

- » Understanding the definition of probability
- » Calculating some probabilities
- » Becoming aware of probability misconceptions

Chapter 1

Seeing Probability in Everyday Life

Probability is part of our everyday lives, from checking the weather and deciding what to wear to looking at the stock market and making predictions to seeing something strange happen and saying, “What are the odds of that?” In this chapter, you explore the ways probability appears in everyday life and learn some important terms you’ll encounter throughout the rest of the book.

Understanding What Probability Means

Probability is a word that is used all the time, but it has a specific meaning in the statistics and probability world. First, you start with a random *event*, such as flipping a coin. Then you have the *outcome*, or results, of each flip: heads (H) or tails (T). The *sample space* (S) is the set of all possible outcomes of the random event. Then you have an event that is a certain result — or subset of results — in S, and these results are labeled with capital letters like A, B, C, and so on. And the probability of that event is noted by $P(A)$, which is read “P of A.” For example, if you flip a coin three times, let A = you get all heads and B = the number of heads is odd. The probability of some event A is the long-term chance that A occurs over many repetitions of your random event. For example, if there is a 40 percent chance of rain, it means that over many days with the same conditions, it rains 40 percent of the time.

Odds and probabilities are different. A *probability* is a number between zero and one, also known as a *proportion*, like 0.50 or 0.01. It is the total number of ways to do a particular item of interest

divided by the total number of ways possible. *Odds* is a ratio, either the odds for a particular outcome or the odds against a particular outcome. The odds for a particular item are the number of ways to get the item of interest divided by the number of ways not to get the item of interest. For example, the probability of rolling a die and getting a 6 is $\frac{1}{6}$, but the odds in favor of getting a 6 are 1 in 5, since there are five ways to not get a 6: 1, 2, 3, 4, 5, and one way to get a 6. The odds against an outcome are the number of ways to not get the outcome (5) divided by the number of ways to get it (1). So it's 5-to-1 odds against getting a 6.

Probability can apply to an individual or to a group. Using the roll of a die as an example, the percentage of time you get a 6 when you roll a die is $1 \div 6 = 16.67$ percent if you roll it an infinite number of times. However, the chance of getting a 6 when you roll a die once is 1 out of 6, or $\frac{1}{6}$. The probability is the same; the interpretation is different. Also, remember that probability applies to the big picture. For example, if the chance of winning from a scratch-off lottery ticket is 1 out of 10, it doesn't mean buying ten tickets guarantees you a win. It means over the course of an infinite number of tickets, 10 percent of them are winners.

This is another way of saying probability is both a long- and short-term value. If the chance of getting a single head on a single roll of a fair die is $\frac{1}{2}$, then the chance is 1 out of 2. But it also means that if you roll the die an infinite number of times, half of your rolls will turn up heads.

Weather is built on long-term probabilities. If the weather reporter announces that there is a 50 percent chance of rain, that means on 50 percent of the days like this one, it rained. Another way to think of it is that there's a 50-50 chance of rain, but that may not help much either. It's more information than nothing, however.



EXAMPLE

Q. Tell whether the following statement is true or false: "Probability is a short-term idea. If you flip a coin ten times, you will get five heads and five tails."

A. False. Probability is a long-term idea. If you flipped the coin an infinite number of times (long-term), you'd get half heads and half tails, but flipping ten times, you could get any combination of ten heads and tails, and they are all equally likely.

1

Tell whether the following statement is true or false: "Probability applies to single individuals only."

2 If you flip a coin three times, what is S ?

3 If you flip a coin three times, which outcomes are in event $A = \{\text{results are all the same}\}$?

4 What are the odds in favor of rolling a 5 or 6 on a fair die?

5 What are the odds against rolling a 1, 2, or 3 on a fair die?

Calculating Probabilities

Different methods exist for finding probabilities of events. One approach is to use the subjective method, which involves using your personal beliefs. For example, you may predict that it will rain based on how it looks outside. Another way is to use a simulation, in which you set up a model that includes probabilities and let it run its course many, many times, and see how many times a certain outcome appears. This method is used by meteorologists to predict events such as where a hurricane will make landfall; it's also used by bracketologists during the NCAA basketball tournament to predict a winner.

You could also use math formulas and simple calculations to figure out probabilities (which you will do throughout this book). Simple calculations work for many problems, especially problems where all the outcomes are equally likely. For example, if you flip a coin once, your possible outcomes are heads or tails. Then, assuming the coin is fair, you have $P(\text{Heads}) = 1$ out of 2, or $\frac{1}{2}$, which is the same for $P(\text{Tails})$. If you roll a die and let A = an even number result, you know $P(A) = P(2, 4, 6) = 3$ out of 6 or $\frac{3}{6} = \frac{1}{2}$.



EXAMPLE

Q. Deciding not to bring your lunch to work because someone will probably go out to eat with you is using what type of probability?

A. Subjective probability; it's based on your beliefs.

6 Assume you flip a fair coin two times. What is the probability you get the same result on both flips?

7 Suppose that you roll two fair dice. What's the probability of getting the same result both times?

Avoiding These Probability Misconceptions

Some ideas about probability seem right but are actually incorrect, and some go against our intuition. First, if you have two outcomes, like heads and tails, their probabilities are both 50 percent, so we say the probability of rolling either option is 50–50. It's 50–50 because the coin is fair, and each outcome is equally likely. But not all two-outcome scenarios play out that way. Just because a sample space S has two outcomes doesn't mean they are both 50–50. For example, consider shooting free throws in basketball. Your chance of making a basket from the free-throw line is probably better than mine, but maybe not as good as a professional basketball player.

Another misconception is that patterns like 1-2-3-4-5-6-7 can't occur randomly (like in the lottery), but of course, they can. Not only that, but they also have the same chance of appearing as any other combination. Yet another misconception is that you can be “on a roll” or “in a slump” when playing casino games. While this may be true when you are a baseball player trying to hit the ball, it's not true that you can be in a slump or on a roll in casino games because trials are independent. That means one play doesn't influence the next.

It's interesting how even picking a number between 1 and 10 at random is not really random unless you use what is called a random number generator (a computer program that comes up with random numbers for you — or for the casinos). If you ask people to pick a number between 1 and 10, fewer people pick 1, 10, and 5 because they are on the ends of the spectrum or directly in the middle. More people pick 3 and 7 as these numbers are in the middle of the lower half and the middle of the upper half. The bottom line is that what you believe is random may not be from a probability standpoint.

The same is true for outcomes of ten coin flips. Some people think you should get something like HTHTHTHTHT if you flip a coin ten times, and you'd never get something like HTTTTTTTTTH. But that's the whole point. These outcomes are equally likely because each flip is fair, and the flips are independent of each other. So, any combination has an equal chance of occurring.



Q. Tell whether the following statement is true or false: “HHHHHHHHHH would be harder to get on ten coin flips than HTHTHTHTHT.”

A. False. They have the same probability.

9 Tell whether the following statement is true or false: “As you come to a stoplight, it’s either red, green, or yellow. That means each color has $\frac{1}{3}$ chance of occurring when you come to it.”

10 Tell whether the following statement is true or false: “People are more likely to choose the number 5 when asked to choose a random number between 1 and 10.”

11 What does it mean for rolls of a single die to be independent?

12 Tell whether the following statement is true or false: “Just because there are two outcomes does not mean the probability is $\frac{1}{2}$ for each one.”

Solutions to Problems in Seeing Probability in Everyday Life

- 1 False. Probability applies to the entire population you are interested in as well. For example, if the chance of getting heads on a coin flip is $\frac{1}{2}$, that means the next flip has a 50 percent chance of being a head, but it also means that if you flip the coin infinitely, you will get 50 percent heads.
 - 2 If you flip a coin three times, $S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$. There are $2 * 2 * 2 = 8$ possible outcomes.
 - 3 $A = \{\text{results are the same on three flips}\} = \{\text{all heads or all tails}\} = \{HHH, TTT\}$.
 - 4 The odds are 2 to 4 because there are two ways to get a 5 or 6 and four ways to not get a 4 or 6.
 - 5 The odds are 3 to 3 because there are three ways to get 1, 2, or 3 and three ways to not get 1, 2, or 3.
 - 6 $S = \{\text{outcomes of two flips}\} = \{HH, HT, TH, TT\}$. $A = \{\text{results are the same on two flips}\} = \{HH, TT\}$. $P(A) = P(HH \text{ or } TT) = \frac{2}{4}$ because all outcomes of coin flips are equally likely.
 - 7 $S = \{\text{outcomes of two dice}\} = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$. There are $6 * 6 = 36$ outcomes. $A = \{\text{results are the same on both dice}\} = \{11, 22, 33, 44, 55, 66\} = \frac{6}{36}$.
 - 8 No. Some probabilities are subjective, which means they depend on the person thinking about the problem. You might say there is a 10 percent chance of rain, while the meteorologist may say it's 20 percent. And someone else might say it's 30 percent. Certain probabilities are mathematical, like coin flips, dice rolls, and card hands, and some aren't.
 - 9 False. Lights are red, yellow, and green for different amounts of time, depending on the intersection, but at a typical stoplight, the green light is on for the longest duration, allowing traffic to flow. The red light is on for a shorter period to indicate that traffic must stop, and the yellow light flashes briefly before the light turns red to warn drivers that the light is about to change.
- 

REMEMBER
- Just because there may be three outcomes in a particular scenario does not mean the probability is $\frac{1}{3}$ for each outcome. Don't assume the probability is evenly split across the number of outcomes as with a fair coin (50–50) or fair six-sided dice ($\frac{1}{6}$ for each number).
- 10 False. According to research, more people pick 3 and 7 than 5. The point is, all ten numbers are not picked with equal probability, even though it may seem that they should. That's why you can't assume that probability-based phenomena are equally likely if human beings are involved in determining the outcomes. If you used a computerized random-number generator, then the outcomes would be equally likely (like lottery number picks).
 - 11 Independent rolls of a die mean the die must be rolled so that the outcomes don't affect each other.
 - 12 True. The two probabilities can be anything as long as they sum to 1.

