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Introduction – Models and Dynamic Systems

This chapter is a presentation of the issues surrounding the design and implementation of automatic control solutions for industrial applications. Currently, there is an increasing demand for setups that are productive and offer a good yield for the provider by means of modern concepts such as “embedded optimization” and “low-cost automation”. In this context, Automation and Applied Informatics, through their theoretical tools, as well as hardware and software resources, contribute toward the development of applications using the existing technologies as well as the emergent ones. Automation makes it possible to describe the evolution of a real process using an abstract mathematical model, and to introduce the necessary methods and mechanisms to design the advanced control systems that ensure an optimal operation of the process. On the other hand, Informatics, due to the hardware material and software tools available and the efficiency of its powerful computing and communication which contribute toward operations involving data processing, model development, design and identification of control methods, is vital for the optimization, surveillance and protection of industrial processes. The different classes of dynamic models (systems) associated with the processes, as well as the most important algorithms, methods and techniques used in the design of the control systems for industrial facilities, are briefly presented here.

1.1. Overview

The design and implementation of digital control systems for industrial applications has, as a primary objective, the valorization of Automatic Control and Informatics tools in order to improve the quality of products, to ensure the safety of personnel, protect industrial facilities and the environment, and decrease as much as possible the production costs.

This objective is a response to the increasing demands in efficiency and autonomous operating requirements of companies, the need to reduce the role of the

operator during process use (facilities, machines and equipment start to communicate with each other using the mechanisms involved in the concept of the *internet of things*) and the augmenting integration of automation into high-performance production technologies.

Two main categories of Industrial Automatic Control that are required are as follows: first, the mathematical formulism that allows the process to be represented by a mathematical model that expresses its dynamics, and second, the concepts and methods used in the control and optimization of the process [OGA 90, KUO 91, AST 97, GEN 97, POP 06].

A mathematical model is set using the laws that govern the functioning of the process or by using a collection of data acquired from the process.

The control algorithms are calculated based on the model, using numerical methods so as to ensure the level of performance desired in terms of the real processes. In the following step, which has become mandatory, an optimal operating point is found in the real operating conditions using an adequate decision criterion for the process.

The digital methods for computing and programming also ensure acquisition and processing of the data in real time and assist the design of control systems, as well as aiding the control and surveillance of industrial applications.

The classic control solution ensures a good response around a nominal operating point. To achieve a more widespread performance around the nominal point, this type of control must be improved using adaptive, robust, predictive, multimodel or intelligent techniques.

Normally, the exploitation of the process takes place within an “admissibility domain”; this operating domain of the process is imposed by technological limitations.

A standard control process configuration is organized in several steps over two important automation levels: the control (execution) level and the supervisory (decision) level. The supervisory level calculates, based on logical user demands, the optimal decision that can be implemented effectively by the action of the control systems at the execution level. In terms of regulation, the process evolves around the nominal operating point (NP) under the effect of the advanced control strategy, while conserving performance, in a prespecified vicinity of the NP.

The supervisory system finds the optimal operating point (OP) in the admissibility domain for the suitable optimization techniques, and the decision of

the supervisor is transferred automatically from NP to OP. Sometimes, the case arises in which the process leaves the admissible operating domain, at the fault point, in which case the abnormal situation is detected and diagnosed so as to eliminate the cause of the fault as soon as possible. After the reconfiguration of the control systems, so that they become more tolerant to faults, the process operates in a normal state [POP 06, GEV 95, BOR 93].

In this context, our work, located in the control level area, proposes the necessary ingredients for the design and implementation of modern control systems as part of industrial applications and is notably directed toward systems engineering specialists.

We must note the complementarity that exists between this work and two books on optimization in engineering sciences [BOR 13, STE 14], which explain the methodology and techniques required for the implementation of the supervisory level in high-performing solutions of control in industrial processes.

1.2. Industrial process modeling

A mathematical model is the abstract representation of the behavior of a process (in this case, industrial). In other words, it is a dynamic system associated with the process that describes its behavior in a simple manner and allows a better understanding of the examined phenomena in order to carry out simulations, thus avoiding the need to conduct experiments that could be costly or get in the way of the process.

The model obtained will always be at best an idealization of the physical reality, considering that a hypothesis had to be made to obtain it. Furthermore, the search for a model can take on various forms, depending on the objective in question, the tools available and the nature and complexity of the phenomenon or process involved. In the engineering sciences domain, at the origin of the estimation of a dynamic model there is a set of equations that uses the conservation principle relative to certain physical generalized quantities: matter, mass, energy, amount of movement and electric charge. This mechanism of model estimation is reflected by the creation of balance equations for each of these quantities when they are implicated [DAU 04, POP 06, POP 11].

Most often, we start by writing the balance equation that expresses the preservation of a quantity W (mass or energy) over a time dt. The general form of this type of equation is the following:

(Variation dW of W in the system) = (entering quantity of W in the system) – (exiting quantity of W in the system) + (accumulated quantity of W) – (consumed quantity of W). For example, in the case of a representative chemical process, this can be written as

$$dW = Q_e dt - Q_s dt + r_f dt - r_c dt \qquad [1.1]$$

or again,

$$\frac{dW}{dt} = Q_e - Q_s + r_f - r_c\,, \qquad [1.2]$$

where r_f and r_c represent, respectively, the direct and reverse reaction rates for the transformation of the value W in the system.

The conservation principle of the global mass of the system can be written as

$$\frac{dM}{dt} = \frac{d(\rho V)}{dt} = \rho_e Q_e - \rho_s Q_s\,, \qquad [1.3]$$

where ρ is the density of the matter in the system, ρ_e is the density of the matter in the entering flow, ρ_s is the density of the matter in the exiting flow, Q_e is the entering flow rate, Q_s is the exiting flow rate, V is the volume of the system and M is the total mass of the system.

For the conservation of each component, it is known that

$$\frac{dn_a}{dt} = \frac{d(c_a V)}{dt} = c_{a,in} Q_e - c_{a,out} Q_s + (r_{af} - r_{ac})V\,, \qquad [1.4]$$

where c_a is the molar concentration of component A in the system, $c_{a,in}$ is the molar concentration of component A in the entering flow, $c_{a,out}$ is the molar concentration of component A in the exiting flow, Q_e is the entering flow rate, Q_s is the exiting flow rate, V is the volume of the system, n_a is the number of moles in the system and r_{af}, r_{ac} are the reaction rates of component A in the direct and reverse direction of the reaction.

For the conservation of energy:

$$\frac{dE}{dt} = \frac{d(U + K + P)}{dt} = h_e \rho_e Q_e - h_s \rho_s Q_s \pm Q_c \pm F_s\,, \qquad [1.5]$$

where E is the total energy of the system, U is the internal energy, K is the kinetic energy, P is the potential energy, h_e is the specific enthalpy of the matter in the entering flow, h_s is the specific enthalpy of the matter in the exiting flow, Q_c is the quantity of heat exchanged between the system and the environment and F_s is the action of the external forces acting on the system.

For the more complex processes, there is also the possibility of a synthetic representation of the main characteristics of the process from equations constructed in a systemic way to a certain degree of precision.

There are two important classes of models used in Automatic Control: state space models (SSMs) based on the structural state concept, and the input–output models (IOMs) based on the concept of causality.

Depending on the nature of the process and on the goal of the user, the balance equations can be easily transformed into a SSM or into an IOM, which express the dynamics of the process [AST 93, CAL 79, KAI 91, KAP 91, COR 13, DAU 04, POP 06, POP 11].

1.3. Model classes

Here, we present the most common models used in the design of control systems: SSMs and IOMs.

1.3.1. *State space models*

SSMs are connected to the structural transformations of a process that manipulates internal variables used in the study of dynamic systems. Let us consider a physical process with an entering quantity of W_i (primary matter) and an exiting quantity W_e (product), which is the result of a set of transformations of mass and/or energy.

In the stationary operating regime of the process, the two quantities are equal to each other:

$$W_{e0} - W_{i0} = 0 \qquad [1.6]$$

a fact that, through the mass or energy balance, explains the state of equilibrium of the process.

Seeing as the consumer's demands can vary, the exiting quantity W_e is variable in time and the state of the process is therefore unbalanced; this results in a dynamic state. Equilibrium returns through the action of the variables that produce accumulation phenomena in the process in question. Let us consider the internal variables of the process gathered in the vector of the state variables *x(t)*, so relation [1.6] becomes

$$\dot{x}(t) = W_i(t) - W_e(t) \,. \qquad [1.7]$$

The near majority of industrial processes are self-stabilizing and therefore an extra negative term *a* is introduced into relation [1.7], and thus it becomes

$$\dot{x}(t) = ax(t) + W_i(t) - W_e(t) \,. \qquad [1.8]$$

By a simple generalization, considering *x(t)* to be a state vector, relation [1.8] becomes

$$\dot{x}(t) = Ax(t) + W_i(t) - W_e(t) \,, \qquad [1.9]$$

where A is the state matrix that normally ensures the stabilizing character of the system through its eigen values with real negative parts.
The change in the entering quantity W_i is considered by the control vector *u* that influences the state variables using weighting elements included in a matrix *B*, and relation [1.9] is written as

$$\dot{x} = Ax + Bu - W_e \,. \qquad [1.10]$$

It is reasonable to consider that the output values gathered into the vector y are determined from state variables or are sometimes directly influenced by the control vector *u via* matrix *D* [CAL 79, FIL 02].

In this case, the process considered is expressed in general through the following state dynamics model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \,. \qquad [1.11]$$

The set (A, B, C, D) characterizes the state space representation of the process, where $u(t) : \mathbb{R} \to \mathbb{R}^l$, $x(t) : \mathbb{R} \to \mathbb{R}^n$ and $y(t) : \mathbb{R} \to \mathbb{R}^p$.

The representation in [1.10] is a multivariable Multi-Input Multi-Output (MIMO) continuous, linear and non-varying system.

For a monovariable SISO system, relation [1.10] is written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c^T x(t) + du(t) \end{aligned}, \qquad [1.12]$$

where b and c are vectors in $\mathbb{R}^n$ and d is a scalar.

The system:

$$\begin{aligned} \dot{x}(t) &= A(t)x + B(t)u \\ y(t) &= C(t)x + D(t)u \end{aligned}, \qquad [1.13]$$

where the matrices A, B, C, D are dependent on time is a continuous, linear and varying system.

The system that is represented by the relations in [1.14] is a nonlinear system:

$$\begin{aligned} \dot{x} &= f(t, x, u) \\ y &= g(t, x) \end{aligned}, \qquad [1.14]$$

where functions f and g are, respectively, made up of n and p components,

$$f : \mathbb{R}x\mathbb{R}^n \to \mathbb{R}^n, \; g : \mathbb{R}x\mathbb{R}^n \to \mathbb{R}^p. \qquad [1.15]$$

The equivalent discrete models are of the following form:

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) \\ y(k) &= C_d x(k) + D_d u(k) \end{aligned} \qquad [1.16]$$

for the linear, non-varying MIMO system and

$$\begin{aligned} x(k+1) &= A_d x(k) + b_d u(k) \\ y(k) &= c_d x(k) + d_d u(k) \end{aligned} \qquad [1.17]$$

for the SISO representation.

The nonlinear discrete model becomes

$$\begin{aligned} x(k+1) &= f(k, x, u) \\ y(k) &= g(k, x) \end{aligned}. \qquad [1.18]$$

In the usual case of linear, non-varying, continuous systems [1.13], the state trajectory is obtained as the unique solution

$$x(t) = e^{A(t)}x(0) + \int e^{A(t-\tau)}Bu(\tau)d\tau \qquad [1.19]$$

with $F(t)$ the continuous function and $F(k)$ the discrete function, respectively.

$$F(t) = e^{At}, t \in \mathbb{R} \quad F(k) = e^{k}, k \in \mathbb{Z} \qquad [1.20]$$

We denote *h* as the sampling period.

There are known relations between the sets of (A, B, C, D) and (A_d, B_d, C_d, D_d), and for the monovariable system, for example, these correspond to the following expressions:

$$A_d = e^{Ah}; B_d = (\int_0^h e^{A\tau}d\tau)b; c_d = c^T; d_d = d\,. \qquad [1.21]$$

In this case, it is easy to evaluate the continuous system response (weighting function) of the Dirac impulse (A, b, c, d)

$$g(t) = c^T F(t)b, t \in \mathbb{R} \qquad [1.22]$$

and for the discrete system,

$$g(k) = c_d^T F(k)b_d, k \in \mathbb{Z}\,. \qquad [1.23]$$

Next, we have looked at the tangent models defined around an operating point, and therefore at non-varying linear systems [CAL 79, KUO 91, WIR 91, DOI 97, DAU 04].

In this case, we can study the relationship between the two system representations of the process models. (A, B, C, D) is the non-varying linear system and $\nu(A)$ is the spectrum of matrix A (the eigen values of matrix A):

$$\nu(A) = \{\nu_1, \nu_2, ..., \nu_n\}\,. \qquad [1.24]$$

The norm of the fundamental matrix (state transition matrix) verifies the following relation:

$$\|F(t)\| = \left\|e^{At}\right\| < Me^{at}, a = \max\{\mathrm{Re}(\nu_i), i = \overline{1,n}\}. \quad [1.25]$$

By applying the Laplace operator, we obtain the transfer matrix of the continuous system:

$$G(s) = C(sI - A)^{-1}B \quad [1.26]$$

which is a rational fractions matrix with real coefficients:

$$G(s) = [g_{ij}(s)] = \left[\frac{D_{ij}(s)}{E_{ij}(s)}\right]. \quad [1.27]$$

We must note that all rational fractions $g_{ij}(s)$ have the same denominator and are strictly proper (the order of $D(s)$ is lower than the order of $E(s)$).

For the discrete system, the transfer matrix is calculated in the same way:

$$G_d(z) = C_d(zI - A)^{-1}B_d \quad [1.28]$$

with irreducible rational fraction elements that are strictly proper:

$$G_d(z) = [g_{ij}(z)] = \left[\frac{D_{ij}(z)}{E_{ij}(z)}\right]. \quad [1.29]$$

The result in the monovariable case is obvious for a continuous system

$$G(s) = c^T(sI - A)^{-1}b \quad [1.30]$$

and for the equivalent discrete system

$$G(z) = c_d^T(zI - A_d)^{-1}b_d. \quad [1.31]$$

Some structural properties of non-varying linear systems are presented below.

A continuous system is asymptotically stable (internal stability) if the spectrum of the matrix $V(A)$ is located in $\mathbb{C}^-$.

A discrete system $\dot{x} = A_d x$ is asymptotically stable if all of the eigen values have a modulus of less than 1, meaning that the spectrum of the matrix $V(A_d)$ is located in the unit circle $C(0,1) = (z \in \mathbb{C}, |z| \leq 1)$.

Consider, for example, a monovariable continuous system,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c(t)x(t) + du(t) \end{aligned}, \qquad [1.32]$$

where the matrix A is of rank n.

The controllability of the system targets the couple (A, b), and therefore the SISO system [1.32] is controllable if the controllability matrix $R = (b, Ab, A^2b, A^3b, ..., A^{n-1}b)$ has a rank of n.

In an equivalent way, a MIMO system is always controllable if matrix $R = (B, AB, A^2B, A^3B, ..., A^{n-1}B)$ has the rank n.

For a controllable system, there is a control algorithm that can change the dynamics of the systems through state transitions. Controllability is linked to the property of the stabilizability of systems, providing strategies for stabilizing an unstable system, and ensuring desired levels of performance, using a state feedback control.

For a continuous system with A of rank n, the realization (A, B, C) must therefore be minimal (the dimension of the state space is n). This condition is reflected in the case of MIMO systems through the transfer matrix:

$$G(s) = [g_{ij}(s)] = \left[\frac{N_{ij}(s)}{D_{ij}(s)}\right], \deg(N_{ij}(s)) < \deg(D_{ij}(s)) = n \qquad [1.33]$$

which must contain real elements and irreducible rational fractions. In the case of SISO systems, this condition is reflected in the transfer function,

$$G(s) = \left[\frac{N(s)}{D(s)}\right], \deg(N(s)) \leq \deg(D(s)) \qquad [1.34]$$

which is an irreducible rational fraction.

The most important consequence that exploits the quality of a minimal realization of the system (A,b,c,d) is to calculate the SSM, which corresponds to the I/O model.

Consider, for example, the transfer function of a SISO system, expressed by the irreducible fraction,

$$G(s)=\frac{b_n s^n+b_{n-1}s^{n-1}+...+b_0}{a_n s^n+a_{n-1}s^{n-1}+...+a_0} \qquad [1.35]$$

and so the corresponding minimal realization is the following:

$$A=\begin{bmatrix} 0 & 1 & 0...\ 0 \\ 0 & 0 & 1...\ 0 \\ -a_0 & -a_1 & .....-a_n \end{bmatrix},$$

$$b^T=(0\quad 1\quad 0\quad 00\quad 0\quad 0\quad 0), \qquad [1.36]$$

$$c^T=(b_0-b_n a_0, b_1-b_n a_1,...,b_{n-1}-b_n a_{n-1}),$$

$$d=b_n$$

The observability property targets the couple (A,C) and the system [1.12] is observable if the matrix $Q^T=(c,Ac,A^2c,A^3c,...,A^{n-1}c)$ has a rank of n for SISO systems, and equivalently, if the observability matrix $Q^T=(C,AC,A^2C,A^3C,...,A^{n-1}C)$ has a rank of n, for MIMO systems.

Observability is linked to the property of detectability; the detectability of the set (A,C) provides the possibility to construct a stable state estimator associated with the original system and notably to find a matrix L for which the spectrum $V(A+LC)$ is located in the domain $\mathbb{C}^-$ for continuous systems, and $V(A_d+LC_d)$ is located in the circle $C(0,1)$ for discrete systems.

The dynamics of the estimator is described in the following equations:

$$\dot{\hat{x}}=(A+LC)\hat{x}+Bu\ , \qquad [1.37]$$

where $\hat{x}(t)$ is the state vector estimated using the observations made over $u(t)$ and $y(t)$.

The SSMs are used in designing the state feedback control. The control *u(t)* is computed through the relation,

$$u(t) = -Kx(t)\,, \qquad [1.38]$$

where matrix K is determined by a pole placement method.

In the case of observer-based control, it is computed in a similar manner:

$$u(t) = -K_e\hat{x}(t) \qquad [1.39]$$

The optimal control uses the integral criterion:

$$I = \int_0^\infty \left[x^T(t)Qx(t) + u^T(t)Ru(t) \right] dt\,, \qquad [1.40]$$
$$Q \geq 0, R > 0$$

where matrices Q and R (symmetric positive-definite) are chosen to ensure the supplementary qualities of the control *u(t)*, which minimizes criterion [1.40] and is expressed as follows:

$$u(t) = -R^{-1}B^T Px(t)\,. \qquad [1.41]$$

In expression [1.41], matrix P, which is symmetric positive-definite, is the solution of the matricial algebraic Riccati's equation:

$$Q + A^T P + PA - PBR^{-1}B^T P = 0\,. \qquad [1.42]$$

For the integrating term from [1.40], we get

$$x^T(t)Qx(t) + u^T(t)Ru(t) \qquad [1.43]$$

which is positive-definite, the expression of matrix K can be calculated employing Lyapunov functions based techniques [CAL 79, KUO 91, WIR 91, DIO 97, DOI 97, DAU 04].

1.3.2. *Input–output models*

The design of IOM is based on mass or energy balance equations, respecting the principle of causality. In order to reach the form needed in the control design, we start with the nonlinear knowledge-based model, which expresses the operating laws

of the process. We continue with the tangential and normalized behavioral model and finally we introduce the control models in the design of the control systems. All these different classes of models are presented later [DAU 04, POP 06, BOR 10].

1.3.2.1. *Knowledge-based model*

This type of model is usually established from the knowledge of the internal laws, which govern the evolution of the process. Their transcription leads to one (or several) analytical expression(s) of a model whose parameters have physical significance.

In process engineering studies, there are some fundamental principles of classical physics: the principle of the conservation of mass, the fundamental principle of mechanics (conservation of the quantity of movement), the first principle of thermodynamics (conservation of energy) and the second principle of thermodynamics (conservation of entropy).

When chemical, biochemical or microbiological transformations are at play, the kinetics of the reactions must be looked at and knowledge of these is vital in establishing a reactor model. Therefore, knowledge-based models mathematically express the phenomena responsible for the process dynamics.

The first phase in obtaining a knowledge-based model involves defining the system and the phenomena that are to be represented within it. The limits of the physical system must be determined, as must the variables that permit the representation and identification of important physical, chemical or biological phenomena (see section 1.2).

The main objective of "knowledge-based" models is, therefore, to describe a phenomenon using a mathematical relation. They remain very close to the real processes enabling a reproduction of the behavior of the process over a large operating domain. In general they are quite complex, with a large number of parameters which characterize the nonlinearity of the process expressed as technological variables [TER 90, FOU 04].

1.3.2.2. *Behavioral model*

A behavioral model is a systemic representation that uses differential equations or transfer functions to help simulate the dynamics of the process. Generally speaking, a behavioral model is expressed in a standard format and its coefficients have physical significance.

The behavioral model can be obtained following a few mathematical transformations carried out on the raw version of the knowledge-based model [DAU 04].

Using inductive reasoning, it can be stated that, around an operating point, small variations in the input of a system can be linked to small variations in the output for a linear model, which is expressed as:

$$\begin{aligned} & a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_1 \frac{dy(t)}{dt} + a_0 y(t) \\ & = b_m \frac{d^m u(t)}{dt^m} + b_{m-1} \frac{d^{m-1} u(t)}{dt^{m-1}} + \ldots + b_1 \frac{du(t)}{dt} + b_0 u(t) \end{aligned} \quad ; \quad [1.44]$$

where coefficient n specifies the order of the system.

This equation is linear with constant coefficients, and the input–output dynamic behavior of the system can be represented simply with a mathematical tool. In order to easily handle relation [1.44], the Laplace transform is used, which enables the passage from a differential equation to an algebraic one.

Here, we shall evaluate the Laplace transform of equation [1.44]. First, we detail the initial conditions: the value of $y(0)$ and of all its derivatives are given, and the value of $u(0)$ is zero (zero action before $t = 0$).

The Laplace transform and its properties can then be used in order to obtain the relation:

$$\begin{aligned} & (a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0) Y(s) - \left[s^{n-1} (a_n y(0)) \right. \\ & + s^{n-2} \left(a_n \frac{dy}{dt}(0) + a_{n-1} y(0) \right) + \ldots \\ & \left. + s^0 \left(a_n \frac{d^{n-1} y}{dt^{n-1}}(0) + \ldots + a_1 y(0) \right) \right] = (b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0) U(s) \end{aligned} \quad . \quad [1.45]$$

The expression in square brackets corresponds to the free response of the system, meaning the response that represents the effect of the initial conditions. The other term, as a factor of $U(s)$, corresponds to the forced response, meaning the response to the input, where the transfer function is defined by considering the case where all of the initial conditions are equal to zero:

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0} \, . \quad [1.46]$$

Equivalent models exist for discrete systems expressed by the difference equation:

$$\begin{aligned} &a_0 y(k) + a_1 y(k-1) + ... + a_n y(k-n) = \\ &b_0 u(k) + b_1 u(k-1) + ... + b_m u(k-m) \end{aligned} \quad [1.47]$$

and, respectively, by a transfer function in z^{-1}:

$$G(z^{-1}) = \frac{Y(z^{-1})}{U(z^{-1})} = \frac{b_0 + b_1 z^{-1} + ... + b_m z^{-m}}{a_0 + a_1 z^{-1} + ... + a_n z^{-n}}. \quad [1.48]$$

It must be noted that generally the term b_0 is equal to zero (no direct transfer of information).

It is clear that a behavioral model does a good job of expressing the structural component of the process, as the differential equation or the transfer function expresses the order of the mathematical model. This type of model offers the possibility to propose an adequate structure for a control model, which is evaluated through process identification.

The behavioral model is then based on the knowledge-based model and used in the evaluation of the dynamics of a real process through simulation. This type of model provides information regarding the structure of the model through a standard differential equation in which the coefficients are normalized.

The behavioral model is especially interesting, on the one hand, through its relation with knowledge-based models and, on the other hand, for its link with control models [DAU 04, POP 11].

The control model is used in the designing of closed-loop systems.

1.3.2.3. *Control model*

The control model is one of the dynamic models, which are obtained through a particular representation of a behavioral model, expressed using parameters usually handled in Automatic Control (gain, time constant, pure delay, etc.) and which must be identified by the process specialist, for the control design of digital systems. The availability of digital resources enables the establishment of automatic estimation algorithms for the parameters of a model through an identification operation. It is important to emphasize that the identification operation can provide more general, high-performing discrete models and holds several advantages over other approaches.

High-performance identification algorithms have been extensively developed as a result of their recursive implementation adapted to real-time identification problems and their application to numerical systems.

The estimation principle for the parameters of the sampled models is based on the prediction error of the model.

An adjustable parameter sampled model is easily implanted into numerical systems. The error between the output of the process *y(t)* and the output $\hat{y}(t)$ predicted by the model, called the prediction error, is used by a parametric adaptation algorithm, which at each sampling moment modifies the parameters of the model so as to minimize this error. Generally, the output is a low-level pseudo-random binary sequence produced by the computer (a succession of rectangular pulses of randomly variable duration). Once the model is obtained, an objective validation can be carried out using statistical tests regarding the prediction error $\varepsilon(t)$. The validation test makes it possible, for a given process, to choose the best algorithm for estimating the parameters and the best model. This direct approach for the identification of dynamic process models has a range of other advantages, such as:

– significant data compression, as at any given moment the recursive algorithms only deal with an input/output pair rather than with all of the input/output data;

– relatively low memory and computational power requirements;

– easy installation onto a microcomputer;

– possible realization of identification systems in real time;

– possibility of tracking slowly varying parameters over time;

– identification of measurable perturbation models.

One of the key elements in the establishment of this approach for the identification of process models is the *parameter adaptation algorithm*, which drives the parameters of the adjustable model at each sampling step. This algorithm has a "recursive" structure, meaning that the new value of the parameters is equal to the most recent value plus a correction term, which depends on the most recent measurements. Usually, a "vector" of the parameters is defined and its components are the different parameters to be identified. Parameter adaptation algorithms have the following structure:

[New estimation of the parameters (vector)] = [Previous estimation of the parameters (vector)] + [Adaptation gain (matrix)] × [Function of the measurement (vector)] × [Function of the prediction error (scalar)]. The vector of the functions of the measurements is called the "*vector of the observations*".

We consider the model in discrete time:

$$\hat{y}(k+1) = \sum_{i=1}^{nA} \hat{a}_i y(k+1-i) + \sum_{i=1}^{nB} \hat{b}_i u(k+1-i) \qquad [1.49]$$

and using the notations

$$\hat{\theta}^T(k+1) = \left[\hat{a}_1, \hat{a}_2, ..., \hat{a}_{nA}, \hat{b}_1, \hat{b}_2, ...\hat{b}_{nB} \right] \qquad [1.50]$$

$$\Phi^T(k) = \left[-y(k), -y(k-1), ..., -y(k+1-nA); u(k), ..., u(k+1-nB) \right] \qquad [1.51]$$

we obtain

$$\hat{y}(k+1) = \hat{\theta}^T(k)\Phi(k), \forall k \in \mathbb{Z}\,. \qquad [1.52]$$

We use a suitable identification method, for example Recursive Least Square (RLS), to estimate the parameters of the model as the solution of the optimization problem:

$$\min_{\hat{\theta}(k)} \{ J(\hat{\theta}(k)) = \sum_{i=1}^{k} \left[y(i) - \hat{\theta}^T(k)\Phi(i-1) \right] \} \qquad [1.53]$$

The solution to problem [1.53] is given in the following recursive relation:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\Phi(k)\varepsilon(k+1) \qquad [1.54]$$

with

$$F(0) = \alpha, I(\alpha > 0); \hat{\theta}(0) = \theta_0 \qquad [1.55]$$

and

$$F(k+1) < F(k)\,. \qquad [1.56]$$

The recursive algorithm [1.54] ensures the convergence of the RLS method, and the estimation error is reduced to zero, asymptotically,

$$\lim_{k \to \infty} \varepsilon(k+1) = 0\,. \qquad [1.57]$$

From equation [1.49], we easily obtain the polynomial form of the model:

$$\hat{y}(z^{-1}) = \frac{\hat{b}(z^{-1})}{\hat{a}(z^{-1})} u(z^{-1}) \qquad [1.58]$$

where the set

$$\begin{aligned} \hat{a} &= 1 + a_1 z^{-1} + \ldots + a_{nA} z^{-nA} \\ \hat{b} &= b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{nB} z^{-nB} \end{aligned} \qquad [1.59]$$

is considered to be the model used for the control design of digital systems [LAN 93, LAN 97, POP 00, POP 01, DAU 04].

For the evaluation of knowledge-based behavioral and control models, linearization around an operating point (tangent systems) for different types of industrial processes is necessary in order to reach the step of designing the system control algorithm in the closed-loop system.

We can cite [LAN 93, GEN 97, POP 06, BOR 11] and simply appeal to the design of model-based systems using the method of pole placement. We consider the structure of the looped system in Figure 1.1, where $(\hat{b}(z^{-1}) / \hat{a}(z^{-1}))$ is the control model, and the set $(R(z^{-1}), S(z^{-1}))$ is the control algorithm.

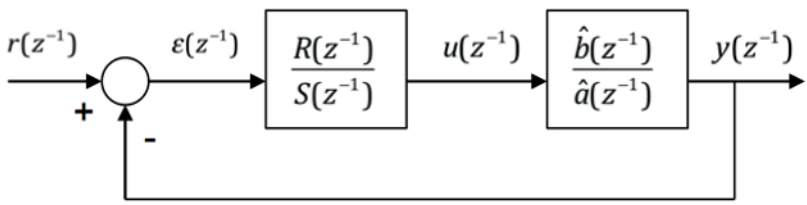

Figure 1.1. *Digital system in a closed-loop system*

We note $P_i(z^{-1})$ the characteristic polynomial that imposes the level of performance. The solution for the Diophantine polynomial equation:

$$P_i\left(z^{-1}\right) \overset{\Delta}{=} \hat{a}\left(z^{-1}\right) S\left(z^{-1}\right) + \hat{b}\left(z^{-1}\right) R\left(z^{-1}\right)$$

provides the unknown polynomials of the controller ($R^*(z^{-1}), S^*(z^{-1})$). This digital control can be improved following adaptive, robust, predictive or multimodel strategies [LAN 97, DAU 04, POP 06, BOR 11].

1.3.2.4. *Decision models*

In this case, the process is considered to be a "black box", and the establishment of the model is based on observations of the inputs/outputs of the process. The inputs of the model are the parameters controlled by regulation and the outputs are considered to be quality indicators. The model obtained must appropriately describe the behavior of the process in its near-stationary operating mode. This class of models mainly involves complex, nonlinear and multivariable industrial processes and is used in the resolution of optimization problems. The solution to these problems represents the optimal decision for the process, introduced as set points for the closed-loop systems, ensuring the optimal operating point. The structure of the decision models is inspired from laws that govern the process, or from a sensitivity analysis of the outputs of the process in relation to the variation of the inputs. The parameters of the model are estimated using experimental techniques (MC-off line), following a protocol of data acquisition and processing. The models are checked and updated whenever it is necessary [CAL 79, TER 00, FIL 09, LUP 13].

This procedure starts with the programming of the experimental data acquisition from the process. After a data processing protocol that chooses the inputs and the output(s) of the model, the structure of the multivariable nonlinear model is stated and the data files are prepared for the identification operation itself. The output of the model represents the order of quality $\hat{z}$ of the process in question and the inputs represent the controlled variables. The general representation of the model is therefore a nonlinear function of unknown parameters $\hat{a}$ and the input vector y:

$$\hat{z} = f(\hat{a}, y)\,, \qquad [1.60]$$

where the estimator $\hat{a}$ of the parameters of the model is calculated using the least squares method over the whole of the experimental dataset. If the data are organized in a matricial–vectorial manner, the estimator $\hat{a}$ is calculated following the relation:

$$\hat{a} = \left(YY^T\right)^{-1} Y^T z\,. \qquad [1.61]$$

Here, Y is the matrix of the observations acquired at the inputs and z is the vector of the observation acquired at the output.

Using an algebraic operator Q, the decision model and the associated constraints are used for building the optimization problem for the process in question:

$$\min\{I(y) = Q\left[\hat{z}(y)\right]\} . \qquad [1.62]$$

In its first part, this work presents the concepts, the methods and the highest performing techniques used in the design of digital systems to specialists in Industrial Automatic Control. In the final part, it presents examples for the implementation of practical applications in the use of industrial processes [CAL 79, TER 98, POP 06].

In Chapter 2, we present the methods of identification (in the closed-loop system) that provide the best levels of performance in the estimation of the digital form of dynamic process models, which are necessary for the design of control systems.