1

Introduction to Structural Optimization

1.1. Introduction

Structural optimization is a topic which affects many different physical domains – particularly solid mechanics – and which it is tricky to characterize, as its formulations may have a number of different aspects. Firstly, a distinction regarding the way in which geometries are parameterized is presented, and secondly, a distribution pertaining to the intrinsic nature of optimization algorithms is established.

Determining the appropriate shape for structural components is a crucially important problem for engineers. In all areas of structural mechanics, the impact of proper design of a part is very significant in terms of its strength, its lifetime and its usage. This is a challenge faced on a daily basis in the sectors of spatial research, aeronautics, the automobile industry, naval competition, fine mechanics, precision mechanics or artwork in civil engineering, and so on. To develop the art of the engineer requires enormous effort to continuously improve techniques for designing structures. Optimization is of primary importance in improving the performance and reducing the weight of aerospace- and automobile machinery, providing substantial energy savings. The different development of computer-aided design (CAD) techniques and optimization strategies is part of this context. There has been keen interest in structural optimization for over thirty years. Whilst it is still too infrequently applied in the conventional techniques used by research centers, it is becoming more widely used as its reliability improves. Having begun with the simplest of problems, the field of application of structural optimization today extends to new and ever more interesting challenges.

To illustrate the evolution of structural optimization techniques, we can arbitrarily split structural optimization into three major groups (or families). In historical terms, each of them has been addressed in order of increasing difficulty and generality.

With sizing optimization, we are only able to modify the dimensions of an object whose shape and topology are fixed. There can be no modification of the geometric model. We speak of a homeomorphic transformation.

Shape optimization involves making changes of shape which are compatible with a predetermined topology. Typical shape optimization modifies the parametric representation of the boundaries of the domain. By moving the boundaries of the domains, we can seek the best solution out of all the structures obtained by homeomorphic transformation of the original object. In this case, it is clear that we can make a change to the transverse dimensions as well as a modification to the object's configuration, but it is certainly not acceptable to modify its connectivity or its nature – in particular, the number of components that it has. The optimal object exhibits the same topology as the original object.

With topology optimization, we can fundamentally change the nature of the object. The "topology" refers to the number and position of the components of the domains. Here, the object's geometry is presented with no prerequisites as to the connectivity of the domains or the components present in the solution. We take no initial information about the topology of the optimal shape.

1.2. History of structural optimization

It was in the early 1960s that Schmit [SCH 60] and Fox [FOX 65] laid the foundations for a modern theory of structural optimization, based on the concepts of mathematical programming and sensitivity analysis. Paradoxically, at the time, "fully stressed design" was the only widely used technique in practice, although it lacked any theoretical justification, other than empiricism and the engineers' intuition. It was Prager and Taylor [PRA 68] who set out variational methods and Lagrangian optimality conditions to justify the criteria of fully stressed design for a class of structural optimization problems. The optimality conditions of the optimization problem were then used directly to construct an iterative resolution algorithm known as the "optimality criteria method".

Originally, structural optimization was mainly limited to the sizing optimization of trusses or gantries. Thus, sizing optimization of structures was the first field of application for optimality criteria. When dealing with the problem of sizing, we look at the transverse sections of the structural elements, though their length and the location of their joints remain fixed. During the late 1960s and early 1970s, optimality criteria were soon adapted to large structures, modeled using the finite-element method (FEM) [VEN 73]. The optimality criteria method produced a few interesting results, and a number of extensions have been presented since the 1970s, including Venkayya's generalized criteria method [VEN 73] or Rozvany and Zhou's [ROZ 91] discretized optimality criteria.

Although, since the 1970s, attention has mainly been focused on sizing, the problem of truss topology has also been studied by Prager [PRA 74], on a very restricted class of structures based on the concept of a truss put forward by Michell in 1904 [MIC 04]. The problem lies in finding the best possible configuration so that the truss or gantry can convey forces to the foundations whilst minimizing a given performance and satisfying the design constraints. Michell's theory [MIC 04] is related to the topology of trusses made of bars of minimal mass. The optimal solution, from Michell's point of view, is composed solely of perpendicular bars, which form a structure whose configuration is optimal for the maximum tensile- and compressive stresses. All the configuration problems studied by Prager later on were solved analytically, so the practical application of topology was very limited. To remedy this shortcoming, Rozvany [ROZ 76] invested a great deal of effort in developing new approaches to solve these configuration problems as automatically as possible.

After that, optimal truss topology was studied in greater depth by Kirsch [KIR 90]. If we impose a very small minimum value for the cross-section, then the "layout" optimization of trusses and gantries can be approached as a conventional sizing problem on a very large scale. The solution is then obtained by application of generalized optimality criteria for a variety of objective functions: compliance, movements, tensions, and eigenvalues [ZHO 91].

Finally, the crux of the problem of truss topology optimization seems to have been identified by Bendsøe *et al.* [BEN 91]. The problem of truss topology is examined with an integral approach combining analysis and design simultaneously. The problem of minimum compliance is transformed into a problem of non-differentiable optimization and then, in the case of a truss of bars, into a linear problem. In this form, it can be solved on very large structures, using non-differentiable optimization methods [BEN 93a], an interior point method [BEN 93b], a dual method [BEC 94], or a penalty/barrier multiplier method [BEN 97].

The three main groups of structural optimization cannot be considered recent concepts, but their integration into biomechanics is – especially topology optimization [FRA 10].

1.3. Sizing optimization

1.3.1. *Definition*

With *sizing optimization*, we can only modify the cross-section or transverse thickness of the components of structure whose shape and topology are fixed. There can be no modification of the geometric model and its features. Figure 1.1 shows a truss made of 17 bars with different cross-sections (circular, rectangular and I-shaped).

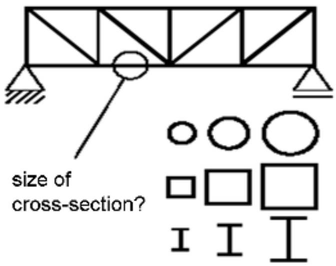


Figure 1.1. *Changing the dimensions whilst preserving the same topology of the section.*

Sizing optimization can be performed by considering the same topology to produce various dimensions. For example, when the cross-section is circular, we merely need to vary the diameters to minimize an objective or several objectives under certain constraints.

1.3.2. *First works in sizing optimization*

The problem of sizing optimization has benefited the most from this research, so optimization of the transverse dimensions is, today, a reliable tool. This problem was also extended to that of flexural elements [FLE 83], to improving performance when subjected to vibration, and to the stability of balance. Besides the transverse dimensions of the structural elements, it is possible to vary their shape. To the best of our knowledge, few works have been devoted to the study of the optimal shape for a truss. In this type of problem, only the location of the structural joints is altered, whilst the topology remains unchanged. Svanberg [SVA 81] was one of the only people to carry out truss shape optimization on the basis of FEM analyses and solving using mathematical programming techniques.

1.3.3. *Numerical application*

1.3.3.1. *Description and modeling of the studied problem*

Figure 1.2a shows a cantilever beam and its I-shaped cross-section in Figure 1.2b. This beam is embedded at one end, and subjected to free vibration. The material from which the beam is made is structural steel, which has a Young's modulus $E = 200,000\ MPa$ and Poisson's ratio of $\nu = 0.3$. The density of the material is $\rho = 7.854 \times 10^{-6}\ Kg / mm^3$. The material exhibits linear elastic isotropic behavior. The length of that beam is: $L = 300\ mm$ and the dimensions of the cross-section are: $B = 60\ mm$, $H = 100\ mm$ and $T = 20\ mm$ (Figure 1.2b).

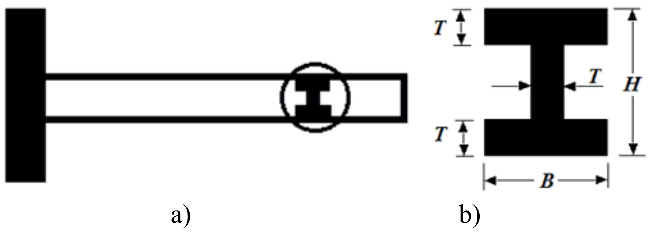


Figure 1.2. *Cantilever beam subject to free vibration.*

The objective of this study is to calculate the first four modes of resonance and then perform sizing optimization on the dimensions of the cross-section. The problem of optimization, therefore, is to minimize the structural volume under the constraint of the first resonance frequency. This problem can be formulated as follows:

$$\begin{aligned} \min \quad & : Volume(B,H,T) \\ s.t. \quad & : f_1(B,H,T) - f_w \leq 0 \\ & : 40 \leq B \leq 100 \\ & : 80 \leq H \leq 160 \\ & : 10 \leq T \leq 30 \end{aligned} \qquad [1.1]$$

where $f_w = 20Hz$ is the maximum value of the first resonance frequency.

1.3.3.2. *Numerical results*

To perform sizing optimization on the ANSYS software, for instance, we consider the dimensions of the cross-section as optimization variables so as to obtain a parameterized model. Then, a direct simulation is performed as the heart of the optimization loop.

1.3.3.2.1. Direct simulation

Figures 1.3a and b show the geometric model and meshing model of the cross-section of the beam under examination. At the start, the mesh is created in 2D using the linear element (PLANE42 - 4-node).

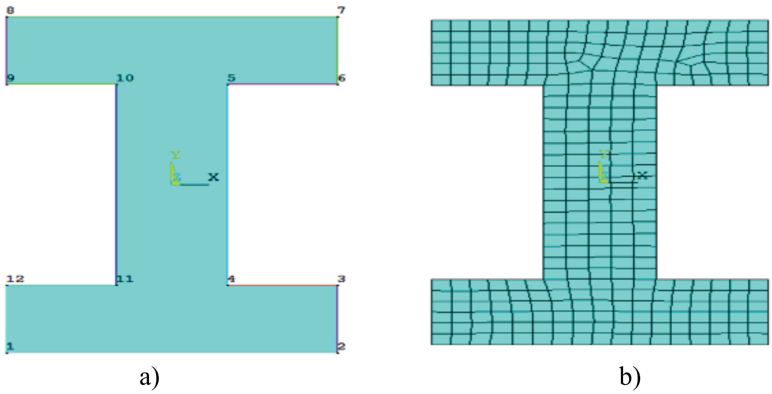


Figure 1.3. *a) Geometric model and b) meshing model of the cross-section. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Next, a 3D model is constructed and meshed, using a linear element SOLID45 - (8-node). Figure 1.4 shows the boundary conditions where one of its ends is fixed.

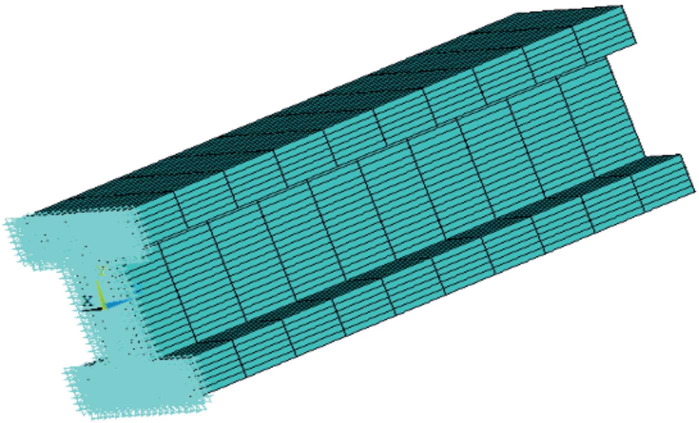


Figure 1.4. *Boundary conditions. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Figure 1.5 shows the first four modes of resonance of the beam at hand. The resulting values of the resonance frequencies are: $f_1 = 14.30\ Hz$, $f_2 = 29.68\ Hz$, $f_3 = 33.73\ Hz$ and $f_4 = 80.14\ Hz$.

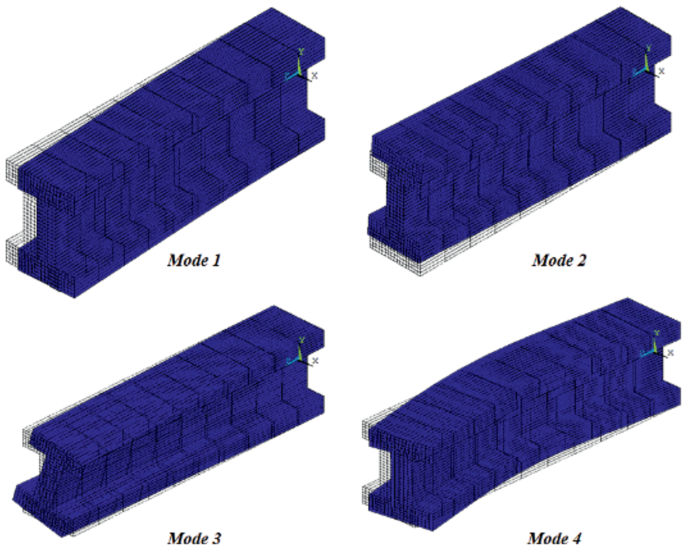


Figure 1.5. *First four modes of resonance. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Figure 1.6 shows the general approach employed in direct FEM simulation in dynamics [KHA 11g].

The general equation in dynamics used in this approach can be written as follows:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F} \qquad [1.2]$$

where $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$ are the matrices of the rigidity, damping and mass, respectively. $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the displacement vectors as a function of time, its first derivative (velocity) and second derivative (acceleration). $\mathbf{F}$ is the external force vector as a function of time. In the case of modal analysis, the damping matrix and the force vector are ignored, which gives us:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0} \qquad [1.3]$$

By solving this equation, we find the resonance frequencies.

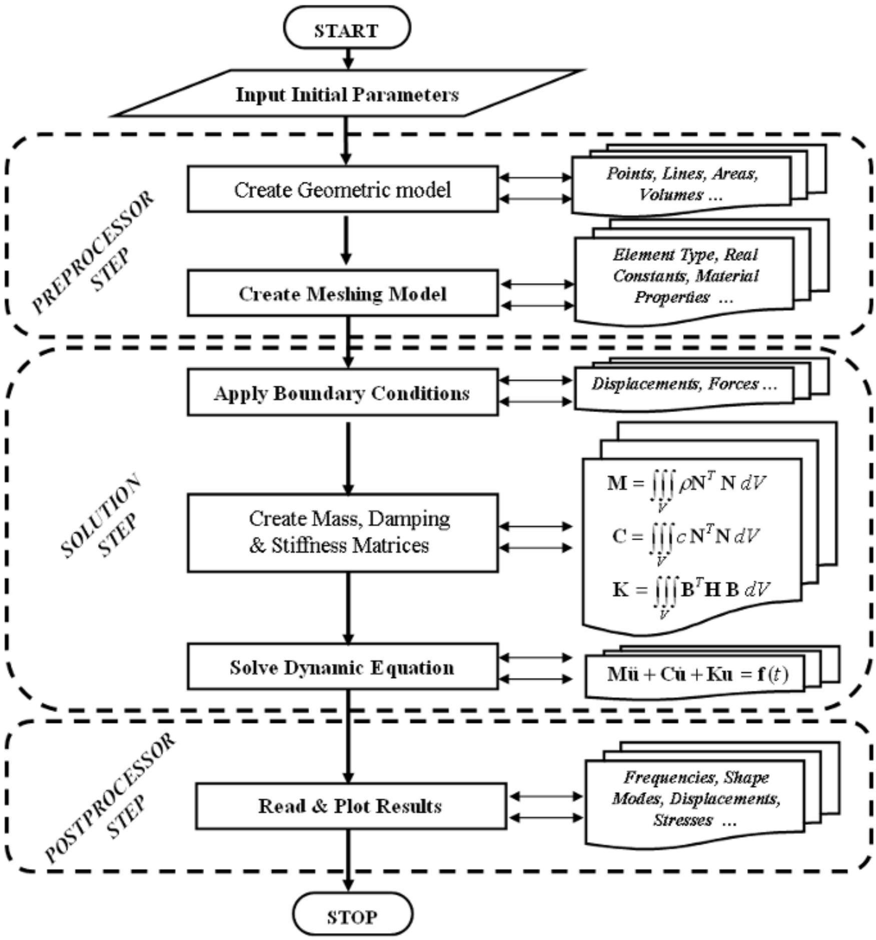
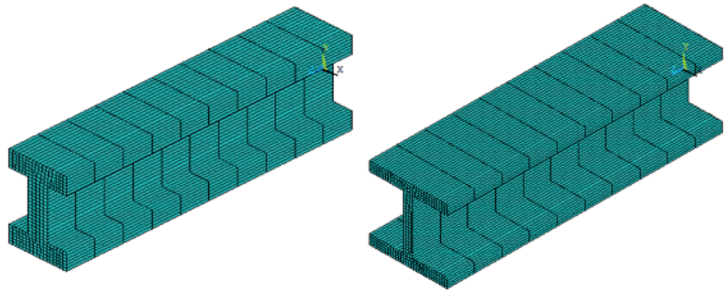


Figure 1.6. *General approach to FEM simulation in dynamics.*

1.3.3.2.2. Sizing optimization

After performing sizing optimization, we obtain a configuration which performs better than the initial configuration. Figures 1.7a and b show the initial- and optimal configurations of the beam under study, respectively.



a) b)

Figure 1.7. *a) Initial configuration and b) optimal configuration. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

At the initial point, the structural volume is equal to: $V_0 = 1,080 \times 10^3 mm^3$, with a value of resonance frequency equal to: $f_1 = 14.30 Hz$. At the optimal point, the structural volume is equal to: $V_{opt} = 699 \times 10^3 mm^3$, with a resonance frequency equal to: $f_1 = 20.01 Hz$. The optimal values of the input parameters are: $B = 84.60\ mm$, $H = 80.56\ mm$ and $T = 10.16\ mm$.

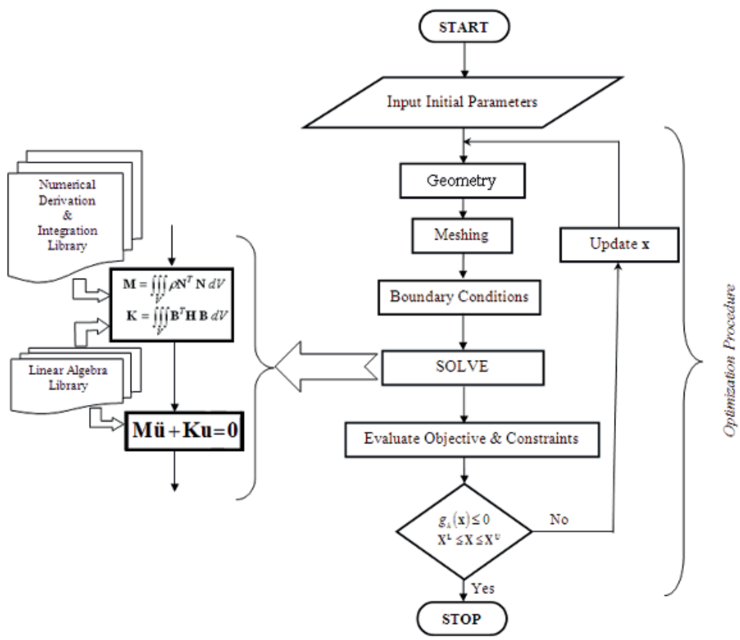


Figure 1.8. *Algorithm used for sizing optimization of the studied beam.*

Figure 1.8 shows the algorithm used for sizing optimization of the studied beam. The optimization loop contains the following steps:

– entering of the input parameters;

– creation of the geometric model;

– creation of the meshing model;

– definition of the boundary conditions;

– solving of the problem for modal analysis;

– convergence test: if the solution converges, STOP; Else, modify the optimization variable vector.

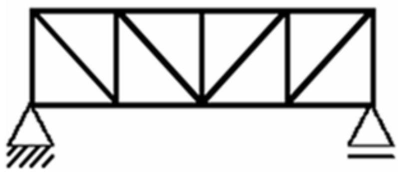
The optimization method used is called the “Curve-Fitting Method” and can be directly employed in the ANSYS Mechanical (APDL) module, up to version ANSYS 13.0. However, we can use the commands from the ANSYS Mechanical (APDL) module in the ANSYS-WorkBench module for recent versions (ANSYS 14.0 and above).

Note that the optimization process has yielded an optimal structure which is lighter, with a higher value of the resonance frequency, which demonstrates the importance of the integration of optimization during the design process to obtain economical, high-performance structures.

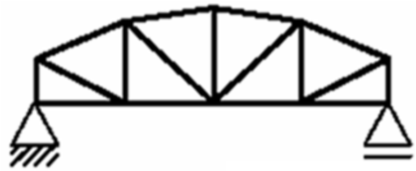
1.4. Shape optimization

1.4.1. *Definition*

With *shape optimization*, it is possible to make changes to the shape, provided they are compatible with a predetermined topology. Conventional shape optimization modifies the parametric representation of the boundaries of the domain. By moving those boundaries, we can try to find the best possible solution out of the set of all the configurations obtained by homeomorphic transformation of the original structure. This being the case, clearly, we can make a change to the transverse dimensions as well as a modification to the configuration of the structure, but it is certainly not acceptable to modify the connectivity or the nature of the structural elements. Figure 1.9 shows three different shapes (Figures 1.9a, 1.9b and 1.9c) for the same topology of a 17-bar truss.



a)



b)

c)

Figure 1.9. *Three different shapes for the same topology of a 17-bar truss.*

In this case, shape optimization involves varying the coordinates of the connecting points between the bars to minimize one or more objectives under certain conditions.

1.4.2. *First works in shape optimization*

Shape optimization of thin, three-dimensional structures began early on, thanks to the work of Zienkiewicz and Campbell [ZIE 73]. Since then, the domain of shape optimization has blossomed to such an extent that it is impossible to offer an exhaustive view, and we refer readers to the detailed reviews offered by Haftka and Gandhi [HAF 86]. The problem is much more complex than sizing, because modifications made to the shape of the external contour lead to modifications of the interior domain. Several difficulties need to be dealt with. The first lies in carrying out the sensitivity analysis, which is significantly more complex than in the case of sizing. We need to link the motions of the interior points in the domain to the boundary variations. This is the problem of determination of the velocity field. Initially limited to transfinite meshes [BRA 84], the technique has been extended to apply to free meshes [BEL 88]. The sensitivity calculation must be carried out semianalytically, which can also lead to significant errors [BAR 88]. The second difficulty lies in mastering the definition of the boundaries of the domain and remeshing the domain regardless of its external contour. Braibant and Fleury [BRA 84] showed that the problem is well-posed and regularized when we use gentle curves such as *B-splines*. Remeshing of the optimized part requires reliable automated meshers. Unfortunately, when the shape changes become very significant, it sometimes becomes difficult to modify the finite-element model without introducing greatly distorted elements and, thus, a very great approximation error. The solution was put forward by Bennet and Botkin [BEN 83]: to prevent this phenomenon, we modify the mesh during the optimization process. Today, we combine automated mesh generation, shape optimization and even error calculation to maintain a constant degree of precision [DUY 94]. To determine the initial shape with no *a priori* knowledge, it is necessary to do away with the parametric representation and be able to do without a shape function to describe the domain. Posing the problem in the form of a distribution of material deals with this criterion, but there was no known general resolution method until Bendsøe and Kikuchi [BEN 88] suggested introducing the idea of porous microstructure and the theory of homogenization in the problem of optimum material distribution.

1.4.3. *Numerical application*

1.4.3.1. *Description and modeling of the studied problem*

Figure 1.10 shows a parameterized plate with the dimensions A = 70 mm, B = 70 mm, C = 50 mm, Ro = 20 mm, Ri = 10 mm, Rc = 30 mm and r = 5 mm. The thickness of that plate is equal to: T = 10 mm.

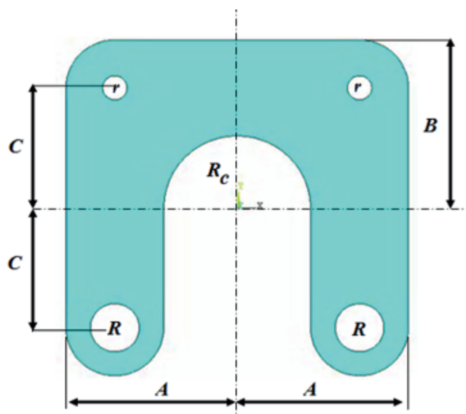


Figure 1.10. *Dimensions of the plate being studied. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

The material in this plate is steel, which has a Young's modulus $E = 200,000 MPa$ and a Poisson's ratio $\nu = 0.3$. The behavior of the material is linear elastic isotropic. The problem of optimization is to minimize the structural volume, subject to the limitation of the maximum value of the von Mises stresses. The problem can be formulated as follows:

$$\begin{array}{ll} \min & : Volume(A, B, R) \\ s.t. & : \sigma_{\max}(A, B, R) - \sigma_w \leq 0 \\ & : 50 \leq A \leq 70 \\ & : 50 \leq B \leq 70 \\ & : 15 \leq R \leq 30 \end{array} \quad [1.4]$$

where σ_w is the allowable stress ($\sigma_w = 10 MPa$).

1.4.3.2. *Numerical results*

To carry out shape optimization on ANSYS, we consider the dimensions *A*, *B* and *Ro* as optimization variables, in order to obtain a parameterized model. Next, a direct simulation is performed as the heart of the optimization loop.

1.4.3.2.1. Direct simulation

Figure 1.11a shows a geometric description and Figure 1.11b shows a meshing model with the boundary conditions. This plate is embedded in the arch of the medium, with radius *Rc*, and subjected to a static pressure on the arches *Ri* of the two lower holes (*P* = 4). The mesh is created in 2D, using the linear element (PLANE42 - 4-node).

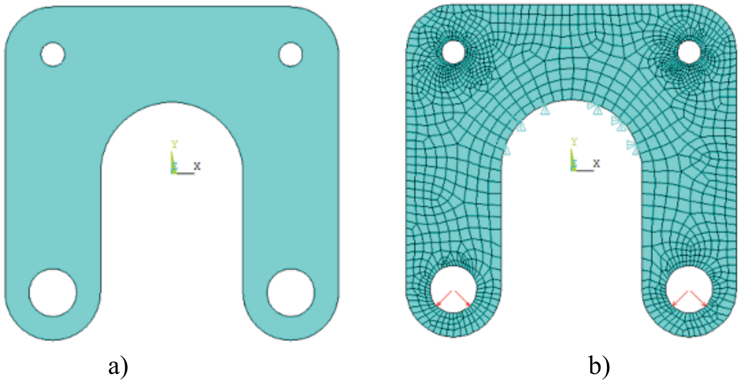


Figure 1.11. *a) Geometric model and b) meshing model with the boundary conditions. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Figure 1.12 shows the general approach used in direct FEM simulation in statics [KHA 11g].

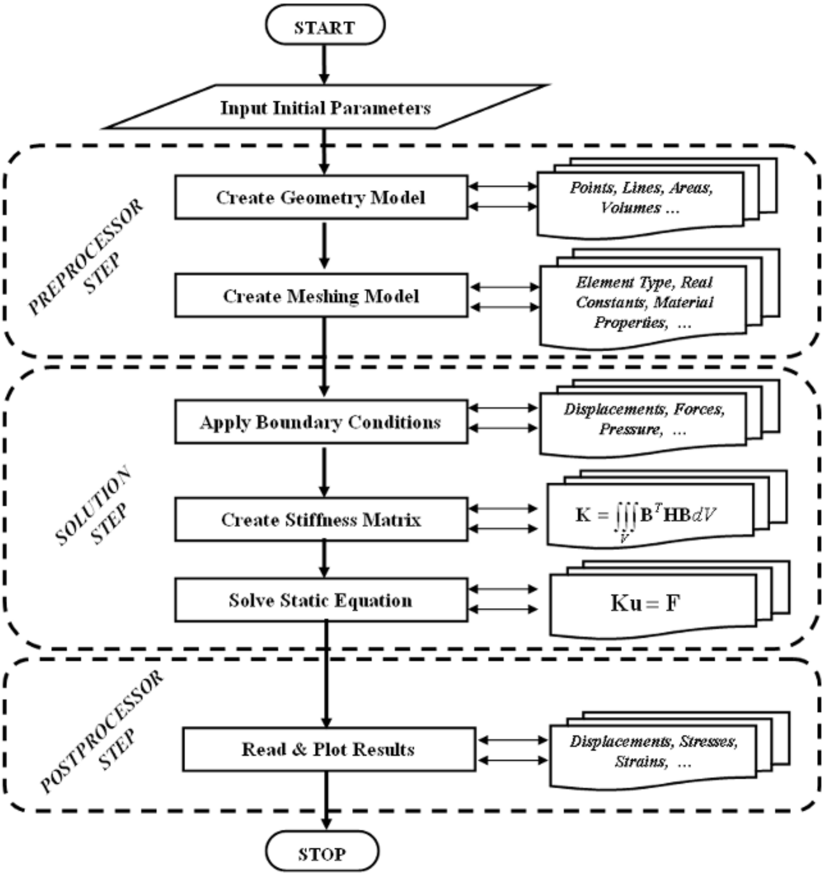


Figure 1.12. *General approach of FEM simulation in statics.*

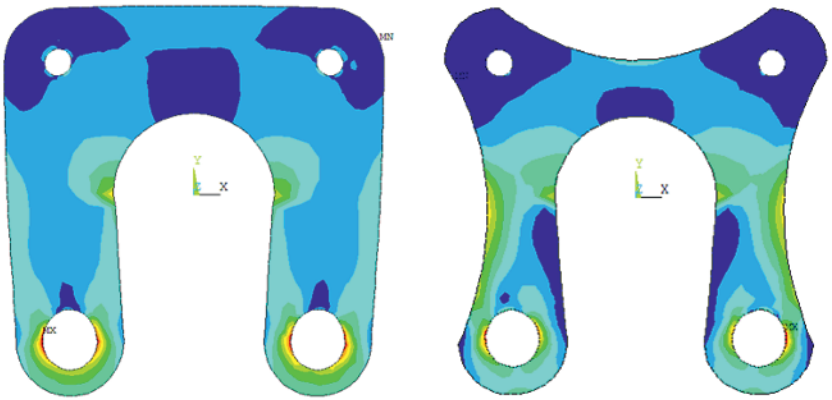
The general equation in statics which is used in this approach can be written thus:

$$\mathbf{Ku} = \mathbf{F} \quad [1.5]$$

where **K**, **u** and **F** are, respectively, the rigidity matrix, the displacement vector and the external force vector. In the case of static analysis, solving equation [1.5] gives the displacement vector and then the different responses (stresses, strains, etc.).

1.4.3.2.2. Shape optimization

Figures 1.13a and b show the distribution of the von Mises stresses in the initial and optimal configurations, respectively.



a) b)

Figure 1.13. *Stress distribution in a) the initial and b) the optimal configurations. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

At the initial point, the structural volume is equal to: $V_0 = 127 \times 10^3 mm^3$ with a maximum value of the von Mises stresses equal to: $\sigma_{\max} = 8.18 MPa$. At the optimal point, the structural volume is equal to: $V_{opt} = 95 \times 10^3 mm^3$, with a maximum value of the von Mises stresses equal to: $\sigma_{\max} = 9.98 MPa$. The optimal values of the input parameters are: $A = 55.13\ mm$, $B = 50.59\ mm$ and $R = 18.77\ mm$.

Figure 1.14 shows the algorithm used for shape optimization of the studied plate. The optimization loop contains the following steps:

– entering of the input parameters;

– creation of the geometric model;

– creation of the meshing model;

– application of the boundary conditions;

– solving of the problem for static analysis;

– convergence testing: if the solution converges, STOP; Else, modify the optimization variable vector.

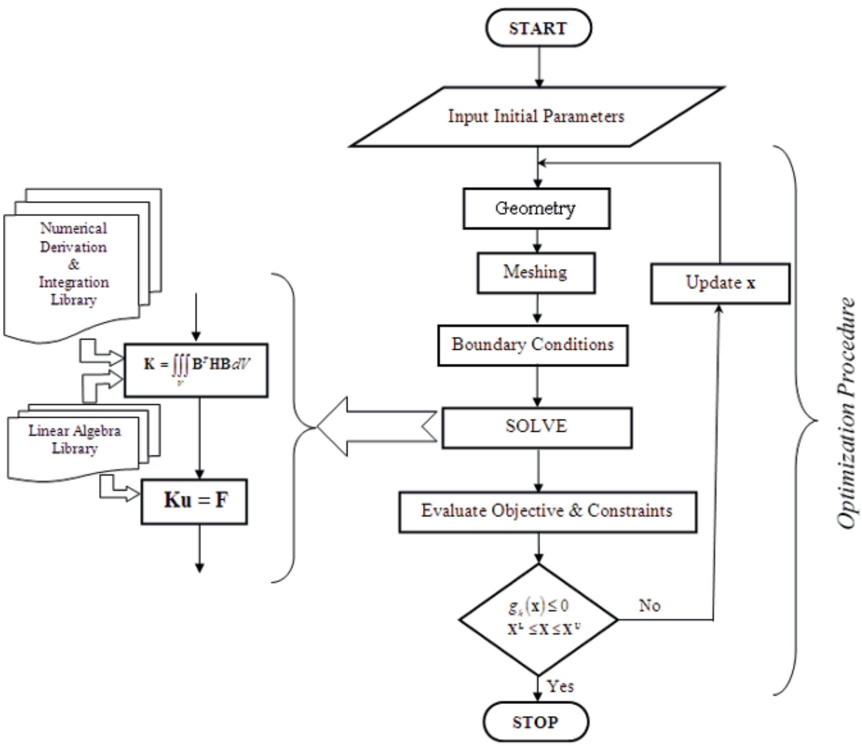


Figure 1.14. *Algorithm used for shape optimization of the studied plate.*

Shape optimization can only modify the boundaries so as to obtain smooth geometry.

1.5. Topology optimization

1.5.1. *Definition*

With *topology optimization*, we can more fundamentally change the nature of the structure. Here, the geometry of the part is optimized with no prerequisites as to the connectivity of the domains or the structural elements present in the solution. Naturally, in order to optimize the topology, in a way, we determine the structure's shape or transverse dimensions of the structure, so certain authors [ROZ 93] also call topology optimization "generalized shape optimization". Figure 1.15a shows a beam with two supports, considered as the initial domain for the topology optimization. Figures 1.15b, c and d are the different resulting topologies.

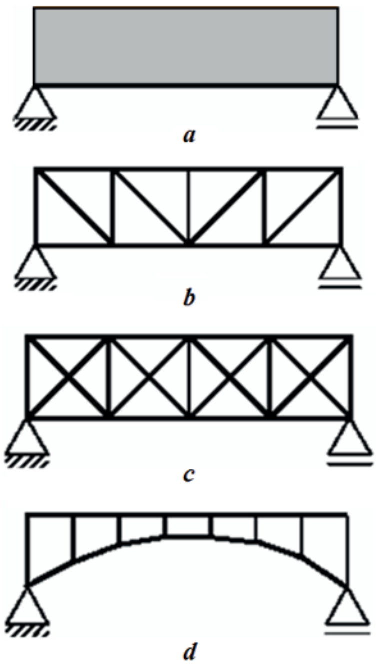


Figure 1.15. *a) Initial domain of the beam, b), c) and d) different resulting topologies.*

The description of the entities (or features) defining the geometry of these structures varies depending on the number of bars. This being the case, we can differentiate the topology by geometric description. The topology of the rectangular cross-section is different to that of the circular (Figure 1.1). The optimization variables are the densities of the materials in the elements making up the structure.

1.5.2. *First works in topology optimization*

The topology of structures can be defined from a number of different points of view: generically, the term "*topology*" denotes the *geometry of position* or *of situation*, also called "*analysis situs*" by Poincaré. In mathematics, the term denotes "*the branch of geometry which studies the qualitative properties and relative positions of geometric entities, regardless of their shape and size*". Two domains with identical topology can be projected onto a single reference domain by a bijective, continuous and differentiable transformation. Whatever the geometric transformations of the reference domain, provided they are continuously differentiable, the topology of all the domains obtained by projection is identical to that of the original domain. In mechanics, the topology of a structure covers the ordering of its component parts and its structural joints, or indeed the connectivity of the domain occupied by the material and, consequently, the number and position of the perforations of the domain. Sometimes, it is common practice to use the term “topology” to apply to other data on relative arrangement, such as the sequence of stacking of folds in a laminated print. The corresponding topology problem, then, would consist of determining the general characteristics of the structure, and topology optimization would aim to make that initial choice as automatically as possible.

1.5.3. *Numerical application*

1.5.3.1. *Description and modeling of the studied problem*

Figure 1.16 shows a beam with two supports, subjected to a static force ($F = 1,000N$). The dimensions of the beam are: $L = 1\ m$ and $H = 0.4\ m$.

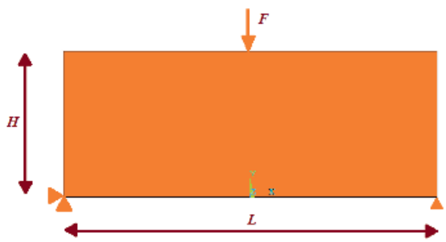


Figure 1.16. *Dimensions of the studied beam.*

The material in this beam is steel, which has a Young’s modulus $E = 200,000MPa$ and a Poisson’s ratio equal to: $\nu = 0.3$. The behavior of the material is linear elastic isotropic. The objective is to perform topology optimization to obtain the best distribution of the materials. The optimization problem is to minimize the compliance of the structure, subject to the volume fraction. This problem can be written in the following form:

$$\begin{aligned} &\text{min} && : C(x_i) \\ &\text{subject to} && : \frac{V(x_i)}{V_0} \leq 50\% \end{aligned} \qquad [1.6]$$

where $x_i \in [0,1]$ is the optimization variable vector representing the densities of the materials in each element of the mesh.

1.5.3.2. *Numerical results*

To carry out topology optimization, we construct the meshing model using the nonlinear element (PLANE82 - 8-node). Figure 1.17 shows the mesh and the boundary conditions.

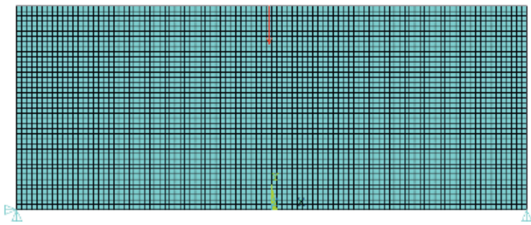


Figure 1.17. *Boundary conditions.*

Using the optimality criteria method, we obtain the topology presented in Figure 1.18.

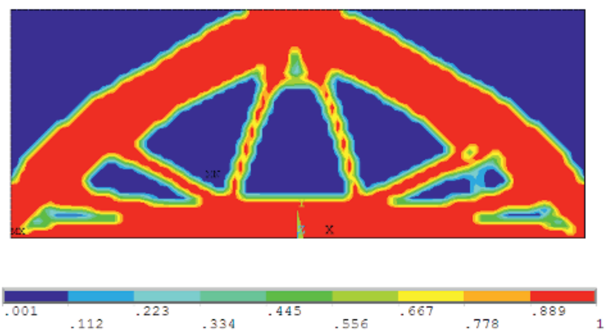


Figure 1.18. *Resulting topology. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Figure 1.19 illustrates the algorithm used for topology optimization of the studied beam. The optimization loop contains the following steps:

– creation of the initial design (geometric- and meshing models);

– solving of problem for static analysis using FEM;

– sensitivity analysis;

– convergence test: If converged, STOP; else, modify the optimization variable vector.

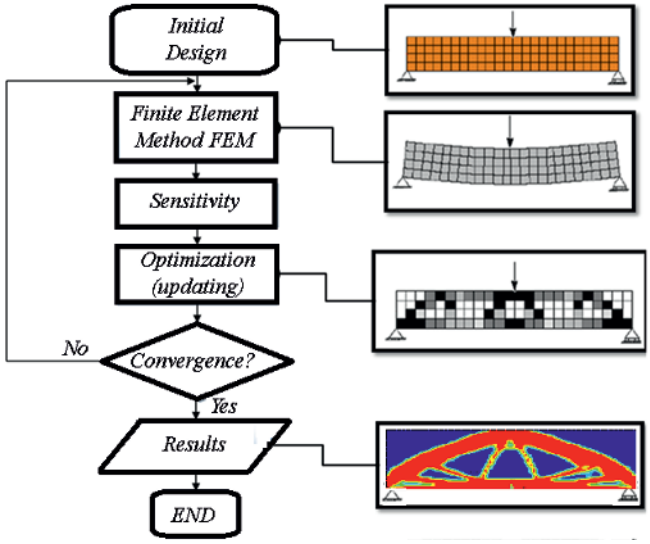


Figure 1.19. *Algorithm used for topology optimization of the studied beam. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Figure 1.20 shows the layout configuration for shape optimization using smooth boundaries.

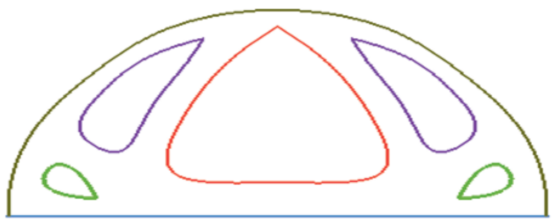


Figure 1.20. *Layout configuration for shape optimization. For a color version of this figure, see www.iste.co.uk/kharmanda2/biomechanics.zip*

Topology optimization produces the silhouette of the studied beam with the best distribution of the materials.

1.6. Conclusion

In this chapter, the principles of the three main groups of structural optimization have been presented, along with numerical applications for each group. In structural design, topology optimization is used as a conceptual phase to obtain an idea of the silhouette of the structure. Next, the detailed phase involves shape optimization so as to achieve smooth geometry and sizing optimization to yield the cross-sections and thicknesses of the structure under study.