Chapter One

First, a word about this chapter. Let's say you're going to learn to swim. You're 5 years old and a little afraid of the water. Your swimming teacher tells you not to be afraid, and picks you up and throws you into the pool!

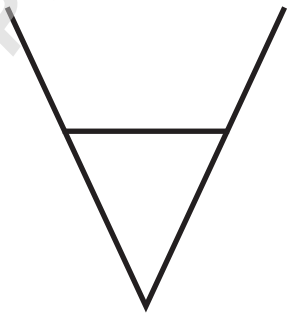
You immediately start thrashing about with your arms and legs. You're really scared, but after a few seconds, you notice that you're not drowning, you're keeping your head above water. In a few more seconds, you've made your way to the side of the pool and you're hanging on to the edge trying to figure out what happened.

You didn't drown because everyone is born with swimming reflexes and instincts. When your teacher threw you in, those reflexes took command and saved you. Now that it's over, you're not as frightened of the water. You've been in the middle of the pool and survived.

This chapter is a little like that first swimming lesson. You may never have studied logic, but you do, in fact, know quite a bit. If you didn't, you could hardly speak, let alone make your way in the world.

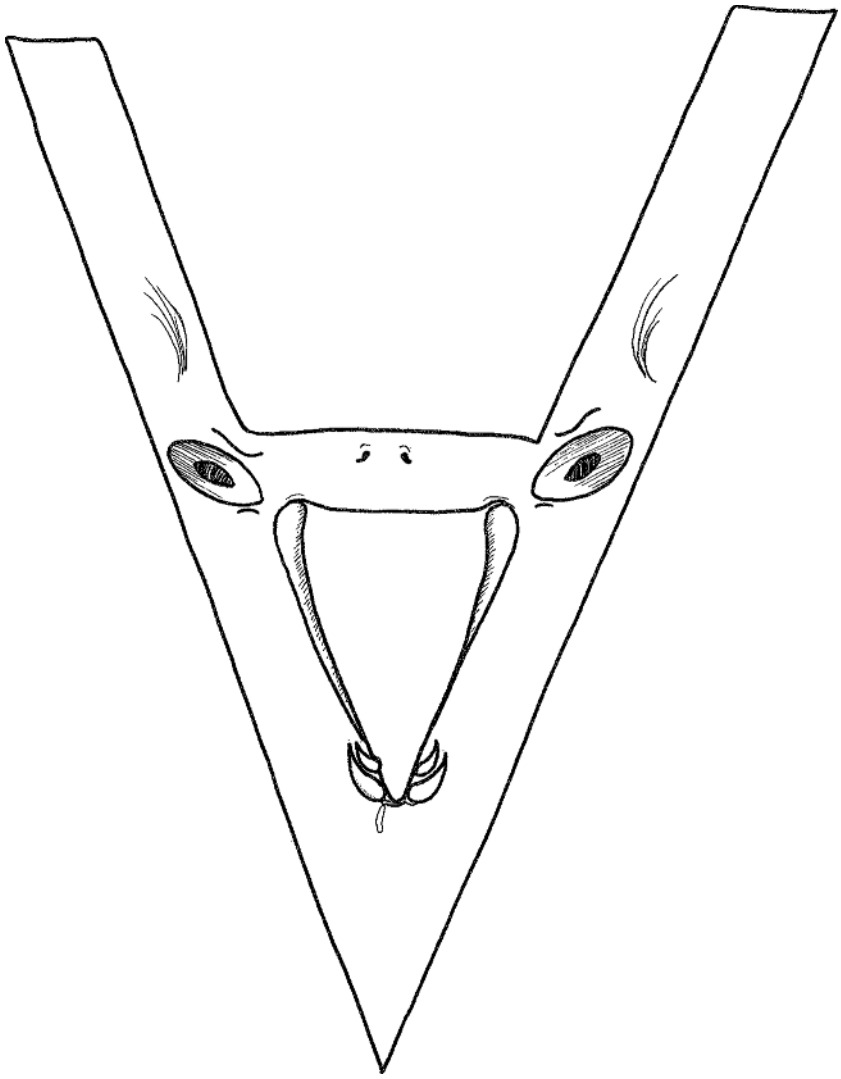
We're going to throw everything at you. You'll be surprised at how easy it is to understand the symbols. It's easy because the logical ideas represented by the symbols are basic ideas that you've worked with all your life.

Logic can seem scary at first. If you don't know what they mean, strange symbols



Sweet Reason: A Field Guide To Modern Logic, Second Edition. James M. Henle, Jay L. Garfield and Thomas Tymoczko.

can appear frightening . . .



But don't panic. The "∀" symbol just means "everything." You'll see how it works in a moment. It's not as mean as it looks.

1.1 Introducing Formal Logic

There was only one catch and that was Catch 22, which specified that a concern for one's own safety in the face of dangers that were real and immediate was the process of a rational mind. Orr was crazy and could be grounded. All he had to do was ask; and

> *as soon as he did, he would no longer be crazy and would have to fly more missions. Orr would be crazy to fly more missions and sane if he didn't, but if he was sane he had to fly them. If he flew them he was crazy and didn't have to but if he didn't want to he was sane and had to.* (Joseph Heller, *Catch-22*)

We begin with connectives, the logical operations that link sentences to each other. We don't have many connectives; they're all familiar to you. You know them as "and", "or", "not", "if . . . then", and "if and only if". Connectives allow us to create complex statements from simple statements. Suppose A and B are statements. Then we'll use

$$A \wedge B$$

to say that both A and B are true. We'll use

$$A \vee B$$

to mean that at least one of A, B is true (A is true or B is true or both are true). We'll use

$$\neg A$$

to mean that A is *not* true. We'll use

$$A \Rightarrow B$$

to mean that if A is true then so is B. And finally we'll use

$$A \Leftrightarrow B$$

to mean that A is true if and only if B is true, that is, A and B have the same truth value.

Let's say we have these statements:

P: George is late to the meeting.
Q: The meeting is in Detroit.
R: George brings a casserole.

Example

How do we say that either George will be late or he'll bring a casserole?
Answer:

$$P \vee R$$

Example

What does $Q \Rightarrow P$ mean?
Answer: If the meeting is in Detroit then George will be late.

Example

Represent the following with symbols: The meeting is in Detroit and either George doesn't bring a casserole or George is late.
Answer: $Q \wedge (\neg R \vee P)$ Note the use of parentheses here. We'll say more about this later.

Exercises Introducing Formal Logic

Odd-numbered solutions begin on page 350

Translate the following sentences using P, Q, and R from above.

1. George is late and the meeting is in Detroit.
2. If the meeting is in Detroit, then George brings a casserole.
3. Either George is late or he does not bring a casserole.
4. George brings a casserole if and only if the meeting is in Detroit.
5. If George does not bring a casserole, he is not late.
6. If the meeting is in Detroit then George brings a casserole, and if George brings a casserole then he is late.
7. The meeting is in Detroit if and only if both George is late and he doesn't bring a casserole.
8. The meeting is in Detroit, and either George is late or he brings a casserole.

Determine the meaning of each of the following sentences.

9. $P \vee R$
10. $R \wedge \neg Q$
11. $Q \Rightarrow P$
12. $R \Leftrightarrow \neg Q$
13. $\neg P \vee (\neg Q \wedge R)$
14. $P \wedge (Q \vee R)$
15. $R \wedge (Q \Rightarrow P)$
16. $Q \vee (\neg P \Leftrightarrow R)$

The Greek philosopher Epimenides is credited with formulating a paradox that has stimulated some of the most important advances in logic from the classical period right up to yesterday afternoon (we guarantee this, no matter when you are reading these words). He, a Cretan, put it this way:

All Cretans are Liars.

Since Epimenides was a Cretan, he was asserting that he is a liar, meaning that what he says is false. So it's false that all Cretans are liars. So maybe he's not a liar. So what he is saying is true? So he is a liar! So it's false! So it's true! Paradox!

The paradox isn't perfect. Epimenides might be a liar, but some Cretans (not Epimenides) could be truth-tellers. But we can refine it.

This sentence is false.

Is it true? If so, then, since what it says is that it's false, it must be a false sentence. But then it must be true. But then it must be false! And so on.

This is the paradox of the Liar. For all its simplicity, it is very deep. Can it be resolved? In the history of logic there have been many proposals . . .

1.2 Constants and Relations

Please accept my resignation. I don't want to belong to any club that will accept me as a member. (Groucho Marx)

We can express more delicate ideas if we set up some symbols to represent individuals and other symbols to represent properties and relations. We'll use some lower case letters to refer to people.

a refers to Jim Henle (a logician)
b refers to Oprah
c refers to Tom Tymoczko (another logician)
d refers to Aristotle (a philosopher, scientist, and logician)
e refers to Hillary Clinton
f refers to Jay Garfield (yet another logician)

We'll use some upper case letters to express particular properties and relationships.

We'll use *W* to say that something is female. We'll write *Wb* to mean that Oprah is female.

We'll use *G* similarly to say that something is male.

We'll use *M* to say that two individuals are married. If we write *Mdc*, for example, then we are saying that Tom Tymoczko and Aristotle are married.

We'll use *P* to represent a relationship among three individuals. *P* will say that the first two individuals are the natural parents of the third. That is, if we write *Pbcd* then we are saying that Oprahand Tom begat Ari (when you've had a little more logic, you can call Aristotle "Ari," too).

Finally, we'll use $=$ to say that two individuals are identical. If we write $e = a$ then we are saying that Hillary Clinton is Jim Henle.

Example

How can we say that both Tom and Jay are male?

Answer: $Gc \wedge Gf$.

Example

What does $Mec \Rightarrow We$ mean?

Answer: If Hillary and Tom are married to each other, then Hillary is female.

Exercises Constants and Relations

Odd-numbered solutions begin on page 350

Write English sentences that express the meanings of these formulas.

1. Wc
2. Mea
3. $d = f$
4. $Pacb$
5. $Pcab$
6. $Pabc$
7. $Wa \wedge Ga$
8. $Ge \Rightarrow \neg Med$

Using only the symbols that have been introduced, write formulas that express the meanings of these sentences.

9. Hillary Clinton is married to Aristotle.
10. Aristotle is male.
11. Aristotle is married to Hillary Clinton.
12. Jim Henle is Oprah
13. Aristotle and Jay Garfield are the parents of Hillary Clinton.
14. Jim Henle is male and Tom Tymoczko is female.
15. Jay Garfield is not married to Jim Henle.
16. If Oprah and Hillary are married then Oprah is male.

The remaining problems concern the following map:

We'll use Nxy to mean that x shares a border with y at more than just a point. For example, Ngh is true because regions g and h are neighbors, but Nkh is false because k and h touch only at the corner. Furthermore, no region will be considered a neighbor of itself.

True or false?

17. Nej
18. $\neg Nah$
19. $Nkh \vee Nhe$
20. $Nbd \wedge Nbc$
21. Ngg
22. $(Ncf \wedge Njf) \wedge \neg Ncj$
23! $\neg Nij \Leftrightarrow \neg Nde$
24! $\neg Nge \Rightarrow (Nag \vee Ngh)$

"During the First World War he [Ernest Harrison] was a naval officer and shaved his mustache. On visiting Cambridge, the Master (not recognizing him) asked him at a dinner whether he was related to 'our dear Ernest Harrison.' Adopting a certain philosophical view of relations (repudiated by Russell) he replied: No."
—J. E. Littlewood, *A Mathematician's Miscellany*

1.3 Quantifiers and Variables

If you call a tail a leg, how many legs has a dog? Five? No, calling a tail a leg don't make *it a leg.* (Abraham Lincoln)

If we say, "Everyone loves ice cream," we aren't talking about anyone in particular. We're making a universal statement. We have logical notation for that. Let's say that Cx means x loves ice cream. Using the individuals of the previous section, Cb would mean that Oprahloves ice cream. Then

$$\forall x Cx$$

means "for all x, x loves ice cream." The "$\forall x$" is a way of discussing all individuals at once.

If we say, "Someone loves ice cream" we again are not talking about a particular person. We're making what we call an existential statement, a statement that something of some kind exists. There's a way to say this in our primitive logical language:

$$\exists x Cx.$$

It means "there is an x such that x loves ice cream."

The x is a variable. It doesn't stand for anyone in particular. If we use a different variable, y, the meaning is the same. Both $\forall x Cx$ and $\forall y Cy$ mean the same thing (they mean that everyone loves ice cream).

Example

How can we say that Hillary is married?
Answer: We say that there is someone who is married to Hillary , that is,

$$\exists x Mxe.$$

Equivalently, we can say $\exists x Mex$, there is someone to whom Hillary is married.

Example

What does $\forall y(Myb \Rightarrow Gy)$ mean?
Answer: It says that every y is such that if y is married to Oprah then y is male. More simply, it says that all of Oprah's spouses are male.

Exercises Quantifiers and Variables

Odd-numbered solutions begin on page 350

Translate each of the following predicate statements into English using the predicate language from the previous section (see chart below).

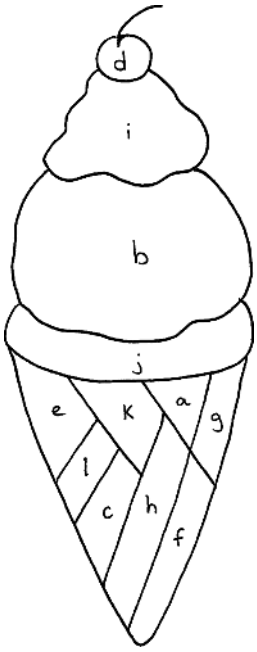
1. $\forall xMxa$
2. $\exists yMya$
3. $\neg\forall yPbfy$
4. $\forall xMbx \vee \exists y\neg Mby$
5. $\exists x(Mxd \wedge Mxb)$
6. $\forall z((z = e) \Rightarrow Wz)$
7. $\neg Gd \Rightarrow \neg\exists yGy$
8. $\forall x(Mxa \Rightarrow Wx)$

a	Jim Henle
b	Oprah
c	Tom Tymoczko
d	Aristotle (aka Ari)
e	Hillary Clinton
f	Jay Garfield
Wx	x is female.
Gx	x is male.
Mxy	x is married to y.
$Pxyz$	x and y are the parents of z.

Translate each of the following sentences into symbolic notation.

9. Either everyone is female or everyone is male.
10. Everyone is either female or male.
11. If Tom Tymoczko is married to someone, then Tom is male.
12. Jay is a bachelor.
13. Hillary is not married to herself.
14. Jim is everyone's mother.
15. Aristotle is married to someone female, or there is a woman who is not married to Aristotle.

16! Hillary is a grandparent.



Remember that we use Nxy to mean x is a neighbor of y and that no region is next to itself. In each of the following, x stands for one of the regions in the ice cream cone above. Find x such that the statement is true.

17. Nxd
18. $Nxi \wedge Nxj$

19. $Nxf \wedge \neg Nxc$
20. $Nxe \wedge (Nxi \vee Nxh)$
21. $Nxb \wedge Nxa$
22. $Nxj \wedge Nxh \wedge \neg Nxg$
23. $Nxe \wedge \neg Nxk$
24. $Nxg \wedge \neg \exists y(Nxy \wedge Nyg)$
25. $\exists y \forall z(Nyx \wedge (Nzx \Rightarrow z = y))$
26. $\forall y(Nxy \Rightarrow Nyb)$

Have you been thinking about the paradox of the Liar? If it keeps you up at night, you have a future in logic.

One proposal to resolve the paradox is this: Perhaps the Liar sentence is neither true nor false. Maybe it has no truth-value at all, or some third, weird truth-value, like "deviant." Then, one might say, there is no paradox. The sentence is just deviant.

But consider the Strengthened Liar paradox:

This sentence is not true.

It's clear that if this sentence is true, we are once again landed into paradox, and that if it is false it's paradoxical as well. Does calling it deviant, or saying that it has no truth value, help?

No. Suppose that it has no truth value, or that it's deviant. Then it's not true, right? But that's what it says! So it *is* true! But it says that it's not! So it is! So it isn't! Back to square one.

1.4 Introducing Informal Logic

> *An autocrat's a ruler that does what th' people wants an' takes th' blame f'r it. A constitootional ixicutive, Hinnissy, is a ruler that does as he dam pleases an' blames th' people.* (Finley Peter Dunne)

You're a first year student. You arrived two weeks ago at Sophist College, the ivy-draped liberal arts institution you dreamed of for years. Two weeks, but you're still floating on air. The academic atmosphere . . . the intellectual giants who are your professors . . . the imposing architecture . . . the excitement of campus life . . . the opportunities you see ahead . . . the challenge of the courses you've just begun . . . everything is as new and as thrilling as you had hoped.

Above all, you're in awe of the older students. They're so confident, so accomplished, so wise, so *cynical.* Well, I suppose there's nothing great about being cynical, except that you have to know a lot to be cynical, don't you? In any case, you relish those bull sessions that last until three in the morning . . . that's where it's at, that's where the world really unfolds, that's where . . .

But then one night the whole wonderful picture collapses. The discussion is about China. You just read that morning about the tight rein the government keeps on people. All you say is, "What they need is some democracy. If they would only let the people rule," and then Cathy jumps on you. Cathy, the junior you admired for her quickness, her assurance – and she seemed to like you.

"What's so terrific about democracy?" she asks. "In a democracy, the people choose, but they make terrible choices. They get freedom in the Balkans and the first thing they do is start shooting at each other. They get the vote in Iraq and they have a civil war.

"We have democracy, right? Well how great is that? We don't protect the environment, our schools are rotten, and we're in debt up to our eyeballs. If democracy is so wonderful, how come only 23 percent of the people vote here?"

You try to cut in. "But democracy has made us the most powerful, the most envied —" But she runs right over you!

"Oh, brother. We're powerful and envied because we're rich, not because of our campaign commercials. And all we do is abuse that power. And anyhow, we don't really have democracy. You know about Washington, D.C.? One of the biggest cities in the country, and they don't have self-government or representation in Congress. Why? Because it's a black city and we're all racists.

"Look at all the democracies in South America: all bankrupt. The only country down there with its act together is Chile, and it took a dictator, Pinochet, to put it on the road to recovery. You know what H.L. Mencken said? He called democracy the form of government that believes that the people know what they want and they deserve to get it – good and hard!"

You're devastated. Your deepest beliefs are in ruins! You can't say a thing because . . . well . . . everything she's saying sort of makes sense. But you still believe in democracy! You know it's right! But then, what's wrong with her arguments? What do you say?

You need to know how to argue!

There are good reasons for learning the art of argument.

First of all, you want to be able to defend your point of view. You want to persuade others. This is certainly true if you're right. And maybe it's useful even if you're wrong.

Secondly, and more nobly, you want to find out what is actually true. There is, perhaps, no better way to get to the bottom of things than to argue. When two skilled debaters engage, the best argument prevails. More often than not the winner is the truth.

Finally, the ability to argue represents power. If you can marshall your thoughts, arrange them in a logical order, and explain them clearly, people will pay attention. If your arguments are understandable and persuasive, you will be influential. *Your* issues, *your* perspectives, *your* proposals will take center stage.

In this book we'll teach you how to argue. We'll do it in stages. We'll start by showing you how to take apart an argument such as Cathy's, diagram it, and attack it. Then we'll show you how to construct your own argument, diagram it, and write it.

* * * * * * * * * * * * *

A word about Cathy. She's sort of unpleasant. Unfortunately, she appears throughout this book; she insisted on it.

But responding to her is a good logical exercise. What's her point, anyway? We'll come back to this, but first we'll think more generally about the task of identifying conclusions.

1.5 Conclusions

> *Joe DiMaggio might have hit in 56 consecutive games, a seemingly unrivaled record, but he never won 33,277 arguments in a row, like Ted Williams, the undisputed champion of contentiousness.* (David Halberstam, *The Teammates*)

The first step in tackling an argument is identifying the conclusion. This is more difficult than it sounds. You would think that anyone going to the trouble of making an argument would make sure we got the point. But that isn't always the case.

Writing is difficult. Writing arguments is especially difficult (as you will soon see). It's not surprising that it's often done poorly. That makes reading arguments a challenge. The key, and it is the key in formal logic too, is language. Unfortunately, while it is easy to say, "I would like to argue that . . ." or "My conclusion is . . ." that is too simple for most writers.

Consider the following three letters to the editor of *The New York Times*, May 11, 2005, responding to a column by Thomas Friedman arguing for an economic boycott of Iran and North Korea if they don't terminate their nuclear programs:

> *It is disturbing that Thomas L. Friedman seems to suggest that the world's most powerful countries (or groups of countries) should simply starve their opponents into submission.*
>
> *First, it would be a blatant violation of international human rights principles. Second, such measures would mostly harm those people (civilians) who have the least power to do anything about the situation in their respective countries.*
>
> *Surely Mr. Friedman does not believe that the leaders of Iran and North Korea are incapable of securing the necessities of life for themselves and their own families, and they have already demonstrated that they care little for the rest of their populations.*
>
> Jessica Crutcher

This is pretty simple. The writer is opposed to a boycott. But note that this conclusion is not explicitly stated. We have to figure that out from the list of negative effects of a boycott.

> *If China pressured North Korea to cease its weapons program by saying to Kim Jong Il, "You will shut down your nuclear weapons program and put all your reactors under international inspection, or we will turn off your lights, cut off your heat and put your*

whole country on a diet," perhaps the United States should insist that China do just that, lest we stop all our imports and bring its production machine to a grinding halt.

Lisa Calef

This letter is clearly in favor a boycott, though again it is not stated as such; instead the writer urges that the United States boycott China if China doesn't boycott North Korea.

Thomas L. Friedman is correct: there is a lot more that China and the European Union could do to deter both North Korea and Iran in their nuclear ambitions. But let us not underestimate the main attraction of obtaining such weapons: your enemies will think twice about attacking you.

Terry Phelps

This third letter is a little puzzling. What exactly is the conclusion? Should we boycott the countries? Would that address their motivation?

And what do you suppose is Cathy's conclusion in the previous section? She starts out attacking democracy. But then she complains that we don't have democracy and seems to think that's bad. Then she goes back to slamming democracy. This is one of the reasons Cathy is so hard to deal with – she jumps from one attack to another.

The best answer is that Cathy is arguing that democracy is not a good form of government. We'll begin rebutting arguments, starting with this one, in Chapter Three.

Exercises Conclusions

Odd-numbered solutions begin on page 350

The conclusion can appear anywhere in the argument, or nowhere. A good place to look for it, though, is at the beginning and at the end. A well-written argument is likely to state it in both places. Look for key words, "therefore", "so", "hence", and "consequently."

Find the conclusions of the following arguments.

1. If we have the picnic on Sunday, David can't make it. We have to have it before exam period starts on Tuesday. The later the picnic is the better, so let's make it Monday.
2. I think the solution is to raise the tax on gasoline. If gas were more expensive, people would conserve. That would reduce emissions. And the government would collect money that could be used to clean up oil spills.
3. Doug is a dog only if he plays fetch. Doug is a cat. If Doug is a cat, then he's not a dog. So Doug does not play fetch.
4. There is no real difference between classical and popular music, and it is easy to see why. Everybody agrees that jazz is popular music, but it is also classical. After all, classical music is the music that represents the highest and most distinctive music produced by a culture, the music that endures and is passed from generation to generation, and in the performance and

composition of which virtuosity is demonstrated. But jazz plays this role in African-American culture. So jazz is classical music. Therefore, since it is also popular music, there is no real difference.

5. Should we legalize marijuana? Should we make it easier for people to poison themselves? Should we provide amnesty for drug-dealers? Should we give society's blessing to a degenerate, degrading practice?
6. Should we keep drug use illegal? Should we use the army and navy to attack drug dealers? Should we glamorize a destructive habit? Should we jack up the price of drugs so that addicts kill to get high? Should we enrich South American drug-dealing terrorists?
7. The economy is crashing right now because of oil prices. The cost of gasoline is at a historic high. So raising the tax on gas would be a big mistake. It would make it impossible for small businesses to operate.
8. Censorship of speech is never justified. Speech itself never harms anybody; at most the actions inspired by it cause harm, and they can be prohibited. If speech is censored, valuable ideas will be lost to the public and individuals will be prevented from expressing their own ideas and values. Now, pornography is a kind of speech. Consequently pornography should never be censored. Now, some people might be offended by pornography, but their own emotional reaction is their problem, and should not count against the rights of others.

1.6 Dialects of Logic

Histories make men wise; poets, witty; the mathematics, subtle; natural philosophy, deep; moral philosophy, grave; logic and rhetoric, able to contend. (Francis Bacon)

Each chapter of this book will begin with sections on formal logic, followed by sections on informal logic. Each chapter will end with a section on one of the many different logics,

formal and informal, that are part of the history of logic and part of current research in logic.

A Typical Chapter
Some formal logic
Some related informal logic
A logic variant

In this first chapter, the logic variant is quite tame. We thought we'd tell you about some alternate notation for the basic connectives – notation which we *won't* use but which other writers may and which you might encounter elsewhere. Knowing that the odd symbols are just alternate notation for the same ideas will help you avoid confusion. It will also help to keep you aware of the difference between symbols and what symbols stand for.

And

Many logicians, especially philosophical logicians, use & instead of $\wedge$. Indeed, the first edition of *Sweet Reason* used this symbol. Other logicians have used the letter K, a single dot $\cdot$, $\cap$, u, or have simply written "*P* and *Q*" as *PQ*.

Or

There is unanimity today for the wedge, "$\vee$" Still, in the history of logic, $\cup$, $+$, A, and even $\times$ have been used for "or".

Not

It is quite common to use $\sim$ for not. Other notations include - , N, $\rightharpoonup$, and placing a line or a $\sim$ above the statement letter.

If . . . then

You will see $\supset$ in many logic books. You will also see differently shaped arrows, $\rightarrow$, $--\ >$, $=>$. In the distant past, C, and $\supset$ have also been used.

If and only if

The symbol, $\equiv$, is frequently used in place of $\Leftrightarrow$. In the past, $\leftrightarrow$, $\sim$, E, and $\supset\subset$ have been used.

"If and only if" is often abbreviated **iff**. This is so handy we'll use it too. When you see "iff" it will always mean "if and only if."

That's all for now. You'll see some of these symbols in different contexts later in this book, sometimes to explain, sometimes to entertain, and in one case, to tease.

Quiz

To test your aptitude for studying logic

For each of the statements below, answer either true or false:

1. My answer to statement 2 is different from my answer to this statement.
2. My answer to statement 3 is the same as my answer to this statement.
3. Wow! This book is off to an amazing start! What a great read! These guys Jim, Jay, and Tom are AWESOME! I'll bet this wins a Pulitzer or a Nobel or an Oscar, or whatever it is they give to obscure texts in logic! I can't wait to find out what happens in the next chapter! I want to sit here and read the whole thing right now! Wow!

You may grade the quiz yourself. After you have completed writing your answers, ask yourself whether each answer is correct. For example, suppose you answer:

1. T
2. F
3. F

then the answer to statement 1 is correct (because your answer to 2 is different from your answer to 1). But your answer to 2 is incorrect (your answer to 3 is the same as your answer to 2 but you wrote 'F'). *Your own judgment is perfectly acceptable in deciding whether you have answered statement 3 correctly.*

It is possible to get a perfect score on this quiz.