

The Mystery of Phi (1.618) and phi (0.618)

WHY IS THE GOLDEN RATIO (1.61803 . . .) of such interest to traders? Known by various names since the ancient Egyptians and Pythagoreans 570–490 BC, the first definition¹ comes from Euclid (325–265 BC); *The Divine Proportion*, or *Divina Proportione*, by Luca Pacioli (1445–1517) was the earliest known treatise devoted to the subject and was illustrated by Leonardo da Vinci, who coined the name *section aurea* or the “golden section.” By any of its historical names, it is of little interest to investors, but investors rarely face the day-to-day battlefield of rapid global market action. Traders on the frontlines, however, live and die on their ability to measure risk and control their capital drawdown. To enter the tough markets of today requires the foresight to call what price a market will move toward. Of equal importance is our need to know where a market should *not* be trading so we can execute a timely exit plan.

So how does an eloquent ratio that has captured people’s interest from the time of antiquity help and influence the decisions a trader must face today? The Golden Ratio (**Figure 1.2**) is a universal law that explains how everything with a growth and decay cycle evolves. Be it the spiral of the solar system, the spiral of a hurricane (**Figure 1.1**), the growth pattern of a nautilus seashell, or the spiral that forms at the start of life as a fern begins to unfurl from the ground on a spring day. Plato (circa 427–347 BC) in the *Republic* asks the reader to “take a line and divide it unevenly.” Under a Pythagorean oath of silence not to reveal the secrets of the mysteries, Plato posed questions in hopes of

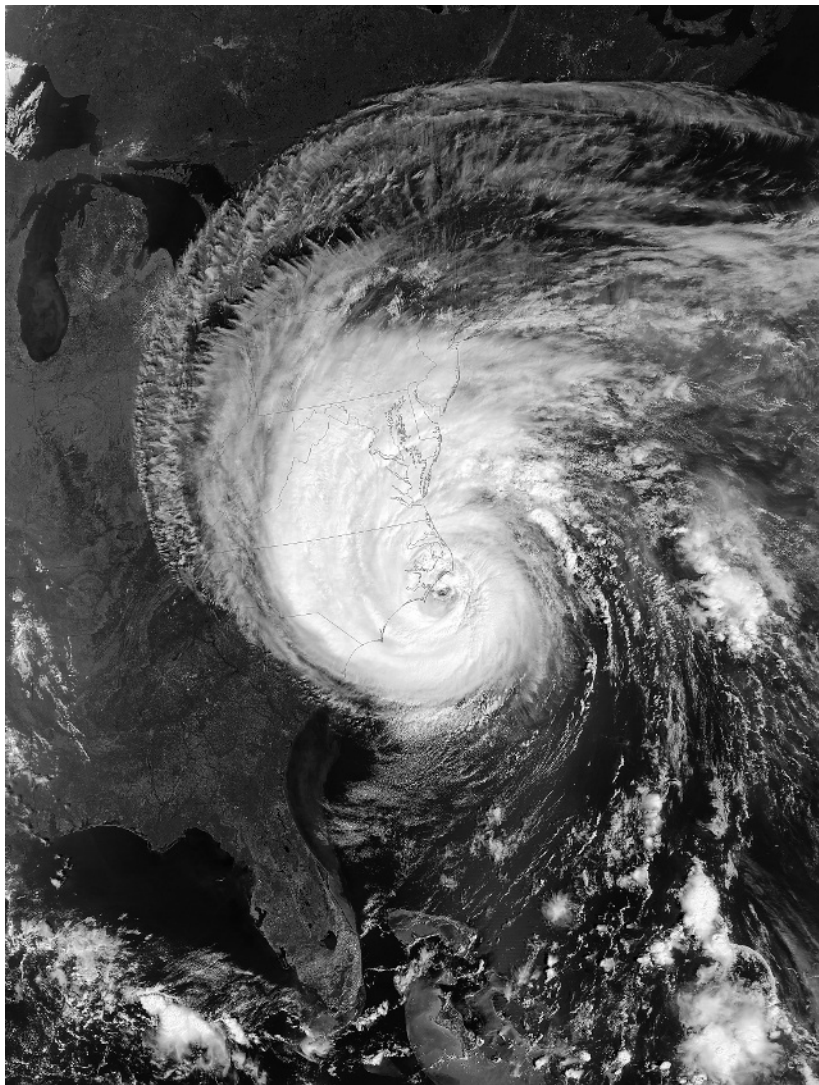


FIGURE 1.1 Hurricane Isabel

Source: NASA

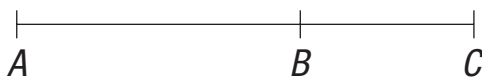


FIGURE 1.2 The Golden Ratio

provoking an insightful response. So why does he use a line, rather than numbers? Why does he ask you to divide it unevenly?

To answer Plato and how this may connect to the needs of all traders today, we first must understand ratio and proportion. All living things from large to small, *including markets*, must abide by the same divine blueprint. As a result, the more traders understand how a ratio called Phi (1.618) influences our lives, the clearer we can look into the face of a grander plan and witness a force acting upon our markets that demands our respect, comprehension, and humility. As a trader's knowledge deepens, he can see in advance a mathematical grid that determines future market movement. But few people truly understand the forces at play and never develop beyond the basic theory that is insufficient to adjust to the natural expansion and contraction cycles found within all markets in both price and time.

Ratio, Means, and Proportion

Ratio (*logos*) is the relation of one number to another, for instance 4 : 8, stated as "4 is to 8." However, proportion (*analogia*) is a repeating ratio that typically involves four terms, 4 : 8 :: 5 : 10, stated as "4 is to 8 is as 5 is to 10." The Pythagoreans called this a four-termed discontinuous proportion. The invariant ratio here is 1 : 2, repeated in both 4 : 8 and 5 : 10. An inverted ratio will reverse the terms, so it can be said, 8 : 4 is the inverse of 4 : 8, and the invariant ratio now is 2 : 1.

Few traders truly grasp the concepts of ratios and become bound by this limitation from doing more advanced work. As example, standing between the two-termed ratio and the four-termed proportion is the three-termed mean, in which the middle term is in the same ratio to the first as the last is to it. The geometric mean between two numbers is equal to the square root of their product. Therefore, it can be said,

the geometric mean of 1 and 9 is $\sqrt{(1 \times 9)}$, which equals 3. This geometric mean relationship is written as $1 : 3 : 9$ or inverted, $9 : 3 : 1$. It can also be written more fully as a continuous geometric proportion where these two ratios repeat the same invariant ratio of $1 : 3$. Thus, $1 : 3 :: 3 : 9$. The 3 is the geometric mean held in common by both ratios. This is the interlacing mathematical glue that binds them together.

The Pythagoreans called this a three-termed continuous geometric proportion. The Golden Ratio has a three-termed continuous geometric proportion. As viewed in Figure 1.2, the Golden Ratio is the ratio that results between two numbers A and C when a line is divided, so that the whole line, AC , has the same ratio to the larger segment, AB , as the larger segment, AB , has to the smaller segment, BC . $AC : AB :: AB : BC$. The geometric mean held in common by both ratios is B or 0.618. As B is the geometric mean of A and C , it can be written as $A : B : C$. We traders will use this ratio to define the mathematical grid a market is forming to build future price swings and pivots.

Pythagoras

Pythagoras (570–490 BC) was born in Ionia on the island of Samos in Greece. He eventually settled in Crotona, a Dorian Greek colony in southern Italy in 529 BC. Pythagoras's thought was dominated by mathematics, and the Pythagorean philosophy can be summarized as: Only through mathematics can anything be proven, and by mathematical limit the unlimited can take form. The thinking of the Pythagoreans was exceedingly advanced for their time in many ways. As a simple example, they knew there must be a substance in the air to allow sound to travel from one person's mouth to another's ears. They knew nothing about atoms or kinetic energy, but already had the conceptual idea. Pythagoras's school gradually formed into a society or brotherhood called the Order of the Pythagoreans. Growing in wealth and power, Pythagoras² was murdered when their meetinghouse was torched. The remaining Pythagoreans were scattered across the Mediterranean. Pythagoras wrote nothing down and none of his followers' original papers on mathematics survived. However, after the attacks on the Pythagoreans at Crotona, they regrouped in Tarentum in southern Italy. Collections of Pythagorean writings on ethics³ show a creative response to their troubles.

In a biography of Pythagoras written seven centuries after Pythagoras's time, Porphyry (AD 233–309) stated the Pythagoreans were divided into an inner circle called the *mathematikoi* (“mathematicians who study all”) and an outer circle called the *akousmatikoi* (“listeners”). Pythagoras's wife Theano and their two daughters led the *mathematikoi*



FIGURE 1.3 Woman Teaching Geometry 1309. It is rare to find a woman teaching Geometry, one of the sacred sciences, as depicted in this painting from 1309. It shows her teaching a group of young monks.

after Pythagoras's death.⁴ The early work of the Pythagoreans on the Golden Ratio might have been lost if it had not been for a woman: Theano, who was a mathematician in her own right. She is credited with having written treatises on mathematics, physics, medicine, and child psychology, although nothing of her writing survives. Her most important work is said to have been a treatise on the principle of the golden mean. In a time when women were usually considered property and relegated to the role of housekeeper or spouse, her work was in keeping with Pythagoras's views that allowed women to function on equal intellectual terms in his society.

Most of the concepts developed by the Pythagoreans gave credit to Pythagoras himself. So it is hard to unravel his specific accomplishments from his followers. But Aristotle, in his *Metaphysica*, sums up the Pythagorean's attitude toward numbers:

The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this subject, but saturated with it, they fancied that the principles of mathematics were the principles of all things.⁵

The Pythagoreans knew just the positive whole numbers, zero, and negative numbers, and the irrational numbers didn't exist in their system. Despite this limitation about irrational numbers, Pythagoras⁶ is believed to have discovered the Golden Ratio through sound—the sound of two hammers of different weights hit an anvil producing a harmonic pitch of divine perfection. This fact, that the origin of the Golden Ratio was discovered through a harmonic ratio, will have implications in the final chapters of this book. It will change how traders should think and apply the Golden Ratio to analyze the markets.

While accounts of Pythagoras's travels differ, historians agree he traveled to many countries to study with the masters of his time. Many historians believed he was an initiate of the ancient Egyptian priests, as was Plato and Euclid. But why is this of interest in the business of trading today's markets? The ancient priests of Egypt believed that only a student well versed in the ancient quadrivium—the study of seven disciplines that included arithmetic, geometry, music, and astronomy—would be able to solve the problems they faced in their life. This philosophy for attaining wisdom is no different than what is

needed to understand the full potential of the Golden Ratio applied to our global market puzzles today. It is only through a study of ratios can one understand the concepts of proportion, symmetry, harmonics, and rhythm. Many traders erroneously assume that markets move in fixed intervals within linear dimensions. *Markets do not move in such a manner.* Future market price swings will expand or contract in measurable ratios derived from *multiple prior price swings*. Markets are living breathing entities, and the industry is stuck with outdated assumptions and applications that we must reevaluate. In order to move forward beyond the elementary, we must revisit the teachings of the past so we can reprogram our thinking.

It is generally accepted that Plato offers the world the earliest written documentation of the Pythagoreans.⁷ In 387 BC, Plato founded an academy in Athens, often described as the first university. Plato loved geometry also. His school included a comprehensive curriculum of biology, political science, philosophy, mathematics, and astronomy. Over the doors to his academy were the words *αγεωμερητος μηδειςεισιτω*, meaning “Let no one destitute of geometry enter my doors.”

It was long thought both Pythagoras and Plato were by-products of Egyptian schooling. No proof to this regard was available until recently. The highly respected Egyptian Egyptologist Dr. Okasha El-Daly has released in his book, *Egyptology: The Missing Millennium*,⁸ proof that the Greek philosopher Pythagoras had been a student of the carefully guarded wisdom of the ancient Egyptian high priests. Until now, this was only suspected to be true. During a dinner in Sharm El-Sheikh, Egypt, with Dr. El-Daly, he commented to me that the ancient Egyptians had such high regard for Pythagoras that El-Daly found Egyptian papyrus in the British Museum vaults documenting the construction of a tomb chamber within the Pyramid of Menkaure.⁹ This is the third pyramid on the Giza plateau and the smallest of the three. Pythagoras did not die in Egypt and is not buried there, but the references to these preparations for an afterlife were thought unheard of except for the Pharaohs and immediate family members themselves. Why was Pythagoras so highly revered?

Here's the heart of what is of interest to us today. The Pythagoreans believed all things were connected. Movement formed vibrations that brought harmonic associations to all things. (Translated to

all pivots within our price data.) It was thought that only through arithmetic could one prove a universal truth. You might say that the Pythagoreans were the first to consider the law of vibration, a concept of great interest to market analysts using the methods of W.D. Gann. Gann is the trader/analyst who correctly called the great crash of 1929 in his annual report written in 1928. Not only did he call the decline, he also recognized the exact bottom one month after the market low. You will begin to understand the law of vibration in the final chapter and why it is of interest to us.

It is useful to know that Gann was also trained in these ancient studies, and the concepts are still valued by Freemasons today as they seek enlightenment and truth. Gann attained the second-highest order of the Scottish Rite, and as a thirty-second degree Freemason he believed similarly to the Pythagoreans that *arithmetic is pure number, geometry is number in space, music is number in time, and astronomy is number in space and time.*

An invisible thread connecting all these fields of study is the ratio 1.618 (Phi) or its inverse reciprocal 0.618 (phi),¹⁰ which proves to be more valuable in our applications of market analysis. These early concepts of the Pythagoreans unlock how multiple market pivots can accurately define the next price swing or inflection price levels that the market will respect. The underlying premise is to understand that markets grow and contract to universal constants that are predictable, measurable, and present in all market time horizons. Through Fibonacci analysis and the ratios of Phi (1.618) and its reciprocal phi (0.618), we can prove markets follow the laws of nature and we in turn must work with these underlying principles rather than force our will on the price data with erroneous assumptions.

Fibonacci

The Fibonacci number series: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, . . . is both additive, as each number is the sum of the previous two, and multiplicative, as each number approximates the previous number multiplied by the golden section ratio. The ratio becomes more precise as the numbers increase. Inversely, any number divided by its smaller neighbor approximates 0.618 or phi ($1/\phi$).

While traders with rudimentary knowledge know they can add, subtract, divide, and multiply Fibonacci ratios to obtain another Fibonacci ratio,¹¹ they often do not apply this knowledge in their chart analysis. As an example, traders during the Nasdaq crash in 2000 and 2001 would have had no means to determine a price target low if they only knew how to calculate price support levels by subdividing the extreme price high and historic low into the ratios of 61.8 percent, 50 percent, and 38.2 percent.

In **Figure 1.4**, we see the monthly chart for the Nasdaq Composite Index. Range *A* begins at the price high and ends at a price low significantly higher than the market low. The price low selected in 1998 was a significant decline, but was not the historic low, which would have made the problem we are about to discuss worse. Traders who simply subdivided the range as defined by range *A* would witness the market falling through the 61.8 percent support zone and not know what to do to identify new targets. A retracement of 61.8 percent is the decline that occurred relative to the full price range selected. Traders and analysts could further subdivide the range between the 61.8 percent level of *A* and the price low first selected to obtain additional Fibonacci ratios.¹² They in turn could find additional support levels by dividing the range marked *C* into the ratios 38.2 percent, 50.0 percent, and 61.8 percent. We could further subdivide range *D*. Each of these additional subdivisions shows that the Nasdaq Composite respected the ratios identified. Any Fibonacci ratio that is added, subtracted, multiplied, or divided will produce another Fibonacci ratio. Many traders know this, but few know how to apply it. Then I raise the question, should the key reversal spike high into the market high have been truncated when the start of the range was defined? This question will be answered and many more as these chapters unfold.

The key observation to draw from the Nasdaq chart in Figure 1.4 is that each price range identified—*A*, *B*, *C*, and range *D*—is subdivided into three ratios, 38.2 percent, 50 percent, and 61.8 percent, relative to its starting and ending price levels. The calculations that result from subdividing each range are entirely dependent on the range first selected, and these ranges require thought as they become critical to our success. We will not be starting at the price high and then using the extreme price low as favored by our industry to apply

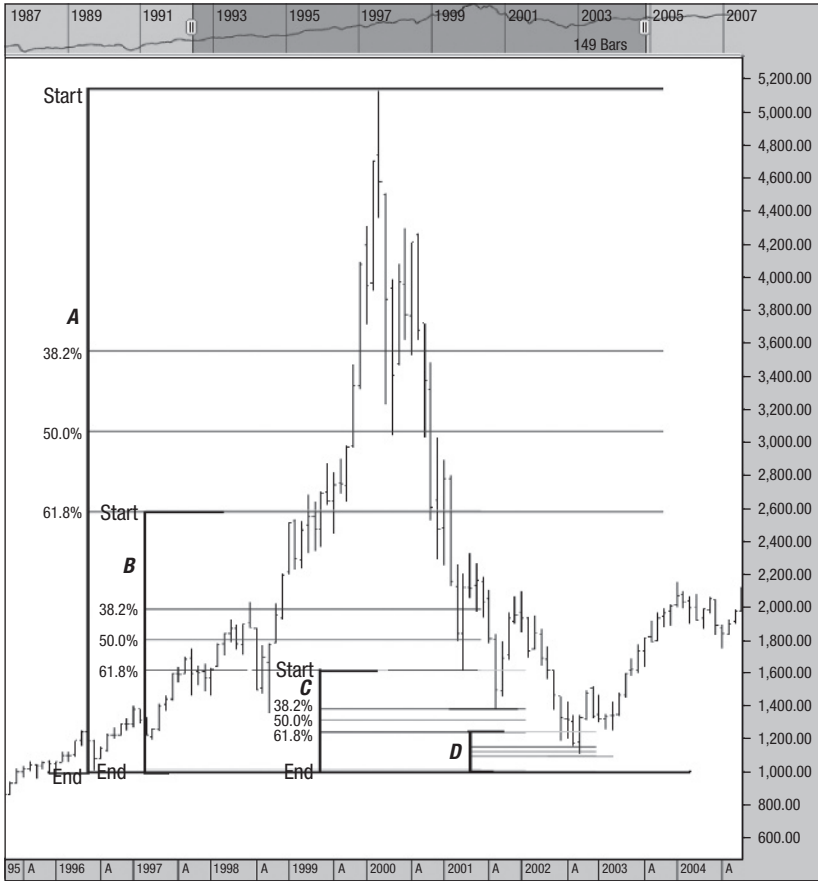


FIGURE 1.4 Nasdaq Composite Index Monthly Chart

Connie Brown, www.aeroinvest.com. Source: Copyright © 2008 Market Analyst Software

Fibonacci analysis. Figure 1.4 is a good introduction but more is involved. The correct ranges to use will be addressed in detail within the next chapter when we focus on the concept of market expansion and contraction.

Leonardo Fibonacci (circa 1170–1250) was not the first to document the Fibonacci number sequence. That honor falls to ancient Hindu mathematicians.¹³



FIGURE 1.5 Leonardo Fibonacci—(Leonardo da Pisa), by Giovanni Paganucci, in the Camposanto di Pisa Cemetery, Italy

Source: Private photograph collection of Connie Brown

Fibonacci might have lost his identity with the number sequence entirely had it not been for Édouard Anatole Lucas¹⁴ (1842–1891), a French mathematician who *rediscovered* the Fibonacci sequence in the late nineteenth century. It was Lucas who attributed the number series to Fibonacci's book *Liber Abaci* (*Book of the Abacus*, 1202), thereby establishing the name of the numbers as the Fibonacci Number Sequence. Lucas is known for defining a formula to find the n th term of the Fibonacci sequence.

Liber Abaci demonstrates routine computations that merchants performed when converting currencies. Fibonacci showed the advantages

of using the Hindu–Arabic number system compared with the Roman numeral system. In hindsight, Fibonacci made things much harder than they needed to be by frequently expressing fractions as unit fractions. As example: *One quarter and one third of a tree lie below ground, a total of twenty-one palmi in length. What is the length of the tree?*²¹⁵ Fortunately, Fibonacci’s book made a huge impact, and as a result, we do not have to trade markets using Roman numerals. But sadly Fibonacci’s studies through Mesopotamia have been lost. The library and museum of Mesopotamian artifacts vanished in the twenty-first century, when the Iraq National Museum in Baghdad was looted and burned after the 2003 Gulf War invasion. In Chapter 4, you will see a valuable artifact within the British Museum of great interest, but an Egyptian Egyptologist was near tears when she told me during my 2007 visit to Cairo that all the manuscripts and papyrus they had in Iraq are now lost forever.

CHAPTER NOTES

Abbreviations are references to the bibliography contained at the back of this book. As example, Leh, 8, 17 refers to the alphabetized bibliography code, Leh, which further describes: Lehner, Mark. *The Complete Pyramids: Solving the Ancient Mysteries*. London: Thames and Hudson Ltd., 1997.

1. The first clear definition of what later became known as the Golden Ratio was given around 300 BC by the founder of geometry, Euclid of Alexandria. Euclid defined a proportion derived from a simple division of a line into what he called its “extreme and mean ratio.” In Euclid’s words:

A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.

—HEATH

2. Hall, 191–223; see also Taylor.
3. Taylor.
4. Hall; Taylor.
5. Aristotle, *Metaphysica*, 1–5.
6. Stroh, 82–85.
7. Robin.

8. El-Daly, 60.

9. Mad, 5. The story of phi begins in the fourth dynasty of Egypt, about 2500 BC. The last of the original great Seven Wonders of the Ancient World are the extraordinary pyramids of Egypt. The Great Pyramid of Khufu (called Cheops by the Greeks), the Pyramid of Khafre (Chephren), Khufu's grandson, and the smallest of the three called the Pyramid of Menkaure, are located on the Giza plateau near Cairo.

Everyone agrees on the meticulous precision with which the pyramids were built. Khufu is oriented to within three feet six inches of an arc from true North. In addition, it is well known that Khufu contains the phi proportion between its sides and base. With only rudimentary arithmetic and only ropes or sticks for measuring, this seems like an improbable result. But we will study this geometry further in Chapter 4.

Khufu has been measured repeatedly since the seventeenth century AD, each time with greater accuracy. A right angle could be constructed by folding a rope with thirteen equally spaced knots, such that three spaces between knots form one side of a triangle, four spaces form another side, and five spaces form the hypotenuse. Several researchers have found murals and papyrus that show knotted ropes. Several pyramids were built on the proportions of the three-four-five triangle. The height, half-base, and slant angle of Khafre have been measured at 143.5 meters, 107.5 meters, and fifty-three degrees ten feet. The ratio of these, $143.5/107.5 = 1.33488$, is very close to $4/3$ as verified by the angle. (Leh, 7) This ratio of 1.33 will take on greater significance later in the book.

10. *Merriam-Webster's Collegiate Dictionary: Tenth Edition* shows that Phi (ϕ) is pronounced, "fi" like *fire*, and not *pie*. The reciprocal phi or $1/(\phi)$ is pronounced, "fee" like *bee*.

11. Fibonacci ratios can be added, subtracted, multiplied, or divided and the result will always be another Fibonacci ratio. As examples:

$$\begin{array}{ll} 0.618 \times 0.618 = 0.382 & 0.618 - 0.382 = 0.236 \\ 0.382 \times 0.618 = 0.236 & 0.382 - 0.236 = 0.146 \\ 0.236 \times 0.618 = 0.146 \text{ etc.} & 0.236 - 0.146 = 0.090 \end{array}$$

12. Ibid.

13. Gies, 57, 110, and Dunlap, 35. The Italian mathematician born 1170 AD in Pisa, Italy, was commonly known as Fibonacci, which was a shortened form of Filius Bonaccio (son of Bonaccio). Fibonacci's book *Liber Abaci* (*Book of the Abacus*) introduced in 1202 the Hindu-Arabic system of numbers to Europe. Fibonacci states in his introduction that he accompanied his father Guilielmo, on an extended commercial mission in Algeria with a group of Pisan merchants. There, he says, his father had him instructed in the Hindu-Arabic numerals and computations. He continued his studies in Egypt. (Cooke, 289) (Fibonacci was recognized as a great mathematician, but his solution to the rabbit problem was overlooked when he determined the number of immortal pairs of adult and baby rabbits each month over a one year interval revealing the Fibonacci number sequence 0, 1, 1, 2, 3, 5, 8, 13, . . . and so on.)

See also Parm and Parm, 85, and Pin, 45. In the classical period of Indian mathematics (400 AD–1200 AD), important scholars like Aryabhata, Brahmagupta, and Bhaskara II made early contributions to the study of the decimal number system, zero, negative numbers, arithmetic, and algebra. Trigonometry was introduced into ancient India through the translations of Greek works, further showing that Fibonacci applied the number sequence to solve the rabbit problem, but he did not discover it.

14. Hark, 276–288.

15. Cooke, 289.