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## Chapter **1**

# What Are Fractions, Decimals, and Percents?

**W**elcome to your journey into the world of fractions! If you pick up this book, you're probably feeling a bit unsure about fractions, and that's completely okay. You're not alone — many people find fractions tricky. The good news is that understanding fractions is like honing any new skill. In the same way that you may have figured out how to balance so that you could ride a bike or caught onto the basic rules of play for a new game, you can grasp the concepts and calculations associated with fractions if you have patience, practice, and the right guidance.

This book is designed to be your friendly companion while you explore fractions and their close cousins, decimals and percents. I give you the basics, breaking down what fractions are and why they matter. I also help you tackle problems one step at a time,

with plenty of examples, tips, and encouragement along the way. My goal is to keep you from ever feeling lost or overwhelmed when working with fractions.

Remember, everyone figures things out at their own pace. So take your time, ask questions, and even make mistakes — you have to do all of those things to get good at anything. And with the help offered in this book, not only can you understand fractions, but you might also discover that they're not so scary, after all.

## Taking a First Look at Fractions

At a basic level, fractions give you a way to represent parts of a whole. Imagine that you have a pizza, and you cut it into eight equal slices. If you eat one slice out of eight, you've eaten one part of the eight slices, or  $\frac{1}{8}$  of the pizza. That's a fraction! Fractions help us understand and work with pieces of something, whether it's food, time, money, or anything else that can be divided into equal sections.

In the following sections, I explain how fractions work to represent parts of the whole, how to read and write fractions for use in calculations, and why fractions are so useful.

### Understanding how fractions work

Fractions use two numbers to show how much of an item you're working with after you divide the whole into equal parts. For example, if you have a chocolate bar that's divided into 4 equal pieces and you eat 3 pieces, the fraction that represents what you ate is  $\frac{3}{4}$ . In this fraction, you use two numbers, 3 and 4, which you write one over the other with a vertical line in between.

Here are the names for these two numbers:

- » **Numerator:** The top number in a fraction (3 in the chocolate bar example), which shows how many of the equal parts you're specifically working with (in this case, you ate them)
- » **Denominator:** The bottom number in a fraction (4 in the chocolate bar example), which shows the total number of equal parts that the whole is divided into

As another example, imagine that you have a rope that's 10 meters long, and you cut off a 7-meter piece. The fraction representing the part of the rope that you cut off is  $\frac{7}{10}$ . The numerator is 7 (because you cut 7 meters), and the denominator is 10 (because the whole rope was 10 meters long).

## Making sense of equivalent fractions

Unlike whole numbers, where different numbers always represent different values (for example, 3 is always less than 5), fractions are a bit more flexible. A pair of fractions can look different, each containing a different *numerator* (top number) and *denominator* (bottom number), and yet their value can be exactly the same.

Different number combinations can represent the same value because fractions rely on the relationship between the numerator and the denominator. For example,  $\frac{1}{2}$  and  $\frac{2}{4}$  might seem different at first glance, but they actually represent the same amount. In both cases, you're talking about half of something — whether it's 1 out of 2 parts, or 2 out of 4 parts.



REMEMBER

Pairs of fractions like this (such as the  $\frac{1}{2}$  and  $\frac{2}{4}$  example) are called *equivalent fractions*. Even though the numerators and denominators are different numbers, the value that they represent is the same. You really need to keep this idea in mind when you add, subtract, and compare fractions! (I go in depth about equivalent fractions in Chapter 4, so flip there if you want all the details.)

## Understanding improper fractions

When you think about fractions, you might think of them as numbers that are less than 1 — that is, numbers that fall between 0 and 1 on the number line. These types of fractions, where the numerator is smaller than the denominator, are called *proper fractions*. For example,  $\frac{4}{5}$  is a proper fraction because 4 is less than 5, so the value it represents is less than 1.

However, fractions can also be equal to or greater than 1, and these types of fractions are called *improper fractions*. In an improper fraction, the numerator is equal to or larger than the denominator, which means that the fraction represents one whole or more.

Improper fractions are common in the real world. For example,  $\frac{5}{4}$  is an improper fraction. It tells us that we have 5 parts, but each whole is made up of only 4 parts. So, if you have  $\frac{5}{4}$  of a cake, you have 1 whole cake and an extra  $\frac{1}{4}$  of another identical cake.

Similarly, if you have 7 meters of fabric divided into 2-meter sections, you have  $\frac{7}{2}$ , which is 3 and a half sections.

## Seeing how improper fractions and mixed numbers relate

In the preceding section, you can explore improper fractions — fractions where the numerator is equal to or greater than the denominator. But in this section, you can find out about another way to express these fractions as mixed numbers. A *mixed number* is simply a combination of a whole number and a proper fraction. It gives you another way of showing the same value that an improper fraction does, but in a form that you can often more easily visualize or understand.

For example, I revisit the improper fraction  $\frac{5}{4}$  that I talk about in the preceding section. This fraction tells you that you have more than one whole, but how much more? To express  $\frac{5}{4}$  as a mixed

number, you break it down into one whole ( $\frac{4}{4}$ ) and the remaining  $\frac{1}{4}$ . So,  $\frac{5}{4}$  is the same as the mixed number  $1\frac{1}{4}$ . Both represent the same amount — 1 whole and  $\frac{1}{4}$  of another whole.

In real life, mixed numbers come up often. Imagine that you're baking (something really sweet) and using 2 full cups of sugar, plus another half cup. Instead of saying that you have  $\frac{5}{2}$  cups, you probably say you have  $2\frac{1}{2}$  cups of sugar. Both expressions are valid, but the mixed number is a little easier to understand in a real-world situation.

Consider another example when measuring length: If you have a board that's 3 full meters, plus an additional  $\frac{1}{10}$  of a meter, you could express that as  $\frac{31}{10}$  of a meter. However, this amount is probably easier to understand if you use the equivalent mixed number  $3\frac{1}{10}$  meters.



REMEMBER

Every mixed number is just another way of writing an improper fraction, and it offers you a clearer picture of how many whole items and how many parts you have than an improper fraction does.

## Seeing Why Decimals Are So User-Friendly

In this section, I want to help you explore a way to express values that fall between whole numbers, allowing you to sidestep using fractions. (See the section “Taking a First Look at Fractions,” earlier in this chapter, for an overview of fractions.) *Decimals* are a more user-friendly alternative to fractions because they align neatly with our base-10 number system, which makes them easier to understand and use in everyday situations.

Although fractions such as  $\frac{1}{2}$  or  $\frac{3}{4}$  represent parts of a whole, decimals express the same ideas, but in a different format. For

example, instead of saying  $\frac{1}{2}$ , you can use the decimal 0.5. Similarly, you can express  $\frac{3}{4}$  as 0.75.



TIP

The digits that fall to the right of the period in a decimal represent the *decimal places*, which I describe in the following two sections.

Decimals are especially handy in contexts such as money, measurements (especially the metric system), and technology, where precision is a key aspect. Whether you calculate the total cost of items at the store, measure ingredients for a recipe, or read a digital scale, decimals offer a straightforward and precise way to represent values that aren't whole numbers. And decimals also bypass many of the difficulties that calculating with fractions can carry (for example, finding a common denominator and simplifying your answer — see Chapter 4 for more on these subtleties of fractions).

In the sections that follow, you can find out how decimals work, how to convert between fractions and decimals, and how to use decimals in various real-world scenarios. And you can discover just how versatile and helpful decimals can be.

## Understanding decimals that have one decimal place

*Decimal place value*, meaning how far each decimal digit is from the decimal point, helps you understand the value of the digits in a decimal number. For decimals that have only one decimal place (that is, one digit to the right of the period), each digit represents *tenths* of the whole.

For example, in the decimal number 0.9, the digit 9 is in the tenths place, where it represents 9 out of 10 equal parts of a whole. So, 0.9 is the same as  $\frac{9}{10}$  of a whole.

Here are a few real-world examples to illustrate how to interpret decimals:

» **Measuring weight:** If you have a bag of nails that weighs 2.3 kilograms, the digit 3 in the tenths place represents  $\frac{3}{10}$

of a kilogram (or 300 grams because one kilogram contains 1000 grams). Thus, the bag of nails weighs 2 kilograms, plus an additional  $\frac{3}{10}$  of a kilogram.

» **Measuring length:** If you have a piece of ribbon that's 3.4 meters long, the digit 4 in the tenths place means  $\frac{4}{10}$  of a meter (or 40 centimeters because one meter contains 100 centimeters). Thus, the ribbon is 3 meters plus another  $\frac{4}{10}$  of a meter in length.

» **Cooking:** If a recipe calls for 1.5 cups of flour, the 5 in the tenths place represents  $\frac{5}{10}$  of a cup, which equals half a cup (because 5 is half of 10). So, 1.5 cups is equal to 1 cup plus an extra half a cup of flour.

The preceding examples show you that decimals use a digit's place value to represent a fraction by applying a format that's similar to whole numbers. This similarity means that you can make calculations and comparisons more easily with decimals than with fractions.

## Understanding decimals that have two decimal places

After understanding decimals that have one decimal place — the tenths place, which breaks a whole into 10 equal parts (see the preceding section) — you can move on to decimals that have two decimal places. These decimals break a whole into 100 equal parts, which are called *hundredths*.

When you have two decimal places, the second digit represents hundredths. For example, 0.01 means 1 out of 100 equal parts of a whole. Similarly, 0.25 means 25 out of 100 equal parts. Think of it like cutting something into 100 tiny pieces and counting how many pieces you have.

Here are some real-world examples of what decimals that have two places represent:

- » **Counting money:** In the United States, people most commonly use decimals that have two places when dealing with money. You can break down every U.S. dollar into 100 cents, where each cent is represented as the decimal \$0.01. So, if you have \$0.25, you have  $\frac{25}{100}$  of a dollar, or 25 cents.
- » **Measuring length:** Another common use for decimals involves measuring length by using the metric system. For example, imagine that you're measuring the thickness of a short stack of paper, like your term paper, and it measures 0.08 centimeters. This means that the paper is  $\frac{8}{100}$  of a centimeter thick, or 8 out of 100 equal parts of a centimeter.
- » **Measuring weight:** Metric weight measurements commonly use decimals. For example, if you weigh a small object and find that it weighs 0.35 kilograms, this indicates that the object weighs  $\frac{35}{100}$  of a kilogram, or 35 out of 100 equal parts of a kilogram.

## Understanding fraction-decimal equivalence

Fractions (flip back to the section “Taking a First Look at Fractions,” earlier in this chapter) and decimals are two contrasting but mathematically equivalent ways of representing parts of a whole. Although they might look different, they both express the same idea: how much of something you have relative to its whole.

And because fractions and decimals express the same ideas, you can convert between them. The following tells you how to convert

- » **From fractions to decimals:** Simply divide the numerator by the denominator. For instance, to convert the fraction  $\frac{3}{5}$  to a decimal, divide 3 by 5, which equals 0.6.

» **From decimals to fractions:** Write the decimal as a fraction that has a denominator based on its place value.

For example, you can write 0.46 as  $\frac{46}{100}$ , which simplifies to  $\frac{23}{50}$ . (You can find out the nuances of how to simplify a fraction in Chapter 4.)

Because fractions and decimals are just two sides of the same coin, you can switch between these forms as needed, making it easier to work with parts of a whole in different contexts.

## Putting Percents into Perspective

You can describe parts of a whole as fractions (discussed in the section “Taking a First Look at Fractions,” earlier in this chapter) and decimals (check out the section “Seeing Why Decimals Are So User-Friendly,” earlier in this chapter); I want to introduce a third way to describe them. You can use *percents*, in the same way that you can use fractions and decimals, to express what part of a whole you have.

For example, you can express a part of a whole as a fraction (for example,  $\frac{1}{2}$ ) or an equivalent decimal (0.5), and you can also express the equivalent amount as a percent (50%). Percents are a useful and often intuitive way to communicate proportions, especially in everyday contexts such as sales discounts, real estate and investment calculations, and statistics and probability.

In the following sections, you can explore how to understand, calculate, and use percents in various situations. And you can make connections between fractions, decimals, and percents while you proceed so that you can see how you can use percents as a powerful tool for comparing and understanding parts of a whole.

## Making sense of percents



REMEMBER

The word *percent* comes from the Latin phrase *per centum*, which literally means *per hundred*. So that can clue you in that percents tell you how many parts you're dealing with specifically, out of 100 equal parts.

For example, if something is 75% of the whole, it means that you're counting 75 out of 100 equal parts. This idea of breaking things into 100 parts makes percents (like decimals) particularly handy for understanding proportions and making comparisons in the real world. For example:

- » **Sales and discounts:** Suppose that a store is having a 20% off sale. If an item originally costs \$50, a 20% discount means that you pay less by taking 20 out of every 100 parts of the original price. To calculate this, you find 20% of \$50, which is \$10:

$$\$50 \times \frac{20}{100} = \frac{1,000}{100} = \$10$$

So the item will now cost \$40:

$$\$50 - \$10 = \$40$$

- » **Test scores:** Imagine that you take a test and score 85%. This score means that you answered 85 out of every 100 questions correctly. If the test had 100 questions, you got 85 of the questions right. If the test had 200 questions (I hope not!), you got 170 correct ( $\frac{85}{100} = \frac{170}{200}$ , and are equivalent fractions; for more on this topic, check out Chapter 4).
- » **Battery life:** How about if your phone shows that it has 60% battery remaining? This percentage means that the phone has over half of its total battery capacity still available; or in other words, 60 out of 100 equal parts of the battery's charge are still available to use.

These examples show how percents show up in everyday life and make it easier to understand and compare different quantities.

# Converting percents to decimals and fractions

Because percents represent parts of a whole — just as fractions and decimals do — when you understand how to convert between these equivalent expressions, you can understand and calculate with these three values more easily. These conversions allow you to switch between different representations of the same value, depending on what form is most convenient and applicable to the situation. Knowing how to move between these forms helps you compare values, perform calculations, and better understand the relationships between different quantities.

## Converting percents to decimals, and vice versa

Percents and decimals are closely related because they both represent parts of a whole using the base-10 system. Converting between these two forms is straightforward and requires only a few simple steps.

To convert a percent to a decimal, follow these steps:

- 1. Remove the percent sign (%).**

If you're working with 75%, write it as simply 75.

- 2. Divide the number by 100.**

You can easily do this calculation by moving the decimal point two places to the left.

For example, 75 becomes 0.75 because  $75 \div 100 = 0.75$ .

To convert a decimal to a percent, follow these steps:

- 1. Multiply the decimal by 100.**

Just move the decimal point two places to the right.

For example, 0.25 becomes 25 because  $0.25 \times 100 = 25$ .

- 2. Add the percent sign (%).**

So the 25 calculated in Step 1 becomes 25%.

## Converting percents to fractions, and the reverse

You can also convert percents into fractions. Because percents and fractions both represent parts of a whole, you can simplify how you calculate and compare values in different contexts.

To convert percents to fractions, follow these steps:

**1. Remove the percent sign.**

If you have 40%, make it simply 40.

**2. Write a fraction that has the percent number over 100.**

For this example, you'd have the fraction  $\frac{40}{100}$ .

**3. Simplify the fraction, if possible.**

For example,  $\frac{40}{100} = \frac{40 \div 20}{100 \div 20} = \frac{2}{5}$ .

Flip to Chapter 4 for more on simplifying fractions.

To convert a fraction to a percent, follow these steps:

**1. Divide the numerator by the denominator to get a decimal.**

For the fraction  $\frac{4}{5}$ ,  $4 \div 5 = 0.8$ .

**2. Multiply the decimal that you calculate in Step 1 by 100.**

In this example,  $0.8 \times 100 = 80$ .

**3. Add the percent sign (%) to the result in Step 2.**

Add the percent sign to 80 to get 80%.