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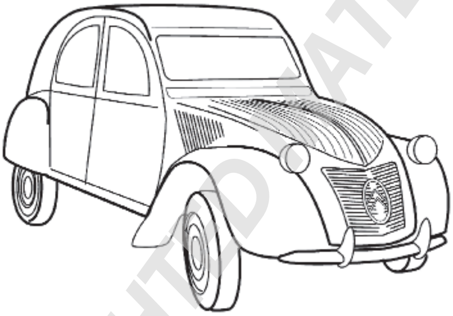
Measurement: A New View



1.1 The Importance of Measurement

> **THERE ARE KNOWN KNOWNS**; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know.

Donald Rumsfeld was thinking of international affairs when he said those words in 2002. But he could have said the same about the electric power system and, in particular, the measurements we make in the power system. He completed his thought with these words:

> … it is the latter category that tends to be the difficult ones.

Regarding measurements in the power system, we agree wholeheartedly! The power system *depends* on measurement. Measurements are used to help the planning, design, operation, and billing for the generation, delivery, and use of electric energy. That situation is surely familiar. It is perhaps less familiar to realize that, in general, the result of *every* measurement made is intended to be a factor in some decision, somewhere. In the power system, measured results generally become part of some mathematical procedure, analyzing the system, for example. *Yet even some of our most familiar measurements are unsuitable for use in mathematics.*

That is one of the "unknown unknowns" of power system measurements. We all tend to have absolute faith in our measurements. Knowing that there exist things that you are not even aware of, and that can matter, is a start to resolving the problem. Over the last half-century, the science of measurement has been steadily advancing until, at this point, we can say that the root cause of measurement problems in the power system has been identified. There is nothing wrong with our instrumentation: The problem lies in our fundamental understanding of the nature of the measurements.

This book is about exposing the unknown unknowns in power system measurements, revealing not only that very few of the measurements we make are actually suitable for the uses we have for them, but also explaining why.

These are strong statements, made relevant by the fact that it *really matters*. Serious money is involved. Worldwide, people spend about 400 billion US dollars for their electricity. Every year.

The power system has experienced a *century-long succession* of measurement problems *for which we have still not found solutions*. This matters now more than at any time in the past, because as the move to renewable resources accelerates and the grid evolves, more measurements will be made, but fewer of them will be subject to human scrutiny. Automated equipment will automatically make use of poor measurements, without any sense of regret.

Recent examples of automated measurement problems have been associated with the tripping of generation, attributed by NERC, the North American Electric Reliability Corporation, to "unknown and unexpected performance issues" according to [1]. Perhaps the expression is not quite right—there have been enough examples that they might now be regarded as "expected" performance issues.

- The Blue Cut fire in Southern California caused more than a GW of solar generation to relay out in 2016. Though the fire did cause some lines to be momentarily tripped [2], none of the affected photovoltaic (PV) sources had been disconnected from the power system.
- In 2017, also in Southern California [3], 900 MW of solar PV generation was "interrupted".
- What must have seemed to be a trend was soon evident. In 2018 [4], there were two "resource interruptions" in Southern California.
- Later that year, a similar problem showed up in England. It started with the loss of less than 300 MW of nearby generation as a result of a lightning strike on

the 400-kV transmission system. Remote sources reduced output, and so much generation was lost that system collapse was avoided only by shedding the load of about a million customers. See [5] for details.[1]

- In 2022 [1], the failure of a surge arrester in Odessa, Texas, caused generation to disconnect across the west of Texas. Much of the generation was associated with inverter-based resources, but not all. In the south of the state, 700 MW of synchronous generation was disconnected. Figure 1.1 shows the locations only of the "unexpected" PV losses.

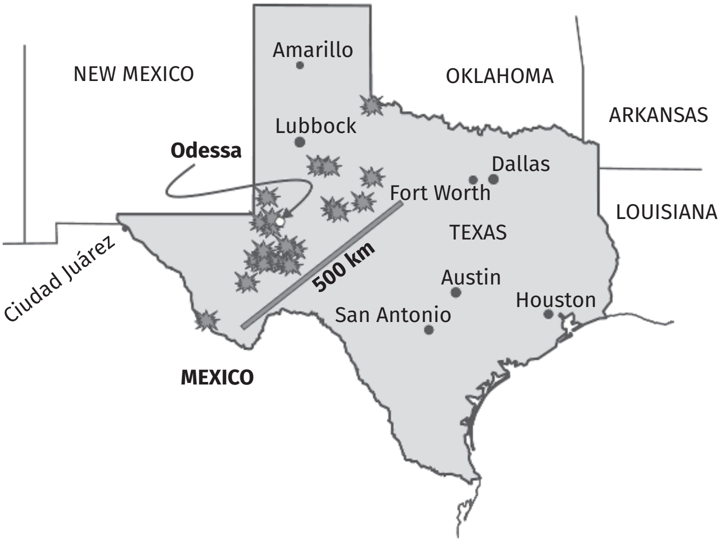


Figure 1.1 Solar PVs disconnected by the Odessa event.

Each of these events involved measurement. At least one of the Blue Cut fire outages resulted from an erroneous frequency measurement. The English blackout got started because of discrepancies between speed measurements. Yet in spite of its importance, we do not teach power system students about measurement. We teach them instead about a power system that can be understood using some "knowledge" of parameters such as voltage or frequency, that knowledge being obtained via some unspecified scheme, evidently involving more hand-waving than the conductor managing the entire choral works of Vivaldi. We need a new view of what measurement is. Our present view has led to too many problems.

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[1] Several UK reports can be found by searching for "Technical Report on the events of 9 August 2019."

Two of the things we measure quite routinely in the power system illustrate the point. One whose name is familiar is *reactive power,* which is such a complicated and fascinating mess that we are obliged to defer it to Chapter 5 before we tackle it. Set that aside for the moment. In this introductory chapter, we will find it instructive to examine *power factor*, the *oldest* measurement *problem* in the power business.

The quantity was defined in 1892. By 1919, measuring it was a serious challenge. The challenge is still unsolved. We examine it because the problems with measuring it are representative of others in power system measurements.

It's not like our community hasn't tried to solve the problem, as we shall see in the next section. There we look not just at the problem of power factor, but also at the efforts people have made (and are still making) seeking a solution. Here, and throughout the book, we assume the reader knows enough about the power system to follow the discussion.

1.2 Power Factor and the AIEE-NELA Committee

Power factor was defined in a paper by John Ambrose Fleming on the subject of transformers [6], as the ratio of the real power to the apparent power in a circuit.

You might think that was straightforward enough, but it seems it wasn't. By 1919, measuring it had become such a *serious* problem that people were talking and writing about it. The problem was not ignored. The community did what it usually does when faced with a serious problem: they formed a committee.

Measuring power factor was problematic for the member companies of NELA, the National Electric Light Association. It is reasonable to assume that in the early 20th century, the electric light was almost a synonym for power system load. Nevertheless, the power factor was not unity at all the loads, and NELA companies had to measure power factor.

John Ambrose Fleming (1849–1945) was born in Lancaster, England. He went to several universities and graduated with First Class Honours in physics and chemistry from Cambridge.

He is credited with inventing the thermionic valve (tube), a diode. An expert in power engineering, he designed the system that Marconi used to accomplish the first transatlantic radio communication.

In a paper presented on November 24, 1892 [6], he created the term "power-factor" (with the hyphen), remarking that "If the currents and pressures were simple sine functions, then the power-factor in that case would be the cosine of the angle of lag of primary current behind the primary terminal potential difference."

Unfortunately, he did *not* say that the cosine was applicable *only* in the case that the "currents and pressures" were well-modeled by sinusoids.

NELA came into existence in 1885, and in 1919, having in those 34 years seen the benefits of having the occasional out-of-town shindig, they were holding their 42nd Convention, in Atlantic City, New Jersey. At the Convention, they announced they were forming a Special Joint Committee (SJC) with AIEE (American Institute of Electrical Engineers, forerunner of IEEE) to look into the matter of defining power factor.

The committee (SJC) was to follow this process [7]:

> First—To consider the purpose to be fulfilled in the use of the term "power factor" in the commercial engineering and scientific aspects of the electrical art.
>
> Second—To offer for the electrical industry in the United States a definition which will definitely and correctly express this purpose and will be suitable for scientific, legal and commercial use.
>
> Third—To study, probably in conjunction with the manufacturers, the best available means of measuring the function thus defined.

The process described (on page 494 of the report) was perfectly appropriate. It recognized the importance of *purpose* as the first thing to establish. Having established a purpose, the committee was then going to find the right *definition*. Having got the definition, they were going to see how to *implement* it. (Remember, this was an era of electromagnetic instrumentation, and it was not just a matter of figuring out a software update, which would be today's solution.)

The SJC was a well-qualified and experienced group of engineers. The chair was Ross McClelland, the chief engineer of the Electric Bond and Share Company of New York. (This was a holding company spun off from General Electric.) The SJC vice-chair was Farley Osgood, the vice-president and general manager of Public Service Electric and Gas in New Jersey. He had been vice-president of AIEE (1914–1916) and would be elected president in 1924. The secretary was S.G. Rhodes, who was the chairman of the AIEE Committee on Instrumentation and Measurements. Arthur Kennelly, who had worked for Edison in West Orange between December 1887 and March 1894, was a member. At the time of the Special Joint Committee, Kennelly was a professor of electrical engineering at both MIT and Harvard.

According to the Annual Report of the AIEE Committee on Instrumentation and Measurements, the "Special Joint Committee canvassed the manufacturing, operating, scholastic and purely engineering branches of the electrical industry and prepared a report of its findings."

Analyzing the large amount of material they obtained took a while! Some of the findings were reported in a survey paper. It was reported that many utilities were having difficulties using power factor as a three-phase quantity [8]. Here are some excerpts:

> Over and over again the power companies send in this complaint, “We can find no practical method of metering power factor.”
>
> One big western company writes, “We are strictly up against it and welcome suggestions which if followed would give us relief.”
>
> An Ohio central station company says, “We hope a commercial kv-a. meter will soon be on the market and believe we should sell kv-a. instead of kw.”

Is there not a sense of despair in these comments?

1.3 The Joint Committee Failure

The Special Joint Committee tried hard but they were unable to deliver the goods. Their finding was that they could not reduce the number of definitions of power factor below two, because in some conditions one definition worked, and in others, a different definition was needed. This result was disclosed at a meeting of AIEE, where it was not much helped by the multitude of people who thought they knew better, and added their own favorite definitions in a series of papers presented at the same meeting.

Definition 1 of the SJC was this:

> Power factor in a polyphase circuit is the ratio of the total watts to the (arithmetical) sum of the volt-amperes in the several phases, each measured to a non-inductive neutral point. This definition may be otherwise expressed as the weighted mean of the individual power factor in the phases (weighted according to the volt-amperes in each phase).

Definition 2 was:

> Power factor in a polyphase circuit is the ratio of the total watts to the vector sum of the volt-amperes in the several phases.

Evans [9] offered a familiar equation, $P.F. = \frac{P}{\sqrt{P^2 + Q^2}}$ that required the measurement of reactive power. That particular measurement was being questioned a decade after the SJC, and is still troublesome (see Chapter 5).

Fortescue [10] had much the same idea, but wanted three-phase measurements, and the inclusion of an “unbalance factor.”

Fechheimer [11] also viewed unbalance as being as important as power factor: "The power factor is the ratio of the true watts to the volt-amperes in the balanced positive sequence systems of volts-and amperes. The unbalance factor is the ratio of the negative sequence amperes to the positive sequence amperes."

Lincoln [12] defined polyphase PF as $P.F. = \frac{EI\cos\varphi}{\sqrt{(EI\sin\varphi)^2 + (EI\cos\varphi)^2}}$, which is a version of the formula of Evans, with the details of the proposed measurement of power and reactive power spelled out on the assumption that the various signals were all well-represented as sinusoids.

Holtz [13] (who was employed by an instrument maker), described an instrument that measured "apparent energy" to derive PF.

It is pointed out that the 2010 edition of IEEE standard 1459 [14] added one more animal to the zoo:

$$P.F. = \frac{\left[1 + \left(\frac{P_H}{P_1}\right)\right] PF_1}{\sqrt{1 + \mathrm{THD}_I^2 + \mathrm{THD}_V^2 + (\mathrm{THD}_I \times \mathrm{THD}_V)^2}}$$

as if harmonic power P_H, fundamental power P_1, and fundamental power factor PF_1 were routinely measured quantities, along with the total harmonic distortions of voltage and current.

The AIEE meeting discussing these alternatives was evidently quite—how shall we say this—invigorating! In an endeavor to bring calm, if not sanity, to the discussion, Professor Vladimir Karapetoff told a story that, while it was not presented as a paper, was considered worthy of being reported in the Journal of the AIEE [15]. We repeat it here verbatim because it conveys some sense of the way the discussions must have been going. In a discussion of a paper by Pratt [16], page 1499, Karapetoff wrote that a student of his told him, "You cannot make one coefficient do all the dirty work!" Words to live by!

A Parable about the Mean Radius of the Ellipse

At the recent annual convention of the American Institute of Electrical Engineers a rather heated discussion took place regarding the definition of the power factor of an unbalanced polyphase system. A Joint Committee's report was presented in which NELA members participated, and two conflicting definitions were presented. Among those who took part in the discussion was Professor Vladimir Karapetoff of Cornell University, who was opposed to the very concept of power factor in application to an unbalanced polyphase system, and called the procedure "pouring new wine into old bottles." To illustrate his point he told the following parable, which created considerable amusement among those present, and helped to swing the consensus of opinion to the rational view advocated.

"In a certain country, at a certain stage of its development, the circle used to be the only curve known, and any objects that could not be made square or rectangular were made of circular cross-section. It came to pass, however, that in the course of industrial development the advantages of a flattened circle or ellipse were gradually realized. At first the forms of the ellipse used departed but slightly from the circle, and

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everyone was speaking about 'the mean radius of the ellipse' without thinking of any exact definition of the term. By and by more oblong forms of the ellipse began to be used, and with them came ambiguities, controversies and even law suits, until it became necessary to appoint a joint committee for the definition of the mean radius of the ellipse.

"The committee began its activities by holding a public hearing to consider the interests of those concerned. First came the representatives of makers of canned meats who stated that the public preferred elliptical boxes of the same height and contents as the former round boxes. So these representatives wanted the mean radius of the ellipse defined as that of a circle of the same area. Next came the makers of labels to go around the sides of the same boxes. They complained that they could not figure out correctly the length of the paper strip for the new elliptical boxes and they wanted the mean radius of the ellipse defined as that of a circle of the same circumference and not of the same area. Finally came the representatives of elliptical pie makers, who wanted two separate definitions for the mean radius of the ellipse. According to them, for selling purposes, the largest radius of an elliptical pie should be defined as its 'equivalent mean arithmetical radius', irrespective of the other dimension. For purposes of taxation the shortest radius should be used and defined as the 'equitable mean geometric radius.'

"The committee was in great difficulty to decide among these proposals, when a shabby looking underpaid college professor appeared and bashfully ventured the opinion that the thing really needed was not a fictitious definition of a mean radius of the ellipse, but a careful study of the actual properties of the ellipse.

"Then, he continued, the area as well as the circumference of the ellipse could be expressed in terms of its dimensions, and each industry could be provided with the needed data. Besides, such an investigation would open the way to further progress in the arts.

"The rest of the committee and the audience did not like this speech; they rent their garments in wrath, and they took the blasphemous man out of their city to be stoned. But they delivered him to the judge and the judge delivered him to the attorney general, to be deported with other radicals and criminals who preached violence."

Today, we have not one or even two but at least *seven* different definitions of power factor, evidently in a continuing endeavor to make one coefficient do if not *all* then at least a *whole lot* of the dirty work. The reason for that situation is an unknown unknown for most of us today, just as it was in 1920.

Just in case you don't think power factor is still an important problem, consider a more recent example of what this bizarre situation leads to. One can hardly do better than examine a paper presented by Brent Hughes, the Manager of Corporate Quality Assurance and Revenue Metering at B.C. Hydro [17]. In the paper, Hughes describes a situation in which a meter-change resulted in a customer incurring a power-factor penalty. (The load had not changed, only the meter.) Looking to find out what was wrong, the power company tried some different meters: they gave results of 0.88, 0.95, and 0.96. All the meters were of approved design, and all were found to be working properly.

The chances are good that this experience could be repeated today in any utility you care to examine. Is that not just *crazy*?

Our present situation is disappointing. More than a century after the AIEE/NELA meeting, the power community has still not achieved the goal of the Special Joint Committee of finding "a definition which will … be suitable for scientific, legal and commercial use." That is a sad state of affairs. The solution will become evident later in the book as the true nature of the problem emerges.

1.4 Three Kinds of Metrology

The AIEE/NELA committee was looking for a definition for three areas of use that they named scientific, legal, and commercial. It is worth reviewing these three, as they still frame the problems today.

All these three branches of metrology are of interest to the *Bureau International des Poids et Mesures*, the BIPM, headquartered in Paris, France.[2] BIPM is the international body that unifies measurements worldwide. They coordinate the maintenance and distribution of the International System of units. The Bureau disseminates such things as reference materials,[3] and they coordinate international comparisons of measurement activities. Their concerns and those of power engineers overlap in legal metrology, commercial metrology and, of course, scientific metrology.

Recall that measurements are made to aid in decision-making, and decisions have very different motivations in different contexts.

Let's start by looking at the middle one in the SJC list: legal use.

1.5 Legal Metrology

Legal metrology is the branch of measurement science that aims to provide a consistent and fair basis for trade, and the imposition of regulations. Measurements

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[2] It is likely that the AIEE/NELA committee was aware of the BIPM, since it was founded in 1875 by the signing of the Convention of the Meter, more than 40 years before the SJC was formed. However, the scope of the BIPM did not include electrical quantities until after a revision of the Convention in 1921, extending its responsibilities. Nevertheless, the SJC correctly identified the three major areas of metrology.

[3] The identification on the beer glass in the preface suggests that it contains a Standard Reference Material, perhaps brewed by an enterprising national metrology institute.

made in this category must be capable of simple and straightforward interpretation and must give unambiguous results.

Legal metrology is not a joke, but in meeting those requirements, it sometimes strays so far from the scientific idea of reality that it comes across that way. It is a lot easier to accept the edicts of legal metrology if you are willing to abandon the unwavering urge to apply logic (and even common sense) that characterizes us power engineers. Legal metrology, after all, gave us the name of the famous Citroën 2CV, featured at the top of this chapter.

This beautiful little car was designed in the early-mid 20th century to bring the car to the farmer in France, and was an outstanding example of purposeful engineering. It was front-wheel drive, with suspension designed to cross even a plowed field. It was likely the first production car designed for radial tires. The 2CV name stood for 2 horsepower (in French). Of course, it was *not* a two-hp engine, except according to legal metrology, aimed at the tax calculation. The French government had a formula for calculating the horsepower rating of engines, based on the product of the number of cylinders, the stroke, and the square of the bore (diameter). The calculation for this engine (initially an overhead-valve 375 cc air-cooled unit) rounded to 2. It was a lot simpler than making a practical measurement!

Other governments had similar formulas. The British formula for taxable horsepower was based on the number of cylinders and their diameter, and made no allowance for their stroke: $\text{hp} = d^2 n/2.5$, where d is the cylinder bore in inches, and n is the number of cylinders. By making long-stroke narrow-bore engines, the tax on the vehicle could be kept low, making the purchase cost lower. But the designs did not leave room for two valves in the cylinder head, and they were rather low-performing side-valve engines. As engines got bigger and better, unbelievably low "official" horsepower numbers were developed for tax purposes.

There are other aspects of legal metrology than the taxing of cars, of course. Broadly speaking, the field is the matter of making sure any and all legal requirements are met by measurements and measuring instruments. This requirement is true even if the measurement involves a formula based on what seem to be ill-chosen quantities.

We are reminded of the remark by Eddington: "*Proof* is an idol before whom the pure mathematician tortures himself. In physics we are generally content to sacrifice before the lesser shrine of *Plausibility*."[4] We would add: In legal metrology, we rely instead on a stamp of approval. This is important, because legal metrology is what gives the ordinary person (that is, us, outside of our field

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[4] Sir Arthur Stanley Eddington, in Gifford Lecture (1927), Edinburgh, as collected in "Science and Mysticism," *The Nature of the Physical World* (1928), 337.

of specialization) the confidence that the weighing scales at the butcher's are trustworthy, so long as he keeps his thumb off the pan. Legal metrology plays an important—if largely unseen—role in many aspects of our lives. There is a considerable infrastructure supporting these activities.

In the electric power system, there are legal requirements on the accuracy of billing meters, for example. Even the details of the algorithms used are sometimes part of the requirement. Since over the territory of a legal jurisdiction, there is probably a significant amount of money involved, that does seem sensible. However, there is no necessity for different legal jurisdictions to share the same requirements, even in this field.

For example, while most utilities bill for delivered energy, several states in India allow the billing of large customers for the time integral of apparent power, rather than the integral of "real" power. The shaded areas in Figure 1.2 show the states in India that use apparent power for billing, according to the website of the Maharashtra State Electricity Distribution Co. at https://www.mahadiscom.in/wp-content/uploads/2018/07/MSEDCL-MTR-Petition-Searchable-Format.pdf (see page 213).

Figure 1.2 Indian States billing for apparent power.

The fact that apparent power is an area of legal metrology means that any of the states could allow this kind of billing or mandate it. As we shall see later in Chapter 4, apparent power is not a trouble-free measurand, but so long as The Authorities can be persuaded to agree and grant the body, *any* scheme is workable. Like power factor, or the early French and British horsepower numbers, apparent power is an example of legal metrology that doesn't work in a scientific sense.

Another example of a legal-metrology requirement is the need to get the "right answer" when measuring signals represented by the somewhat unusual test waveforms for voltage and current shown in Figure 1.3.

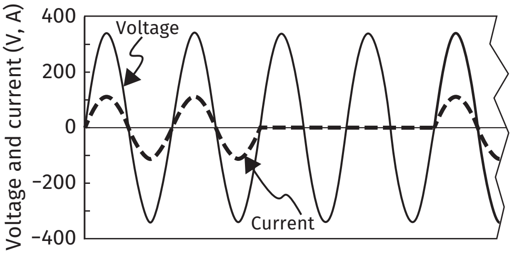

Figure 1.3 Test waveform for "integral cycle" measurement.

The test waveform shows a voltage at 240 Vrms and a current waveform at 80 A for two cycles and then zero for two cycles. This four-cycle waveform repeats indefinitely. For the sake of discussion, assume the voltage has a frequency of 60 Hz.

Calculated cycle-by-cycle, the results for power would be (from the left): 19.2 kW, 19.2 kW, 0, 0. An electromagnetic meter measuring this waveform would "average" these results, and give a reading of $(19.2 + 19.2 + 0 + 0)/4 = 9.6$ kW. An electromagnetic billing meter integrating the power value over a long time would produce a bill for energy based on this value.

Because of the same kind of averaging, an electromagnetic ammeter would indicate a current of 40 A. However, a "true rms" value for the current would consider that the fundamental frequency over which the rms is to be calculated is only 15 Hz, a quarter of the power frequency. By its definition, the rms quantity assumes a periodic signal, and the period in this example occupies four of the power-frequency cycles. The "true rms" instrument would give a value of 56.6 A for the current.

The "averaged" apparent power calculated by the results of the electromagnetic ammeter is $40 \times 240 = 9.6$ kVA. That is old-style legal metrology. The "averaged cycle-by-cycle rms" would be $(19.2 + 19.2 + 0 + 0)/4 = 9.6\,\text{kVA}$, numerically the same. Both of these are numerically the same as the power.

The "true rms" apparent power would be $56.6 \times 240 = 13.6\,\text{kVA}$. The power, which for power engineers means the average power over a cycle (or some integral number of cycles), would be the same as for the electromagnetic instrument, 9.6 kW. Since this is much smaller than the "true rms" apparent power, some calculations used by power engineers would suggest there was reactive power somewhere. The power factor would not be unity.

One might reasonably ask which calculation is correct. The best way to think of it is that each is "correct" in its own way. The "true rms" is correct from the point of view of the perfectionist engineer. The "averaged" apparent power is correct in that it gives no hint at reactive power, which is in agreement with the idea of a resistive load. As far as billing for "real" power is concerned, it makes no difference which method is used to calculate apparent power.

The usual rule in billing metrology (that is, legal metrology) would say that the "averaged" rms values of the single-cycle method are "correct." The averaged rms values are thus declared valid for billing. It seems that such a calculation serves no "scientific" purpose, however, since it is neither rms nor average, but a mixture of each. To accept the "truth" of the "true" rms value, however, one would have to be oblivious to the fact that the power system does not typically have a 15-Hz fundamental and that, therefore, the measurement of the "true rms" responds to no scientific purpose either.

While the calculation of the energy delivered by means of the "true" rms (over four power-frequency cycles) also gives the correct value and may seem more "scientific," the method does give the misleading impression that there is reactive power in the system. That is more a problem with our conceptualization and interpretation of "apparent power" and "reactive" than with the use of rms. More on that in Chapter 5.

1.6 Scientific Metrology

Scientific metrology aims to provide measurements that accurately describe the world around us in a manner consistent with our understanding of the way the physical world "works." Our concern is, of course, primarily with electric power, but the ideas involved include more mechanical and chemical aspects of measurement, for example.

The BIPM has produced two documents that we should be aware of. We will be referring to them throughout the book. One is the Guide to the Expression of

Uncertainty in Measurement [18], and the other is the International Vocabulary of Metrology [19]. These are known as the GUM and the VIM. (We have no idea why the GUM is not the GEUM. Perhaps "E" got added late.) These two documents are authoritative and informative, but neither could be described as an easy read. Nevertheless, they are valuable to have on hand, and they are available as free downloads from the BIPM. We will expand on some of the material in these guides in this book. And because these guides mostly reflect the interests of scientific measurements, this book will augment the information they give. In some sense our focus is broader, even though we consider only power system applications.

Consider the words below from page 1 of the GUM. These are from the statement of scope:

> **1.2** This *Guide* is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity—the measurand—that can be characterized by an essentially unique value. If the phenomenon of interest can be represented only as a distribution of values or is dependent on one or more parameters, such as time, then the measurands required for its description are the set of quantities describing that distribution or that dependence.

There are several important pieces of information buried there. First, the GUM is interested in measuring a *well-defined quantity*. That seems like a reasonable thing, and maybe it is for the GUM. But frankly, that kind of attitude has got us almost nowhere when it comes to solving the various problems of power system measurements. The Special Joint Committee of AIEE and NELA, whose efforts were described in the Introduction, engaged in a process of first understanding the various purposes to which power factor values were put, and then "To offer for the electrical industry in the United States *a definition* which will definitely and correctly express this purpose and will be suitable for scientific, legal and commercial use" (italics added).

Their failure was that they could not serve the applications they uncovered with less than two definitions, and their efforts led to a sort of black-hole effect: many other definitions started orbiting theirs. Today it is clear that power factor is best regarded as legal metrology. This use was noted by Waldo Lyon as long ago as 1933. In [20], he wrote:

> … circuit calculations are usually simpler if the loads are determined by their active and reactive powers rather than by their power factors. That is, as far as circuit analysis is concerned, power factor is a quantity whose retirement need scarcely be noted.
>
> There is, however, another and important use for power factor. There is a need for just such a blanket factor when specifying the character of power

> loads from a commercial standpoint, that is, when writing specifications or power rates. In that case it does not seem necessary that power factor should have a rigorous scientific definition such as has been given potential, current, and power, and can be given reactive power.

We bring this up here because Lyon recognized that legal metrology was not always connected to a rigorous scientific definition. That is a relatively unknown unknown. *Power factor is not a well-defined scientific measurement*. Power engineers, like the writers of the GUM, always expect to rely on definitions. While that seems sensible, for power system measurements, it might not work. We have problems that demand a different solution, as we shall see in Chapters 4 and 5.

Another bit of information in the GUM scope statement is the expectation of a result having a unique value that characterizes the measurand. That amounts to a one-to-one relationship, in essence, a requirement for being able to reverse the process of measurement, knowing what the original must have been. If only one unique result can characterize the quantity measured, we can then reasonably infer that only one *thing* could produce that particular result. Such a situation is rare in the power system, as we shall see.

Finally, we see that the GUM expects the definition to contain enough information to allow the result to account for such things as time- or temperature-dependence. That is not unreasonable, but it may be hard to achieve in a power system whose steady state is never quite steady. We should keep this lack of steady state at the back of our minds. It will come back to haunt us later.

1.7 Commercial Metrology

Commercial metrology (or “technical metrology” or “industrial metrology”) borrows from both legal metrology and scientific metrology. It does not call for suspension of disbelief in the way that legal metrology sometimes does, but is often a more practical aspect of metrology than some scientific metrology. Simpson (at what was then called the US National Bureau of Standards, now the National Institute of Standards and Technology) [21] describes the field this way, “This class includes those measurements made to assure dimensional compatibility, conformation to design specifications necessary for proper function or, in general, all measurements made to insure fitness for intended use of some object.” To facilitate quality measurements in commerce and industry, the national metrology institutes of the world produce (or test) things such as reference materials and gauge blocks, thereby connecting the industrial user of measurements with the standards that represent the BIPM-level of accuracy. The adoption of metric standards is a commercial metrology consideration.

We will see more about how measurements can be "traced" to such standards when we look at *uncertainty* in Appendix A at the end of the book.

1.8 A New View Needed

It doesn't take a crystal ball to see that we are *not* going to find the solution by repeating century-old discussions. The Special Joint Committee brought considerable expertise to the power factor problem: if they couldn't find a definition, we are not going to. It's no good sweeping the matter under the carpet, either. The problem is not going away. Instead, we must make a fundamental change in the way we view measurements in power systems. The fact is, we have been looking at these problems the wrong way. The three purposes of metrology are part of the new view of measurement we need.

Percy Williams Bridgman (1882–1961) was born in Cambridge, Massachusetts, and seemingly never left town. He went to Harvard, stayed there through his PhD work, and taught there, becoming an assistant professor in 1919 and a full professor in 1926.

He was awarded the Nobel Prize in Physics for his work on the physics of high pressures. His most influential book, *The Logic of Modern Physics*, predates the Nobel award: it was published in 1927. In this, and his several other books, the mind of a thoughtful scientist is revealed. Robert Oppenheimer, of the Manhattan Project, studied under him at Harvard, and said that he "found Bridgman a wonderful teacher because he never really was quite reconciled to things being the way they were and he always thought them out." One of his books is entitled *The Way Things Are*.

In 1961, aged 79, Bridgman committed suicide. He had been suffering from cancer for some time. The words of his suicide note ("It isn't decent to make a man do this thing himself. Probably this is the last day I will be able to do it myself.") are sometimes quoted in the debate over assisted suicide.

Another way of looking at measurement was initiated by Percy Bridgman, a Nobel laureate in physics. He considered the developments that have taken place in physics since Einstein's relativity theories, and *because* of them. We all know that Einstein tied space and time together, and had interesting ideas based on the notion that the velocity of light is a constant. That was a new view for physicists. Bridgman was concerned that physicists had been caught unawares by this new view. He had no problem with the new view *per se*, but he *was* bothered by physicists being caught unawares. Bridgman wrote a book about the matter [22]. On page 4, he wrote:

> Now here it seems to me is the greatest contribution of Einstein. Although he himself does not explicitly state or emphasize it, I believe that a study of what he has done will show that he has essentially modified our view of what the concepts useful in physics are and should be.

Bridgman's book was influential in clarifying a then-new view in physics, which goes by the name "operationalism." As it happens, almost all the measurements in the power system reflect his work, though they have been made that way accidentally, and without any awareness of operationalism or of Bridgman.

And, strange to tell, they are of a kind not much considered in the work of the BIPM, or by measurement theorists, because few scientific measurements are operational.

Our new view *must* include understanding operationalism. For many readers, operationalism will be another unknown unknown. We review operational measurements in Chapter 4, but before that, we review the kind of scientific measurement that we power engineers regard as our default method. The measurements are called "representational," and they are the subject of Chapter 3.

1.9 Philosophy

What Einstein did was clearly a success in his field, and in Bridgman's view, it maintained much of the *status quo* in a satisfactory way. Toward the end of Bridgman's book (page 175), he added a comment regarding "physical intuition":

> What Einstein really did, therefore, was to demand that even when space-time is warped by the presence of a gravitational field, those physical phenomena which can be described in terms of differential equations continue to be described by linear differential equations of the second order; that is, that nature continues to be describable in terms of a causality concept, with propagation phenomena, and a simple energy function. The consequences of a guess like this about the properties of nature *appeal to our physical intuition* as being worth following out, and of course we know the experimental *justification*.[5] (Italics added.)

James Clerk Maxwell (1831–1879) was born in Scotland, and was what the British call "a bright lad," keen to learn how things work.

He entered Edinburgh University in 1847 (at age 16), and went to Cambridge in late 1850, graduating in 1854. In 1856, he became Chair of the Natural Philosophy department of Marischal College in Aberdeen. From there, he went to King's College, London, as Chair of their Natural Philosophy department. In 1865, he returned to his family home in Scotland, but was brought back to Cambridge as the first Cavendish professor in 1871.

Here, as well as managing the Cavendish Laboratory, he completed work on his Treatise on Electricity and Magnetism, work that was to secure his place in the history of science.

Maxwell died at age 48 of cancer. His mother had died at the same age and from the same kind of cancer. He is buried in Scotland, and memorialized in Westminster Abbey.

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[5] Einstein began with a universe containing the same sort of experimental problem that we face: there was no way for the existing view to explain the precession of the orbit of Mercury.

A major aspect of the problem with power system measurements is that while they *do* appeal to our physical intuition, unlike the results from relativity, they give experimental results that are far short of deserving to be called *justification*.

Understanding why this is the case involves what might be called a “new kind of conversation” for most power engineers. This book is part of that conversation. We started by acknowledging that *philosophy* has an important role in measurement theory.

Philosophy seems to be a word that scares some people. After all, a philosopher is the kind of person who will try to convince us that, logically, a heavier-than-air machine cannot possibly fly.

We don’t want to put you off, but a certain amount of philosophizing is unavoidable. Philosophy is just about doing some rational thinking, sometimes even thinking about what is involved in thinking. We can do this!

We have seen diagrams like Figure 1.4 as an explanation of the need for a new theory. At first glance, the diagram seems to capture important relationships. But in fact the diagram is missing some very important parts.

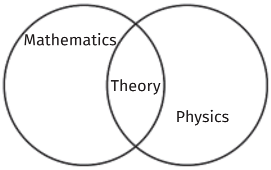

Figure 1.4 Venn diagram.

In particular, the involvement of the scientist or engineer is not represented, and that is a fatal omission for the diagram.

The question of the role of the scientist in relation to physics has probably been part of the field of philosophy for some thousands of years. Of particular importance to us is something that Maxwell said in 1870. For one thing, it identifies an error in the diagram.

The insight we have from Maxwell is indeed philosophical.

1.10 The Conceptual and the Physical

In 1870, Maxwell gave an address to the British Association.[6] It is reprinted in *Nature* [23]. The BA had become deeply involved in standardizing electrical

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[6] The full name is actually the British Association for the Advancement of Science, but the usual way to refer to it is the BA. Coincidentally, the BA was founded in 1831, the year of Maxwell’s birth.

measurements by the second half of the century. In 1870, Maxwell was the incoming president.

Near the end of the address, he wanted to underscore the difference between mathematics and the physical world. He said one is an *operation of the mind* and the other is a *dance of molecules*. The statement clearly separates the conceptual aspects from the physical, the mathematics from the real world. But Maxwell did *not* connect them by overlapping them and labeling the overlap "theory." He specifically kept them separate.

We hope you find Maxwell's comparison memorable, because the concept of the separation of the physical and the conceptual is at the foundation of the act of measurement, and that concept really *must* be in your mind when you're thinking about (and therefore whenever you are discussing) measurement. It matters to us, thinking about power system measurements, because it draws a distinction between what something *is* and what someone *knows* about it.

That distinction is the starting point for thinking about measurement. Measurement is about getting some information (*conceptual* stuff) about something that is of the real world (the usual name for a *physical* place).

That is a philosophical thought, but it is not very challenging. The field of philosophy distinguishes between many aspects of study. The two that are relevant to our thinking at this point are named *ontology* and *epistemology*. Ontology is to do with *existence*, what something *is*. Epistemology is about knowledge, its nature, and how it is that we think we *know* what something is. From time to time in the book, we may need to recall these words, but it really doesn't matter *exactly* what these labels mean. As far as we are concerned, the important point for us is maintaining in our minds that there is a distinction between them: between what something *is*, and what we think we *know* about it. *What we think we know is the result of measurement*.

Measured information is something you can file under epistemology. As we shall see later, it is a very rare measurement that allows you to make any ontological inference. And yet surely the purpose of a measurement is to learn something about the world, in order to support a decision of some sort.

Given the purpose of measurement, that real/conceptual separation might seem strange. We are saying that if you have just used a voltmeter to measure a voltage, you still do not know what the voltage *is*! It makes no difference whether the voltmeter is an electromechanical one or a digital one, or how accurate the manufacturer claims the instrument is. You don't have any ontological information.

That is not to say that the exercise of making a measurement is pointless. The purpose of a measurement—any measurement—is to obtain a result that contributes to some kind of decision-making. One might want to increase or decrease a voltage, or decide whether to build a new power line. The fact that the result of a measurement doesn't actually tell you what something *is* will be further discussed in Chapters 2, 3, and 4. There you will find what you *do* learn as the result of a measurement.

Perhaps we should add one further consideration to the matter of philosophy. We separate the physical and the conceptual. But if we don't get the conceptual aspects of a measurement *right*, the measurement stands a good chance of being a challenge to implement and giving a meaningless result. We know of reputable engineers who have convinced themselves that measuring is just a matter of sampling in some fashion, and there are perhaps one or two even who think the idea of "foam on the beer" is useful. The philosophical part of measurement is considered in the first part of this book because it creates a foundation for what follows, but even in this first chapter, the reader has already seen a practical aspect of the matter: The three categories of metrology do not rely on the same philosophical ideas.

Since we assume the reader already knows the power engineering, the mathematics, and the rest, we'll have to explore only the philosophy together. There will be some philosophizing in the first few chapters, as we have seen. The whole book depends on the concepts discussed here and in the next few chapters. But remember, a book is a teacher with infinite patience: it shows no irritation if it has to be read more than once. So if you feel the need to read some of this more than once, just remember that we have all been in that position at some time or other. And once you have internalized the concepts in these early chapters, the rest of the book will be straightforward.

1.11 Conceptual and Physical—More History

Keeping the conceptual and the physical separated in the mind is important to understanding the results of measurements. This consequence of the separation of physical and conceptual was viewed as an obvious necessity by the reviewer [24][7] of a book by John Ambrose Fleming (the person who named power factor). The reviewer wrote a long and rambling review (so long it had to be published in four episodes during October of 1889).

He was discussing what we might call dimensional analysis when he wrote the following:

> We approach this mysterious subject with trepidation, for, as a glance at the 14th Report of the Association for the Improvement of Geometrical Teaching, of all books in the world, will show, some people think apples may be multiplied by pennies with perfect rectitude. We prefer, however,

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[7] The author of this review is not identified in the journal. However, the writing style, the kind of humor, and the clear facility demonstrated in dealing with Maxwell's equations, both in their original form and the "cleaner-looking" vector notation, make it reasonably certain that Oliver Heaviside wrote the review.

> to adhere to the traditions of our youth. When we wish to know the price of a dozen apples at two pence each we do not operate upon the apples or the coins themselves; we take the number of apples and multiply it by the number of pence each costs, and divide the number obtained by the number of pence in a shilling. We even go so far as to say length squared is not area, and that all mathematical operations are performed on abstract numbers only.

There really *was* an Association for the Improvement of Geometrical Teaching. Its aim was expressed clearly in the name of the club. At the time, the teaching of geometry was planned as a stultifyingly conservative learning of the work of Euclid. Practically *any* change to that would have represented an improvement. But it is hard to imagine that this Association was really proposing to teach the performing of mathematical operations on "concrete quantities."

There is actually no mention of apples being multiplied by pennies in the 14th Report mentioned by the reviewer, but the reviewer was using these words to make a point. In the report, there *is* a paper by a Professor A. Lodge, in which appears the following (on page 48):

> The distributive law is satisfied: for example,
>
> $$\begin{aligned} 2\text{ feet} + 3\text{ feet} &= (2+3)\text{ feet} \\ &= 5\text{ feet} \end{aligned}$$

This seems unexceptionable. But on the following page is written

> $$\begin{aligned} \text{Thus } 4\text{ feet} \times 2\text{ yards} &= 8 \times 1\text{ foot} \times 1\text{ yard} \\ &= 24 \times 1\text{ foot} \times 1\text{ foot} \\ &= 24\text{ square feet} \\ &= 24 \times 12\text{ inches} \times 12\text{ inches} \\ &= 3{,}456\text{ square inches,} \\ &\quad\text{etc.} \end{aligned}$$

To us, that "equation" multiplying concrete quantities such as feet and yards seems incredible. We would certainly not object to saying that the rectangle whose sides measured 4 feet and 2 yards had an area of 3,456 square inches. But our minds would rebel at multiplying feet by yards, or as Heaviside wrote, apples by pennies.[8]

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[8] At least the president of the association (one R.B. Hayward, M.A., F.R.S., author of a book on solid geometry) evidently had a sense of humor. After one rather difficult paper had been read, he thanked the speaker, and remarked that

> he had been struck rather with awe at the long words which had been introduced. They had been mere words to him, but he hoped that in future, with [the author's] help, they would be able to attach some meaning to them.

But we should not feel superior to those ancestors of ours who made this mistake. *We make exactly the same mistake ourselves when we write things such as "The voltage is sinusoidal."*

A voltage cannot *be* sinusoidal: the sinusoid is a mathematical function, and although a voltage may generate a display on the oscilloscope that greatly resembles the graph of a sinusoid, the voltage *is* not sinusoidal. The fact that we make this kind of mistake as an automatic response to our observations is probably a tribute to our human instinct, and the limited range of our formal education.

And we power engineers go even further. We talk about the angle between the voltage and the current. Does such an idea differ in any significant way from multiplying feet by yards? It does differ: what we say is *worse*. At least feet and yards have the same dimension: length. Because current and voltage are different things, when we talk of an angle between them, are we not guilty of *subtracting* apples from pennies?

It is very important to get this straight. We will probably continue to be lax in our speech and maybe even in our writing, but in our minds, we must get this stuff clear. Our failure to do so leads to a good deal of confusion and wrong-headed thinking.

Mistakes of this kind still occur. Even the folk who should know better sometimes get it wrong. For example, the IEC Standard on the phasor measurement unit (PMU) [25] contains these words (page 13):

> The synchrophasor measurand is a complex number that can be represented in polar coordinates as:
>
> $$X(t) = \left(\frac{X_m(t)}{\sqrt{2}}, \phi(t) \right)$$

The problem here is that the measurand *is* a physical thing. The GUM makes that very clear on page 1:

> **1.2** This *Guide* is primarily concerned with the expression of uncertainty in the measurement of a well-defined physical quantity—the measurand—that can be characterized by an essentially unique value.

A complex number is not a physical quantity. The physical quantity is a voltage or current, and on the oscilloscope with a horizontal axis linear in time, their images look sinusoidal. The sinusoid is mathematical, and one way to represent the sinusoid is to use the exponential notation. That notation eventually became associated with the word "phasor," though the linkage was never approved by any authority [26]. Though many engineers are so familiar with our mathematical notations that they conflate the physical and the conceptual, they are mistaken in doing so. (A widely-held but erroneous concept of what a phasor *is* has adversely affected the performance of the Phasor Measurement Unit. This topic is discussed in Chapter 8.)

Power factor is not a problem that must remain obscure for ever. Once it is understood *why* it presents a measurement problem, it can be dealt with appropriately. Reactive power is another quantity whose measurement seems beyond understanding at present. The PMU could perform much better than it does today. The solution to all these issues rests on a better appreciation of what measurement is all about.

The next two chapters introduce the separating of physical and conceptual things as a relevant and important part of our new view.

1.12 The Magical Role of the A/D Converter

Digital technology makes it simple to identify the point in the process of measurement at which the physical and the conceptual are separated. Suppose we are measuring a voltage. It makes no difference for our purposes at this moment whether the voltage is AC or DC, because we are just going to consider the case of an idealized A/D converter to get one idealized sample. The A/D converter has as its input a physical voltage. For the time being, we can ignore the details of what happens inside the A/D converter, and just observe that the output of the converter is a pulsating voltage. We interpret that varying voltage to be a stream of ones and zeroes that represent the value of the input voltage at the instant of the sample.

The A/D converter has therefore converted the input signal, which is a *physical* thing, into an output signal that we interpret as a string of symbols. Symbols are *conceptual* things. The A/D converter has wedged apart the conceptual and the physical, and given us in the conceptual domain a numerical value for something physical. The A/D converter has moved information from the domain of dancing molecules to the domain of the mind. Certainly, the output exists as a physical voltage, but the *information* about the voltage is now accessible for further treatment.

The input to the A/D converter is a physical quantity, and its output is a representation of that in the conceptual domain. We have achieved step one in the process of the measurement of a power system quantity. But information about the physical quantity is contained in that one sample, and in the way that sample relates to others coming from the A/D converter, and also to the way that the string of numbers from this converter relates to the string of numbers from other A/D converters representing other phases or other quantities. Measuring is about conceptualizing and dealing with these relationships. Mathematics is the way we deal with them.

1.13 A Perspective of This Book

The depth of our problem in power system measurements is such that it is no exaggeration to say that the system itself is not properly understood and modeled. Important parameters such as power, reactive power, and frequency do not always present sensible measurement results. The purpose of this book is to light the path to better results. In essence, we aim to elucidate what our community has been missing. Measurements are made for a purpose, and we are making too many measurements that are not fit for their purpose.

There are many ways to measure power factor, and later we will see the same is true of reactive power and frequency. As power engineers, we sense that for measurements there should be one "right answer," and when we don't get that one right answer, we seek a new definition. Our hope is that we can eventually find the right definition for what we are measuring. Such measurements are of a kind named *representational*. Measurements of this kind underlie most scientific work. Understanding them is very important, and they are the subject of Chapter 3. But we discover that the search for a "right" definition does not always work, and we have to prepare to diverge from the expected path after that discovery.

This is not just a matter of tidying up a few loose ends. We might be no more intelligent than the three or four generations of power engineers who have tackled the issue, and we are likely no better educated, as far as these problems are concerned. It takes the first five or six chapters of this book to explain the whole situation. But in the end, we find the situation is one we can deal with, if we are prepared to revise our perspective.

But the fact is that *measurement theory* has made great strides in the last few decades, and understanding *that* will lead to solutions.

1.14 Chapter Summary: History

1) Measurement is a complicated process that has presented challenges to the power engineer for over a century. We are mistaken if we imagine that there are now simple solutions.
2) As early as 1870, the separation of the physical and conceptual aspects of doing science was recognized and remarked upon by Maxwell.
3) Power factor was a serious measurement problem about 25 years after it was first defined in 1892. A significant effort to resolve the problem was made.
4) There are now about seven different definitions for power factor, almost all dating to meetings held in 1919 and 1920. They don't often give the same result.
5) Several of the outage events described by NERC as "unknown and unexpected" are blamed on measurement problems, as are similar outages in the United

Kingdom. These events have led to the disconnection of significant amounts of generation.

1.15 Chapter Summary: Takeaways

1) Our philosophical view of all measurement begins with recognizing that the physical and the conceptual are to be kept separate in the mind. Measurement is a bridge from the physical to the conceptual.
2) There are three kinds of measurement: legal, commercial, and scientific. They have different aims.
   a) Legal metrology is concerned with measurements concerning trade. They must lead to straightforward interpretation and must give unambiguous results. They do not have to make sense from the scientific point of view, but they do have to be "approved."
   b) Commercial metrology aims at ensuring dimensional compatibility by producing and sharing things like reference materials and gauge blocks whose properties can be traced back to the high-quality measurements of BIPM.
   c) Scientific measurements are about understanding the way the physical world works. For scientists and engineers, this sort of metrology is the "received wisdom." Scientific measurements are always subject to experimental validation and verification.

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