1

Basic Concepts and Balances

1.1. Thermal energy and the first law of thermodynamics

Let us recall that the first law of thermodynamics is a law of *energy conservation*. It introduces the "internal energy" *U*, which represents the sum of the energies (kinetic and potential) of the system.

Assuming that there is no mass exchange and that we are considering a closed system that is subject to a thermodynamic transformation between two states, initial (1) and final (2), the variation in internal energy, *U(2) – U(1)*, is the sum of the following two terms:

– the macroscopic works performed, $W_{1\rightarrow 2}$; generally this is the work of the pressure forces;

– the energy exchange between the system and its outside: $Q_{1\rightarrow 2}$.

This is reflected by: $\Delta U = U(2) - U(1) = W_{1\rightarrow 2} + Q_{1\rightarrow 2}$

We can therefore deduce a formal definition of the thermal energy (or heat) that is exchanged by the system between the initial state defined by (1) and the final state defined by (2): $Q_{1\rightarrow 2} = U(2) - U(1) - W_{1\rightarrow 2}$

For the thermodynamic systems studied, the work involved is generally due to pressure forces. The work is then given by the integral of these forces:

$$W_{1\rightarrow 2} = -\int_{V1}^{V2} p\mathrm{d}V$$

For the systems encountered in heat transfer analyses, volumes are generally constant (*isochoric systems*). The pressure forces are therefore not at work.

Consequently: $W_{1\to 2} = 0$

For such systems, thermal energy is therefore given by the variation in the system's internal energy: $Q_{1\to 2} = U(2) - U(1)$

This relation between the variation in a system's internal energy and the heat received (or yielded) by this system is, *inter alia*, used to determine the *specific heats* (also known as *heat capacities* or *sensible heats*) of different materials, C_p. This is known as *calorimetry*.

Indeed, for the closed systems considered in calorimetry, the internal energy is linked to the mass of the system by: $U = M\,C_p\,(T - T^*)$,

where T^* is a given reference temperature.

Thus, if we consider a mass, M, of a given material with heat capacity C_p, to which an amount of heat, Q, is supplied in a closed system, then the variation in its internal energy, U, is given by: $Q = U(2) - U(1)$

i.e. $Q = M\,C_p\,(T_2 - T_1)$

To determine C_p experimentally, one can then measure M, Q, T_1 and T_2 during the course of an experiment, in which an amount of heat, Q, is supplied to the mass, M, in a closed system (calorimeter). Knowledge of M, Q, T_1 and T_2 will therefore enable C_p to be calculated:

$$C_p = \frac{Q}{M(T_2 - T_1)}$$

1.2. Thermal energy and the second law of thermodynamics

The second law of thermodynamics is a *law of evolution*. It introduces the "entropy" state function, which may be interpreted as a *measure of the disorder* of the matter within the system.

At the macroscopic scale, and for a reversible transformation, the elementary variation, δS, of the entropy function, S, is defined as the ratio of the quantity of heat exchanged over the temperature of the system:

$$\delta S = \frac{\delta Q_{rev}}{T}$$

The expression of the amount of heat exchanged can then be induced as follows:

$$\delta Q_{rev} = T\delta S$$

Hence: $Q_{rev} = \int_1^2 \delta Q_{rev} = \int_1^2 TdS$

For an isothermal transformation (at constant temperature), going from an initial equilibrium state (1) to a final equilibrium state (2), we obtain:

$$Q_{rev} = T\int_1^2 dS$$

i.e.: $Q_{rev} = T \cdot \Delta S$

$$Q_{rev} = T \cdot \left[S(2) - S(1)\right]$$

The thermal energy (or heat) exchanged by a given system is therefore associated with a positive variation in the system entropy.

Thus, whenever a system *is heated* (Q_{rev} positive), we can expect *an increase in its entropy*. This corresponds to an increase in the disorder of the matter constituting the system; the particles are in a more excited state after receiving thermal energy.

Likewise, *a decrease in the system entropy* will require heat extraction, which *corresponds to a cooling* (Q_{rev} negative).

1.3. For an energy and mass accounting: balances

With a view to examining energy or mass transfers within a system in detail, it is necessary to define a way to follow up the quantities transferred. As in accounting, we are led to define balances-sheets of energy and matter flows and exchanges between systems.

To be more specific, consider the system represented in Figure 1.1, which exchanges enregy and mass with the outside. The inputs (e_i: $1 \leq i \leq n$) *can represent either energy or matter flows. Likewise*, the system admits m outputs (s_j: $1 \leq j \leq m$).

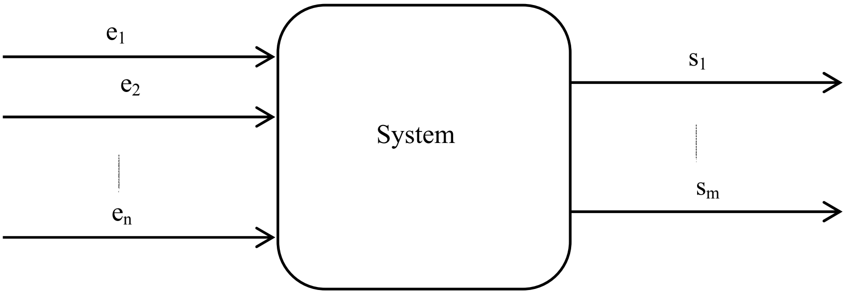

Figure 1.1. *Balances on a system*

1.3.1. *Accounting principles for system inputs and outputs*

The system inputs and outputs may indicate arrivals (inputs) or departures (outputs) of mass or energy.

1.3.1.1. *Mass inputs and outputs accounting*

Generally, these are mass flows entering or exiting the system. They can therefore be counted in terms of amounts of mass, arriving in or leaving the system by unit of time. In practise, these are:

- – *mass flows*, counted in kg/hr, or any other equivalent unit;
- – *volume flows*, counted in m^3/hr, or any other equivalent unit;
- – *molar flows*, counted in moles/hr, or any other equivalent unit.

1.3.1.2. *Illustration: calculating the mass entering a reactor*

Consider the reactor shown in Figure 1.2, which enables product *C* to be produced from inputs *A* and *B*.

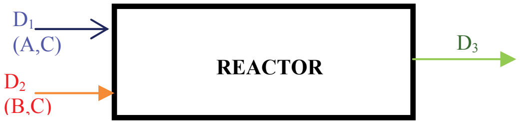

Figure 1.2. *Inputs of mass into a reactor. For a color version of this figure, see www.iste.co.uk/benallou/energy1.zip*

Flows D_1 and D_2 are given in kmol/hr:

$D_1 = 112$ kmol/h

$D_2 = 100$ kmol/h

D_1 consists of the binary mixture, *(A, C)*, with a mole fraction, $x_A = 0.85$.

D_2 consists of the binary mixture, *(B, C),* with a mole fraction, $xB = 0.95$.

Question

Calculate D_A, D_B and D_C, which represent the amounts of *A*, *B* and *C*, resepctively, entering the reactor per unit time.

Solution

We have: $D_A = x_A D_1$

$D_B = x_B D_2$ $D_c = (1-x_A) D_1 + (1-x_B) D_2$

NUMERICAL APPLICATION.–

$D_1 = 112$ kmol/h $D_2 = 100$ kmol/h
$x_A = 0.85$ $x_B = 0.95$
$D_A = 0.85 \times 112 = 95.2$ kmol/hr

$D_B = 0.95 \times 100 = 95$ kmol/hr $D_c = 0.15 \times 112 + 0.05 \times 100 = 21.8$ kmol/hr

1.3.1.3. *Energy inputs and outputs accounting*

Each flow of mass, entering or exiting the system, transports with it an amount of energy represented by its *enthalpy*. We know that this enthalpy can be determined with respect to a reference temperature, T^*. Thus, a flow, e_i, arriving in the system at a temperature, T_i, transports an amount of energy per unit of time, E_i, given by:

$$E_i = e_i C_{pi} (T_i - T^*),$$

whereby C_{pi} is the specific heat of e_i at T_i.

In the same way, the energy transported by the flow, s_j, exiting the system at temperature, T_j, is represented by the enthalpy, E_j, given by:

$$E_j = s_j C_{pj} (T_j - T^*),$$

whereby C_{pj} is the specific heat of s_j at T_j.

Thus, the energy flows entering or exiting the system can be counted in kcal/hr, or in any other equivalent unit.

NOTE – PRACTICAL CALCULATIONS.– We are often interested in the same flow, *D*, that enters the system at a temperature, T_1, and exits it at a temperature, T_2; in this case, the expression of the difference between the energy input transported by the flow, *D* (E_D), and the energy output transported by *D*, (S_D), makes it possible to get rid of temperature T^*:

$$E_D - S_D = DC_p(T_1 - T^*) - DC_p(T_2 - T^*) = DC_p(T_1 - T_2)$$

Thus: $E_D - S_D = DC_p(T_1 - T_2)$

1.3.1.4. *Illustration: energy gain in a dryer*

Air enters a dryer at a flow rate *V* and a temperature $T_E = 25°C$, and leaves it at $T_S = 65°C$.

Questions

1) Calculate the mass flow rate D of air through the dryer (in kg/h).

2) Calculate, in kcal/mn, the energy gain of the flow, *D*, during this operation.

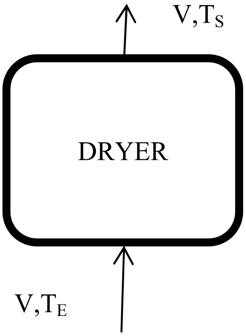

Figure 1.3. *Dryer*

Data:

$T_E = 25°C$

$T_S = 65°C$

$V = 5\ m^3/mn$

$C_p = 0.6\ cal/g°C$

$\rho = 650\ g/m^3$

Solutions

1) Mass flow rate of air

$D = V\rho$

NUMERICAL APPLICATION.–

$\rho = 650\ g/m^3$
$V = 5\ m^3/mn$

$D = (5 \times 60)\ 0.65$

$D = 195\ kg/h$

2) Energy gain of D

Using E_D and S_D to denote the enthalpies of D entering and exiting, respectively, the dryer, the energy gain, G_D, of the flow, D, is given by:

$$G_D = E_D - S_D = DC_p(T_S - T_E)$$

whereby: $D = V\rho$.

Therefore: $G_D = V\rho C_p(T_S - T_E)$

NUMERICAL APPLICATION.–

$V = 5\ m^3/mn$
$\rho = 650\ g/m^3$
$C_p = 0.6\ cal/g°C$
$T_E = 25°C$
$T_S = 65°C$
$G_D = 5 \times 650 \times 0.6 \times (65–25)$ **$G_D = 78$ kcal/min**

1.3.2. *Accumulation in the system*

When the flows entering a system are *greater* than those exiting it, we say that *mass accumulates* in the system. Indeed, the difference between incoming and outgoing flows is simply stored in the system.

This reasoning *applies to both mass and energy*. It also applies when the incoming flows are *less* than the outgoing flows. In this case, we will speak of negative accumulation, which in fact represents a decrease in the mass or energy between system input and output.

Thus, the term "accumulation", be it positive or negative, of mass or energy in a system is expressed by *the variation* of the amount of mass (or energy) contained in the system. Mathematically, this variation is represented by *the differential, with respect to time,* of the amount of matter (or energy) contained in a system.

1.3.2.1. *Accumulation of mass*

A system, of volume V and density ρ has a mass given by: $M = \rho V$

The accumulation of mass in the system is therefore the differential of M with respect to time: $A = \dfrac{\partial M}{\partial t} = \dfrac{\partial (\rho V)}{\partial t}$

In the case where ρ is constant, the term representing accumulation of mass in the system becomes: $A = \rho \dfrac{\partial V}{\partial t}$

NOTE.– If, instead of looking at the accounting of the mass in the system, we focus on the number of moles, the reasoning remains the same, and the "accumulation" term is then expressed as the variation, with respect to time, of the number of moles, N, of the system: $A = \dfrac{\partial N}{\partial t}$

In the same way, if the accounting is established on a volumetric basis, the "accumulation" term is expressed as the variation, with respect to time, of the system volume: $A = \dfrac{\partial V}{\partial t}$

1.3.2.2. *Illustration: accumulation of mass in a tank*

The tank shown in Figure 1.4 has a cross-section area S. This tank is used to mix inputs A and B.

The flows, D_A and D_B, of *A* and *B* respectively, are such that the height, *h*, of the liquid in the tank follows a linear law during the tank filling phase:

$h(t) = k_1 t + k_2$ where k_1 and k_2 are given constants.

Question

Calculate the amount of mass that accumulates in the tank per unit of time during the filling phase.

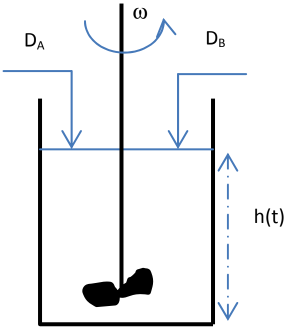

Figure 1.4. *Mixing tank*

Data:

$S = 1.57\ \text{m}^2$

$\rho = 1{,}115\ \text{kg/m}^3$

$k_1 = 0.01\ \text{m/mn}^2$

Solution

The mass of matter in the tank is given by: $M = \rho V = \rho hS$

Therefore: $M(t) = \rho S (k_1 t + k_2)$

Hence:

$$A = \frac{dM}{dt} = k_1 \rho S$$

NUMERICAL APPLICATION.– $A = 0.01 \times 1{,}115 \times 1.57$ **A = 17.51 kg/min**

1.3.2.3. *Energy accumulation*

Consider a volume V containing matter at a given temperature T. The density and the specific heat of the matter are considered as being respectively ρ and C_p; the internal energy, E, of this system is given by: $E = \rho V C_p (T - T^*)$

where T^* is a reference temperature.

The energy accumulation in the system is therefore given by the variation, with respect to time, of the system's internal energy; that is (T^* being a constant):

$$A = \frac{\partial E}{\partial t} = \frac{\partial (\rho V C_p T)}{\partial t}$$

Very often, pressure and volume are constant, thus: $A = \frac{\partial E}{\partial t} = \rho V C_p \frac{\partial T}{\partial t}$

The product, $\rho V C_p$, is sometimes known as the *thermal capacitance*.

1.3.2.4. *Illustration: energy accumulation in a tank*

The stirred tank of volume V, shown in Figure 1.5, contains a liquid having a density ρ and a specific heat C_p. The liquid is heated by an electrical resistance such that its temperature varies linearly with time: $T = at + b$

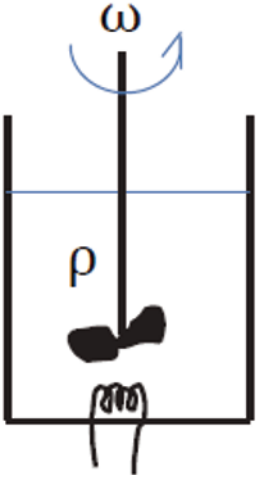

Figure 1.5. *Heating tank*

Question

Calculate the amount of energy accumulating in the tank.

Data:

$\rho = 1{,}200$ kg/m^3

$C_p = 1.1$ kcal/kg

$V = 0.96$ m^3

$a = 2$°C/mn

Solution

The enthalpy of the liquid is given by:

$$E = \rho V C_p (T - T^*)$$

Therefore: $E(t) = \rho V C_p (at + b - T^*)$

Energy accumulating in the tank is then given by:

$$A = \frac{dE}{dt} = \rho V a C_p$$

NUMERICAL APPLICATION.– $A = 1{,}200 \times 0.96 \times 2 \times 1.1$ **A = 2,534.kcal/min**

1.3.3. *Generation in a system*

In several situations, mass and/or energy are created from within a system. As an example, nuclear reactors generate large quantities of energy and so is the case with exothermal chemical reactions.

Generation of mass can also stem from chemical reactions where inputs are consumed (negative generation) yielding a new product (positive generation).

1.3.3.1. *Generation of mass*

In the case where the components entering the system react with each other to give another product, generation can be counted in terms of the number of moles or in terms of the mass generated.

This generation is generally calculated from the reaction rate. Two formulations of this rate are generally used:

– r: the mass rate of reaction, expressed as the mass of the product generated per unit volume and per unit time.

– R: the molar rate of reaction, expressed as the number of moles of the product generated per unit volume and per unit time.

Thus, the generation term can take one of the following forms:

$$G = r\,V \text{ or: } G = R\,V$$

1.3.3.2. *Illustration: generation of mass in a reactor*

A stirred tank of cross section area S is used to manufacture a product C from A and B through an incomplete chemical reaction between inputs A and B according to the equation:

$$A+B \rightleftharpoons C$$

This reaction is of order 2.

The reaction rate gives the mass of C produced per unit time and per unit volume, as follows: $r_c = k\,\rho_A\ \rho_B$

where ρ_A and ρ_B are the mass concentrations of A and B.

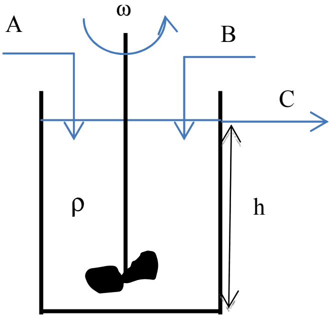

Figure 1.6. *Stirred reactor*

A follow up of the mass concentrations of A and B in the tank during reaction show a linear dependance over time as follows:

$$\rho_A = \rho_{A0} - k_1\,t \qquad \rho_B = \rho_{B0} - k_2\,t$$

where ρ_{A0} and ρ_{B0} are the values of the initial mass concentrations of A and B in the tank.

Question

Calculate the amount of *C* generated during the first thirty minutes of reaction.

Data:

Initially there is no C in the tank: at $t = 0$: $\rho_{c0} = 0$.
$\rho_{A0} = 450$ g/liter $\rho_{B0} = 600$ g/liter
$h = 50$ cm $S = 20\ \text{dm}^2$
$k_1 = 15$ g/(liter-minute) $k_2 = 20$ g/(liter-minute)
$k = 10^{-4}$ liter/(g-minute)

Solution

Generation of C is given by: $G_c(t) = r_c(t)\ V$

Where: $r_c(t) = k\ \rho_A(t)\quad \rho_B(t)$

Hence: $G_c(t) = k\ V\ \rho_A\ (t)\ \rho_B\ (t)$

Or: $G_c(t) = k\ h\ S\ (\rho_{A0} - k_1\ t)\ (\rho_{B0} - k_2\ t)$

Hence the amount of *C* generated during the first thirty minutes:

$$G_{30} = \int_0^{30} G_c(t)dt$$

$$G_{30} = khS\int_0^{30} (\rho_{A0} - k_1 t)(\rho_{B0} - k_2 t)dt$$

$$G_{30} = khS\left[\frac{k_1 k_2}{3} t^3 \rho_{A0}\rho_{B0} - (k_2\rho_{A0} + k_1\rho_{B0})\frac{t^2}{2}\rho_{A0}\rho_{B0} t\right]_0^{30}$$

NUMERICAL APPLICATION.–

$$G_{30} = 10^{-4}(5)(20)\left[\frac{(15)(20)}{3}(30)^3 - (20*450 + 15*600)\frac{(30)^2}{2} + (450)(600)(30)\right]$$

G_{30} = 27 kg

1.3.3.3. *Energy generation*

We say that there is energy generation in the system when there is creation or dissipation of energy during the operation of this system. We know that this occurs when:

– an electric heating resistance is placed in the system;

– an exothermic reaction is taking place in the system. In this case, the amount of energy created is given by the heat of the reaction ($\Delta H > 0$);

– the system is handling an endothermic reaction. In this case, the system needs an energy input for its operation. The amount of energy consumed is given by the heat of the reaction ($\Delta H < 0$);

– a nuclear reaction takes place in the system, etc.

The amount of energy generated is often related to its *generation density*, which is also known as the *generation power density*, P.

Thus, the amount of energy generated per unit time is given by:

$$G = P\,V,$$

where V is the system volume.

1.3.3.4. *Illustration: energy generation in a reactor*

An exothermic chemical reaction occurs in the reactor shown in Figure 1.7:

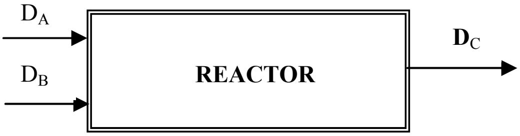

Figure 1.7. *Inputs of mass into a reactor*

The reaction enables the product, C, to be produced from inputs A and B, according to the equation:

$$A + B \longrightarrow C \quad \Delta H = 1{,}250 \text{ cal/mole}$$

Question

Knowing that the total reaction (total consumption of inputs) requires a time, τ, calculate the energy generation power, in kW, in the reactor: π.

<u>*Data:*</u>

$D_A = D_B = 110$ kmol/hour.

Solution

The generation power π is given by the amount of energy generated during the course of the reaction, divided by the time that elapses in order for the reaction to take place.

Given that the reaction occurs during a time *t*, then:

$$\pi = \frac{Q}{t}$$

with: $Q = N\Delta H$,

whereby N is the number of moles of *C* produced by the reaction. As the reaction is total, N is equal to the number of moles of *A* (or *B*) entering the reactor. Given that the flow rate of A into the reactor is D_A, N is given by:

$$N = t\, D_A$$

Hence: $Q = \tau\, D_A\, \Delta H$

Therefore the generation power is: $Q = D_A\, \Delta H$

NUMERICAL APPLICATION.–

$D_A = 110$ kmol/hour
$\Delta H = 1{,}250$ cal/mole

π = 110*1,250 kcal/hour **π = 159 kW**

1.3.4. *Balance equation*

DEFINITION.– We will call "mass balance" ("energy balance" respectively) the equation reflecting the conservation of mass (energy respectively) in a given system.

This equation is written:

Accumulation = Input – Output + Generation

Using the same notations as in figure 1.1, the balance equation can then be rewritten as follows:

$$A = \sum_{i=1}^{n} e_i - \sum_{j=1}^{m} s_j + G$$

where A and G are, respectively, accumulation and generation.

1.3.4.1. *Illustration: mass balance on a reservoir*

Let us consider a water tower designed to deliver water to homes in a village. The reservoir of this tower is supplied by a well with a flowrate D_1.

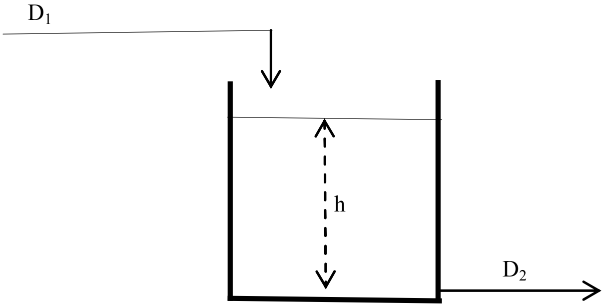

Figure 1.8. *Water supply reservoir*

D_1 is a function of time that represents the amount of water extracted from the well and delivered to the reservoir per unit of time.

D_2 is a function of time that represents the water withdrawal rate to supply the village.

Questions

1) Show that the mass balance on this reservoir may be expressed in the form:

$$\rho S \frac{dh}{dt} = D_1 - D_2,$$

whereby S is the cross-section area of the reservoir and ρ the water density.

2) Analyze the security risks for the following different situations:

a) $D_1 - D_2 > 0$.

b) $D_1 - D_2 < 0$.

c) $D_1 - D_2 = 0$.

Solutions

1) Mass balance

The balance equation is given by:

$$A = \text{Input} - \text{Output} + G$$

where A is the accumulation of water in the reservoir and G is generation.

Given that there is no reaction, $G = 0$

Accumulation is determined by differentiating the mass of water contained in the reservoir with respect to time.

The water mass in this reservoir is $M(t) = \rho\, S\, h$, where S is the cross-section area of the reservoir and h is the height of water.

Consequently, accumulation is given by: $A = \dfrac{d(\rho S h)}{dt}$

The balance equation is then: $\dfrac{d(\rho S h)}{dt} = D_1 - D_2$

For constant ρ and S within the field of operation, the balance equation reduces to:

$$\rho S \frac{dh}{dt} = D_1 - D_2$$

Thus, water accumulation in the reservoir may be positive or negative, depending on the sign of the difference, $D_1 - D_2$.

2) Risk analysis

a) $D_1 - D_2 > 0$

In this case, the accumulation is always positive. Consequently, the amount of water in the reservoir will continue to increase over time, which could result in an overflow of water. It is therefore necessary to control the height, *h*, so that it does not exceed the maximum height permitted.

b) $D_1 - D_2 < 0$

In this case, the accumulation is always negative. The reservoir will therefore empty over time, which presents the risk of an interruption in the water supply to the population. Thus, as for the previous case, it will be necessary to control the height, *h*, so that it never falls below a minimum value.

c) $D_1 - D_2 = 0$

In this case, accumulation is nil. The reservoir will therefore function without emptying or filling: the water height in the reservoir remains constant. This situation is ideal, but it would require controling flows D_1 and D_2 to ensure $D_1 = D_2$ over the course of time.

NOTE.– In this example, D_1 and D_2 are considered in mass terms (i.e. in kg/hr, for example). If these flowrates were known in volume terms (in liters/hr, for example), then the mass flows would be obtained by multiplying the volume flows by the density, ρ.

i.e.: $D_1 = \rho d_1$ and $D_2 = \rho d_2$

The balance equation will then become: $S\frac{dh}{dt} = d_1 - d_2$

1.3.4.2. *Illustration: energy balance of an electric water heater*

Consider the domestic hot water production system represented in Figure 1.9.

This system makes it possible to heat water from inlet temperature T_1 to service temperature T_2 by means of an electrical resistance with power, *P*.

As the water heater is well insulated from the outside, we will assume, as a first approximation, that there are no heat losses.

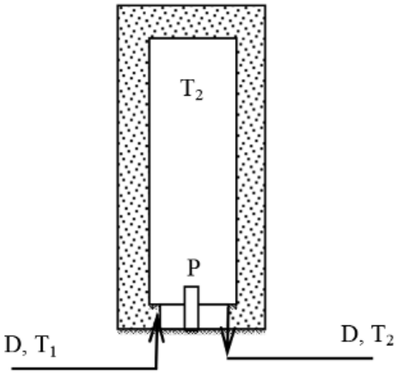

Figure 1.9. *Balance on an electric water heater*

Questions

1) Develop the energy balance equation.

2) Infer the differential equation which determines the evoltion of T_2 over time.

Solutions

1) The energy balance is written:

A = Input – Output + G

The difference between the energy input and output is:

$$\text{Input} - \text{Output} = D\, C_p\, (T_1 - T_2)$$

Moreover, the "accumulation" term is given by: $A = \rho ShC_p \frac{dT_2}{dt}$

The generation, in turn, is equal to the heating power: $G = P$

Hence the heat balance equation is: $\rho ShC_p \frac{dT_2}{dt} = DC_p(T_1 - T_2) + P$

2) Differential equation for T_2

Dividing the energy balance equation by ρShC_p, we infer the following differential equation for T_2: $\frac{dT_2}{dt} = \frac{1}{\rho ShC_p}\left[DC_p(T_1 - T_2) + P\right]$

1.4. Fluxes and flux densities

DEFINITION.– We will call the "flux ϕ" of a parameter G (mass, energy) propagating along a given direction Δ, the amount of G crossing an infinite surface perpendicular to Δ per unit time.

1.4.1. *Energy fluxes*

An energy flux is an *amount of energy per unit time*. It can therefore correspond to a variation in internal energy (or enthalpy) per unit time.

An energy flux actually represents a power of the transfer taking place. Consequently, energy fluxes will be expressed in calories per second (cal/s), in kilocalories per hour (kcal/hr), in watts (W) or in kilowatts (kW), etc.

1.4.2. *Mass fluxes*

A mass flux is a *flow of matter* that enters or exits a system *per unit time.* It can correspond to a *mass per unit time* or *a number of moles per unit time*.

Mass fluxes will therefore be expressed in kilograms per hour (kg/hr) or in moles per minute (moles/min), etc.

1.4.3. *Flux densities*

DEFINITION.– We will call a "flux density φ" a flux per unit of surface area.

Consequently, a mass flux density is a mass or a number of moles *per unit time and per unit surface*.

The mass flux density will therefore be expressed in kg/(hm^2) or in moles/(hm^2), etc. Likewise, an energy flux density corresponds to *energy per unit time and per unit area*.

The heat flux density will therefore be expressed in kcal/(hm^2) or in kW/m^2, etc.

1.4.3.1. *Illustration: simultaneous transfer of mass and energy*

Let us consider a common operation in the manufacture of beverages: the preparation of sugar water. This preparation is made in a stirred tank where a flow, *d*, of glucose syrup is dissolved in water.

To accelerate the dissolution, the tank is heated by steam (see Figure 1.10), which enters the tank jacket, condenses by yielding its latent heat to the tank, then leaves the jacket as condensed water.

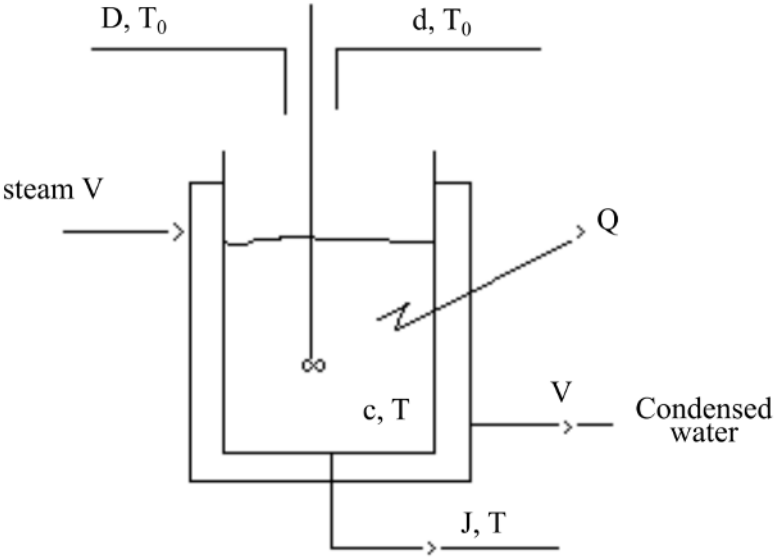

Figure 1.10. *Production of sweet juice*

<u>Detailed legend to Figure 1.10:</u>

D is the mass flow rate of water entering the tank, temperature T_0.

J is the mass flow rate of sugar water produced, temperature T.

C is the mass fraction of sugar in the tank.

C_o is the mass fraction of sugar in the glucose syrup, flow d.

Q designates the heat losses.

V is the steam mass flow rate. This same flow is converted to condensed water.

Λ is the latent heat of condensation of saturated steam.

H_{100} is the enthalpy of condensed water, expressed at the point of condensation.

h is the liquid height in the stirred tank.

S is the tank cross-section area.

Questions

1) Establish the overall mass balance that reflects the overall conservation of mass in the mixing tank.

2) Establish the sugar-specific mass balance.

3) Establish the heat balance of this tank.

Solutions

1) Overall mass balance

As there is no generation of mass in the tank (G = 0), the balance is written:

Accumulation = Inputs – Outputs

In order to express the "accumulation" term, it is necessary to know the expression of the total mass, *M,* in the tank. *M* is given by: $M = \rho V = \rho S h$

The "accumulation" term is therefore given by (ρ and S being constant):

$$A = \rho S \frac{dh}{dt}$$

The overall mass input is: $E = D + d$

The mass output is represented by the flow, *J*.

Hence, the overall mass balance equation is: $\rho S \frac{dh}{dt} = D + d - J$

2) Specific mass balance

The specific mass takes care of sugar conservation.

The amount of sugar in the tank is given by:

(total mass) x (mass fraction of sugar) = $(\rho S h)\, C$

Therefore, we can express the “accumulation” term as follows:

$$A = \frac{d(\rho ShC)}{dt} = \rho S \frac{d(hC)}{dt}$$

In addition, the sugar entering the tank is: $E = d\ C_0$

And the sugar exiting the tank is given by: $S = J\ C$

The “generation” term is nil.

The sugar-specific balance equation is then: $\rho S \frac{d(hC)}{dt} = dC_0 - JC$

3) Heat balance

This balance can also be written in the form:

Accumulation = Inputs – Outputs + Generation

As there is no energy generation in the system, the balance equation becomes:

Accumulation = Inputs – Outputs

Let us express the different terms of this equation:

a) Accumulation: in order to express the “accumulation” term, it is necessary to know the expression of the amount of energy in the tank. The latter is represented by the enthalpy of the liquid present in the tank. It is given by: $H = MC_p(T - T^*)$, where T^* is a reference temperature.

Replacing *M* with its expression $(M = \rho V = \rho Sh)$, we obtain the expression of the amount of energy in the tank: $H = \rho ShC_p(T - T^*)$

Thus the “accumulation” term of energy in the tank is given by:

$$A = \frac{dH}{dt} = \frac{d\left[\rho ShC_p(T - T^*)\right]}{dt}$$

i.e. (ρ, C_p and S being constant): $A = \rho SC_p \frac{d(hT)}{dt}$

b) The “energy inputs”

The system admits the following inputs:

i) The enthalpies associated with the flows, *D* and *d*

$$H_D = DC_{pD}(T_0 - T^*) \quad \text{and} \quad H_d = dC_{pd}(T_0 - T^*)$$

ii) The input associated with the steam

$$H_V = H_{V0} + V\Lambda,$$

where H_{V0} is the enthalpy of the liquid at vaporization point and Λ is the latent heat of vaporization.

Thus, the “energy inputs” term is written:

$$E_{in} = H_D + H_d + H_{V0} + V\Lambda$$

Hence: $E_{in} = D\,C_{pD}(T_0\text{-}T^*) + d\,C_{pd}(T_0\text{-}T^*) + H_{V0} + V\Lambda$

c) The “energy inputs”

The system admits the following outputs:

i) The heat losses: *Q*

ii) The enthalpy H_J associated with the flow *J*.

$H_J = J\,C_{pJ}(T\text{-}T^*)$

iii) The output H_W associated with the condensed water

$H_W = H_{V0}$

Thus, the “energy outputs” term is written:

$$E_{out} = Q + H_J + H_{V0}$$

Consequently, the term $E_{in} – E_{out}$ may be expressed as follows:

$$E_{in} - E_{out} = H_D + H_d + H_{V0} + V\Lambda - Q - H_J - H_{V0}$$

$$E_{in} - E_{out} = D\,C_{pD}(T_0\text{-}T^*) + d\,C_{pd}(T_0\text{-}T^*) - J\,C_{pJ}(T\text{-}T^*) + V\Lambda - Q$$

$$E_{in} - E_{out} = (D\,C_{pD} + d\,C_{pd})\,T_0 - J\,C_{pJ}T - (D\,C_{pD} + d\,C_{pd} - J\,C_{pJ})\,T^* + V\Lambda - Q$$

Yet, in the reference state, the term $(DC_{pD} + dC_{pd} - JC_{pJ})T^*$ is nil.

Therefore: $E_{in} - E_{out} = (D\,C_{pD} + d\,C_{pd})\,T_0 - J\,C_{pJ}T + V\Lambda - Q$

Thus, the energy balance is written: *(A = E – S)*:

$$\rho SC_p \frac{d(hT)}{dT} = (DC_{pD} + dC_{pd})T_0 - JC_{pJ}T + V\Lambda - Q$$

1.5. Operating states

Prior to establishing the balances on a system, it is important to define its running mode or operating state. We can distinguish two types of operating states:

- Steady state.
- Transient state.

1.5.1. *Steady state*

DEFINITION.– Steady state is defined as a running condition where all system variables are constant over time.

It is a condition reached by a system after an operating time long enough to enable all of its variables to stabilize and no longer vary over the course of time. We can also say that the system has reached a stationary state, or an equilibrium state.

1.5.2. *Transient state*

DEFINITION.– The transient (or dynamic) state corresponds to the condition whereby at least one of the system variables varies with time.

It is the condition in which systems evolve during their start-up phase, or during the shutdown of an installation. We then talk about a dynamic state or a variable regime.

If, for example, we consider the general balance equation on the reservoir presented in Figure 1.8: $\rho S \frac{dh}{dt} = D_1 - D_2$

– *In steady state*: all of the variables are constant over time. In particular h is constant. Consequently: $\frac{dh}{dt} = 0$

The balance equation in steady state therefore becomes: $D_1 - D_2 = 0$

– *In transient state*: $\frac{dh}{dt} \neq 0$

The balance equation is a differential equation which gives the variation of the height of the liquid in the reservoir:

$$\frac{dh(t)}{dt} = \frac{D_1(t) - D_2(t)}{\rho S}$$

1.5.2.1. *Illustration: transient and steady states energy balances of a water heater*

We will consider the water heater presented in Figure 1.9.

Questions

1) Assuming that steady state has been reached, establish the equation that reflects the overall energy balance of the water heater.

2) Deduce therefrom the power, *P*, needed in order to heat the flow, *D*, from 18°C to 70°C.

3) After reaching steady state, we decide to turn off the water heater. We would like to have a clear idea of the way the temperature of the water inside the tank will vary after switching the device off. In this perspective, you are asked to establish the differential equation governing the variation in this temperature over the course of time, and to give the boundary condition that would be necessary for its integration.

Data:

$D = 300$ kg/day	$\rho = 1{,}000$ kg/m^3	$C_p = 1.2$ kcal/(kg °C)
$T_2 = 70$ °C	$T_1 = 18$ °C	

Solutions

1) Overall energy balance at steady state

The general equation of the energy balance of the water heater presented in Figure 1.9 is written (see section 1.3.4.2):

$$\rho h S C_p \frac{dT_2}{dt} = D C_p \left(T_1 - T_2\right) + P$$

At steady state, this equation becomes:

$$D\ C_p\ (T_1 - T_2) + P = 0$$

2) Power needed to heat the flow D from T_1 to T_2

Solving the energy balance equation for *P* yields:

$$P = D\ C_p\ (T_2 - T_1)$$

NUMERICAL APPLICATION.–

Power needed to heat a flow, *D*, of water from 18°C to 70°C.

D = 300 kg/day C_p = 1.2 kcal/(kg °C)

T_1 = 18 °C T_2 = 70 °C

$$P = \frac{300*1{,}2}{24*3600}*4{,}18*(70-18)$$

P = 906 W

3) Differential equation expressing the variation of temperature with time

The balance equation, in transient state, is given by:

$$\rho S h C_p \frac{dT_2}{dt} = D C_p (T_1 - T_2) + P$$

It enables the prediction of temperature variations as a function of any variations in *D*, T_1 or *P*:

$$\frac{dT_2}{dt} = \frac{1}{\rho S h C_p}\left[D C_p (T_1 - T_2) + P\right]$$

In the case of interest to us, the water heater has been switched off; therefore, $P = 0$, while T_1 = 18 °C = constant and T_2 (0) = 70 °C.

Thus, from the moment the water heater is switched off ($t = 0$), the differential equation that governs temperature variation over time is given by:

$$\frac{dT_2}{dt} = \frac{DC_p(18 - T_2)}{\rho ShC_p},$$

with the boundary condition: $T_2(0) = 70$ °C.

1.6. Transfer area

1.6.1. *What does the transfer area represent?*

DEFINITION.– We will use "transfer area" to refer to the surface through which the mass and/or energy are exchanged.

1.6.2. *Illustration: transfer area in a heat exchanger*

A tubular heat exchanger consists of a large tube (of diameter *D* and length *L*), which constitutes the outer wall, or envelope, of the device. A number, *n*, of tubes (all having the same diameter, *d*, and length, *L*) are placed in this envelope (see Figure 1.11).

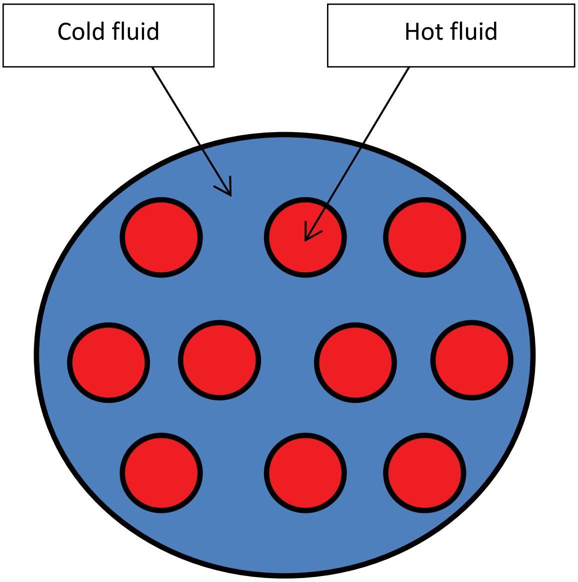

Figure 1.11. *Cross-section of a tubular exchanger*

Inside the large tube (envelope), circulates a fluid we wish to heat (“cold fluid”) using the heat that would be released by a “hot fluid”, circulating in the small tubes.

In addition, the large tube is in direct contact with the outside. This results in heat losses to the atmosphere.

Questions

1) Calculate the transfer area, S_a, relative to heat exchange between the large tube and the atmosphere.

2) Determine, as a function of n, the transfer area, S_n, relative to heat exchange between the fluid circulating in the large tube and the n small tubes.

3) Determine n such that S_n is twice as big as S_a.

Data:

$D = 10$ cm

$d = 2$ cm

$L = 0.5$ m

Solutions

1) External transfer area

$$S_a = \pi DL$$

NUMERICAL APPLICATION.–

$D = 0.1$ m　　　$L = 0.5$ m

$S_a = 0.1 \text{ x } 0.5 \text{ x } \pi$　　　$S_a = 0.157 \text{ m}^2$

2) Transfer area with the n tubes

The exchange surface of a tube is:

$$S_1 = \pi dL$$

Hence, for n tubes:

$$S_n = n\ \pi dL$$

3) Value of n

$$S_n = 2\ S_a$$

i.e.: $n\pi dL = 2\pi DL$.

Hence: $n = 2D/d$.

NUMERICAL APPLICATION.–

$n = 2 \times 10/2$ **n = 10 tubes**

1.6.3. *Illustration: transfer area inferred from a technical drawing*

A mechanical spare part of a pump consists of two solids, S_1 and S_2, which have a common contact surface, noted S_E. This part is mounted such that during operation of the pump, the solid, S_1, is at a temperature, T_1, that is much greater than the temperature, T_2, of S_2.

Question

From the technical drawings of the spare part (plan view and left-side view), identify the transfer area, S_E, between S_1 and S_2 by hatching it in the left-side view, then calculate S_E.

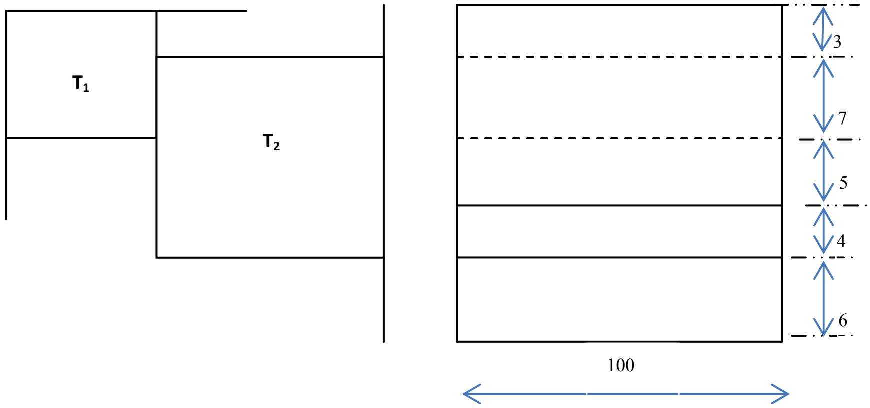

Figure 1.12. *Technical drawings of a pump spare part*

Solution

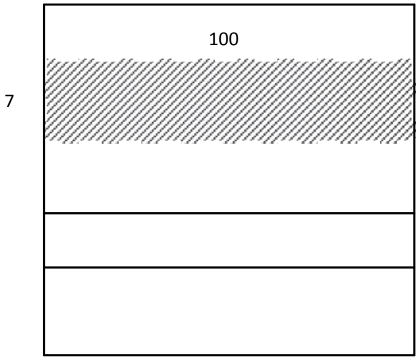

Figure 1.13. *Exchange surface (hatched)*

NUMERICAL APPLICATION.–

$S_E = 7 \text{ x } 100$ **$S_E = 700 \text{ mm}^2$**

1.7. Driving potential difference

As we will see in the following chapters, the concept of driving potential difference (DPD) is very important in the quantification of mass or energy transfers. This concept is developed by similarity to electricity. Indeed, let us recall that in electrical circuits, the flux of electrons, or the electric current, *i*, is determined by the potential difference, ΔV, (difference between equipotentials V_1 and V_2) and by the electrical resistance, *R*:

$$i = \frac{\Delta V}{R} = \frac{V_2 - V_1}{R}$$

More generally speaking, for a system described by equipotentials P_1 and P_2, we define the driving potential difference: $\Delta P = P_2 - P_1$

This concept can be applied to energy exchanges as well as to mass transfers. The following sections define the heat transfer potential difference (HPD) and the mass transfer potential difference (MPD) based on differences in temperature or concentration equipotentials. These transfer potential differences are sometimes referred to as driving forces because they constitute the "engine" necessary for a transfer to take place. Moreover, they are instrumental in determining the heat and mass fluxes transferred between systems.

1.7.1. *Heat transfer potential difference*

Let us consider two systems (or parts of the same system); one at temperature T_1 and the other at temperature T_2.

Heuristically, we know that heat will flow only if $T_1 \# T_2$ and that the quantitative importance of this flow will depend on how large the difference between $T_1 \# T_2$ is. While these practical observations will be confirmed in the next chapter, they are directly related to the definition of the *heat transfer potential difference.*

DEFINITION.–

The *heat transfer potential difference* (HPD) refers to the temperature difference (or gradient) between the two systems (or parts of the same system):

$$HPD = \Delta T = T_2 - T_1 \text{ Or: } \mathrm{HPD}\big|_x = \left.\frac{\partial T}{\partial x}\right|_x \mathrm{d}x$$

From the definition, we deduce that: $\mathrm{T}\big|_{x+dx} = \mathrm{T}\big|_x + \mathrm{HPD}\big|_x$

1.7.1.1. *Illustration: HPD and transfer area for a tank*

For the mixing tank represented in Figure 1.10, we assume that at steady state (see section 1.4.3.1), the temperature of the liquid in the tank is T_T and that the liquid height in this tank is h.

Questions

Calculate the heat potential difference (HPD) between the liquid of the tank and the heating wall and determine the heat transfer area.

Data:

Tank cross-section:	$S = 1{,}256.6\ \mathrm{cm}^2$
$T_T = 65°C$	$h = 70\ \mathrm{cm}$

Solutions

1) Heat transfer potential difference

The heating wall is in contact with a condensing vapor. Since condensation *always occurs at a constant temperature* (condensing temperature), the temperature of the heating wall can be assumed to be equal to the condensing temperature.

Thus: $T_c = 100°C$.

The HPD is then given by: HPD = $T_c - T_T$

NUMERICAL APPLICATION.–

$T_c = 100°C$ $T_T = 65°C$

HPD = 100 – 65 **HPD = 35 °C**

2) Heat transfer area

The heat transfer area is equal to the contact surface between the liquid and the heating wall.

The tank radius being *R* and referring to liquid height by *h*, this contact surface is given by:

$$S_E = 2\pi Rh$$

Yet: $R = \sqrt{\frac{S}{\pi}}$

Hence: $S_E = 2h\sqrt{\pi S}$

NUMERICAL APPLICATION.–

h = 70 cm S = 1,256.6 cm^2

$S_E = 2 * 70 * \sqrt{1,256,6 * \pi}$ **S_E = 8,796 cm^2**

1.7.1.2. *Illustration: elementary DPT and exchange surface*

Consider the metal bar represented in Figure 1.14. The temperature of this bar is a function of *x*, *y* and *z*.

We will consider the volume element of this bar, between *x* and *x* + *dx*, defined by *dx*, *dy* and *dz* (see Figure 1.14).

The cross-section of the bar, located at *x*, is at temperature θ_x, whilst the cross-section located at *x* + *dx* is at temperature θ_{x+dx}.

We suppose that the temperature profile along the bar decreases as a function of x; we then have $\theta_x > \theta_{x+dx}$ and the energy will therefore propagate in the direction $x \rightarrow x + dx$.

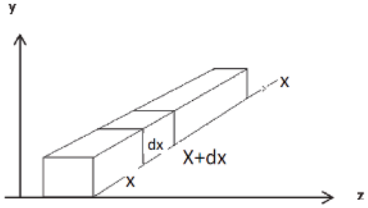

Figure 1.14. *Volume element of a metal bar*

Questions

1) Determine the expression of the heat transfer area in the direction x, S_x.

2) What is the expression of the heat transfer potential difference (HPD) in the direction x?

Solutions

1) The exchange surface, S_x, is given, quite simply, by:

$$S_x = dy\,dz$$

2) The exchange potential difference in the direction x is the temperature gradient in the direction x; i.e.:

$$\left.HPD\right|_x = \left.\frac{\partial\theta}{\partial x}\right|_x dx$$

1.7.2. *Mass transfer potential difference*

Let us now consider two parts of a system (or two systems); one is at concentration c_1 and the other is at concentration c_2.

DEFINITION.–

We will use "*mass transfer potential difference* (MPD)" to refer to the concentration difference or gradient:

$$MPD = \Delta c = c_2 - c_1 \text{ or } \left.MPD\right|_x = \left.\frac{\partial c}{\partial x}\right|_x dx$$

From the definition, we deduce that: $c\big|_{x+dx} = c\big|_{x} + \mathrm{MPD}\big|_{x}$.

NOTES.– Concentrations, *c,* in the MPD definition can be expressed in molar or mass terms:

- In terms of molar concentrations, c is expressed in mol/liter or in $\mathrm{kmol/m^3}$ or in any other equivalent units.

- For a problem posed in terms of mass concentrations, ρ (g/liter or $\mathrm{kg/m^3}$), the MPD is given by: $\mathrm{MPD} = \Delta\rho = \rho_2 - \rho_1$

– In terms of mole fractions x, (x = number of moles of *A*/total number of moles):

$$\mathrm{MPD} = \Delta x = x_2 - x_1$$

– In terms of mass fractions, ω (ω = mass of *A*/total mass):

$$\mathrm{MPD} = \Delta\omega = \omega_2 - \omega_1$$

1.7.2.1. *Illustration: MPD in an ultrafiltration cell*

Ultrafiltration is one of the most important seawater desalination techniques. It consists of passing seawater through *reverse osmosis* cells.

A reverse osmosis cell is composed of two compartments subject to different pressures ($P_1 > P_2$), separated by a *semi-permeable membrane,* that is, a membrane which is permeable to water and impermeable to salt (see Figure 1.14).

1) If $P_1 = P_2$, the system will have a natural evolution towards a new steady state, where the concentrations of H_2O in the two compartments will tend towards the same value, *m*, the equilibrium concentration. Such an evolution is known as osmosis.

2) If $P_1 > P_2$, the system is forced to operate unnaturally. Water will pass from the first compartment (which has a low H_2O concentration) to the second compartment (which has a higher H_2O concentration). This type of (unnatural) operation is only possible under pressure ($P_1 > P_2$). It makes it possible to recover the fresh water in compartment 2, while the salt remains trapped in compartment 1.

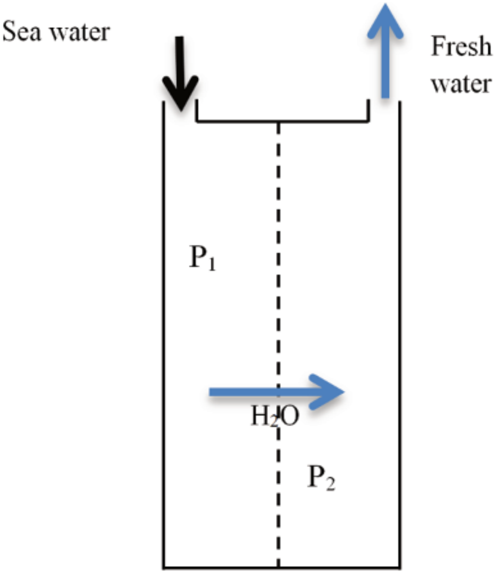

Figure 1.15. *Reverse-osmosis cell*

This mode of operation is known as *reverse osmosis*, simply because it does the opposite of osmosis.

We will focus on the case where $P_1 = P_2$.

Questions

1) Calculate the mass transfer area between the two compartments.

2) Calculate the driving potential difference based on mass concentrations of water.

Data:

Cell diameter: $D = 30$ cm

Cell height: $H = 1$ m

Density of compartment 1: $\rho_1 = 1{,}025$ g/liter

Density of compartment 2: $\rho_2 = 1{,}000$ g/liter

Salt content of compartment 1: $\rho_1^{\mathrm{NaCl}} = 35\,\mathrm{g/liter}$

Salt content of compartment 2: $\rho_2^{\mathrm{NaCl}} = 0{,}001\,\mathrm{g/liter}$

Solutions

1) Mass transfer area

The mass transfer area in this case is the semi-permeable membrane surface. It is given by:

$$S_E = D \times H$$

N.B.–

$S_E = (0.3)(1)$ **$S_E = 0.3\ \mathrm{m}^2$**

2) Driving potential difference

The problem is posed in terms of mass concentrations; the MPD is then given by: $\mathrm{MPD}^{H_2O} = \Delta\rho^{H_2O} = \rho_2^{H_2O} - \rho_1^{H_2O}$

Yet $\rho_1^{H_2O}$ and $\rho_2^{H_2O}$ are unknown, but we do know ρ_1^{NaCl} and ρ_2^{NaCl}.

a) Calculating $\rho_1^{H_2O}$

For a volume, V, of compartment 1, the total mass is $\rho_1 V$ and the masses of H_2O and NaCl are such that: $m_1^{H_2O} + m_1^{NaCl} = \rho_1 V$

Therefore: $m_1^{H_2O} + \rho_1^{H_2O} V = \rho_1 V - \rho_1^{NaCl} V$

Hence: $\rho_1^{H_2O} = \rho_1 - \rho_1^{NaCl}$

b) Calculating $\rho_2^{H_2O}$

Likewise, for a volume V of compartment 2, the masses of H_2O and NaCl are such that: $m_2^{H_2O} + m_2^{NaCl} = \rho_2 V$

Therefore: $m_2^{H_2O} = \rho_2 V - m_2^{NaCl}$

Or: $m_2^{H_2O} = \rho_2 V - \rho_2^{NaCl} V$

i.e.: $\rho_2^{H_2O} = \rho_2 - \rho_2^{NaCl}$

Therefore: $\mathrm{MPD}^{H_2O} = \Delta\rho^{H_2O} = \rho_2^{H_2O} - \rho_1^{H_2O}$

$$\mathrm{MPD}^{H_2O} = \rho_2^{H_2O} - \rho_1^{H_2O} = \left(\rho_2 - \rho_1\right) + \left(\rho_1^{NaCl} - \rho_2^{NaCl}\right)$$

NUMERICAL APPLICATION.–

$\rho_1 = 1{,}025$ g/liter $\rho_2 = 1{,}000$ g/liter

$\rho_1^{NaCl} = 35\ \text{g/liter}$ $\rho_2^{NaCl} = 0.001\ \text{g/liter}$

$$\mathrm{MPD}^{H_2O} = \left(1{,}000 - 1{,}025\right) + \left(35 - 0.001\right)$$

$\mathrm{MPD}^{H_2O} = -25 + 34.999$ $\mathrm{MPD}^{H_2O} = 9.999\ \text{g/liter}$

1.8. Exercises and solutions

EXERCISE 1.1. *Balance in steady state*

We will consider the perfectly stirred reservoir represented in Figure 1.16, where flows, d_1, d_2 and d_3, have constant concentrations, c_1, c_2 and c_3, respectively. In addition, d_1 and d_2 are constant.

Questions

1) Assuming that the liquid height in the reservoir is constant, show that the system is operating at steady state.

2) In this case, what must the value of d_3 be?

3) Which condition makes it possible to have $c_3 = 2c_2$?

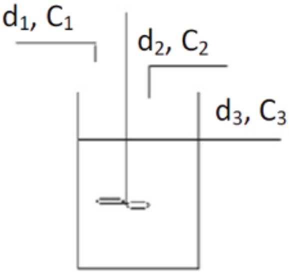

Figure 1.16. *Stirred tank*

Data:

$d_1 = 0.5$ m^3/hr

$d_2 = 1.5$ m^3/hr

Solutions

1) The height h is constant

An overall balance on the tank gives: $\rho S \dfrac{dh}{dt} = d_1 + d_2 - d_3$

The height, *h*, being constant, we have: $\dfrac{dh}{dt} = 0$

Subsequently: $d_3 = d_1 + d_2$

d_1 and d_2 being constant, d_3 is constant.

Thus, as all of the variables are constant, the system is then at steady state.

2) Value of d_3

At steady state, we will obtain: $d_3 = d_1 + d_2$

3) Condition enabling us to obtain $c_3 = 2c_2$

At steady state, we have: $d_1 c_1 + d_2 c_2 = d_3 c_3$

Hence: $c_3 = (d_1 c_1 + d_2 c_2)/d_3$

Supposing $c_3 = 2c_2$, we obtain: $2c_2 d_3 = d_1 c_1 + d_2 c_2$

i.e.: $c_2 = \dfrac{d_1 c_1}{2d_3 - d_2}$

EXERCISE 1.2. *Cooling an electronic circuit*

Your first job is at a plant that manufactures electronic components. Your supervisor informs you that your team is currently designing an electronic circuit, the operation of which releases a heat energy flux, ϕ_S. He also informs you that the correct operation of this electronic circuit relies on how this amount of energy could be conveyed out of the circuit. To do this, the circuit will need to be equipped with a metal radiator, the role of which is to dissipate this energy to the outside.

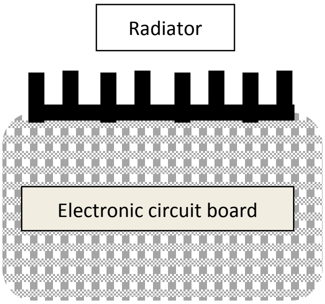

Figure 1.17. *Printed circuit board comprising an energy dissipator*

Assuming that the flux, ϕ_D, of energy dissipated by the radiator to the outside is proportional to its heat transfer area with the surrounding environment, S_R and to the heat transfer potential difference (HPD) between the radiator and the surrounding environment, your supervisor asks you to:

Questions

1) Give the expression of ϕ_D as a function of the radiator temperature, the ambient temperature, S_R, and a proportionality factor that we will denote K.

2) Determine the value of the heat transfer area, S_R, in order to ensure correct operation of the electronic circuit.

Your supervisor gives you the following data:

$\phi_S = 50$ Watts; $T_R = 150$ °C; $T_a = 25$ °C; K = 15 kcal/hm^2°C

Solutions

1) Expression of ϕ_D

We have assumed that ϕ_D is proportional to the exchange surface, S_R, and to the heat transfer potential difference HPD. We can therefore write:

$$\phi_D = KS_R(T_R - T_a)$$

2) Value of the exchange surface, S_R

In order to assure a correct operation of the semiconductor, it is necessary that the energy released by the circuit be conveyed out. We thus should have: $\phi_D = \phi_S$

Hence: $KS_R(T_R - T_a) = \phi_S$ i.e.: $S_R = \dfrac{\phi_S}{K(T_R - T_a)}$

NUMERICAL APPLICATION.–

$\phi_S = 50$ Watts $T_R = 150$ °C $T_a = 25$ °C $K = 15$ kcal/hm^2°C

$$S_R = \frac{50 * 3{,}600}{15 * 1{,}000 * 4{,}18(150 - 25)}$$

$\mathbf{S_R = 0{,}023\ m^2}$

EXERCISE 1.3. *Balance in transient state*

During a visit that you pay to a rural area, the engineer in charge of the development of this area asks for your help to tackle a problem that he is unable to solve. The problem concerns the fact that the municipality’s drinking water supply is being cut off from time to time without a clear reason. He informs you that all of the municipality’s homes, businesses and services are supplied by a water tower, represented by the reservoir shown in Figure 1.18.

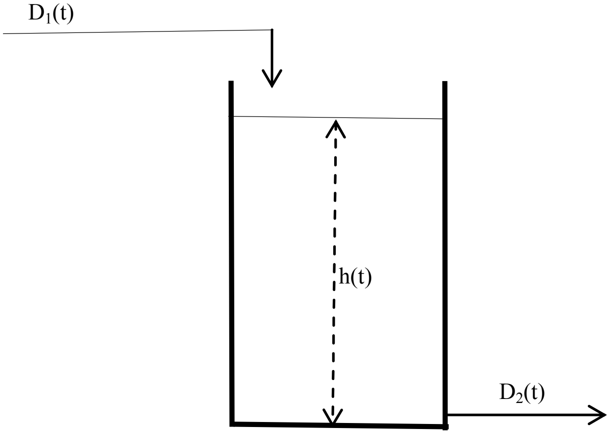

Figure 1.18. *Water-tower reservoir*

The flow D_1 is delivered by a pump extracting water from the town well to continuously feed the water tower. D_1 (t) follows a parabolic law, which has been modeled by: $D_1(t) = a\ b\ t^2$

whereby:

t and D_1 are expessed respectively in hours and in m^3/hr

a is a constant to be determined, expressed in m^3/h^3

b is a known constant, without dimensions

The flow, D_2, corresponds to water extraction by different users. It has been determined as a function of the consumption of the different homes, businesses and services of the rural town. The modeling of this consumption, between 0 hours and 24 hours, led to an exponential law, given by:

$$D_2(t) = kte^{0,1t}$$

whereby t, D_2 and k are expressed respectively in hours, m^3/hr and m^3/h^2

The town's engineer informs you that, at least once every 24 hours, there is no more water left in the tower, leading to an interruption in the public service. You then decide to help the town's engineer analyze the problem. You advise him to answer the following questions:

Questions

1) Give the differential equation that governs the mass balance on the reservoir.

2) Determine the relation between k and a so that dh/dt becomes nil at $t = 0$ *hrs* and at $t = 24$ *hrs*.

3) Show that the equation, $\frac{dh}{dt} = 0$, admits a third root between 0 hrs and 24 hrs.

4) Show that the height, *h(t)*, of the liquid in the reservoir passes by a minimum, h_{min}, and determine t_{min}.

5) Establish the table of variations of *h(t)* and plot its curve between 0 hrs and 24 hrs.

6) By integrating the differential equation, give the evolution, as a function of time, of the height, *h(t)*, of the liquid in the reservoir. We will assume that at $t = 0$, $h(0) = h_0$.

7) How can an interruption in the public service be avoided? Calculate h_{min} in this case.

Data:

$b = e^{2.4}$; $k = 240$ m^3/h^2; Tower tank radius: $R = 8$ m

Solutions

1) Differential equation that governs the mass balance

$$S\frac{dh}{dt} = D_1(t) - D_2(t)$$

Yet: $D_1(t) = a\,b\,t^2$ and $D_2(t) = kte^{0.1t}$ i.e.: $S\frac{dh}{dt} = abt^2 - kte^{0.1t}$

2) Relation between k and a so that dh/dt is nil at t = 0 hrs and at t = 24 hrs

We have: $S\frac{dh}{dt} = t(abt - ke^{0.1t})$

At *t = 0 hrs*: $\frac{dh}{dt} = 0$

At *t = 24 hrs*: $\frac{dh}{dt} = 0 \rightarrow abt - ke^{0.1t} = 0 \rightarrow 24ae^{2.4} - ke^{2.4} = 0$

i.e.: $a = \frac{k}{24}$

NUMERICAL APPLICATION.– $k = 240$ **a = 10**

3) Third root of dh/dt

The expression of *dh/dt* is: $S\frac{dh}{dt} = t(10e^{2.4}t - 240e^{0.1t})$

Therefore $t = 0$ and $t = 24$ are roots of the equation $\frac{dh}{dt} = 0$. Let us locate the third root of this equation:

At $t = 1\text{hr}$: $10e^{2,4}t - 240e^{0,1t} = 0 < 0$.

At $t = 3\text{hrs}$: $30e^{2,4}t - 240e^{0,3t} = 0 > 0$.

At $t = 2.5\text{hrs}$: $25e^{2,4}t - 240e^{0,25t} = 0 < 0$.

As *dh/dt* is continuous, it therefore becomes nil between 2.5 hrs and 3 hrs.

Therefore: *dh/dt admits a third root, t_{min}*, situated between 2.5 hrs and 3 hrs.

4) Minimum, h_{min}, of $h(t)$ and calculation of t_{min}

The table of the signs of dh/dt shows that:

– The derivative of $h(t)$ is nil at t_{min}, situated between 2.5 hrs and 24 hrs.

– This derivative is negative for $t < t_{min}$ and becomes positive for $t > t_{min}$.

Therefore: $h(t)$ admits a minimum at t_{min}.

T	0 hrs	2.5 hrs	t_{min}		24 hrs
$\frac{dh}{dt}$	0	–	0	+	0

Table 1.1. *Signs of* $\frac{dh}{dt}$

The calculation of t_{min} can be carried out by solving the equation, $10e^{2.4}t - 240e^{0.1t} = 0$ between 2.5 hrs and 3 hrs. Of course, any of the methods for solving non-linear equations can be used (Newton-Raphson, dichotomy, etc.). Nevertheless, we prefer to continue with the trial and error approach by pursuing sign analyses of dh/dt.

Approximate calculation of t_{min} by pursuing analysis of the sign of dh/dt:

– $t = 3$hrs: $30e^{2.4} - 240e^{0.3} > 0$.

– $T = 2.5$hrs: $25e^{2.4} - 240e^{0.25} < 0$.

– $T = 2.9$hrs: $29e^{2.4} - 240e^{0.29} < 0$.

– $T = 2.915$hrs: $29.15e^{2.4} - 240e^{0.2915} > 0$.

– $T = 2.914$hrs: $29.14e^{2.4} - 240e^{0.2914} > 0$.

– $T = 2.913$hrs: $29.13e^{2.4} - 240e^{0.2913} < 0$.

We therefore retain: $t_{min} = 2.9135$ hrs.

5) Table of variations and curve of $h(t)$ between 0 hrs and 24 hrs

T	0hrs	2.5hrs	t_{min}		24hrs
$\frac{dh}{dt}$	0	–	0	+	0
$h(t)$	$h(0)$	↘	h_{min}	↗	$h(24)$

Table 1.2. *Variations in h(t)*

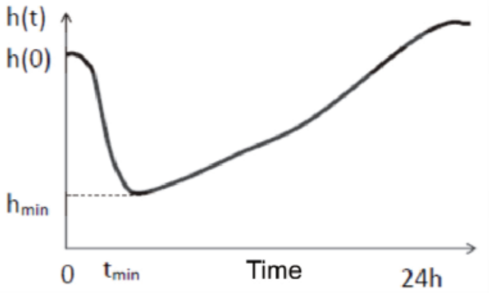

Figure 1.19. *Sketch of th curve of h(t)*

6) Height of the liquid in the reservoir as a function of time

The differential of *h* is given by: $\frac{dh}{dt}=\frac{1}{S}t(abt-ke^{0.1t})$. Integrating this differential equation gives *h(t)*:

$$h(t)-h(0)=\frac{1}{S}\int_0^t t(abt-ke^{0.1t})dt$$

$$h(t)-h(0)=\frac{1}{S}\left[\frac{ab}{3}t^3-\frac{k}{0.01}(0.1te^{0.1t}-e^{0.1t})\right]_0^t$$

i.e: $h(t)-h(0)=\frac{1}{S}\left[\frac{ab}{3}t^3-\frac{k}{0.01}(0.1te^{0.1t}-e^{0.1t}+1)\right]$

7) Condition to avoid a break in public service

For there not to be a break in service, it is necessary that: $\forall t:\ h(t)>0$.

This condition can also be expressed in a more conservative way by $h_{min}>0$.

$$h_0+\frac{1}{S}\left\{\frac{ab}{3}t_{min}^3-\frac{k}{0.01}\left[\left(0.1t_{min}-1\right)e^{0.1t_{min}}+1\right]\right\}>0$$

$$h_0>\frac{1}{\pi R^2}\left\{\frac{k}{0.01}\left[1+\left(0.1t_{min}-1\right)e^{0.1t_{min}}\right]-\frac{ab}{3}t_{min}^3\right\}$$

NUMERICAL APPLICATION.– $b = e^{2.4}$ $k = 240\ \mathrm{m^3/h^2}$ $R = 8\ \mathrm{m}$ $a = 10$ $t_{\min} = 2.9135\ \mathrm{hr}$

$$h_0 > \frac{1}{25\pi}\left\{\frac{240}{0.01}\left[1+\left(0.29135-1\right)e^{0.29135}\right]-\frac{10e^{2.4}}{3}\left(2.9135\right)^3\right\}$$

The condition is therefore that $h_0 > 4.66$ m

EXERCISE 1.4. *Calorimetry*

The first law of thermodynamics makes it possible to show that the variation in a system's heat energy is linked to the variation of its internal energy through the relation:

$$Q_{1\to 2} = U(2) - U(1)$$

We wish to use this relation to determine the specific heats (or heat capacities), C_p, of liquids. To achieve this, we use a device that enables an amount, Q, of heat to be transferred to a known mass, M, of liquid.

We will assume that the device is perfectly insulated from the outside and that it does not participate in the heat exchange, such that the heat transferred to the fluid uniquely serves to increase its internal energy.

Initially, the device contains a mass, M, that is at temperature T_1. In final equilibrium state, the temperature of the liquid is T_2.

Questions

1) Give the expressions of the internal energies, *U(1)* and *U(2)*, as a function of M, of the specific heat (assumed to be independent of temperature) and of the temperatures T_1, T_2 and T^* (reference temperature).

2) Deduce therefrom an expression for the heat capacity, C_p, as a function of M, Q and temperatures, T_1 and T_2.

3) In this question, we will consider that heat is supplied to the system by an electrical resistance of power P placed in the device. We will assume that electricity is converted into heat with an efficiency $\eta_1 = 90\%$.

a) Calculate the amount of energy supplied to the system, if the resistance is powered up for a duration Δt.

b) Determine C_p as a function of P and the other parameters of the problem.

c) Calculate C_p.

Data:

$M = 1$ kg $\quad P = 500$ W

$T_1 = 20$ °C $\quad T_2 = 85$ °C $\quad \Delta t = 30$ minutes

Solutions

1) Internal energies U(1) and U(2)

$$U(1) = MC_p(T_1 - T^*) \quad \text{and} \quad U(2) = MC_p(T_2 - T^*)$$

2) Heat capacity, C_p, as a function of M, Q, T_1 and T_2

The first law of thermodynamics gives: $Q_{1\to2} = U(2) - U(1)$

Hence: $Q = MC_p(T_2 - T^*) - MC_p(T_1 - T^*)$

$$Q = MC_p(T_2 - T_1)$$

Hence the expression of C_p is: $C_p = \dfrac{Q}{M(T_2 - T_1)}$

3) Energy is supplied to the system by an electrical resistance of power, P

a) Amount of heat supplied to the system

This amount *Q* is given by: $Q = \eta_1 P \Delta t$

NUMERICAL APPLICATION.–

$\eta_1 = 0.9$

$P = 500$ W

$\Delta t = 30$ minutes

$Q = 0{,}9 * 500 * 30 * 60$ $\quad$ ***Q* = 810 kJ**

b) C_p as a function of *P* and the other parameters

This function is obtained by eliminating *Q* from the following two expressions:

$$C_p = \frac{Q}{M(T_2 - T_1)} \quad \text{and} \quad Q = \eta_1 P \Delta t$$

Hence: $$C_p = \frac{\eta_1 P\Delta t}{M(T_2 - T_1)}$$

c) Calculating C_p

NUMERICAL APPLICATION.–

$\eta_1 = 0.9$ $P = 500$ W $M = 1$ kg

$T_1 = 20$ °C $T_2 = 85$ °C $\Delta t = 30$ minutes

$C_p = 12.46$ kJ/kg°C

EXERCISE 1.5. *Energy balance of an electric water heater*

The electric water heater of Figure 1.20 enables the water to be heated from temperature T_1 to temperature T_2; energy being supplied by an electrical resistance of power, P, placed at the bottom of the water heater.

We will assume that electricity is converted into heat with a yield η.

The water heater being well insulated from the outside, we will assume as a first approximation that heat losses are nil.

We wish to know the evolution of temperature, T_2, as a function of the temperature at the input of the water heater, T_1.

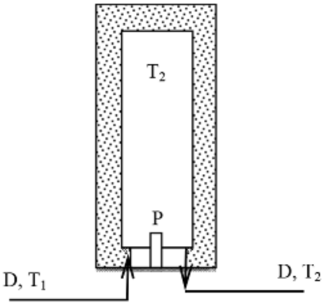

Figure 1.20. *Electric water heater*

Questions

1) Give the expressions of the energy fluxes entering and exiting the water heater.

2) Give the expression of the “energy accumulation” term in the water heater.

3) Give the differential equation governing the variations in temperature, T_2.

4) Give the evolution of T_2 as a function of time.

5) Show that the curve of $T_2(t)$ admits a slope at the origin equaling: $\frac{\eta P}{\rho V C_p}$

6) Show that the curve of $T_2(t)$ admits an asymptote at: $T_1 + \frac{\eta P}{\rho V C_p}$

7) Plot the curve of $T_2(t)$.

8) We will now assume that heat losses are proportional to the Thermal Potential Difference (TPD = $T_2 - T_a$), where T_a is the ambient temperature, such that the expression of the heat losses flux to the outside is given by: $\varphi_L^S = K(T_2 - T_a)$

a) What becomes of the differential equation that governs the variations of T_2?

b) Determine θ, α and β in such a way that this equation be put in the form:

$$\frac{d\theta}{dt} = -\alpha\theta + \beta$$

c) By analogy with the solution developed in the fourth question, give the expression of $T_2(t)$ with heat losses.

d) In this case, what are the slope at the origin and the asymptote at infinity?

Solutions

1) Expressions of the fluxes entering and exiting the water heater

a) The incoming fluxes

i) By the flow, D, at temperature T_1: $\phi_D^E = DC_p(T_1 - T^*)$

ii) By the electrical resistance: $\phi_P^E = \eta P$

b) The outgoing fluxes

i) By the flow, D, at temperature T_2: $\phi_D^S = DC_p(T_2 - T^*)$

ii) By heat losses: $\phi_L^S = 0$

2) The “energy accumulation” term in the water heater:

$$A = \frac{d}{dt}\left[MC_p(T_2 - T^*)\right],$$

whereby M is the mass of water in the heater: $M = \rho V$.

i.e.: $A = \frac{d}{dt}\left[\rho V C_p (T_2 - T^*)\right]$

ρ, V, Cp, T_2 and T^* being constant, A becomes: $A = \rho V C_p \frac{dT_2}{dt}$

3) Differential equation that governs the variation in temperature, T_2

This equation is obtained from the energy balance, in transient state, which is expressed as follows: $A = \phi_D^E + \phi_P^E - \phi_D^S - \phi_L^S$

i.e.: $\rho V C_p \frac{dT_2}{dt} = D C_p (T_1 - T_2) + \eta P$

Hence the differential equation governing T_2 is:

$$\frac{dT_2}{dt} = -\frac{D}{\rho V}(T_2 - T_1) + \frac{\eta P}{\rho V C_p}$$

4) Evolution of T_2 as a function of time

$T_2(t)$ is obtained by integrating the differential equation. For greater simplicity, we define parameters α, β and θ as follows:

$$\alpha = \frac{D}{\rho V}, \ \beta = \frac{\eta P}{\rho V C_p}, \ \theta = T_2 - T_1$$

The differential equation becomes: $\frac{d\theta}{dt} = -\alpha\theta + \beta$

This equation admits a general solution in the form:

$$\theta = K_1 e^{-\alpha t} + K_2$$

If we substitute this general solution into the differential equation, we obtain:

$$-\alpha K_1 e^{-\alpha t} = -\alpha\left(K_1 e^{-\alpha t} + K_2\right) + \beta$$

Hence: $\alpha K_2 = \beta$ i.e.: $K_2 = \frac{\beta}{\alpha}$

Thus: $\theta = K_1 e^{-\alpha t} + \frac{\beta}{\alpha}$

In order to determine K_1, we use the fact that initially ($t = 0$) the water in the heater is cold ($T_2 = T_1$). Therefore $\theta(0) = 0$

Hence: $K_1 = -\frac{\beta}{\alpha}$

The complete solution of the differential equation is therefore:

$$\theta = \frac{\beta}{\alpha}(1 - e^{-\alpha t}) \quad \text{i.e.: } T_2(t) = T_1 + \frac{\beta}{\alpha}(1 - e^{-\alpha t}) \quad \text{Or: } T_2(t) = T_1 + \frac{\eta P}{DC_p}(1 - e^{-\alpha t})$$

5) Slope at the origin of $T_2(t)$

$$\frac{dT_2}{dt}(0) = -\frac{D}{\rho V}\left[T_2(0) - T_1(0)\right] + \frac{\eta P}{\rho V C_p}$$

$$T_2(t) = T_1 + \frac{\eta P}{DC_p}(1 - e^{-\alpha t}) \quad \rightarrow \quad T_2(0) = T_1$$

Hence: $\frac{dT_2}{dt}(0) = \frac{\eta P}{\rho V C_p}$

6) Asymptote of $T_2(t)$

$$T_2(t) = T_1 + \frac{\eta P}{DC_p}(1 - e^{-\alpha t}) \quad \text{Thus: } \lim_{t \to \infty} T_2(t) = T_1 + \frac{\eta P}{DC_p}$$

Moreover: $\lim_{t \to \infty} \frac{dT_2}{dt} = -\frac{D}{\rho V}(\frac{\eta P}{DC_p}) + \frac{\eta P}{\rho V C_p}$ i.e.: $\lim_{t \to \infty} \frac{dT_2}{dt} = 0$

Therefore, $T_2(t)$ does admit an asymptote at $T_2 = T_1 + \frac{\eta P}{\rho V C_p}$

7) Curve of $T_2(t)$

t	0		∞
$\frac{dT_2}{dt}$	$\frac{\eta P}{\rho V C_p}$	$+$	0
$T_2(t)$	T_1	↗	$T_1 + \frac{\eta P}{D C_p}$

Table 1.3. *Variations in T_2 as a function of time*

Hence the curve of $T_2(t)$:

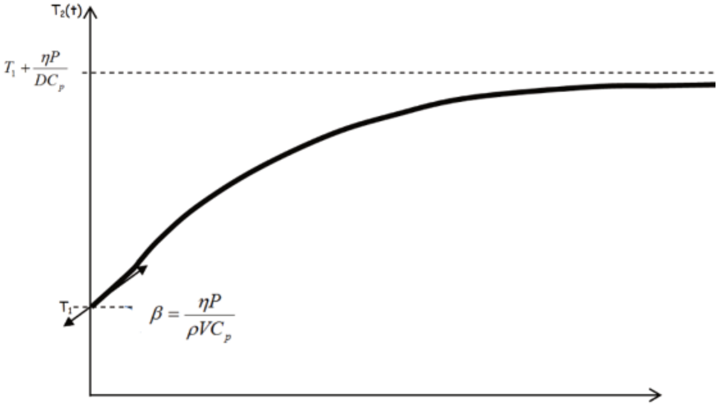

Figure 1.21. *Curve of $T_2(t)$*

8) Case where the water heater is subject to heat losses proportional to the TPD

a) Differential equation with heat losses

The energy balance equation, in transient state, is, in this case:

$$A = \phi_D^E + \phi_P^E - \phi_D^S - \phi_L^S$$

with: $\phi_L^S = K(T_2 - T_{a)}$

i.e.: $MC_p \frac{dT_2}{dt} = DC_p(T_1 - T_2) - K(T_2 - T_a) + \eta P$

or:

$$\rho VC_p \frac{dT_2}{dt} = -(DC_p + K)T_2 + DC_p T_1 + KT_a + \eta P$$

b) Differential equation with heat losses

The differential equation that governs the variation in T_2 is written:

$$\rho VC_p \frac{dT_2}{dt} = -(DC_p + K)T_2 + (DC_p + K)T_1 + \eta P - K(T_1 - T_a)$$

or:

$$\frac{dT_2}{dt} = -\frac{DC_p + K}{\rho VC_p}(T_2 - T_1) + \frac{\eta P - K(T_1 - T_a)}{\rho VC_p}$$

This equation may be put in the form:

$$\frac{d\theta}{dt} = -\alpha\theta + \beta$$

if $\theta = T_2 - T_1$, $\alpha = \frac{DC_p + K}{\rho VC_p}$ and $\beta = \frac{\eta P - K(T_1 - T_a)}{\rho VC_p}$

c) Expression of $T_2(t)$ with heat losses

$$\theta = \frac{\beta}{\alpha}(1 - e^{-\alpha t})$$

i.e.: $T_2(t) = T_1 + \frac{\beta}{\alpha}(1 - e^{-\alpha t})$

Or: $T_2(t) = T_1 + \frac{\eta P - K(T_1 - T_a)}{DC_p + K}\left[1 - e^{-\frac{DC_p + K}{\rho VC_p}t}\right]$

d) Slope at the origin and asymptote with heat losses

The slope at the origin is: $\beta = \dfrac{\eta P - K(T_1 - T_a)}{\rho V C_p}$.

The asymptote at infinity is: $T_1 + \dfrac{\eta P - K(T_1 - T_a)}{D C_p + K}$.

EXERCISE 1.6. *Starting up an electric water heater*

You have just bought an electric water heater having a useful volume *V*, equipped with an electrial resistance of power *P*.

Your plumber has just finished installing the water heater, completing all of the connections to the water network. He then plugs it into the electrical mains.

At this point, you ask him:

– “So, when would we be able to have the first shower?”

He replies:

– “Considering that you need to wait at least until the water temperature in the heater reaches T_2, you would need to wait for at least a full six hours.”

Your mother then asks you:

– “Is it true it would take that long?”

Diplomatically, you reply:

– “Our plumber is an experienced man, he is no doubt right.”

But in reality, you do not know at all, and deep down you feel that you would like to obtain a scientific proof. You then decide to use your knowledge of heat transfer to calculate the time needed based on the equations that govern the system’s operation. To do this, you begin by defining your system as the water contained in the water heater.

You then define an initial (the total mass of water, ρV, is at temperature T_1) and a final (the water is at $T_2 = T_1 + \Delta T$) state.

Questions

1) Applying the first law of thermodynamics, establish the equation linking T_2 to P.

2) You wish to verify the validity of the equation obtained in the first question by adopting a balance approach.

a) Identify the different terms of the energy balance equation: inputs, outputs, generation and accumulation.

b) Establish the balance equation.

3) What is the time, Δt, needed to have a temperature increase of ΔT.

4) Was the plumber right? Explain.

Data:

$V = 150$ liters	$\rho = 1$ kg/liter	$C_p = 1$ kcal/kg °C
$P = 1.5$ kW	$T_1 = 20$ °C	$T_2 = 60$ °C

Electricity is converted into heat with a yield $\eta = 85\%$

Solutions

1) Equation linking T_2 to P

The first law of thermodynamics enables the variation in internal energy, $U(2) - U(1)$, to be linked to the heat, $Q_{1\to 2}$, received by the system between the initial and the final states:

$$Q_{1\to 2} = U(2) - U(1)$$

Thus: $Q_{1\to 2} = \eta P \Delta t$,

whereby Δt is the time elapsed between the initial and the final states.

Moreover, internal energies are given by:

$U(1) = \rho V C_p (T_1 - T^*)$, whereby T^* is a reference temperature.

$U(2) = \rho V C_p (T_2 - T^*)$

Hence: $U(2) - U(1) = \rho V C_p (T_2 - T_1)$

Consequently: $\eta P \Delta t = \rho V C_p (T_2 - T_1)$

2) Balance approach

The energy balance is written generally, as follows:

$$A = \sum_{i=1}^{n} e_i - \sum_{j=1}^{m} s_j + G$$

a) Identifying the different terms of the balance equation

– Incoming flux: 0.

– Outgoing flux: 0.

– Fluxes generated: ηP.

– Accumulated flux: $\frac{d}{dt}(\rho V C_p T)$.

b) Establishing the balance

The balance equation is then written:

$$\rho V C_p \frac{dT}{dt} = \eta P$$

Or: $\eta P\, dt = \rho\, V\, C_p\, dT$

Integrating between 0 and Δt: $\eta P\, \Delta t = \rho\, V\, Cp\, \Delta T$

3) Determining Δt

$$\Delta t = \frac{\rho V C_p}{\eta P} \Delta T$$

4) Calculating Δt and explaining the plumber's opinion

NUMERICAL APPLICATION.–

V = 150 liters	ρ = 1 kg/liter	C_p = 1 kcal/(kg °C)	
P = 1.5 kW	T_1 = 20 °C	T_2 = 60 °C	η = 85%.

Hence: $\Delta t = \frac{(1)(150)(1*4.18)}{(0.85)(1.5)}(60-20)$ Δt = 5.46 hours

The plumber was therefore not far off the mark.

EXERCISE 1.7. *Balance on a heat exchanger*

A heat exchanger (see Figure 1.22) is a device for heating a flow, D_1, of a given fluid, using the energy supplied by another flow, D_2, of a hot fluid; this heat exchange occurring of course by ensuring the two fluids inside the exchanger do not mix.

Having a constant flow, D_1, the fluid to be heated enters the exchanger at a known temperature, t_1. It should exit the exchanger at t_2, which is also known.

Figure 1.22. *Heat exchanger. For a color version of this figure, see www.iste.co.uk/benallou/energy1.zip*

Likewise, the heating fluid is of constant flow, D_2. It enters the exchanger at a known T_1 and leaves it at T_2, which is unknown.

Questions

1) Give, as a function of a reference temperature, T^*, the expressions of the energy fluxes entering and exiting the exchanger.

2) Based on a heat balance at steady state, determine the expression of T_2 as a function of the parameters of the problem.

3) Calculate T_2.

4) What is the heat flux exchanged between the two fluids (in kW)?

Data:

$D_1 = 1.5$ T/hr $C_{p1} = 1$ cal/g °C $t_1 = 18$ °C $t_2 = 80$ °C

$D_2 = 1.2$ T/hr $C_{p2} = 1.15$ cal/g °C $T_1 = 95$ °C

Solutions

1) Energy fluxes entering and exiting the exchanger

Incoming fluxes:

– For the flow, D_1: $\phi^E_{D_1} = D_1C_{p1}(t_1 - T^*)$.

– For the flow, D_2: $\phi^E_{D2} = D_2C_{p2}(T_1 - T^*)$.

Outgoing fluxes:

– For the flow, D_1: $\phi^S_{D_1} = D_1C_{p1}(t_2 - T^*)$.

– For the flow, D_2: $\phi^S_{D2} = D_2C_{p2}(T_2 - T^*)$.

2) Expression of T_2

The steady state heat balance is written: $\phi^E_{D_1} + \phi^E_{D2} = \phi^S_{D_1} + \phi^S_{D2}$

Or: $\phi^E_{D1} - \phi^S_{D1} = \phi^S_{D2} - \phi^E_{D2}$

i.e.: $D_1C_{p1}(t_1 - t_2) = D_2C_{p2}(T_2 - T_1)$

From which we derive the expression of T_2: $T_2 = T_1 + \dfrac{D_1C_{p1}}{D_2C_{p2}}(t_1 - t_2)$

3) Calculating T_2

NUMERICAL APPLICATION.–

$D_1 = 1.5$ T/hr $D_2 = 1.2$ T/hr $C_{p1} = 1$ cal/(g °C) $C_{p2} = 1.15$ cal/(g °C)

$t_1 = 18$ °C $t_2 = 80$ °C $T_1 = 95$ °C

$T_2 = 95 + \dfrac{1.5*1}{1.2*1.15}(18 - 80)$ $T_2 = 95 - 67.4$ **T_2 = 27.6 °C**

4) Exchanged heat flux

$$\phi = D_1C_{p1}(t_2 - t_1)$$

NUMERICAL APPLICATION.–

$\phi = 1\,500 * 1 * (80 - 18)$ **ϕ = 107 kW**

EXERCISE 1.8. *Diluting a concentrate*

A reservoir is used to dilute a flow, *d*, of a concentrate, by mixing it with a flow of water, *D*. The flow, *d*, of concentrate contains a solute with a mass concentration, ρ_e. The diluted flow, D_S, is to have a mass concentration ρ_s.

Questions

1) Calculate the flow, $D_{S,}$ leaving the reservoir, if the height, *h*, of the solution in this reservoir was to remain constant.

2) Calculate the mass concentration, ρ_s, of the liquid exiting the reservoir.

Data:

$d = 50$ litres/hr $D = 150$ litres/hr $x_e = 650$ g/liter

Solutions

1) Flow, $D_{S,}$ to keep h constant

The overall mass balance is written as follows: $S\frac{dh}{dt} = D + d - D_S$

h is constant → $\frac{dh}{dt} = 0$

Hence: $D_S = D + d$

NUMERICAL APPLICATION.–

$D_S = 150 + 50$ **D_S = 200 liters/hour**

2) Mass concentration ρ_s

The solute-specific mass balance is written, at steady state:

$$\rho_e\, d = \rho_s\, D_S$$

Hence: $\rho_S = \frac{\rho_e d}{d + D}$

NUMERICAL APPLICATION.–

$$x_S = \frac{650(50)}{50+150} \qquad \boldsymbol{\rho_s = 162.5 \text{ g/liter}}$$

EXERCISE 1.9. *Preparing sweet juice*

You work at a fruit-juice plant.

The production department needs to prepare a flow, *S*, of sweet water with a mass concentration equal to ρ_s.

Your supervisor suggests assuring this preparation by recovering production surplus from the neighboring workshop. This surplus is available in the form of a flow, *E*, of slightly sweet water (mass concentration, ρ_E).

The flow, *E*, recovered from the neighboring workshop, must be mixed with a flow *C* of sugar concentrate having a mass concentration ρ_C.

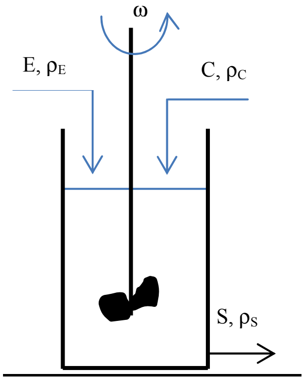

Figure 1.23. *Stirred mixing tank*

Your supervisor asks you to prepare answers to the following issues.

Questions

1) What is the amount of sugar that may be recovered from the flow, *E?*

2) In steady state, what is the relation between *S*, *C* and *E*?

3) Using a sugar specific mass balance, determine the expression of ρ_S as a function of *E*, *C*, ρ_E and ρ_C.

4) To assess the possible variations of ρ_S as a function of the flows *E* and *C*, the expression found in the third question can be considered as a function with two variables, $\rho_S(E,C)$:

a) Determine $\frac{\partial \rho_S}{\partial E}$

b) Given that ρ_E has to be less than ρ_C, what can you deduct therefrom?

c) Calculate $\lim\limits_{E \to \infty} \rho_S$. How can this result be interpreted?

d) Calculate $\lim\limits_{C \to \infty} \rho_S$. How can you interpret this result?

5) What is the condition on ρ_E, so that ρ_S is greater than $\rho_S^* = 450$ g/liter? Is this condition satisfied in the present case?

Your supervisor provides you with the following data:

$E = 325$ liters/hour	$\rho_E = 150$ g/liter
$C = 100$ liters/hour	$\rho_C = 650$ g/liter
$S = 425$ liters/hour.	

Solutions

1) Amount of sugar that may be recovered from E

The amount of sugar contained in the flow *E* is: $Q_S = \rho_S E$.

2) Relation between S, C and E

At steady state: $S = E + C$

3) Expression of ρ_S

The sugar specific mass balance is written, at steady state, as follows:

$$\rho_S\, S = \rho_E\, E + \rho_C\, C$$

Substituting for S: $\rho_S\,(E + C) = \rho_E\, E + \rho_C\, C$

Thus: $\rho_S = \frac{E}{E+C}\rho_E + \frac{C}{E+C}\rho_C$

4) Variations of ρ_S as a function of flows E and C

a) Determining $\dfrac{\partial \rho_S}{\partial E}$

$$\rho_S(E,C) = \frac{E\rho_E + C\rho_C}{E+C}$$

$$\frac{\partial \rho_S}{\partial E} = \frac{\rho_E(E+C) - (E\rho_E + C\rho_C)}{(E+C)^2}$$

$$\frac{\partial \rho_S}{\partial E} = \frac{C(\rho_E - \rho_C)}{(E+C)^2}$$

b) ρ_E being less than ρ_C, then: $\rho_E - \rho_C < 0$

Consequently: $\dfrac{\partial \rho_S}{\partial E} < 0$

ρ_S is therefore a decreasing function of E.

c) Calculating $\lim\limits_{E\to\infty} \rho_S$

$$\rho_S = \frac{E\rho_E + C\rho_C}{E+C} \qquad \lim_{E\to\infty} \rho_S = \rho_E$$

This result is logical since the properties of the flow E will overwhelm the rest.

d) Calculating $\lim\limits_{C\to\infty} \rho_S$

$$\rho_S = \frac{E\rho_E + C\rho_C}{E+C} \qquad \lim_{C\to\infty} \rho_S = \rho_C$$

This result shows that mass concentration of *S* cannot ever exceed that of the concentrate, *C*.

5) Condition on ρ_E so that $\rho_S > \rho_S^*$

$$\rho_S(E,C) = \frac{E\rho_E + C\rho_C}{E+C}$$

$$\rho_S > \rho_S^* \quad \Rightarrow \quad \frac{E}{E+C}\rho_E + \frac{C}{E+C}\rho_C > \rho_S^*$$

Hence: $\rho_E E + \rho_C C > \rho_S^*(E+C)$

i.e.: $\rho_E > \dfrac{E+C}{E}\rho_S^* - \dfrac{C}{E}\rho_C$

Condition satisfied?

NUMERICAL APPLICATION.–

E = 325 liters/hour

C = 100 liters/hour

ρ_E = 150 g/liter

ρ_C = 650 g/liter

S = 425 liters/hour

ρ_S* = 450 g/liter

$$\rho_E > \frac{425}{325}(450) - \frac{100}{325}(650)$$

The condition is then $\rho_E >$ **388.5 g/liter**.

But for the present case ρ_E = 150 g/liter. The condition is therefore not satisfied.

EXERCISE 1.10. *Production of soft drinks*

During production of the MaroCola© soft drink, a sweet juice is prepared by mixing a flow, *d*, of sugar syrup in a stirred tank with a flow, *D*, of pure water. In order to accelerate dissolution, the tank is heated by steam. Figure 1.24 presents the layout of the tank used for this purpose. The definition of the different parameters is as follows:

D is the mass flow of water at input, temperature T_0

J is the mass flow of sweet water produced, temperature T

C is the mass fraction of sugar in the tank (kg of sugar/kg of solution)

C_0 is the mass fraction of sugar in the flow of syrup (kg of sugar/kg of syrup)

V is the mass flow of steam. Same as that of condensed water

Λ is the latent heat of condensation of the saturated steam

H100 is the mass enthalpy of condensed water, expressed at the point of condensation

Q denotes the flux of heat losses

h denotes the liquid height in the stirred tank

S is the cross-section of the tank

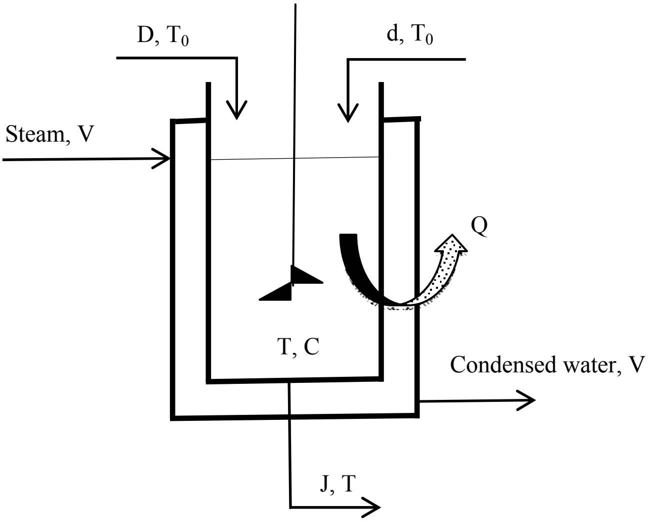

Figure 1.24. *Production of sweet juice*

Questions

Your supervisor asks you to:

1) Establish, in steady state, the relation that must exist between *D*, *d* and *J*, so that the liquid height in the reservoir remains constant.

2) Deduce therefrom the value of *d* in steady state.

3) Determine, under these conditions, the mass fraction, *C*, of the juice, *J*, produced as a function of the other parameters of the problem.

4) Calculate *C*.

5) What would be the necessary energy flux in steady state?

6) Deduce therefrom the flow of steam to be used in steady state.

Your assistant technician has already gathered the available data for you:

Q, flux of heat losses: $Q = 1{,}500$ W

Λ, latent heat of condensation of the saturated steam: $\Lambda = 640$ kcal/kg

J, mass flow of sweet water to be produced: $J = 2.055$ tons/day

T, temperature of J and of the liquid in the tank: $T = 60$°C

D, mass flow of water at input: $D = 1.8$ tons/day

T_0, temperature of D: $T_0 = 22$°C

C_0, mass fraction of sugar in the syrup flow, d: $C_0 = 0.85$ kg of sugar/kg of syrup

C_{pD}, specific heat of the water: $C_{pD} = 1$ cal/g°C

C_{pd}, specific heat of the syrup, d: $\mathrm{C_{pd}} = 1.3$ cal/g°C

C_{pJ}, specific heat of the sweet water, J: $C_{pJ} = 1.18$ cal/g°C

Solutions

1) Mass inputs and outputs

a) Overall inputs and outputs

- The overall inputs are D and d.

- There is only one overall output, J.

b) The sugar inputs and outputs

- One sugar input: dC_0.

- One sugar output: JC.

2) Mass balances in transient state

Two types of material balances are to be considered:

– the overall balance;

– the sugar-specific balance.

a) Overall mass balance in transient state

It is written as follows: $A = \sum_{i=1}^{n} e_i - \sum_{j=1}^{m} s_j + G$

As there is no generation of mass in the system, we have $G = 0$.

Moreover, the "accumulation" term is given by: $A = \frac{dM}{dt} = \rho S \frac{dh}{dt}$,

where S is the reservoir cross-section: $S = \pi R^2$

Hence the overall mass balance in transient state is: $\rho S \frac{dh}{dt} = D + d - J$

b) The sugar-specific mass balance in transient state

As there is no sugar generation in the system, the sugar-specific balance is written:

Accumulation of sugar in the tank = Sugar inputs – Sugar outputs

Moreover, the "accumulation" term is given by: $A = \frac{d(M_S)}{dt}$,

where M_S is the mass of sugar contained in the tank at time t. M_S is given by:

$$M_S = C\,M = C\,\rho\,V = C\,S\,h\,\rho$$

Hence: $A = \rho S \frac{d(hC)}{dt}$

The sugar inputs are:

– For D: 0.

– For d: $C_0 d$.

The only output is J, the sugar content of which is: $C\,J$.

Thus, the sugar-specific balance equation, in transient state, is written as:

$$\rho S \frac{d(hC)}{dt} = C_0 d - CJ$$

3) Value of d at steady state

At steady state, the *overall balance* equation becomes:

$$D + d - J = 0$$

Hence: $d = J - d$

NUMERICAL APPLICATION.–

$d = 2.55 - 1.8$ **d = 0.75 tons/day**

4) Concentration, C, of the juice produced as a function of the other parameters

At steady state, the specific balance becomes: $C_0\, d - C\, J = 0$

Hence:

$$C = C_0 (1 - \frac{d}{J})$$

5) Calculating C

NUMERICAL APPLICATION.–
$C_0 = 0.85$ kg of sugar/kg of syrup
$J = 2.55$ tons/day
$d = 0.75$ tons/day

$$C = 0.85(1 - \frac{0.75}{2.55})$$

***C* = 0.71 kg of sugar/kg of juice**

6) The energy inputs and outputs

a) The energy inputs

They are conveyed by the flows, *D*, *d* and *V*:

– For D: $E_D = D\, C_{pD}\, (T_0 - T^*)$.

– For d: $E_d = d\, C_{pd}\, (T_0 - T^*)$.

– For V: $E_V = V\, \Lambda$.

b) The energy outputs

They are conveyed by the flow, *J*, and by the heat losses, *Q*:

– For *J*: $E_J = J\,C_{pJ}\,(T - T^*)$.

– The flux of the heat losses is: *Q*.

7) Energy balance in transient state

It is written:

$$A = \sum_{i=1}^{n} e_i - \sum_{j=1}^{m} s_j + G$$

The “generation” term is nil.

The “accumulation” term is the derivative with respect to time of the enthalpy of the liquid contained in the reservoir. The latter is given by:

$$E_M = M\,C_{pJ}\,(T - T^*) = \rho\,V\,C_{pJ}\,(T - T^*) = \rho\,S\,h\,C_{pJ}\,(T - T^*)$$

Hence:

$$A = \frac{dE_M}{dt} = \rho S C_{pJ} \frac{d}{dt} h(T - T^*)$$

Thus, the energy balance equation, in transient state, is written:

$$A = E_D + E_d + E_V - E_J - Q$$

$$A = D\,C_{pD}\,(T_0 - T^*) + d\,C_{pd}\,(T_0 - T^*) + V\,\Lambda - J\,C_{pJ}\,(T - T^*) - Q$$

$$A = (D\,C_{pD} + d\,C_{pd})T_0 - (D\,C_{pD} + d\,C_{pd})T^* + V\,\Lambda - J\,C_{pJ}\,T + J\,C_{pJ}\,T^* - Q$$

$$A = (D\,C_{pD} + d\,C_{pd})T_0 - (D\,C_{pD} + d\,C_{pd} - J\,C_{pJ}\,)T^* + V\,\Lambda - J\,C_{pJ}\,T - Q$$

Yet:

$$J\,C_{pJ} = D\,C_{pD} + d\,C_{pd}$$

Therefore:

$$A = (D\ C_{pD} + d\ C_{pd})T_0 - J\ C_{pJ}\ T + V\ \Lambda - Q$$

Hence, the balance equation in transient state:

$$\rho S C_{pJ} \frac{d}{dt} h(T - T^*) = \left(DC_{pD} + dC_{pd}\right)T_0 - JC_{pJ}T + V\Lambda - Q$$

8) Flux of energy necessary in steady state

The heat balance in steady state is written:

$$\left(DC_{pD} + dC_{pd}\right)T_0 - JC_{pJ}T + V\Lambda - Q = 0$$

From this, one can draw the expression of the flux that would be necessary. The latter is given by the product, $V\Lambda$:

$$\phi = V\,\Lambda = Q + J\ C_{pJ}\ T - (D\ C_p D + d\ C_p d)\ T_0$$

NUMERICAL APPLICATION.–

$Q = 1{,}500$ W

$d = 0.75$ tons/day	$C_{pd} = 1.3$ cal(/g°C)	$J = 2.55$ tons/day
$C_{pJ} = 1.18$ cal/(g°C)	T = 60°C	D = 1.8 tons/day
$C_{pD} = 1$ cal/(g°C)	$T_0 = 22$°C	

$$\phi = 1.5 + \frac{2.55*1000}{24*3600}*1.18*4.18*60 - \left(\frac{1800}{24*3600}*4.18 + \frac{750}{24*3600}*1.3*4.18\right)*22$$

ϕ = 7.28 kW

9) The steam rate to be used

The steam rate, V, is obtained from the heat balance equation, as follows:

$$V = \frac{Q + JC_{pJ}T - \left(DC_{pD} + dC_{pd}\right)T_0}{\Lambda}$$

NUMERICAL APPLICATION.–

$\Lambda = 640$ kcal/kg

$\phi = 7.28$ kW

$$V = \frac{\phi}{\Lambda}$$

$$V = \frac{7.28}{640 * 4.18} * 3600 * 24$$

$V =$ **235 kg/day**

EXERCISE 1.11. *Air-conditioning/heating a room*

We will consider the computer room represented in Figure 1.25.

We wish to establish the heat balance of this room, with a view to determining the compensation, Q, that the air-conditioning/heating system would need to provide in order to maintain the room at constant temperature, T.

We will assume that the heat loss through the floor of the room is constant and equal to Q_S.

The room admits several windows in contact with the outside environment, the heat loss of which is noted Q_F.

The walls and ceiling of the room are also in contact with the outside. Their overall energy losses can be represented by the flux, Q_M.

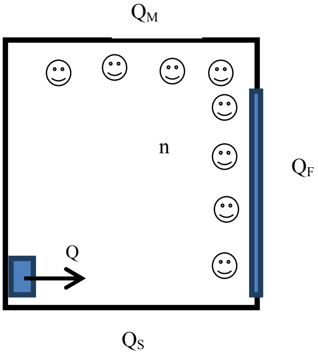

Figure 1.25. *Computer room*

We wish to establish this balance during the work period, when a number, n, of individuals are at their workstations; the door to the room is closed during this period, such that there are no losses through the door.

Questions

The manager of the computer center asks you to:

1) Give the expressions of the generated fluxes.

2) Identify the energy fluxes entering and exiting the system.

3) Establish the heat balance of the room at steady state.

4) Deduce therefrom the expression that Q must satisfy in stationary state.

The manager of the computer center informs you that:

– Each pairing (person-computer) exudes a heat flux p_c.

– There are as many computers in the room as there are people.

Solutions

1) Expressions of the generated flux

$$Q_G = Q + Q_n$$

Whereby: $Q_n = n\, p_c$

i.e.: $Q_G = Q + n\, p_c$

2) Flux of energy, entering and exiting the system

– Entering fluxes: 0.

– Exiting fluxes: Q_M, Q_F and Q_S.

3) Heat balance of the room in steady state

$$Q_M + Q_F + Q_S = Q + n\, p_c$$

4) Expression of Q at steady state

$$Q = Q_M + Q_F + Q_S - n\, p_c$$

EXERCISE 1.12. *Calculating the heating fluid flow*

We wish to heat a constant flow, D_1, of a given fluid, using a flow, D_2, of a hot fluid.

D_1 enters the exchanger at a known temperature, t_1, It is to be heated to a temperature, t_2, which is also known.

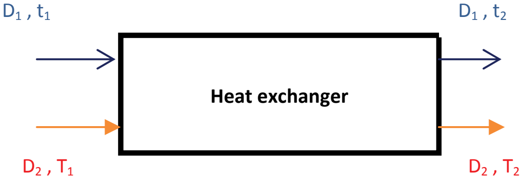

Figure 1.26. *Heat exchanger. For a color version of this figure, see www.iste.co.uk/benallou/energy1.zip*

The heating fluid, of constant flow, D_2, which is to be determined, enters the exchanger at temperature T_1 and leaves the exchanger at temperature T_2.

Questions:

1) Using a heat balance in steady state, determine D_2 as a function of the parameters of the problem.

2) Calculate D_2.

3) Calculate the flux of heat exchanged between the two fluids.

Data:

$D_1 = 0.5$ T/hr $Cp_1 = 0.8$ cal/(g °C) $Cp_2 = 1$ kcal/(kg °C)

$t_1 = 20$ °C $t_2 = 95$ °C $T_1 = 95$ °C $T_2 = 80$ °C

Solutions

1) Expression of D_2 in steady state

The heat balance in steady state is written as follows:

$$D_1 C_{p1} (t_1 - T^*) + D_2 C_{p2} (T_1 - T^*) = D_1 C_{p1} (t_2 - T^*) + D_2 C_{p2} (T_2 - T^*)$$

Hence: $D_2 C_{p2} (T_1 - T_2) = D_1 C_{p1} (t_2 - t_1)$

i.e.: $D_2 = \frac{C_{p1}}{C_{p2}} \frac{t_2 - t_1}{T_1 - T_2} D_1$

2) Calculating D_2

NUMERICAL APPLICATION.–

$D_1 = 0.5$ T/hr $C_{p1} = 0.8$ cal/(g °C)

$t_1 = 20$ °C $t_2 = 95$ °C

$C_{p2} = 1$ cal/(g °C) $T_1 = 95$ °C $T_2 = 80$ °C

$D_2 = \left(\frac{0.8}{1}\right)\left(\frac{95-20}{95-80}\right)0.5$ **D_2 = 2 T/hr**

3) Heat flux transferred between the two fluids

The flux of the heat exchanged between the two fluids may be calculated from one of the following expressions:

$$\phi = D_2 C_{p2} (T_1 - T_2)$$

Or:

$$\phi = D_1 C_{p1} (t_2 - t_1)$$

We will retain:

$$\phi = D_2 C_{p2} (T_1 - T_2)$$

NUMERICAL APPLICATION.–

$D_2 = 2$ T/hr $C_{p2} = 1$ cal/(g °C)

$T_1 = 95$ °C $T_2 = 80$ °C

$\phi = 2{,}000 \times 1 \times (95 - 80)$ **ϕ = 30,000 kcal/hr**

EXERCISE 1.13. *Mass balance on a reactor*

We will consider a reactor in which a chemical reaction takes place between two binary fluids. The first binary fluid has a flow D_1 composed, essentially, of component *A* and traces of component *C* (mass fraction, ω_{A1}). The second has a flow $D_{2,}$ composed, essentially, of component *B* and an inert component (mass fraction, ω_{A2}).

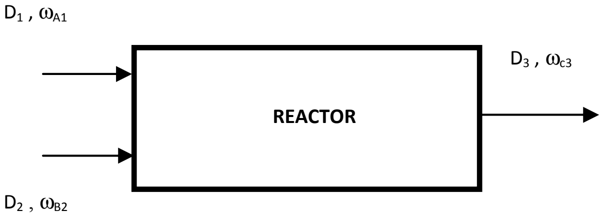

Figure 1.27. *Chemical reactor*

The chemical reaction within the reactor is first-order and complete. It gives rise to product *C*.

Questions

1) Identify the fluxes, j_E, "mass input" and j_S, "mass output".

2) Calculate in kilograms per hour the amounts of *A* and *B* that enter the reactor.

3) Using a global mass balance, calculate D_3.

Data:

$D_1 = 150$ kg/hr	$\omega_{A1} = 0.92$
$D_2 = 200$ kg /hr	$\omega_{B2} = 0.46$

Solutions

1) Mass fluxes, entering and exiting

– Entering flux: $j_E = D_1 + D_2$.

– Exiting flux: $j_S = D_3$.

2) Amounts of A and B entering the reactor

Amount of *A*: $Q_A = D_1\ \omega_{A1}$

NUMERICAL APPLICATION.–

$Q_A = 150 \times 0.92$ **$Q_A = 138$ kg/hr**

Amount of *B*:

$$Q_B = D_2\ \omega_{B2}$$

NUMERICAL APPLICATION.–

$Q_B = 200 \times 0.46$ **$Q_B = 92$ kg/hr**

3) Calculating D_3

The overall balance in steady state gives, directly, the expression of D_3:

$$D_3 = D_1 + D_2$$

NUMERICAL APPLICATION.–

$D_3 = 150 + 200$ **$D_3 = 350$ kg/hr**

1.9. Reading: seawater desalination

In desert areas close to the coast where water resources are scarce, seawater desalination is used to meet the drinking water needs of the populations and possibly the productive sectors.

1.9.1. *Level of purification*

Seawater desalination is a process that makes it possible to obtain drinkable water from seawater. The freshwater quality, of course, depends on the objective: for beverage use, drinkable water will be required. For use in agriculture or industry, it will be possible to use water that is to a certain extent freshwater. Obviously, the more extensive the purification, the higher the production cost of the freshwater.

1.9.2. *Water sources used*

The waters used do not necessarily come from the sea. In reality, the source used is that located closest to the place of use: brackish waters from lakes or rivers, as well as surface waters or groundwaters are also used as raw inputs.

1.9.3. *Water characteristics according to the source*

The waters are characterized by their salt content. For seawater, this content is around *35 g/liter*. The salt content of course differs according to the sea or ocean under consideration. Thus, in the Atlantic and the Pacific, it is around *30 g/liter*, in the Mediterranean it is around *36 g/liter*; and in the Persian Gulf and the Red Sea, it exceeds *40 g/liter*.

1.9.4. *Several techniques*

There are several processes used to extract freshwater from seawater or brackish water. The most commonly used are *distillation*, *electrodialysis* and *reverse osmosis*. As we briefly explained in the illustration presented in section 1.7.2.1, reverse osmosis is based on a pressurized ultrafiltration through membranes that are said to be “semi-permeable”; in other words these membranes have micropores (“micro-holes”) that are so small that they block the passage of salts and only let water molecules through.

The distillation-based procedures are used above all when there is steam available as a subproduct from another process. We then speak of cogeneration.

1.9.5. *Energy cost: the decisive factor*

The cost price of desalinated water depends directly on the amount of energy consumed by the desalination process used. The theoretical minimum amount of energy required for desalination is that which would be obtained in an isentropic operation. This minimum is of the order of $565\ Wh/m^3$.

We know, however, that it will not be possible to achieve this minimum amount in practice.

As an example, reverse osmosis, which is currently the least energy-consuming technique, requires around *ten times this minimum*. Indeed, the average energy consumption for reverse osmosis desalination is $4 - 5\ kWh/m^3$.

Nevertheless, reverse osmosis remains the most frequently used procedure at present, given that its energy consumption remains low compared to that of competing processes. These include a set of techniques based on distillation:

– *Flash distillation* in one or more stages (with single or multiple effects) provides water that presents relatively good characteristics. Its energy consumption is quite high: between *10 and 15 kWh/m*^{3}.

– *Vapor-compression distillation*: its average energy cost is between *6 and 8 kWh/m*^{3}.

1.9.6. *A promising outlook*

There are other techniques that may prove of interest, depending on future developments. These are techniques whereby energy supply is covered by solar- or wind-powered systems.

These include, in particular:

– *Distillation using a solar concentrator* where the energy needed for the water to evaporate is supplied by a set of parabolic mirrors that concentrate the solar radiation onto one focal point: the boiler. The brackish water passing into the focal point evaporates. Its subsequent condensation provides the desired freshwater.

– *Distillation in solar collectors*, where the brackish water is introduced directly into the chambers composing the flat-plate collectors. The latter may be multi- or single-stage. In multi-stage collectors, brackish water is fed into the top of a staircase-shaped absorber. Whilst heating up, the water fed to the first step on the staircase flows towards the second step, and so on. During the course of this passage from one stage (step) of the absorber to the next, the water temperature continues to increase and evaporation begins. The droplets are then recovered by the glazing, which leads the collected condensates to a collector which will supply desalinated water. In this type of system, the solar energy is transformed directly into heat and the condensation forms directly on the glazing. Solar distillers such as this present the advantage of being relatively simple to build and operate. However, the fact that these systems do not use solar radiation concentration means that the temperature levels achieved remain limited. As a result, productivity is modest, but this could be adequate for freshwater production for the needs of just one or a few homes.

– *Electrodialysis*, where the separation between water and salt is assured by an electric current, which migrates the ions to the electrodes. This procedure is ideal where low-cost electricity is available, such as electricity generated from solar photovoltaic fields.

– *Thus, the coupling of electrodialysis with solar generated electricity* presents great potential for the future.

– *Likewise, wind generators can be paired with reverse-osmosis*, where the energy required for reverse osmosis is fully or partially provided by a wind farm. This is the ideal solution for windy regions where large quantities of freshwater can be generated.