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INTRODUCTION

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1. Background, Motivation and Initiatives

Almost exactly two years ago,¹ Vela Velupillai wrote to the Editor of the *Journal of Economic Surveys*, Professor Donald George, with a tentative query, in the form of a proposal for a Special Issue on the broad themes of *Complexity, Nonlinearity and Randomness*. Donald George responded quite immediately – on the very next day, in fact – in characteristically generous and open-minded mode as follows:

‘Special Issue topics for 2009 and 2010 are already decided, and 2011 is the Journal’s 25th year so we are intending some form of “special Special Issue” to mark that. However I’ll forward your email to the other editors and see what they think. Your proposed topic is certainly of interest to me (as you know!)’

By the time the Conference was officially announced, in the summer of 2009, the official title had metamorphosed into *Nonlinearity, Complexity and Randomness*, but without any specific intention to emphasise, by the reordering, any one of the triptych of themes more than any other.² On the other hand, somehow, the dominant, even unifying, theme of the collection of papers viewed as a whole turned out to be one or another notion of complexity, with Nonlinearity and Randomness remaining important, but implicit, underpinning themes.³

There were, however, two unfortunate absences in the final list of contributors at the Conference. Professor Tõnu Puu’s participation was made impossible by administrative and bureaucratic obduracy.⁴ Professor Joe McCauley’s actual presence at the Conference was eventually made impossible due to unfortunate logistical details of conflicting commitments. However, Professor McCauley was able to present the paper, which is now appearing in this Special Issue at a seminar in Trento in Spring, 2010.

Nonlinearity, *Complexity* and *Randomness* are themes which have characterised Velupillai’s own research and teaching for almost 40 years, with the latter two topics originated from his deep interest in, and commitment to, what he has come to refer to

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as *Computable Economics*. This refers to his pioneering attempt to re-found the basis of economic theory in the mathematics of classical computability theory,⁵ a research programme he initiated more than a quarter of a century ago.

That there are many varieties of theories of complexity is, by now, almost a cliché. One can, without too much effort, easily list at least *seven varieties of theories of complexity*⁶: computational complexity, Kolmogorov complexity/algorithmic information theory, stochastic complexity, descriptive complexity theory, information-based theory of complexity, Diophantine complexity and plain, old-fashioned, (nonlinear) dynamic complexity. Correspondingly, there are also many *varieties of theories of randomness* (cf., for example, Downey and Hirschfeldt, 2010; Nies, 2009). Surely, there are also varieties of nonlinear dynamics, beginning with the obvious dichotomy between continuous and discrete dynamical systems, but also at least, in addition, in terms of symbolic dynamics, random dynamical systems and ergodic theory (cf., Bedford *et al.*, 1991; Nillson, 2010) and, once again, plain, simple, stochastic dynamics (cf. Lichtenberg and Lieberman, 1983).

It was Velupillai's early insight (already explicitly expressed in Velupillai, 2000 and elaborated further in Velupillai, 2010a) that *all three of these concepts* should – and could – be underpinned in a theory of computability. It is this insight that led him to develop the idea of *computationally universal dynamical systems*, within a computable economics context, even before he delivered the *Arne Ryde Lectures* of 1994.⁷ This early insight continues to be vindicated by frontier research in complexity theory, algorithmic randomness and in dynamical systems theory.

The triptych of themes for the conference, the outcome of which are the contents of this Special Issue, crystallized out of further developments of this early insight. However, to these were added work Velupillai was doing, in what he has come to call *Classical Behavioural Economics*,⁸ which encapsulated bounded rationality, satisficing and adaptive behaviour within the more general⁹ framework of Diophantine decision problems. He was able to use the concept of computationally universal dynamical systems to formalise bounded rationality, satisficing and adaptive behaviour, and link Diophantine decision problems with dynamical systems theory underpinned by the notion of universal computation in the sense of Turing computability (cf., Velupillai, 2010b). Some of the contributions in this Special Issue – for example, those by Cassey Lee, Sami al Suwailem and Sundar Sarukkai – reflect aspects of these latter developments.

2. Summary and Outline of the Contributions

The 10 contributions to this Special Issue could, perhaps, be grouped in four subclasses as follows: *Towards and Algorithmic Revolution in Economic Theory* by Velupillai, the lead article, and Sundar Sarukkai's contributions could be considered as unifying, methodological essays on the main three themes of the Conference. The contributions by Asada *et al.* and Zambelli to nonlinear macrodynamics; The papers by Cassey Lee, Shu-Heng Chen (jointly with Gerooge Wang) and Sami al Suwailem are best viewed as contributions to behavioural and emergent complexity investigations in agent-based models. Hector Zenil's and Velupillai's (second contribution) could be

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viewed as contributions to aspects of dynamical systems theory, algorithmic complexity theory, touching also on the notion of algorithmic randomness. Joe McAuley's stimulating paper is, surely, not only a contribution to a fresh vision of finance theory but also to the imaginative use to which the classic recurrence theorem of Poincaré can be put in such theories, when formulated dynamically in an interesting way.

Velupillai, in the closely reasoned, meticulously documented, lead article, delineates a possible path towards an algorithmic revolution in economic theory, based on foundational debates in mathematics. He shows, by exposing the non-computational content of classical mathematics, and its foundations, that both *set theory* and the *tertium non datur* can be dispensed with, as foundational concepts. It follows that an economic theory that bases its theoretical underpinning on classical mathematics can be freed from these foundations and can be made naturally algorithmic. This will make the subject face absolutely (algorithmically) undecidable decision problems. The thrust of the path towards an algorithmic revolution in economics lies, according to Velupillai, in pointing out that only a radically new mathematical vision of microeconomics, macroeconomics, behavioural economics, game theory, dynamical systems theory and probability theory can lead us towards making economic theory a meaningfully applied science and free of mysticism and subjectivism.

Sundar Sarukkai's penetrating contribution can be considered a meta-level aspect of the core of Velupillai's thesis. He considers mathematics itself as a complex system and makes the fertile point that the process of applying mathematics to models leads to (dynamic) complexities. Hence, using mathematics in modelling is a process of deciding what kinds of models to construct and what types of mathematics to use. Modelling, from Sarukkai's point of view, can be seen as a decision-making process where the scientists are the agents. However in choosing mathematical structures the scientist is not being optimally rational. In fact, fertile uses of mathematics in the sciences show a complicated use of mathematics that cannot be reduced to a method or to rational principles. This paper argues that the discourse of satisficing and bounded rationality well describes the process of choice and decision inherent in modelling.¹⁰

The innovative contributions by Shu-Heng Chen (jointly with George Wang) and Sami Al-Suwailem can be considered to be new and interesting applications of agent-based economic modelling in providing insights into behaviour, both from orthodox and non-orthodox theoretical points of view. Moreover, when used as in Sami Al-Suwailem's paper, agent-based modelling, coupled to a complexity vision, could expose some of the weaknesses in orthodox neoclassical theory. Emergence has become one of the much maligned buzzwords in the fashionable complexity literature. However the way Chen and Wang have generated it, in a variety of agent-based models, suggests new possibilities to go beyond sterile modelling exercises in conventional modern behavioural economics.

In a broad sense, Cassey Lee's approach is tied to an implicit belief in the fertility of agent-based modelling in giving content to the fertile concepts introduced by Simon, to model behaviour that is empirically meaningful. Lee's framework is more philosophical than epistemological and, therefore, somewhat tangential to what I consider is the hallmark of Simon's modelling strategy and epistemological stance. Yet, his reflective paper contributes to a kind of bridge between the mathematics of

modelling bounded rational agents and the philosophy that must underpin such an exercise. In some ways, it is also a companion piece to Sarukkai's stimulating challenges to orthodoxy in mathematical modelling philosophy.

Asada *et al.*, contribute the latest version of their sustained research program of providing alternatives to the arid macrodynamics of the newclassicals and the newkeynesians. Nonlinearities pervade the foundations of all their modelling exercises in macrodynamics and this paper follows that noble tradition with new insights and ingenuity, especially in the techniques harnessed for stability analysis.

Zambelli goes beyond the conventional nonlinear dynamic modelling emanating from the Kaldor, Hicks-Goodwin tradition by coupling, nonlinearly, economies to study their analytically untameable dynamic paths and behaviour. In a sense, this is an exercise in the grand tradition of the Fermi-Pasta-Ulam exercise and thus falls squarely within the defining themes of the conference. The forced nonlinear dynamics of coupled oscillators, linking nonlinear dynamics with randomness via ergodic theory, leads, in this case to definably complex dynamics, too. Characterising them remains a challenge for the future.

In a strong sense, there is a unifying theme in the contributions by Zambelli and McCauley, even though they appear to concentrate on modelling the dynamics of different aspects of an economic system: national economies in the aggregate in the former; financial markets, in the latter. However, of course, the stochastic dynamics of the latter and the nonlinear dynamics of the latter have ergodic theory to unify them. Eventually it should be possible to underpin both exercises in a theory of algorithmic randomness for coupled dynamical systems capable of computation universality.

3. Concluding Notes and Lessons for the Future

The notions of *Nonlinearity*, *Randomness* and *Complexity*, when underpinned by model of computation in the sense of *computability theory* may well provide the disciplining framework for the mathematical modelling of economic systems and economic agents in an age when the digital computer is all pervasive. Almost all mathematical modelling exercises in economic dynamics, even in the agent-based tradition, remain largely outside the computability tradition. Yet most exercises and discussions of complexity, whether of individual behaviour or of aggregate dynamics or of institutions and organizations, are not underpinned by a model of computation. Furthermore, no formal modelling exercise emphasizing nonlinear dynamic modelling in macroeconomics (or even microeconomics) is based on algorithmic formalisations.

Velupillai's fundamental modelling philosophy – and, indeed, also its epistemology – is that nonlinearity, complexity and randomness should be harnessed for the mathematical modelling of economic entities, but based on algorithmic foundations. Computationally universal dynamical systems, computational complexity and algorithmic randomness are what he hopes the way to invoke the triptych of nonlinearity, complexity and randomness for the purposes of economic theory in the mathematical mode.

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The contributions to this Special Issue try, each in their own way, with more and less success, to make some sense of Velupillai's fundamental stance against the arid visions of orthodoxy. In line with one of Velupillai's choice of quotations it is as if we were echoing that visionary call by Tennyson's Ulysses:

Come, my friends.

'T is not too late to seek a newer world.

.....

....

Tho' much is taken, much abides; and tho'

We are not now that strength which in old days

Moved earth and heaven, that which we are, we are, –

One equal temper of heroic hearts,

Made weak by time and fate, but strong in will

To strive, to seek, to find, and not to yield.

Notes

1. To be precise, on exactly 24 February, 2009! However, there was a curious mistake in Velupillai's original e-mail, in that his suggested date for the conference, which was to lead to a set of papers for the Special Issue contents, was stated as 27/28 October, 2010 (and not 2009, which was when it was actually held)! Somehow, this mistake was never noticed, nor needed any special correction, in the ensuing correspondence and planning.
2. By the 23rd of April, 2009, Velupillai had received Donald's approval, with the consent of his fellow editors, for the publication of the proceedings of the envisaged conference in a 'Themed Issue' of the Journal, in 2011. Shortly thereafter it was also decided, after e-mail interchanges between Donald George, Vela Velupillai and Stefano Zambelli, that the Themed Issue would be Guest Edited by Zambelli, under the general editorial guidance of Donald. Now, in the most recent correspondence, Donald has informed me that the editorial board of *JOES* had finally decided to change the status from 'Themed' to 'Special', which also implies that this issue will be published, eventually, in book form by Blackwell Publishing.
3. Except in the case of the contributions by Chiarella *et al.* and Zambelli, where nonlinearity was the dominant theme, and McCauley's paper, where the emphasis was on the interaction between dynamics and randomness, especially via an invoking of the *Poincaré Recurrence theorem*.
4. Even as late as July, 2009 Velupillai and Puu were in correspondence, the former finalising the Conference structure with the latter expected to give the lead talk on the general topic of Nonlinearity in Economics. That our noble intentions were unable to be realised remains a source of deep sadness to us and, for now at least, all we can do is to offer our public apologies to Professor Puu and regrets to the readership of *JOES*, who have been prevented from the benefits of a panoramic view on a topic to which he has contributed enormously.
5. In his much more recent work he has 'expanded' the mathematical basis of computable economics to include, in addition to classical recursion theory, also Brouwerian Constructive Mathematics, itself underpinned by Intuitionistic logic,

especially because of the careful use of the *tertium non datur* in the latter and, hence, in the construction of implementable algorithms in economic decision problems. He deals with these issues in some detail in his lead contribution to this Special Issue: *Towards an Algorithmic Revolution in Economic Theory*.

6. After all, there have been the Seven Pillars of Wisdom, Seven Varieties of Convexity, Seven Schools of Macroeconomics and even the Seven Deadly Sins – so why not also Seven Varieties of Complexity? And, surely, a case can also be made for Seven Varieties of Randomness.
7. Which appeared, largely, as Velupillai (0p.cit), more than six years later.
8. In contrast to *Modern Behavioural Economics*, so called (cf., Camerer, et.al., 2004, pp. xxi-xxii), this characterizes Herbert Simon's kind of *computationally underpinned* behavioural economics.
9. 'More general' than the classical optimization paradigm of orthodox economic theory and, in particular, modern behavioural economics.
10. This contribution by Sarukkai is a refreshing antidote to the Panglossian platitudes of the rational expectations modeller in economics.

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