency. This allows simultaneous transmission and reception, unlike, for instance, a pair of walkie-talkies that use a single frequency. In the cell phone, the analog voice signal is converted to a digital signal via the

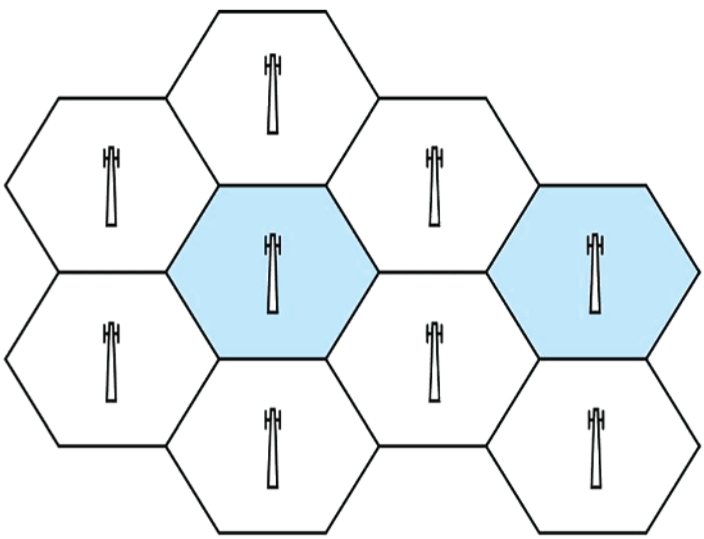

FIGURE 1.9 Hexagonal cell tower grid. The two shaded grids can use the same frequencies.

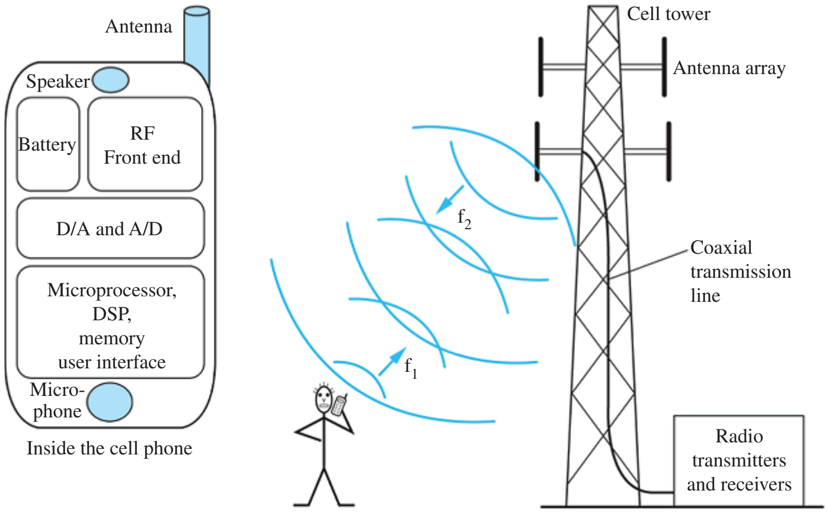

FIGURE 1.10 Typical cellular telephone system.

analog-to-digital (A/D) converter. Digital signals can be compressed and broadcast in a variety of communication schemes[1.7] that enable many users to occupy the same system at once. The digital signal processing (DSP) block handles very high-speed signal calculations, typically over 100 million instructions per second (MIPS), with speeds reported more than 1000 MIPS. The microprocessor handles other operations, including the user interface (display for output and keypad for input) and accessing memory (perhaps using a "flash card" containing phone numbers or other information). The radio frequency (RF) front end amplifies the weak received RF signal and converts it to a lower frequency required by the rest of the electronics in the cell phone. The front end must also raise the frequency of an output signal to the desired RF transmission frequency and boost its power. Finally, the front end must separate the functions of reception and transmission, which use the same antenna. The cell phone antenna must be small and unobtrusive. At the tower, each vertical bar is typically an array of patch antennas.

Understanding the signal processing and the information handling required for a cellular system is a worthy objective; it is expected that these topics will be covered in some of the students' other courses. But where does electromagnetics come into play in a cellular system? Certainly, the physical

[1.7] For instance, time division multiple-access (TDMA), frequency division multiple-access (FDMA), code division multiple-access (CDMA).

operation of microelectronic devices is governed by the laws of electromagnetics. Here are some other ways in which electromagnetics plays a role:

- Waves propagate in transmission lines such as coaxial cables (Chapter 2)
- Efficient signal handling requires impedance matching of transmission lines (Chapter 2).
- Waves propagate in space and through material media (Chapters 5 and 6).
- Communications between towers may employ fiber optics and optical components (Chapter 7).
- Waves are radiated and received by antennas (Chapter 8).

Besides cellular phone systems, other wireless communications systems include direct broadcast satellite (DBS) systems for television, GPS service for navigation, and RF identification (RFID) tags for inventory control and item tracking. Understanding any of these wireless systems requires a solid foundation in electromagnetics.

1.5 WAVE FUNDAMENTALS

Toss a stone in a quiet pool and observe the ripples traveling radially away from the point of impact. These water waves travel, or propagate, at a particular velocity and carry energy with them. The medium itself (water) only bobs up and down. Other types of waves include sound waves, mechanical waves traveling as ripples in a rope, waves in a stretched slinky, and, of course, light traveling as electromagnetic waves.[1.8]

Consider a sinusoidal wave traveling in the k direction. A general equation for such a wave is

$$\boxed{A(k, t) = A_0 e^{-\alpha k} \cos(\omega t - \beta k + \phi)} \tag{1.7}$$

Here, $A(k,t)$ represents the value of the wave at some point k at a specified time t. The *amplitude*, $A_0 e^{-\alpha k}$, is made up of the initial amplitude at $k = 0$, A_0, and an exponential term to account for attenuation of the wave as it propagates. The *phase* inside the cosine argument consists of three parts: ωt, where ω is the *angular frequency* ($\omega = 2\pi f$), with units of radians per second (rad/s); βk, where β is the *phase constant*[1.9] ($\beta = 2\pi/\lambda$) with units of rad/m, and the *phase shift* ϕ.

[1.8] Physicists theorized the existence of gravitational waves. Verification arrived in 2015, with detection of gravity waves by the LIGO (Laser Interferometer Gravitational-Wave Observer).

[1.9] Another term often used in place of phase constant is *wave number* (most often represented by k). While phase constant and wave number quite often mean the same thing, they can be different for some guided wave structures.

EXAMPLE 1.3

A wave propagating in the $+k$ direction is given by

$$A(k,t) = 10\cos(\omega t - \beta k)$$

where $\omega = 1000\pi$ rad/s and $\beta = \pi$ rad/m. Determine the wave frequency and wavelength.

Since $\omega = 2\pi f$,

$$f = \frac{\omega}{2\pi} = \frac{1000\pi \text{ rad/s}}{2\pi \text{ rad/cycle}} = 500\frac{\text{cycles}}{\text{s}} = 500 \text{ Hz}$$

Next, $\beta = 2\pi/\lambda$, so

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi \text{ rad/cycle}}{\pi \text{ rad/m}} = 2\frac{\text{m}}{\text{cycle}} = 2 \text{ m}$$

Assume the phase shift ϕ is zero, and the wave equation when $k = 0$ becomes

$$A(0,t) = A_0 e^{-\alpha(0)}\cos(\omega t - \beta(0) + 0) = A_0\cos(\omega t) \tag{1.8}$$

This is plotted[1.10] in Figure 1.11 for $A_0 = 0.80$ and $f = 1$ MHz. A characteristic of a sine or cosine wave is that it repeats every 2π radians (or 360°). Put another way, we have $\cos(\omega t) = 1$ for $\omega t = n2\pi$, where $n = 0; 1; 2 \ldots$ The period

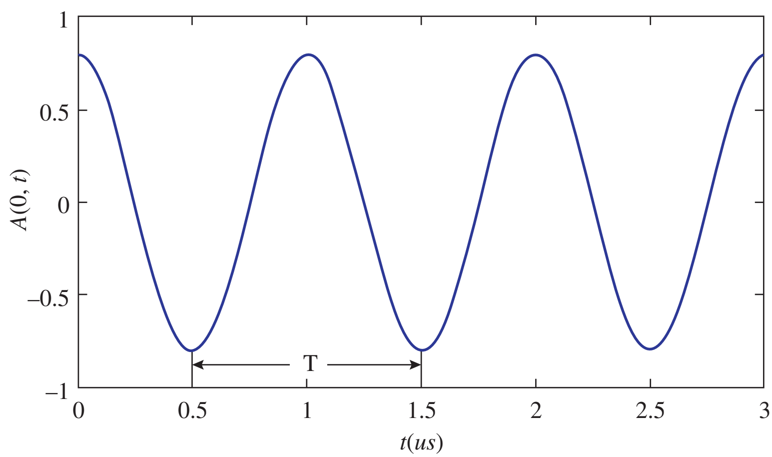

FIGURE 1.11 Plot of $A(0,t) = A_0\cos(\omega t)$ with $A_0 = 0.80$, $c = 3 \times 10^8$ m/s, and $f = 1$ MHz.

[1.10] The MATLAB programs for this and the other plots in this section are provided at the companion website.

T indicated in Figure 1.11 is the time elapsed for one cycle, or $\omega T = (1)2\pi$. Thus,

$$\boxed{T = \frac{2\pi}{\omega} = \frac{1}{f}} \tag{1.9}$$

The phase shift ϕ is now reinserted into equation (1.8),

$$A(0, t) = A_0 \cos(\omega t + \phi) \tag{1.10}$$

This is plotted in Figure 1.12 with $\phi = -45°$. This trace leads the original one by 45°.

Returning to $\phi = 0$, the behavior of the wave versus position k can be studied at time $t = 0$. Assuming the wave is in a lossless medium (like vacuum), there is no attenuation. The attenuation constant α equals zero and $e^{-\alpha k}$ equals 1. So,

$$A(k, 0) = A_0 e^{-0(k)} \cos(\omega(0) - \beta k + 0) = A_0 \cos(-\beta k) = A_0 \cos(\beta k) \tag{1.11}$$

This is plotted in Figure 1.13. Once again there is a 2π radian repeat cycle, or $\cos(\beta k) = 1$ when $\beta k = n2\pi$. One cycle is a wavelength long, or $\beta\lambda = (1)2\pi$. This is rearranged to give the relation between wavelength and phase constant,

$$\boxed{\beta = \frac{2\pi}{\lambda}} \tag{1.12}$$

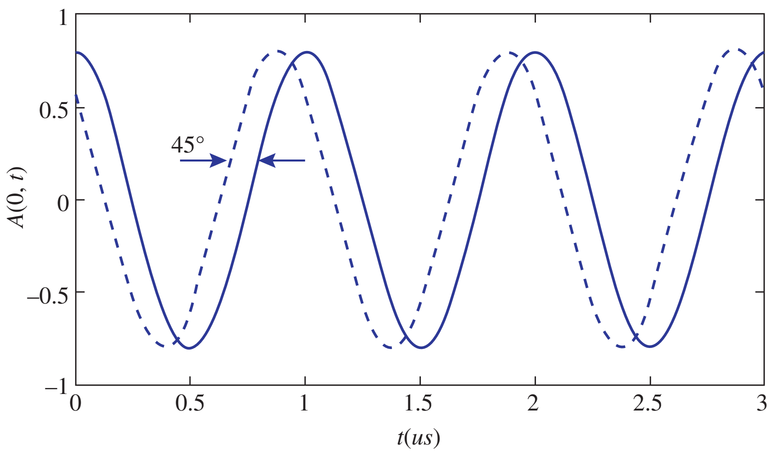

FIGURE 1.12 Plot of $A(0,t) = A_0\cos(\omega t + \phi)$ with $A_0 = 0.80$ and $f = 1$ MHz. The solid trace is with zero phase shift ($\phi = 0°$), while the dashed trace is for $\phi = -45°$.

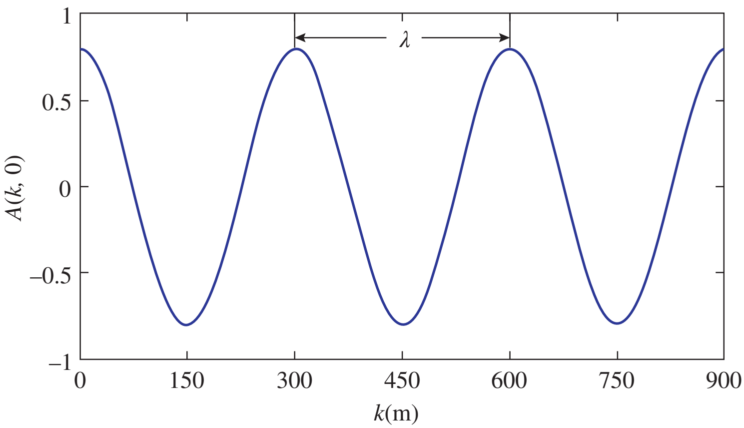

FIGURE 1.13 Plot of $A(k,0) = A_0\cos(-\beta k)$ with $A_0 = 0.80$ and $f = 1$ MHz. The distance between peaks is a wavelength λ.

Reinserting attenuation:

$$A(k,0) = A_0 e^{-\alpha k}\cos(-\beta k) \tag{1.13}$$

Arbitrarily assuming attenuation constant $\alpha = 0.50$ Np/m, the wave amplitude decreases with k as seen in Figure 1.14. The rate of decrease is given by the *attenuation constant* α, in nepers per meter (Np/m), where the

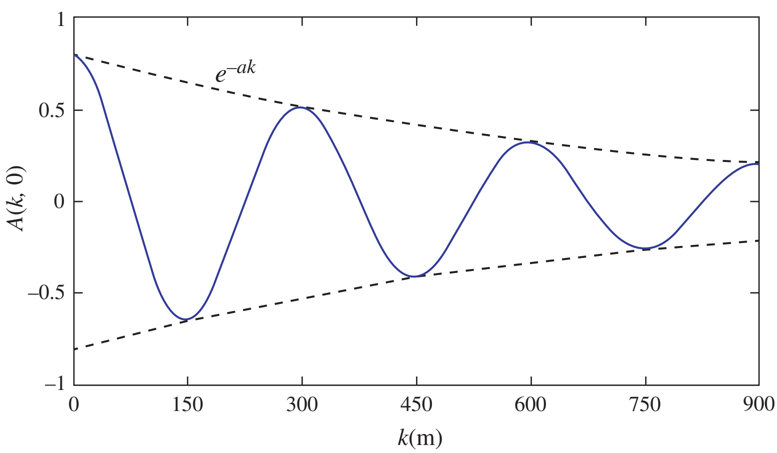

FIGURE 1.14 Plot of $A(k,0) = A_0\, e^{-\alpha k}\cos(-\beta k)$ with $A_0 = 0.80$, $f = 1$ MHz, and an arbitrarily chosen attenuation constant $\alpha = -0.50$ Np/m.

neper is a dimensionless unit. Chapter 6 will describe how the properties of the medium through which the wave travels determine the attenuation constant.

Wave propagation information is also gleaned from the wave equation of (1.7). Assuming a lossless medium ($\alpha = 0$) and letting $\phi = 0$, the wave equation becomes

$$A(k,t) = A_0 e^{-0(k)} \cos(\omega t - \beta k + 0) = A_0 \cos(\omega t - \beta k) \tag{1.14}$$

This is plotted in Figure 1.15 at progressive times. For each trace, a dot has been added representing a constant phase point on the wave. As time increases, the phase point is moving in the $+k$ direction, so this is a $+k$ traveling wave. The phase point moves like a surfer riding a wave. How fast is the surfing phase point, and hence the wave, traveling? Consider the phase

$$\omega t - \beta k = C \tag{1.15}$$

where C is an arbitrary constant representing a constant phase point as demonstrated by the dots in Figure 1.15. Taking the time derivative of both sides of this expression,

$$\omega - \beta \frac{dk}{dt} = 0 \tag{1.16}$$

which is rearranged as

$$\frac{dk}{dt} = \frac{\omega}{\beta} \tag{1.17}$$

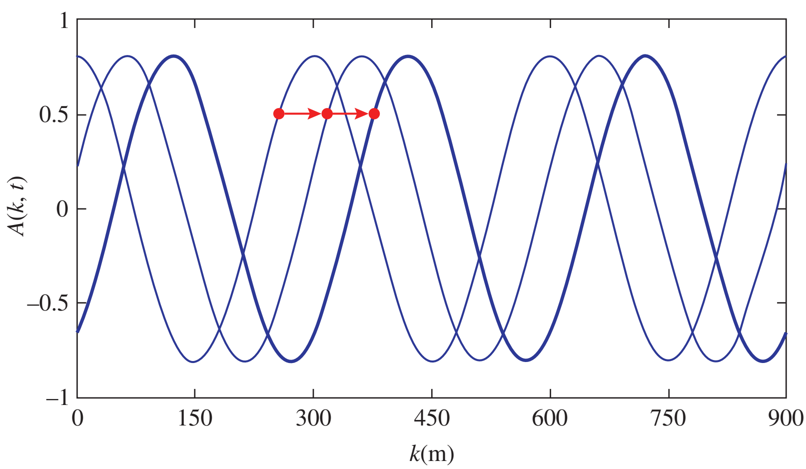

FIGURE 1.15 Plot of $A(k,t) = A_0\cos(\omega t - \beta k)$ with $A_0 = 0.80$ and $f = 1$ MHz. A constant phase point travels at a speed $u_p = \lambda f$.

Here, *dk/dt* is the *phase velocity*, u_p. Also,

$$\frac{\omega}{\beta} = \frac{2\pi f}{2\pi/\lambda} = \lambda f \tag{1.18}$$

thus finding the well-known expression relating wave velocity to wavelength and frequency,

$$u_p = \lambda f \tag{1.19}$$

The phase velocity is determined by the permeability and permittivity of the medium, as will be seen in Chapter 5.

Drill 1.5 Determine the phase velocity of the wave in Example 1.3. (*answer:* 1 km/s)

A MATLAB program to create a movie illustrating traveling wave motion is provided on the companion website.

EXAMPLE 1.4

In Chapter 3, electric field intensity is represented by E, in units of volts per meter. Given a 100 MHz electric field with 1 V/m amplitude propagating in the $+k$ direction in air, write a wave equation in the form of equation (1.7).

It is reasonable to assume air is a lossless medium, so attenuation $\alpha = 0$. Since frequency is 100 MHz, the angular frequency is

$$\omega = 2\pi f = \left(\frac{2\pi \text{ radians}}{\text{cycle}}\right)\left(\frac{100 \times 10^6 \text{Hz}}{1}\right)\left(\frac{\text{cycle}}{\text{Hz} \cdot \text{s}}\right) = 200\pi \times 10^6 \,\frac{\text{radians}}{\text{s}}$$

Also, since the speed of light c is approximately 3×10^8 m/s, the wavelength is $\lambda = c/f = 3$ m. The phase constant $\beta = 2\pi/\lambda = 2\pi/3$ (rad/m). Therefore,

$$E(k,t) = 1\cos(200\pi \times 10^6 t - (2\pi/3)k + \phi)\frac{\text{V}}{\text{m}}$$

Determining phase shift ϕ requires more information. If, for instance, it is known that $E(0,0) = 1$ (V/m), then

$$\begin{aligned} E(0,0) &= 1\cos(200\pi \times 10^6(0) - (2\pi/3)(0) + \phi)\frac{\text{V}}{\text{m}} \\ &= 1\cos(\phi)\frac{\text{V}}{\text{m}} = 1\,\frac{\text{V}}{\text{m}} \end{aligned}$$

And therefore $\phi = 0°$. The wave equation becomes

$$E(k,t) = 1\cos(200\pi \times 10^6 t - (2\pi/3)k)\,\frac{\text{V}}{\text{m}}$$

■

Drill 1.6 Suppose a propagating electric field is given by

$$E(k,t) = 34e^{-0.002k}\cos(2\pi \times 10^9 t - 10\pi k + 45°)\ \frac{\text{V}}{\text{m}}$$

Find (a) the initial amplitude at $k = 0$, (b) the attenuation constant, (c) the wave frequency, (d) the wavelength, and (e) the phase shift in radians. (*answers*: (a) 34 V/m, (b) 0.002 Np/m, (c) 1 GHz, (d) 0.20 m, (e) $\pi/4$ rad)

1.6 PHASORS

Many electromagnetic applications involve fields that vary sinusoidally with position and time. Such *time-harmonic* fields are encountered in a host of communications applications, and of course, all AC circuitry is sinusoidal. In addition, repeated pulses of information may be treated as a Fourier Series of sinusoidal waves.

A time-harmonic signal can be represented as a *phasor*, which gives the magnitude and phase of a sinusoidal wave. The advantage of doing so is that the time factor is removed from the analysis, and time derivatives and integrals become simple algebraic exercises. Phasors are based on the use of complex numbers.[1.11] At a fixed point in time, the value of a sinusoidal wave can be represented versus position by a polar plot of its amplitude r and phase θ. This polar plot can be superimposed onto a set of *real* (*Re*) and *imaginary* (*Im*) axes, as shown in Figure 1.16. The complex value can be written $re^{j\theta}$, or as is evident from the figure,

$$re^{j\theta} = r\cos\theta + jr\sin\theta \tag{1.20}$$

The relation between the polar form of the complex value ($re^{j\theta}$) and the rectangular form ($r\cos\theta + jr\sin\theta$) is given by Euler's[1.12] identity:

$$e^{j\theta} = \cos\theta + j\sin\theta \tag{1.21}$$

Note that $\cos\theta$ is the real part of $e^{j\theta}$, or $\cos\theta = Re[e^{j\theta}]$, and $\sin\theta$ is the imaginary part, or $\sin\theta = Im[e^{j\theta}]$. It is also customary to employ a shorthand polar form, or

$$re^{j\theta} = r\underline{|\theta} \tag{1.22}$$

[1.11] See Appendix C? or Companion Website for a summary of complex numbers.

[1.12] The name Euler is pronounced "oiler."

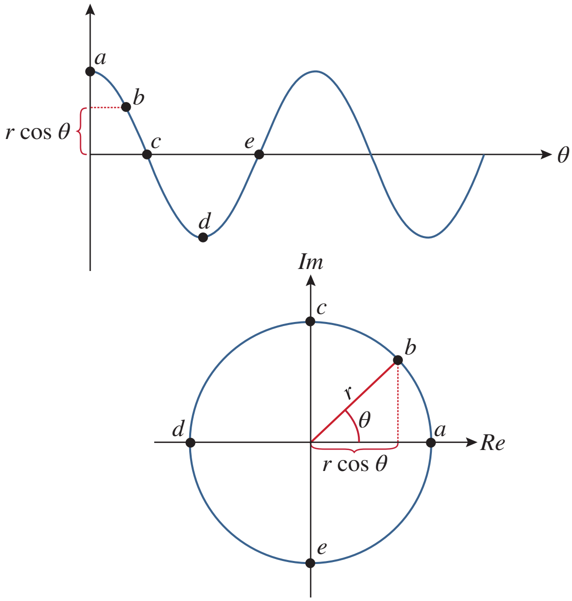

FIGURE 1.16 Cosine function plot synchronized with a polar plot.

EXAMPLE 1.5

Convert the polar form expression $e^{j\pi/2}$ to rectangular form.

Applying Euler's identity equation (1.21) where $\theta = \pi/2$,

$$e^{j\pi/2} = \cos(\pi/2) + j\sin(\pi/2) = 0 + j = j$$

Drill 1.7 Convert the polar form expression $e^{j\pi}$ to rectangular form. (*answer:* -1)

To see how a time-harmonic signal can be represented by a phasor, consider a general wave in the $+k$ direction as given by equation (1.7)

$$A(k, t) = A_0 e^{-\alpha k} \cos(\omega t - \beta k + \phi)$$

Using Euler's identity, this can be written

$$A(k, t) = \mathrm{Re}[A_0 e^{-\alpha k} e^{j(\omega t - \beta k + \phi)}] \tag{1.23}$$

This can be reorganized as

$$A(k, t) = \mathrm{Re}[A_S(k) e^{j\omega t}] \tag{1.24}$$

where the phasor form of the wave is

$$A_S(k) = A_0 e^{-\alpha k} e^{-j\beta k} e^{j\phi} \tag{1.25}$$

The phasor, written with an s subscript,[1.13] has amplitude $A_0 e^{-\alpha k}$ and phase $\phi - \beta k$. The time dependence term, $e^{j\omega t}$, for this time-harmonic wave is removed from the phasor form. To convert the phasor back to instantaneous form, the $e^{j\omega t}$ term is reinserted and the real part taken.

A MATLAB program in the companion website creates an animation comparing location of a point on a polar plot with its corresponding location on a cosine wave.

EXAMPLE 1.6

Convert the instantaneous quantity $A(t) = 16\cos(\pi \times 10^6\, t + \pi/3)$ to a phasor.

$$A(t) = 16\cos(\pi \times 10^6 t + \pi/3) = \mathrm{Re}[16\, e^{j\pi/3} e^{j(\pi\times 10^6)t}]$$

and the phasor is

$$A_S = 16\, e^{j\pi/3}$$

This phasor is a constant and not a function of k, so A_S is used rather than $A_S(k)$. ■

Drill 1.8 Convert the instantaneous quantity $A(x, t) = A_0 \sin(4\pi \times 10^8\, t + 2x)$ to a phasor. (*answer:* $A_S(x) = A_0 \mathrm{e}^{\,j(2x - \pi/2)} = -jA_0 \mathrm{e}^{j2x}$)

EXAMPLE 1.7

Convert the phasor $A_S = j5e^{\,j\pi/4}$ to an instantaneous quantity.

The expression for A_S is inserted into equation (1.24):

$$A(t) = Re[A_S e^{j\omega t}] = Re[j5e^{j\pi/4} e^{j\omega t}]$$

Recalling that $e^{j\pi/2} = j$,

$$\begin{aligned} A(t) &= Re[5e^{j\pi/2} e^{j\pi/4} e^{j\omega t}] = \mathrm{Re}[5e^{j3\pi/4} e^{j\omega t}] \\ &= Re[5e^{j(\omega t + 3\pi/4)}] = 5\cos(\omega t + 3\pi/4) \end{aligned}$$

■

Drill 1.9 Convert the phasor $A_S = 10e^{-j\pi/4}$ to an instantaneous quantity. (*answer:* $A(t) = 10\cos(\omega t - \pi/4)$)

[1.13] The student may recall the s-domain in circuit analysis, where $s = j\omega$.

SUMMARY

- The spatial distribution of a property is known as a field. The field concept is useful for dealing with forces that act at a distance, such as gravity and electromagnetic interactions.
- Electric charge produces an electric field. Electric field lines start at a positive charge and end at a negative charge.
- Electric current produces a magnetic field. Magnetic field lines form closed loops.
- A capacitor stores energy in the electric field, while an inductor stores energy in the magnetic field.
- Time-varying electric and magnetic fields are linked, and the combination is termed the electromagnetic field.
- An electromagnetic wave carries energy and propagates at the speed of light. In vacuum, the frequency f and wavelength λ are related to the speed of light c by $c = \lambda f$.
- Many electromagnetic concepts are demonstrated in a wireless communications system, including wave propagation, antennas, RF circuits, and electromagnetic interference.
- The general sinusoidal wave representation is

$$A(k,t) = A_0 e^{-\alpha k}\cos(\omega t - \beta k + \phi)$$

where A_0 is the amplitude at $k = 0$ and α is the attenuation constant. The cosine argument is the phase, with angular frequency ω (radians/s), phase constant β (radians/m), and phase shift ϕ (radians).
- Using Euler's equation and phasors can simplify analysis for time harmonic problems by separating the time and the position dependencies. The phasor for

$$A(k,t) = A_0 e^{-\alpha k}\cos(\omega t - \beta k + \phi)$$

is

$$A_S = A_0 e^{-\alpha k} e^{-j\beta k} e^{j\phi}$$

PROBLEMS

1.1 ELECTROMAGNETIC FIELDS

1.1 (a) Calculate the product $\sqrt{\mu_0 \varepsilon_0}$. Use Table D.2 to reduce the units to only meters and seconds. (b) Now calculate $1/\sqrt{\mu_0 \varepsilon_0}$. Comment on your result.

1.2 THE ELECTROMAGNETIC SPECTRUM

1.3 NUMERIC CONSIDERATIONS

1.2 How much energy is associated with a photon of red light at 690 nm?

1.3 Consider the spectrum in Figure 1.8. In a vacuum, what is the wavelength range associated with X-band?

1.4 What wavelength, in cm, corresponds to the common microwave oven-operating frequency of 2.45 GHz? Assume air may be treated as vacuum for this problem.

1.4 WIRELESS COMMUNICATIONS

1.5 WAVE FUNDAMENTALS

1.5 A wave propagating in the $-k$ direction is given by

$$A(k, t) = 20\cos(\omega t + \beta k)$$

where $\omega = 100$ rad/s and $\beta = 2$ rad/m. Determine: (a) wave frequency, (b) wavelength, (c) wave velocity.

1.6 **MATLAB:** Figure 1.11 was generated by a MATLAB code available at the companion website. (a) Modify this code to redraw the plot with four cycles instead of three. (b) Further modify the code to add a 90° phase shift.

1.7 **MATLAB:** Figure 1.14 was generated by a MATLAB code available at the companion website. Change the attenuation constant to 0.5 Np/m and replot including grid lines.

1.8 An electric field propagating in the $+k$ direction is given by

$$E(k, t) = 10\, e^{-0.10k} \times \cos\left(\pi \times 10^7 t - \pi k + \frac{\pi}{4}\right) \frac{\text{V}}{\text{m}}$$

(a) Determine the attenuation constant, the wave frequency, the wavelength, the phase velocity, and the phase shift. (b) How far must the wave travel before its amplitude is reduced to 1.0 V/m?

1.6 PHASORS

1.9d Convert the phasor form expression $e^{-j\pi/2}$ to rectangular form.

1.10 Express $(e^{j\theta} + e^{-j\theta})/2$ in rectangular form.

1.11 Convert the phasor $A_S = 4 + j3$ to an instantaneous quantity.

1.12 What is the phasor for the propagating electric field expression of Problem 1.8?