PART ONE

Introduction to Bonds

Part One describes fixed-income market analysis and the basic concepts relating to bond instruments. The analytic building blocks are generic and thus applicable to any market. The analysis is simplest when applied to plain vanilla default-free bonds; as the instruments analyzed become more complex, additional techniques and assumptions are required.

The first two chapters of this section discuss bond pricing and yields, moving on to an explanation of such traditional interest rate risk measures as modified duration and convexity, followed by a discussion of floating-rate notes (FRNs). Chapter 3 looks at spot and forward rates, the derivation of such rates from market yields, and the yield curve.

Yield-curve analysis and the modeling of the term structure of interest rates are among the most heavily researched areas of financial economics. The treatment here has been kept as concise as possible, at just two chapters. The References section at the end of the book directs interested readers to accessible and readable resources that provide more detail.

CHAPTER 1

The Bond Instrument

Bonds are the basic ingredient of the U.S. debt-capital market, which is the cornerstone of the U.S. economy. All evening television news programs include a slot during which the newscaster informs viewers where the main stock market indexes closed that day and where key foreign exchange rates ended up. Financial sections of most newspapers also indicate at what yield the Treasury long bond closed. This coverage reflects the fact that bond prices are affected directly by economic and political events, and yield levels on certain government bonds are fundamental economic indicators. The yield level on the U.S. Treasury long bond, for instance, mirrors the market's view on U.S. interest rates, inflation, public-sector debt, and economic growth.

The media report the bond yield level because it is so important to the country's economy—as important as the level of the equity market and more relevant as an indicator of the health and direction of the economy. Because of the size and crucial nature of the debt markets, a large number of market participants, ranging from bond issuers to bond investors and associated intermediaries, are interested in analyzing them. This chapter introduces the building blocks of the analysis.

Bonds are debt instruments that represent cash flows payable during a specified time period. They are essentially loans. The cash flows they represent are the interest payments on the loan and the loan redemption. Unlike commercial bank loans, however, bonds are tradable in a secondary market. Bonds are commonly referred to as *fixed-income* instruments. This term goes back to a time when bonds paid fixed coupons each year. Today that is not necessarily the case.

Asset-backed bonds, for instance, are issued in a number of tranches—related securities from the same issuer—each of which pays a different fixed or floating coupon. Nevertheless, this is still commonly referred to as the fixed-income market.

In the past, bond analysis was frequently limited to calculating *gross redemption yield,* or *yield to maturity.* Today basic bond math involves different concepts and calculations. These are described in several of the references for Chapter 3, such as Ingersoll (1987), Shiller (1990), Neftci (1996), Jarrow (1996), Van Deventer (1997), and Sundaresan (1997). This chapter reviews the basic elements. Bond pricing, together with the academic approach to it and a review of the term structure of interest rates, are discussed in depth in Chapter 3.

In the analysis that follows, bonds are assumed to be *default free.* This means there is no possibility that the interest payments and principal repayment will not be made. Such an assumption is entirely reasonable for government bonds such as U.S. Treasuries and U.K. gilt-edged securities. It is less so when you are dealing with the debt of corporate and lower-rated sovereign borrowers. The valuation and analysis of bonds carrying default risk, however, are based on those of default-free government securities. Essentially, the yield investors demand from borrowers whose credit standing is not risk-free is the yield on government securities plus some *credit risk* premium.

The Time Value of Money

Bond prices are expressed "per 100 nominal"—that is, as a percentage of the bond's face value. (The convention in certain markets is to quote a price per 1,000 nominal, but this is rare.) For example, if the price of a U.S. dollar–denominated bond is quoted as 98.00, this means that for every \$100 of the bond's face value, a buyer would pay \$98. The principles of pricing in the bond market are the same as those in other financial markets: the price of a financial instrument is equal to the sum of the *present values* of all the future cash flows from the instrument. The interest rate used to derive the present value of the cash flows, known as the *discount rate,* is key, since it reflects where the bond is trading and how its return is perceived by the market. All the factors that identify the bond—including the nature of the issuer, the maturity date, the coupon, and the currency in which it was issued—influence the bond's discount rate. Comparable bonds have similar discount rates. The following sections explain the traditional approach to bond pricing for plain vanilla instruments, making certain assumptions to keep the analysis simple. After that, a more formal analysis is presented.

Basic Features and Definitions

One of the key identifying features of a bond is its *issuer,* the entity that is borrowing funds by issuing the bond in the market. Issuers generally fall into one of four categories: governments and their agencies; local governments, or municipal authorities; supranational bodies, such as the World Bank; and corporations. Within the municipal and corporate markets there are a wide range of issuers that differ in their ability to make the interest payments on their debt and repay the full loan. An issuer's ability to make these payments is identified by its *credit rating.*

Another key feature of a bond is its *term to maturity:* the number of years over which the issuer has promised to meet the conditions of the debt obligation. The practice in the bond market is to refer to the term to maturity of a bond simply as its *maturity* or *term.* Bonds are debt capital market securities and therefore have maturities longer than one year. This differentiates them from money market securities. Bonds also have more intricate cash flow patterns than money market securities, which usually have just one cash flow at maturity. As a result, bonds are more complex to price than money market instruments, and their prices are more sensitive to changes in the general level of interest rates.

A bond's term to maturity is crucial because it indicates the period during which the bondholder can expect to receive *coupon payments* and the number of years before the *principal* is paid back. The principal of a bond—also referred to as its *redemption value, maturity value, par value,* or *face value*—is the amount that the issuer agrees to repay the bondholder on the maturity, or *redemption,* date, when the debt ceases to exist and the issuer redeems the bond. The coupon rate, or *nominal rate,* is the interest rate that the issuer agrees to pay during the bond's term. The annual interest payment made to bondholders is the bond's *coupon.* The *cash amount* of the coupon is the coupon rate multiplied by the principal of the bond. For example, a bond with a coupon rate of 8 percent and a principal of $1,000 will pay an annual cash amount of $80.

A bond's term to maturity also influences the volatility of its price. All else being equal, the longer the term to maturity of a bond, the greater its price volatility.

There are a large variety of bonds. The most common type is the *plain vanilla,* otherwise known as the *straight, conventional,* or *bullet* bond. A plain vanilla bond pays a regular—annual or semiannual—fixed interest payment over a fixed term. All other types of bonds are variations on this theme.

In the United States, all bonds make periodic coupon payments except for one type: the *zero-coupon.* Zero-coupon bonds do not pay any coupon. Instead, investors buy them at a discount to face value and redeem them at par. Interest on the bond is thus paid at maturity, realized as the difference between the principal value and the discounted purchase price.

Floating-rate bonds, often referred to as *floating-rate notes* (FRNs), also exist. The coupon rates of these bonds are reset periodically according to a predetermined benchmark, such as 3-month or 6-month LIBOR (London interbank offered rate). LIBOR is the official benchmark rate at which commercial banks will lend funds to other banks in the interbank market. It is an average of the offered rates posted by all the main commercial banks and is reported by the British Bankers Association at 11.00 hours each business day. For this reason, FRNs typically trade more like money market instruments than like conventional bonds.

A bond issue may include a provision that gives either the bondholder or the issuer the option to take some action with respect to the other party. The most common type of option embedded in a bond is a *call feature.* This grants the issuer the right to "call" the bond by repaying the debt, fully or partially, on designated dates before the maturity date. A *put provision* gives bondholders the right to sell the issue back to the issuer at par on designated dates before the maturity date. A *convertible bond* contains a provision giving bondholders the right to exchange the issue for a specified number of stock shares, or equity, in the issuing company. The presence of embedded options makes the valuation of such bonds more complicated than that of plain vanilla bonds.

Present Value and Discounting

Since fixed-income instruments are essentially collections of cash flows, it is useful to begin by reviewing two key concepts of cash flow analysis: discounting and present value. Understanding these concepts is essential. In the following discussion, the interest rates cited are assumed to be the market-determined rates.

Financial arithmetic demonstrates that the value of $1 received today is not the same as that of $1 received in the future. Assuming an interest rate of 10 percent a year, a choice between receiving $1 in a year and receiving the same amount today is really a choice between having $1 a year from now and having $1 plus $0.10—the interest on $1 for one year at 10 percent per annum.

The notion that money has a time value is basic to the analysis of financial instruments. Money has time value because of the opportunity to invest it at a rate of interest. A loan that makes one interest payment at maturity is accruing *simple interest.* Short-term instruments are usually such loans. Hence, the lenders receive simple interest when the instrument expires. The formula for deriving *terminal,* or *future,* value of an investment with simple interest is shown as (1.1).

$$FV = PV(1 + r) \tag{1.1}$$

where
FV = the future value of the instrument
PV = the initial investment, or the present value, of the instrument
r = the interest rate

The market convention is to quote *annualized* interest rates: the rate corresponding to the amount of interest that would be earned if the investment term were one year. Consider a three-month deposit of $100 in a bank earning a rate of 6 percent a year. The annual interest gain would be $6. The interest earned for the ninety days of the deposit is proportional to that gain, as calculated below:

$$I_{90} = \$6.00 \times \frac{90}{365} = \$6.00 \times 0.2465 = \$1.479$$

The investor will receive $1.479 in interest at the end of the term. The total value of the deposit after the three months is therefore $100 plus $1.479. To calculate the terminal value of a short-term investment—that is, one with a term of less than a year—accruing simple interest, equation (1.1) is modified as follows:

$$FV = PV\left[1 + \mathrm{r}\left(\frac{\text{days}}{\text{year}}\right)\right] \tag{1.2}$$

where FV and PV are defined as above,
r = the annualized rate of interest
days = the term of the investment
year = the number of days in the year

Note that, in the sterling markets, the number of days in the year is taken to be 365, but most other markets—including the dollar and euro markets—use a 360-day year. (These conventions are discussed more fully later in the chapter.)

Now consider an investment of $100, again at a fixed rate of 6 percent a year, but this time for three years. At the end of the first year, the investor will be credited with interest of $6. Therefore for the second year, the interest rate of 6 percent will be accruing on a principal sum of $106. Accordingly, at the end of year two, the interest credited will be $6.36.

This illustrates the principle of *compounding:* earning interest on interest. Equation (1.3) computes the future value for a sum deposited at a compounding rate of interest:

$$FV = PV\left(1+r\right)^{n} \tag{1.3}$$

where *FV* and *PV* are defined as before,

r = the periodic rate of interest (expressed as a decimal)
n = the number of periods for which the sum is invested

This computation assumes that the interest payments made during the investment term are reinvested at an interest rate equal to the first year's rate. That is why the example stated that the 6 percent rate was *fixed* for three years. Compounding obviously results in higher returns than those earned with simple interest.

Now consider a deposit of \$100 for one year, still at a rate of 6 percent but compounded quarterly. Again assuming that the interest payments will be reinvested at the initial interest rate of 6 percent, the total return at the end of the year will be

$$\begin{aligned} &100 \times \left[\left(1+0.015\right)\times\left(1+0.015\right)\times\left(1+0.015\right)\times\left(1+0.015\right)\right] \\ &= 100 \times \left[\left(1+0.015\right)^{4}\right] = 100 \times 1.6136 = \$106.136 \end{aligned}$$

The terminal value for quarterly compounding is thus about \$0.13 more than that for annual compounded interest.

In general, if compounding takes place m times per year, then at the end of n years, mn interest payments will have been made, and the future value of the principal is computed using the formula (1.4).

$$FV = PV\left(1+\frac{r}{m}\right) \tag{1.4}$$

As the example above illustrates, more frequent compounding results in higher total returns. **FIGURE 1.1** shows the interest rate factors corresponding to different frequencies of compounding on a base rate of 6 percent a year.

This shows that the more frequent the compounding, the higher the annualized interest rate. The entire progression indicates that a limit can be defined for continuous compounding, i.e., where m = infinity. To define the limit, it is useful to rewrite equation (1.4) as (1.5).

FIGURE 1.1 *Impact of Compounding*

Interest rate factor = $\left(1+\frac{r}{m}\right)^{m}$

COMPOUNDING FREQUENCY	INTEREST RATE FACTOR FOR 6%
Annual	$(1+r) = 1.060000$
Semiannual	$\left(1+\frac{r}{2}\right)^{2} = 1.060900$
Quarterly	$\left(1+\frac{r}{4}\right)^{4} = 1.061364$
Monthly	$\left(1+\frac{r}{12}\right)^{12} = 1.061678$
Daily	$\left(1+\frac{r}{365}\right)^{365} = 1.061831$

$$
\begin{aligned}
FV &= PV\left[\left(1+\frac{r}{m}\right)^{m/r}\right]^{rn} \\
&= PV\left[\left(1+\frac{1}{m/r}\right)^{m/r}\right]^{rn} \\
&= PV\left[\left(1+\frac{1}{w}\right)^{n}\right]^{rn}
\end{aligned}
\tag{1.5}
$$

where

$w = m/r$

As compounding becomes continuous and m and hence w approach infinity, the expression in the square brackets in (1.5) approaches the mathematical constant e (the base of natural logarithmic functions), which is equal to approximately 2.718281.

Substituting e into (1.5) gives us

$$FV = PVe^{rn} \tag{1.6}$$

In (1.6) e^{rn} is the *exponential function* of *rn.* It represents the continuously compounded interest rate factor. To compute this factor for an interest rate of 6 percent over a term of one year, set r to 6 percent and n to 1, giving

$$e^{rn} = e^{0.06\times 1} = \left(2.718281\right)^{0.06} = 1.061837$$

The convention in both wholesale and personal, or retail, markets is to quote an annual interest rate, whatever the term of the investment, whether it be overnight or 10 years. Lenders wishing to earn interest at the rate quoted have to place their funds on deposit for one year. For example, if you open a bank account that pays 3.5 percent interest and close it after six months, the interest you actually earn will be equal to 1.75 percent of your deposit. The actual return on a three-year building society bond that pays a 6.75 percent fixed rate compounded annually is 21.65 percent. The quoted rate is the annual one-year equivalent. An overnight deposit in the wholesale, or *interbank,* market is still quoted as an annual rate, even though interest is earned for only one day.

Quoting annualized rates allows deposits and loans of different maturities and involving different instruments to be compared. Be careful when comparing interest rates for products that have different payment frequencies. As shown in the earlier examples, the actual interest earned on a deposit paying 6 percent semiannually will be greater than on one paying 6 percent annually. The convention in the money markets is to quote the applicable interest rate taking into account payment frequency.

The discussion thus far has involved calculating future value given a known present value and rate of interest. For example, \$100 invested today for one year at a simple interest rate of 6 percent will generate $100 \times (1 + 0.06) = \$106$ at the end of the year. The future value of \$100 in this case is \$106. Conversely, \$100 is the present value of \$106, given the same term and interest rate. This relationship can be stated formally by rearranging equation (1.3) as shown in (1.7).

$$PV = \frac{FV}{\left(1+r\right)^{n}} \tag{1.7}$$

Equation (1.7) applies to investments earning annual interest payments, giving the present value of a known future sum.

To calculate the present value of an investment, you must prorate the interest that would be earned for a whole year over the number of days in the investment period, as was done in (1.2). The result is equation (1.8).

$$PV = \frac{FV}{\left(1 + r \times \frac{\text{days}}{\text{year}}\right)} \tag{1.8}$$

When interest is compounded more than once a year, the formula for calculating present value is modified, as it was in (1.4). The result is shown in equation (1.9).

$$PV = \frac{FV}{\left(1 + \frac{r}{m}\right)^{mn}} \tag{1.9}$$

For example, the present value of \$100 to be received at the end of five years, assuming an interest rate of 5 percent, with quarterly compounding is

$$PV = \frac{100}{\left(1 + \frac{0.05}{4}\right)^{(4)(5)}} = \$78.00$$

Interest rates in the money markets are always quoted for standard maturities, such as overnight, "tom next" (the overnight interest rate starting tomorrow, or "tomorrow to the next"), "spot next" (the overnight rate starting two days forward), one week, one month, two months, and so on, up to one year. If a bank or corporate customer wishes to borrow for a nonstandard period, or "odd date," an interbank desk will calculate the rate chargeable by interpolating between two standard-period interest rates. Assuming a steady, uniform increase between standard periods, the required rate can be calculated using the formula for *straight line* interpolation, which apportions the difference equally among the stated intervals. This formula is shown as (1.10).

$$r = r_1 + \left(r_2 - r_1\right) \times \frac{n - n_1}{n_2 - n_1} \tag{1.10}$$

where

r = the required odd-date rate for n days
r_1 = the quoted rate for n_1 days
r_2 = the quoted rate for n_2 days

Say the 1-month (30-day) interest rate is 5.25 percent and the 2-month (60-day) rate is 5.75 percent. If a customer wishes to borrow money for 40 days, the bank can calculate the required rate using straight line interpolation as follows: the difference between 30 and 40 is one-third that between 30 and 60, so the increase from the 30-day to the 40-day rate is assumed to be one-third the increase between the 30-day and the 60-day rates, giving the following computation

$$5.25\% + \frac{(5.75\% - 5.25\%)}{3} = 5.4167\%$$

What about the interest rate for a period that is shorter or longer than the two whose rates are known, rather than lying between them? What if the customer in the example above wished to borrow money for 64 days? In this case, the interbank desk would extrapolate from the relationship between the known 1-month and 2-month rates, again assuming a uniform rate of change in the interest rates along the maturity spectrum. So given the 1-month rate of 5.25 percent and the 2-month rate of 5.75 percent, the 64-day rate would be

$$5.25 + \left[(5.75 - 5.25) \times \frac{34}{30}\right] = 5.8167\%$$

Just as future and present value can be derived from one another, given an investment period and interest rate, so can the interest rate for a period be calculated given a present and a future value. The basic equation is merely rearranged again to solve for r. This, as discussed later, is known as the investment's *yield.*

Discount Factors

An n-period discount factor is the present value of one unit of currency that is payable at the end of period n. Essentially, it is the present value relationship expressed in terms of \$1. A discount factor for any term is given by formula (1.11).

$$d_n = \frac{1}{(1+r)^n} \tag{1.11}$$

where n = the period of discount

For instance, the five-year discount factor for a rate of 6 percent compounded annually is

$$d_5 = \frac{1}{(1+0.06)^5} = 0.747258$$

The set of discount factors for every period from one day to 30 years and longer is termed the *discount function.* Since the following discussion is in terms of *PV,* discount factors may be used to value any financial instrument that generates future cash flows. For example, the present value of an instrument generating a cash flow of \$103.50 payable at the end of six months would be determined as follows, given a six-month discount factor of 0.98756:

$$PV = \frac{FV}{(1+r)^n} = FV \times d_n = \$103.50 \times 0.98756 = \$102.212$$

Discount factors can also be used to calculate the future value of a present investment by inverting the formula. In the example above, the six-month discount factor of 0.98756 signifies that \$1 receivable in six months has a present value of \$0.98756. By the same reasoning, \$1 today would in six months be worth

$$\frac{1}{d_{0.5}} = \frac{1}{0.98756} = \$1.0126$$

It is possible to derive discount factors from current bond prices. This process can be illustrated using the set of hypothetical bonds, all assumed to have semiannual coupons, that are shown in **FIGURE 1.2**, together with their prices.

The first bond in Figure 1.2 matures in precisely six months. Its final cash flow will be \$103.50, comprising the final coupon payment of \$3.50 and the redemption payment of \$100. The price, or present value, of this

FIGURE 1.2 *Hypothetical Set of Bonds and Bond Prices*

COUPON	MATURITY DATE	PRICE
7%	7-Jun-01	101.65
8%	7-Dec-01	101.89
6%	7-Jun-02	100.75
6.50%	7-Dec-02	100.37

bond is \$101.65. Using this, the six-month discount factor may be calculated as follows:

$$d_{0.5} = \frac{101.65}{103.50} = 0.98213$$

Using this six-month discount factor, the one-year factor can be derived from the second bond in Figure 1.2, the 8 percent due 2001. This bond pays a coupon of \$4 in six months and, in one year, makes a payment of \$104, consisting of another \$4 coupon payment plus \$100 return of principal.

The price of the one-year bond is \$101.89. As with the 6-month bond, the price is also its present value, equal to the sum of the present values of its total cash flows. This relationship can be expressed in the following equation:

$$101.89 = 4 \times d_{0.5} + 104 \times d_1$$

The value of $d_{0.5}$ is known to be 0.98213. That leaves d_1 as the only unknown in the equation, which may be rearranged to solve for it:

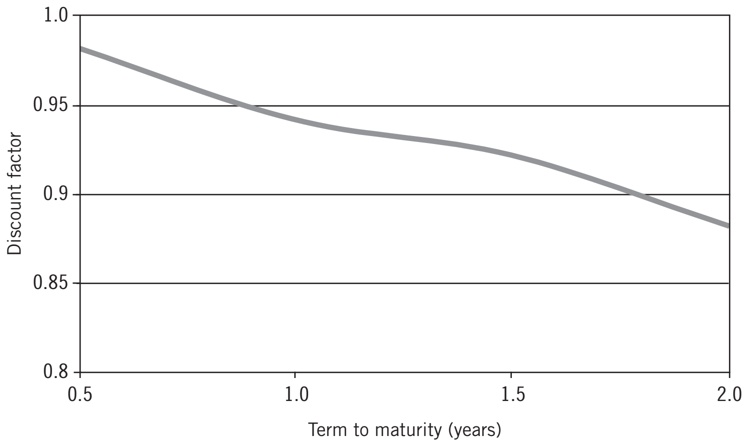
$$d_1 = \left[\frac{101.89 - 4\left(0.98213\right)}{104}\right] = \frac{97.96148}{104} = 0.94194$$

The same procedure can be repeated for the remaining two bonds, using the discount factors derived in the previous steps to derive the set of discount factors in **FIGURE 1.3**. These factors may also be graphed as a continuous function, as shown in **FIGURE 1.4**.

FIGURE 1.3 *Discount Factors Calculated Using Bootstrapping Technique*

COUPON	MATURITY DATE	TERM (YEARS)	PRICE	D(N)
7%	7-Jun-10	0.5	101.65	0.98213
8%	7-Dec-10	1.0	101.89	0.94194
6%	7-Jun-11	1.5	100.75	0.92211
6.50%	7-Dec-11	2.0	100.37	0.88252

FIGURE 1.4 *Hypothetical Discount Function*



This technique of calculating discount factors, known as bootstrapping, is conceptually neat, but may not work so well in practice. Problems arise when you do not have a set of bonds that mature at precise six-month intervals. Liquidity issues connected with individual bonds can also cause complications. This is true because the price of the bond, which is still the sum of the present values of the cash flows, may reflect liquidity considerations (e.g., hard to buy or sell the bond, difficult to find) that do not reflect the market as a whole but peculiarities of that specific bond. The approach, however, is still worth knowing.

Note that the discount factors in Figure 1.3 decrease as the bond's maturity increases. This makes intuitive sense, since the present value of something to be received in the future diminishes the further in the future the date of receipt lies.

Bond Pricing and Yield: The Traditional Approach

The discount rate used to derive the present value of a bond's cash flows is the interest rate that the bondholders require as compensation for the risk of lending their money to the issuer. The yield investors require on a bond depends on a number of political and economic factors, including what other bonds in the same class are yielding. Yield is always quoted as an annualized interest rate. This means that the rate used to discount the cash flows of a bond paying semiannual coupons is exactly half the bond's yield.

Bond Pricing

The *fair price* of a bond is the sum of the present values of all its cash flows, including both the coupon payments and the redemption payment. The price of a conventional bond that pays annual coupons can therefore be represented by formula (1.12).

$$
\begin{aligned}
P &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots\ldots \frac{C}{(1+r)^N} + \frac{M}{(1+r)^N} \\
&= \sum_{n=1}^{N} \frac{C}{(1+r)^n} + \frac{M}{(1+r)^N}
\end{aligned}
\tag{1.12}
$$

where

P = the bond's fair price

C = the annual coupon payment

r = the discount rate, or required yield

N = the number of years to maturity, and so the number of interest periods for a bond paying an annual coupon

M = the maturity payment, or par value, which is usually 100 percent of face value

Bonds in the U.S. domestic market—as opposed to international securities denominated in U.S. dollars, such as USD Eurobonds—usually pay semiannual coupons. Such bonds may be priced using the expression in (1.13), which is a modification of (1.12) allowing for twice-yearly discounting.

$$
\begin{aligned}
P &= \frac{\frac{C}{2}}{\left(1+\frac{1}{2}r\right)} + \frac{\frac{C}{2}}{\left(1+\frac{1}{2}r\right)^2} + \frac{\frac{C}{2}}{\left(1+\frac{1}{2}r\right)^3} + \ldots\ldots + \frac{\frac{C}{2}}{\left(1+\frac{1}{2}r\right)^{2N}} + \frac{M}{\left(1+\frac{1}{2}r\right)^{2N}} \\
&= \sum_{n=1}^{2N} \frac{\frac{C}{2}}{\left(1+\frac{1}{2}r\right)^n} + \frac{M}{\left(1+\frac{1}{2}r\right)^{2N}} \\
&= \frac{C}{r}\left[1 - \frac{1}{\left(1+\frac{1}{2}r\right)^{2N}}\right] + \frac{M}{\left(1+\frac{1}{2}r\right)^{2N}}
\end{aligned}
\tag{1.13}
$$

Note that $2N$ is now the power to which the discount factor is raised. This is because a bond that pays a semiannual coupon makes two interest payments a year. It might therefore be convenient to replace the number of years to maturity with the number of interest periods, which could be represented by the variable *n*, resulting in formula (1.14).

$$P = \frac{C}{r}\left[1 - \frac{1}{\left(1+\frac{1}{2}r\right)^{n}}\right] + \frac{M}{\left(1+\frac{1}{2}r\right)^{n}} \tag{1.14}$$

This formula calculates the fair price on a coupon payment date, so there is no *accrued interest* incorporated into the price. Accrued interest is an accounting convention that treats coupon interest as accruing every day a bond is held; this accrued amount is added to the discounted present value of the bond (the *clean price*) to obtain the market value of the bond, known as the *dirty price.* The price calculation is made as of the bond's *settlement date,* the date on which it actually changes hands after being traded. For a new bond issue, the settlement date is the day when the investors take delivery of the bond and the issuer receives payment. The settlement date for a bond traded in the *secondary market*—the market where bonds are bought and sold after they are first issued—is the day the buyer transfers payment to the seller of the bond and the seller transfers the bond to the buyer.

Different markets have different settlement conventions. U.S. Treasuries, for example, normally settle on "T + 1": one business day after the trade date; T. Eurobonds, on the other hand, settle on T + 3. The term *value date* is sometimes used in place of settlement date; however, the two terms are not strictly synonymous. A settlement date can fall only on a business day; a bond traded on a Friday, therefore, will settle on a Monday. A value date, in contrast, can sometimes fall on a non-business day—when accrued interest is being calculated, for example.

Equation (1.14) assumes an even number of coupon payment dates remaining before maturity. If there are an odd number, the formula is modified as shown in (1.15).

$$P = \frac{C}{r}\left[1 - \frac{1}{\left(1+\frac{1}{2}r\right)^{2N+1}}\right] + \frac{M}{\left(1+\frac{1}{2}r\right)^{2N+1}} \tag{1.15}$$

Another assumption embodied in the standard formula is that the bond is traded for settlement on a day that is precisely one interest period before the next coupon payment. If the trade takes place between coupon

EXAMPLE: *Calculating Consideration for a U.S. Treasury*

The consideration, or actual cash proceeds paid by a buyer for a bond, is the bond's total cash value together with any costs such as commission. In this example, consideration refers only to the cash value of the bond.

What consideration is payable for $5 million nominal of a U.S. Treasury, quoted at a price of 99-16?

The U.S. Treasury price is 99-16, which is equal to 99 and 16/32, or 99.50 per $100. The consideration is therefore:

$$0.9950 \times 5{,}000{,}000 = \$4{,}975{,}000$$

If the price of a bond is below par, the total consideration is below the nominal amount; if it is priced above par, the consideration will be above the nominal amount.

dates, the formula is modified. This is done by adjusting the exponent for the discount factor using ratio *i*, shown in (1.16).

$$i = \frac{\text{Days from value date to next coupon date}}{\text{Days in the interest period}} \tag{1.16}$$

The denominator of this ratio is the number of calendar days between the last coupon date and the next one. This figure depends on the day-count convention (see below) used for that particular bond. Using *i*, the price formula is modified as (1.17) (for annual-coupon-paying bonds; for bonds with semiannual coupons, *r*/2 replaces *r*).

$$P = \frac{C}{\left(1+r\right)^{i}} + \frac{C}{\left(1+r\right)^{1+i}} + \frac{C}{\left(1+r\right)^{2+i}} + \ldots\ldots + \frac{C}{\left(1+r\right)^{n-1+i}} + \frac{M}{\left(1+r\right)^{n-1+i}} \tag{1.17}$$

where the variables *C, M, n,* and *r* are as before

As noted above, the bond market includes securities, known as zero-coupon bonds, or *strips,* that do not pay coupons. These are priced by setting C to 0 in the pricing equation. The only cash flow is the maturity payment, resulting in formula (1.18)

$$P = \frac{M}{\left(1+r\right)^{N}} \tag{1.18}$$

where M and r are as before and N is the number of years to maturity.

EXAMPLE: *Zero-Coupon Bond Price*

A. Calculate the price of a Treasury strip with a maturity of precisely five years corresponding to a required yield of 5.40 percent.

According to these terms, $N = 5$, so $n = 10$, and $r = 0.054$, so $r/2 = 0.027$. $M = 100$, as usual. Plugging these values into the pricing formula gives

$$P = \frac{100}{\left(1.027\right)^{10}} = \$76.611782$$

B. Calculate the price of a French government zero-coupon bond with precisely five years to maturity, with the same required yield of 5.40 percent. Note that French government bonds pay coupon annually.

$$P = \frac{100}{\left(1.054\right)^{5}} = 76.877092$$

Note that, even though these bonds pay no actual coupons, their prices and yields must be calculated on the basis of *quasi-coupon* periods, which are based on the interest periods of bonds denominated in the same currency. A U.S. dollar or a sterling five-year zero-coupon bond, for example, would be assumed to cover 10 quasi-coupon periods, and the price equation would accordingly be modified as (1.19).

$$P = \frac{M}{\left(1+\frac{1}{2}r\right)^{n}} \tag{1.19}$$

It is clear from the bond price formula that a bond's yield and its price are closely related. Specifically, the price moves in the opposite direction from the yield. This is because a bond's price is the net present value of its cash flows; if the discount rate—that is, the yield required by investors—increases, the present values of the cash flows decrease. In the same way, if the required yield decreases, the price of the bond rises. The relationship between a bond's price and any required yield level is illustrated by the graph in **FIGURE 1.5**, which plots the yield against the corresponding price to form a convex curve.

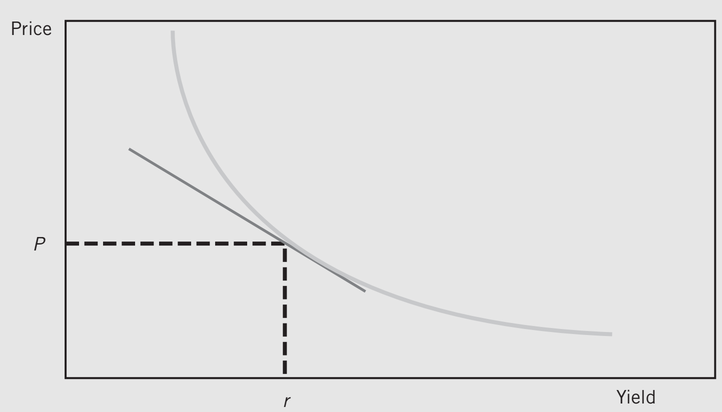
Bond Yield

The discussion so far has involved calculating the price of a bond given its yield. This procedure can be reversed to find a bond's yield when its price is known. This is equivalent to calculating the bond's *internal rate of return,* or *IRR,* also known as its yield to maturity or gross redemption yield (also *yield to workout*). These are among the various measures used in the markets to estimate the return generated from holding a bond.

Summary of the Price/Yield Relationship

- ❑ At issue, if a bond is priced at par, its coupon will equal the yield that the market requires, reflecting factors such as the bond's term to maturity, the issuer's credit rating, and the yield on current bonds of comparable quality.

FIGURE 1.5 *The Price/Yield Relationship*



- ❑ If the required yield rises above the coupon rate, the bond price will decrease.
- ❑ If the required yield falls below the coupon rate, the bond price will increase.

In most markets, bonds are traded on the basis of their prices. Because different bonds can generate different and complicated cash flow patterns, however, they are generally compared in terms of their yields. For example, market makers usually quote two-way prices at which they will buy or sell particular bonds, but it is the yield at which the bonds are trading that is important to the market makers' customers. This is because a bond's price does not tell buyers anything useful about what they are getting. Remember that in any market a number of bonds exist with different issuers, coupons, and terms to maturity. It is their yields that are compared, not their prices.

The yield on any investment is the discount rate that will make the present value of its cash flows equal its initial cost or price. Mathematically, an investment's yield, represented by *r*, is the interest rate that satisfies the bond price equation, repeated here as (1.20).

$$P = \sum_{n=1}^{N} \frac{C_n}{\left(1+r\right)^n} + \frac{M}{\left(1+r\right)^n} \tag{1.20}$$

Other types of yield measure, however, are used in the market for different purposes. The simplest is the *current yield,* also know as the *flat, interest,* or *running yield.* These are computed by formula (1.21).

$$rc = \frac{C}{P} \times 100 \tag{1.21}$$

where *rc* is the current yield.

In this equation the percentage for *C* is not expressed as a decimal. Current yield ignores any capital gain or loss that might arise from holding and trading a bond and does not consider the time value of money. It calculates the coupon income as a proportion of the price paid for the bond. For this to be an accurate representation of return, the bond would have to be more like an annuity than a fixed-term instrument.

Current yield is useful as a "rough and ready" interest rate calculation; it is often used to estimate the cost of or profit from holding a bond for a short term. For example, if short-term interest rates, such as the one-week or three-month, are higher than the current yield, holding the bond is said to involve a *running cost.* This is also known as *negative carry* or *negative funding.* The concept is used by bond traders, market makers, and leveraged investors, but it is useful for all market practitioners, since it represents the investor's short-term cost of holding or funding a bond. The yield to maturity (YTM)—or, as it is known in sterling markets, gross redemption yield—is the most frequently used measure of bond return. Yield to maturity takes into account the pattern of coupon payments, the bond's term to maturity, and the capital gain (or loss) arising over the remaining life of the bond. The bond price formula shows the relationship between these elements and demonstrates their importance in determining price. The YTM calculation discounts the cash flows to maturity, employing the concept of the time value of money.

As noted earlier, the formula for calculating YTM is essentially that for calculating the price of a bond, repeated as (1.12). (For the YTM of bonds with semiannual coupon, the formula must be modified, as in (1.13).) Note, though, that this equation has two variables, the price P and yield r. It cannot, therefore, be rearranged to solve for yield r explicitly. In fact, the only way to solve for the yield is to use numerical iteration. This involves estimating a value for r and calculating the price associated with it. If the calculated price is higher than the bond's current price, the estimate for r is lower than the actual yield, so it must be raised. This process of calculation and adjustment up or down is repeated until the estimates converge on a level that generates the bond's current price.

To differentiate redemption yield from other yield and interest rate measures described in this book, it will be referred to as rm. Note that this section is concerned with the *gross* redemption yield, the yield that results from payment of coupons without deduction of any withholding tax. The *net redemption yield* is what will be received if the bond is traded in a market where bonds pay coupon *net*, without withholding tax. It is obtained by multiplying the coupon rate C by (1 – marginal tax rate). The net redemption yield is always lower than the gross redemption yield.

The key assumption behind the YTM calculation has already been discussed—that the redemption yield rm remains stable for the entire life of the bond, so that all coupons are reinvested at this same rate. The assumption is unrealistic, however. It can be predicted with virtual certainty that the interest rates paid by instruments with maturities equal to those of the bond at each coupon date will differ from rm at some point, at least, during the life of the bond. In practice, however, investors require a rate of return that is equivalent to the price that they are paying for a bond, and the redemption yield is as good a measurement as any.

EXAMPLE: *Yield to Maturity for Semiannual-Coupon Bond*

A bond paying a semiannual coupon has a dirty price of \$98.50, an annual coupon of 3 percent, and exactly one year before maturity. The bond therefore has three remaining cash flows: two coupon payments of \$1.50 each and a redemption payment of \$100. Plugging these values into equation (1.20) gives

$$98.50 = \frac{1.50}{\left(1+\frac{1}{2}rm\right)} + \frac{101.50}{\left(1+\frac{1}{2}rm\right)^2}$$

Note that the equation uses half of the YTM value *rm* because this is a semiannual paying bond.

The expression above is a quadratic equation, which can be rearranged as $98.50x^2 - 1.50x - 101.50 = 0$, where $x = (1 + rm/2)$.

The equation may now be solved using the standard solution for equations of the form

$$ax^2 + bx + c = 0$$

There are two solutions, only one of which gives a positive redemption yield. The positive solution is

$$\frac{rm}{2} = 0.022755, \text{ or } rm = 4.551\%$$

YTM can also be calculated using mathematical iteration. Start with a trial value for *rm* of r_1 = 4 percent and plug this into the right-hand side of equation 1.20. This gives a price P_1 of 99.050, which is higher than the dirty market price P_M of 98.50. The trial value for *rm* was therefore too low.

Next try r_2 = 5 percent. This generates a price P_2 of 98.114, which is lower than the market price. Because the two trial prices lie on either side of the market value, the correct value for *rm* must lie between 4 and 5 percent. Now use the formula for linear interpolation

$$rm = r_1 + \left(r_2 - r_1\right)\frac{P_1 - P_M}{P_1 - P_2}$$

Plugging in the appropriate values gives a linear approximation for the redemption yield of *rm* = 4.549 percent, which is near the solution obtained by solving the quadratic equation.

A more accurate approach might be the one used to price interest rate swaps: to calculate the present values of future cash flows using discount rates determined by the markets' view on where interest rates will be at those points. These expected rates are known as *forward* interest rates. Forward rates, however, are *implied,* and a YTM derived using them is as speculative as one calculated using the conventional formula. This is because the real market interest rate at any time is invariably different from the one implied earlier in the forward markets. So a YTM calculation made using forward rates would not equal the yield actually realized either. The zero-coupon rate, it will be demonstrated later, is the true interest rate for any term to maturity. Still, despite the limitations imposed by its underlying assumptions, the YTM is the main measure of return used in the markets.

Calculating the redemption yield of bonds that pay semiannual coupons involves the semiannual discounting of those payments. This approach is appropriate for most U.S. bonds and U.K. gilts. Government bonds in most of continental Europe and most Eurobonds, however, pay annual coupon payments. The appropriate method of calculating their redemption yields is to use annual discounting. The two yield measures are not directly comparable.

It is possible to make a Eurobond directly comparable with a U.K. gilt by using semiannual discounting of the former's annual coupon payments or using annual discounting of the latter's semiannual payments. The formulas for the semiannual and annual calculations appeared as (1.13) and (1.12), respectively, and are repeated here as (1.22) and (1.23).

$$P_d = \frac{C}{\left(1+\frac{1}{2}rm\right)^2} + \frac{C}{\left(1+\frac{1}{2}rm\right)^4} + \frac{C}{\left(1+\frac{1}{2}rm\right)^6} + \dots + \frac{C}{\left(1+\frac{1}{2}rm\right)^{2N}} + \frac{M}{\left(1+\frac{1}{2}rm\right)^{2N}} \tag{1.22}$$

$$P_d = \frac{C/2}{\left(1+rm\right)^{\frac{1}{2}}} + \frac{C/2}{\left(1+rm\right)} + \frac{C/2}{\left(1+rm\right)^{\frac{3}{2}}} + \dots + \frac{C/2}{\left(1+rm\right)^N} + \frac{\mathrm{M}}{\left(1+rm\right)^N} \tag{1.23}$$

Consider a bond with a dirty price—including the accrued interest the seller is entitled to receive—of $97.89, a coupon of 6 percent, and five years to maturity. **FIGURE 1.6** shows the gross redemption yields this bond would have under the different yield-calculation conventions.

These figures demonstrate the impact that the coupon-payment and discounting frequencies have on a bond's redemption yield calculation. Specifically, increasing the frequency of discounting lowers the

FIGURE 1.6 *Yield and Payment Frequency*

DISCOUNTING	PAYMENTS	YIELD TO MATURITY
Semiannual	Semiannual	6.500
Annual	Annual	6.508
Semiannual	Annual	6.428
Annual	Semiannual	6.605

calculated yield, while increasing the frequency of payments raises it. When comparing yields for bonds that trade in markets with different conventions, it is important to convert all the yields to the same calculation basis.

It might seem that doubling a semiannual yield figure would produce the annualized equivalent; the real result, however, is an underestimate of the true annualized yield. This is because of the multiplicative effects of discounting. The correct procedure for converting semiannual and quarterly yields into annualized ones is shown in (1.24).

EXAMPLE: *Comparing Yields to Maturity*

A U.S. Treasury paying semiannual coupons, with a maturity of 10 years, has a quoted yield of 4.89 percent. A European government bond with a similar maturity is quoted at a yield of 4.96 percent. Which bond has the higher yield to maturity in practice?

The effective annual yield of the Treasury is

$$rm_a = \left(1 + \frac{1}{2} \times 0.0489\right)^2 - 1 = 4.9498\%$$

Comparing the securities using the same calculation basis reveals that the European government bond does indeed have the higher yield.

a. General formula

$$rm_a = \left(1 + \text{interest rate}\right)^m - 1 \tag{1.24}$$

where m = the number of coupon payments per year

b. Formulas for converting between semiannual and annual yields

$$rm_a = \left(1 + \tfrac{1}{2} rm_s\right)^2 - 1$$

$$rm_s = \left[\left(1 + rm_a\right)^{\frac{1}{2}} - 1\right] \times 2$$

c. Formulas for converting between quarterly and annual yields

$$rm_a = \left(1 + \tfrac{1}{4} rm_q\right)^4 - 1$$

$$rm_q = \left[\left(1 + rm_a\right)^{\frac{1}{4}} - 1\right] \times 4$$

where rm_q, rm_s, and rm_a are, respectively, the quarterly, semiannually, and annually discounted yields to maturity.

The market convention is sometimes simply to double the semiannual yield to obtain the annualized yields, despite the fact that this produces an inaccurate result. It is only acceptable to do this for rough calculations. An annualized yield obtained in this manner is known as a *bond equivalent yield.* It was noted earlier that the one disadvantage of the YTM measure is that its calculation incorporates the unrealistic assumption that each coupon payment, as it becomes due, is reinvested at the rate rm. Another disadvantage is that it does not deal with the situation in which investors do not hold their bonds to maturity. In these cases, the redemption yield will not be as great. Investors might therefore be interested in other measures of return, such as the equivalent zero-coupon yield, considered a true yield.

To review, the redemption yield measure assumes that

- ❑ the bond is held to maturity
- ❑ all coupons during the bond's life are reinvested at the same (redemption yield) rate

Given these assumptions, the YTM can be viewed as an *expected* or *anticipated* yield. It is closest to reality when an investor buys a bond on first issue and holds it to maturity. Even then, however, the actual realized yield at maturity would be different from the YTM because of the unrealistic nature of the second assumption. It is clearly unlikely that all the coupons of any but the shortest-maturity bond will be reinvested at the same rate. As noted earlier, market interest rates are in a state of constant flux, and this would affect money reinvestment rates. Therefore, although yield to maturity is the main market measure of bond levels, it is not a true interest rate. This is an important point. Chapter 2 will explore the concept of a true interest rate.

Another problem with YTM is that it discounts a bond's coupons at the yield specific to that bond. It thus cannot serve as an accurate basis for comparing bonds. Consider a two-year and a five-year bond. These securities will invariably have different YTMs. Accordingly, the coupon cash flows they generate in two years' time will be discounted at different rates (assuming the yield curve is not flat). This is clearly not correct. The present value calculated today of a cash flow occurring in two years' time should be the same whether that cash flow is generated by a short- or a long-dated bond.

Floating Rate Notes

Floating rate notes (FRNs) are bonds that have variable rates of interest; the coupon rate is linked to a specified index and changes periodically as the index changes. An FRN is usually issued with a coupon that pays a fixed spread over a reference index; for example, the coupon may be 50 basis points over the six-month interbank rate. Since the value for the reference benchmark index is not known, it is not possible to calculate the redemption yield for an FRN. The FRN market in countries such as the United States and United Kingdom is large and well-developed; floating-rate bonds are particularly popular with short-term investors and financial institutions such as banks.

The rate against which the FRN coupon is set is known as the *reference rate*. In the United States market, FRNs frequently set their coupons in line with the Treasury bill rate. The spread over the reference note is called the *index spread*. The index spread is the number of basis points over the reference rate; in a few cases the index spread is negative, so it is subtracted from the reference rate.

Generally the reference interest rate for FRNs is the London interbank offered rate or LIBOR. An FRN will pay interest at LIBOR plus a quoted margin (or spread). The interest rate is fixed for a one-, three- or six-month period and is reset in line with the LIBOR fixing at the end of the interest period. Hence at the coupon reset date for a sterling FRN paying six-month LIBOR + 0.50%, if the LIBOR fix is 7.6875%, then the FRN will pay a coupon of 8.1875%. Interest therefore will accrue at a daily rate of £0.0224315.

In theory, on the coupon reset date, an FRN will be priced precisely at par. Between reset dates it will trade very close to par because of the way in which the coupon is reset. If market rates rise between reset dates, an FRN will trade slightly below par; similarly, if rates fall, the paper will trade slightly above. (Note that changes in the credit quality of the issuer, exemplified by a downgrade in credit rating, will impact the price severely in between coupon reset dates). This makes FRNs very similar in behavior to money market instruments traded on a yield basis, although of course FRNs have much longer maturities. Investors can opt to view FRNs as essentially money market instruments or as alternatives to conventional bonds. For this reason, one can use two approaches in analyzing FRNs. The first approach is known as the *margin method*. This calculates the difference between the return on an FRN and that on an equivalent money market security. There are two variations on this, simple margin and discounted margin.

The simple margin method is sometimes preferred because it does not require the forecasting of future interest rates and coupon values. *Simple margin* is defined as the average return on an FRN throughout its life compared with the reference interest rate. It has two components: a *quoted margin* either above or below the reference rate, and a capital gain or loss element which is calculated under the assumption that the difference between the current price of the FRN and the maturity value is spread evenly over the remaining life of the bond. Simple margin uses the expression at (1.25).

$$\text{Simple margin} = \frac{\left(M - P_d\right)}{\left(100 \times T\right)} + M_q \tag{1.25}$$

where

P_d is $P + AI$, the dirty price
M is the par value
T is the number of years from settlement date to maturity
M^q is the quoted margin

A quoted margin that is positive reflects yield for an FRN that is offering a higher yield than the comparable money market security.

At certain times, the simple margin formula is adjusted to take into account any change in the reference rate since the last coupon reset date. This is done by defining an adjusted price, which is either:

$$AP_d = P_d + \left(re + QM\right) \times \frac{N_{sc}}{365} \times 100 - \frac{C}{2} \times 100$$

or

$$AP_d = P_d + \left(re + QM\right) \times \frac{N_{sc}}{365} \times P_d - \frac{C}{2} \times 100 \tag{1.26}$$

where

AP_d is the adjusted dirty price
re is the current value of the reference interest rate (such as LIBOR)
$C/2$ is the next coupon payment (that is, C is the reference interest rate on the last coupon reset date plus M_q)
N_{sc} is the number of days between settlement and the next coupon date

EXAMPLE: *Simple margin*

An FRN with a par value of £100, a quoted margin of 10 basis points over six-month LIBOR is currently trading at a clean price of 98.50. The previous LIBOR fixing was 5.375%. There are 90 days of accrued interest, 92 days until the next coupon payment and five (10) years from the next coupon payment before maturity. Therefore we have :

$$
\begin{aligned}
P_d &= 98.50 + \frac{90}{365} \times 5.375 \\
&= 99.825
\end{aligned}
$$

We obtain T as shown:

$$T = 10 + \frac{92}{365} = 10.252$$

Inserting these results into (1.25) we have the following simple margin:

$$
\begin{aligned}
\text{Simple margin} &= \frac{100 - 99.825}{100 \times 10.252} + 0.0010 \\
&= 0.00117
\end{aligned}
$$

or 11.7 basis points.

The upper equation in (1.26) ignores the current yield effect: all payments are assumed to be received on the basis of par, and this understates the value of the coupon for FRNs trading below par and overstates the value when they are trading above par. The lower equation in (1.26) takes account of the current yield effect.

The adjusted price AP_d replaces the current price P_d in (1.25) to give an *adjusted simple margin*. The simple margin method has the disadvantage of amortizing the discount or premium on the FRN in a straight line over the remaining life of the bond rather than at a constantly compounded rate. The discounted margin method uses the latter approach. The distinction between simple margin and discounted margin is exactly the same as that between simple yield to maturity and yield to maturity. The discounted margin method does have a disadvantage in that it requires a forecast of the reference interest rate over the remaining life of the bond.

The discounted margin is the solution to Equation (1.27) shown below, given for an FRN that pays semiannual coupons.

$$P_d = \left\{ \frac{1}{\left[1 + \frac{1}{2}\left(re + DM\right)\right]^{days/year}} \right\}$$
$$\times \left\{ \frac{C}{2} + \sum_{t=1}^{N-1} \frac{\left(re * + QM\right) \times 100/2}{\left[1 + \frac{1}{2}\left(re * + DM\right)\right]^t} + \frac{M}{\left[1 + \frac{1}{2}\left(re * + DM\right)\right]^{N-1}} \right\} \qquad (1.27)$$

where

DM is the discounted margin

re is the current value of the reference interest rate

*re** is the assumed (or forecast) value of the reference rate over the remaining life of the bond

M_q is the quoted margin

N is the number of coupon payments before redemption

Equation (1.27) may be stated in terms of discount factors instead of the reference rate. The *yield to maturity spread* method of evaluating FRNs is designed to allow direct comparison between FRNs and fixed-rate bonds. The yield to maturity on the FRN (*rmf*) is calculated using (1.27) with both (*re* + *DM*) and (re* + DM) replaced with *rmf*. The yield to maturity on a reference bond (*rmb*) was shown earlier in this chapter. The yield to maturity spread is defined as:

Yield to maturity spread = *rmf* − *rmb*.

If this is positive the FRN offers a higher yield than the reference bond.

Accrued Interest

All bonds except zero-coupon bonds accrue interest on a daily basis that is then paid out on the coupon date. As mentioned earlier, the formulas discussed so far calculate bonds' prices as of a coupon payment date, so that no accrued interest is incorporated in the price. In all major bond markets, the convention is to quote this so-called clean price.

Clean and Dirty Bond Prices

When investors buy a bond in the market, what they pay is the bond's *all-in* price, also known as the dirty, or *gross price,* which is the clean price of a bond plus accrued interest.

Bonds trade either *ex-dividend* or *cum dividend.* The period between when a coupon is announced and when it is paid is the ex-dividend period. If the bond trades during this time, it is the seller, not the buyer, who receives the next coupon payment. Between the coupon payment date and the next ex-dividend date the bond trades cum dividend, so the buyer gets the next coupon payment.

Accrued interest compensates sellers for giving up all of the next coupon payment even though they will have held their bonds for part of the period since the last coupon payment. A bond's clean price moves with market interest rates. If the market rates are constant during a coupon period, the clean price will be constant as well. In contrast, the dirty price for the same bond will increase steadily as the coupon interest accrues from one coupon payment date until the next ex-dividend date, when it falls by the present value of the amount of the coupon payment. The dirty price at this point is below the clean price, reflecting the fact that accrued interest is now negative. This is because if the bond is traded during the ex-dividend period, the seller, not the buyer, receives the next coupon, and the lower price is the buyer's compensation for this loss. On the coupon date, the accrued interest is zero, so the clean and dirty prices are the same.

The net interest accrued since the last ex-dividend date is calculated using formula (1.28).

$$AI = C \times \left[\frac{N_{xt} - N_{xc}}{\text{Day Base}}\right] \tag{1.28}$$

where

AI = the next accrued interest

C = the bond coupon

N_{xc} = the number of days between the *ex-dividend* date and the coupon payment date

N_{xt} = the number of days between the *ex-dividend* date and the date for the calculation

Day Base = the day-count base (see the following)

When a bond is traded, accrued interest is calculated from and including the last coupon date up to and excluding the value date, usually the settlement date. Interest does not accrue on bonds whose issuer has defaulted.

As noted earlier, for bonds that are trading ex-dividend, the accrued coupon is negative and is subtracted from the clean price. The negative accrued interest is calculated using formula (1.29).

$$AI = -C \times \frac{\text{days to next coupon}}{\text{Day Base}} \tag{1.29}$$

Certain classes of bonds—U.S. Treasuries and Eurobonds, for example—do not have ex-dividend periods and therefore trade cum dividend right up to the coupon date.

Day-Count Conventions

In calculating the accrued interest on a bond, the market uses the day-count convention appropriate to that bond. These conventions govern both the number of days assumed to be in a calendar year and how the days between two dates are figured. **FIGURE 1.7** shows how the different conventions affect the accrual calculation. **FIGURE 1.8** is a summary of the calculation for each of the conventions.

In these conventions, the number of days between two dates includes the first date but not the second. Thus, using actual/365, there are 37 days between August 4 and September 10. The last two conventions assume 30 days in each month, no matter what the calendar says. So, for example, it is assumed that there are 30 days between February 10 and March 10. Under the 30/360 convention, if the first date is the 31st, it is changed to the 30th; if the second date is the 31st and the first date is either the 30th or the 31st, the second date is changed to the 30th. The 30E/360 convention differs from this in that if the second date is the 31st, it is changed to the 30th regardless of what the first date is.

FIGURE 1.7 *Accrued Interest, Day-Count Conventions*

Actual/365	$AI = C \times$ actual days to next coupon payment/365
Actual/360	$AI = C \times$ actual days to next coupon/360
Actual/actual	$AI = C \times$ actual days to next coupon/actual number of days in the interest period
30/360	$AI = C \times$ days to next coupon, assuming 30 days in a month/360
30E/360	$AI = C \times$ days to next coupon, assuming 30 days in a month/360

FIGURE 1.8 *Accrued interest day-count convention rules*

CONVENTION	RULES
Actual/actual	The actual number of days between two dates is used. Leap years count for 366 days, non-leap years count for 365 days.
Actual/365 fixed	The actual number of days between two dates is used as the numerator. All years are assumed to have 365 days.
Actual/360	The actual number of days between two dates is used as the numerator. A year is assumed to have 12 months of 30 days each.
30/360	All months are assumed to have 30 days, resulting in a 360-day year. If the first date falls on the 31st, it is changed to the 30th. If the second date falls on the 31st, it is changed to the 30th, but only if the first date falls on the 30th or the 31st.
30E/360	All months are assumed to have 30 days, resulting in a 360-day year. If the first date falls on the 31st, it is changed to the 30th. If the second date falls on the 31st, it is changed to the 30th.
30E+/360	All months are assumed to have 30 days, resulting in a 360-day year. If the first date falls on the 31st, it is changed to the 30th. If the second date falls on the 31st, it is changed to the 1st and the month is increased by one.