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Introduction to Structural Analysis

The teaching objectives for this chapter are as follows:

– the role of structural analysis teaching;

– the concept of a structure;

– the development of structural analysis methods;

– the distinction between categories of structures;

– the calculation of a statically indeterminate structure.

This chapter is descriptive and gives a general presentation of the preliminary aspects of statically determinate structure analysis. In the first part, we present the concept of a structure, the objectives to be achieved during structural analysis teaching and the history of its development. In the second part, we look at structural classification based on the structural dimension. Finally, we give the calculation of the degree of static indeterminacy of the structures.

1.1. Introduction

The primary role of structural study and analysis is to determine the internal actions and the support reactions of a structure subjected to mechanical loads, imposed deformations and settlements of supports. An action can mean either a force and/or a moment. In the same way, a deformation can mean a displacement and/or a rotation.

Structures are classified into two broad categories: (1) statically determinate structures and (2) statically indeterminate structures. The three static equilibrium equations are used to analyze statically determinate structures. In this case, the support reactions can be determined using only static equations. As a result, internal actions, such as a bending moment, a torsion moment, a shear force, and a normal force, can be deduced using the internal equilibrium equations.

On the contrary, for statically indeterminate structures equilibrium equations are not sufficient to calculate the unknowns of the problem. This means that the number of unknowns (the support reactions) is strictly greater than that of the equations. The difference between the number of unknowns of the problem and the equations is called the degree of static indeterminacy of the system or structure.

1.2. Concept of a structure

The word "structure" in the field of mechanics indicates any solid body that satisfies the following conditions:

1) the presence of a material characterized by mechanical properties;

2) the structure studied has a geometry or a form;

3) the structure is linked to the external environment through the supports;

4) the body is subjected to external loading.

The definition of a structure is clearly projected in Figure 1.1.

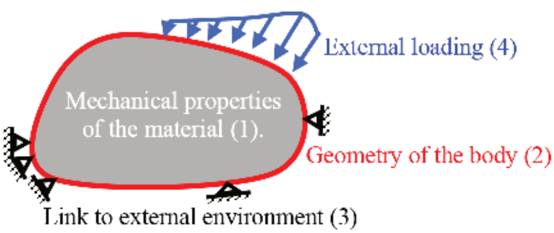


Figure 1.1. *The conditions for defining a structure*[1]

1.3. Structural analysis

Structural analysis is the calculation of responses when a structure is subjected to a set of external loads. In general, the structural response is characterized by the determination of internal actions or deflections at any point in the structure. To achieve this goal, it is necessary to use a mathematical method, an experimental test, or an analytical or numerical model.

1 All of the figures in this chapter are available to view in full color at www.iste.co.uk/khalfallah/analysis1.zip.

In addition, the main role of structural analysis is to study structural performance when it is subjected to the effects of the external environment, such as a set of external actions, a movement of the support or a temperature change. The common features of structural performance, which are important during the design phase, are as follows:

– internal forces: axial force, shear force, bending moment and torsion moment;

– support reactions;

– the deflections that occur after external loads are applied.

The classical methods of structural analysis have calculation limitations that depend in general on the geometry of the structure or the applied loading (standard problems). Recently, the evolution of computing machines has led to an enormous development in methods of analysis, especially matrix methods.

The purpose of the two categories of calculation methods is to arrive at a structural design that responds to criteria of resistance and economy, simultaneously.

1.4. History of structural analysis

Structural design and analysis is an ancient art and many civilizations have contributed to its development. For example, the Egyptian pyramids were built around 2000 BC with a civilization of the time.

The first constructions were based on empirical experiments and rules constituting the fundamental basis of structural analysis. In this subject, the first principles of statics appeared toward 300–400 BC. Thereafter, the Romans contributed to the evolution of structural analysis by using stone in constructions up to 500 AD and by developing new forms of construction in arches and vaults.

During the Middle Ages (500–1500), the Greeks and Romans collaborated in the development of structural analysis through the construction of cathedrals. During the Renaissance period, Leonardo Da Vinci (1452–1519) described the first theories of structures. However, Galileo (1564–1642) initiated the theory of mechanics of materials by studying the breaking of cantilever beams. Then Hooke (1635–1703) established the law of material elasticity for the first time. Johann Bernoulli (1667–1748) formulated the principle of virtual displacements. Daniel Bernoulli (1700–1782) studied elastic curves and strain energy in bending. Navier (1785–1836) initiated statically indeterminate structural analysis, and then Coulomb (1736–1806) published his work on the resistance of materials, in which he analyzed flexed beams.

The noble period of structural analysis is the 19th Century. During this period, the theoretical foundations of the mechanics of materials and structures were widely developed. We can distinguish the following developments:

1) the principles of the analysis of statically determinate truss systems, Whipple (1804–1888);

2) the theorem of three moments, Clapeyron (1799–1864);

3) the theorem of reciprocal displacements, Maxwell (1831–1879);

4) the study of influence lines, Mohr (1835–1918);

5) Castigliano's theorem (1847–1884);

6) the slope deflection method, Maney (1888–1947);

7) the moment distribution method, Cross (1885–1959);

8) the relaxation method, Southwell (1888–1970).

Several researchers in structural mechanics subsequently participated in the development of matrix and modern analysis methods. In addition, the 19th Century saw the development of new materials, new techniques and structures with complex shapes.

Over the past few decades, there have been considerable developments in the field of digital mechanics after the revolution in computer science and digital methods.

1.5. Classification of structures

Choosing which type of structure to use depends on several factors, such as the operating characteristics, the condition of resistance, the economy criterion, the aesthetics and the availability of the construction material. It is worth emphasizing that the loading system plays a major role in selecting the type of structure.

The structures used in civil engineering can be divided into six categories. This classification is based on the structural dimension and the law of stress distribution in the structure. For these reasons, they can be classified as follows.

1.5.1. *Plane trusses*

The plane trusses are systems made up of several interconnected bars at the articled joints. The bars of a truss are subjected only to traction or compression.

When the elements of a truss and the load are applied to a single plane, the truss is called a plane (Figure 1.2).

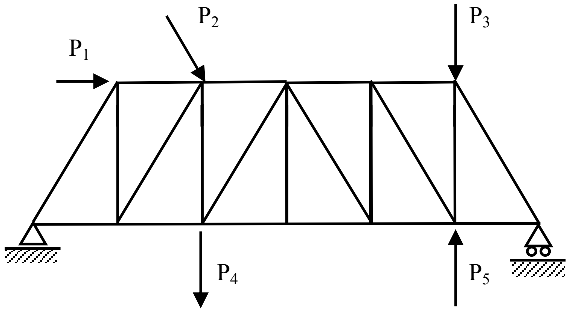


Figure 1.2. *Truss*

In this type of structure, it is only the nodal loads that can be considered in the calculation and distributed loads can't be considered.

In the same context, we can distinguish cable structures that are subject only to pure traction. This category is used in the construction of bridges or in the floors of special structures (Figure 1.3).

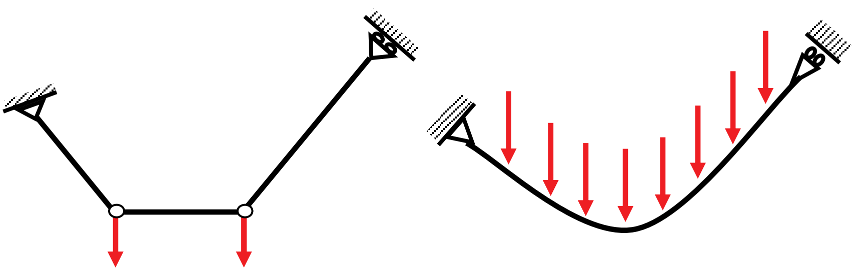


Figure 1.3. *Membranous structures*

In general, analyzing a truss cannot be established by a consideration of the plane due to the size of the structure and the applied loading system. In this case, it can be studied as a spatial structure and the load is distributed according to the three geometrical dimensions. Similar to plane systems, external forces generate only normal stresses in the bars of the truss (Figure 1.4).

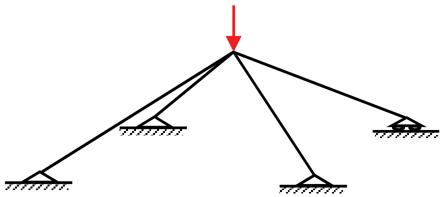


Figure 1.4. *Spatial trusses*

1.5.2. *Beams*

A beam is a rectilinear structural element. In general, beams are solicited by loads applied on the same geometric plane and perpendicular to the middle axis. In this type of structure, the applied loads cause bending and shearing forces. The normal force is neglected when the applied loads are perpendicular to the axis of the beam (Figure 1.5).

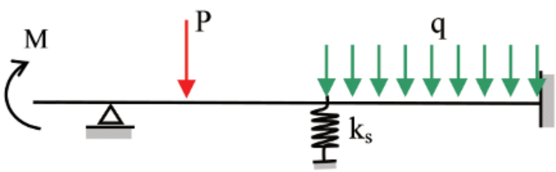


Figure 1.5. *Beams*

The internal actions at any section are generally a bending moment, a tangential force and a normal force (Figure 1.6).

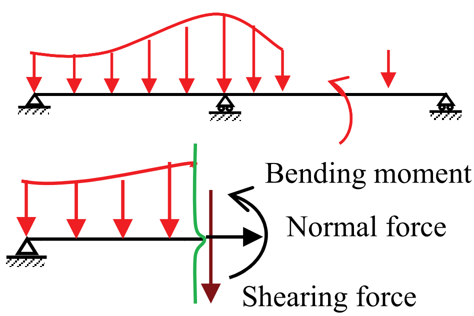


Figure 1.6. *Internal actions*

1.5.3. *Frames*

Frames are systems composed of several rectilinear and/or oblique elements interconnected by rigid joints. In this case, external loads can be applied to the joints and on structural elements. In plane frames, the elements and the loads are linked to a single plane. The structural elements are acted on by bending, shear force and normal force due to the external loading system (Figure 1.7).

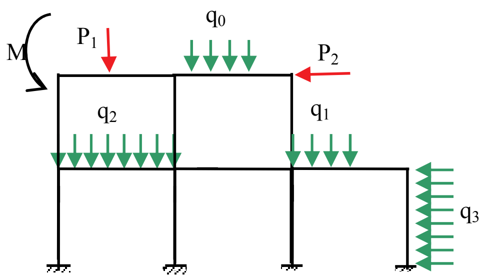


Figure 1.7. *Frames*

Space frames are the most appropriate in civil engineering buildings. The loading system is applied along the structural elements and at the joints. Generally, the loads generate bending, shear, normal force and torsion as internal actions. (Figure 1.8).

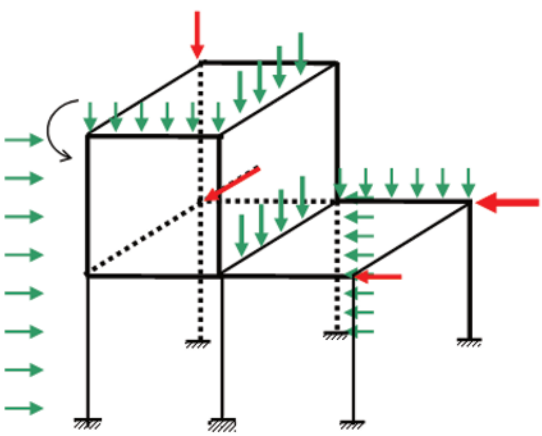


Figure 1.8. *Three-dimensional frames*

1.5.4. *Crossbeams*

The crossbeams are composed of a layer of rectilinear elements arranged on the same plane constituting a series of beams. The loading system is applied perpendicular to the structural plane. The internal loads that can be envisaged are bending, shear, normal force and torsion. The crossbeams are designed to bear heavy loads of slabs, sports halls, auditoriums, etc. (Figure 1.9).

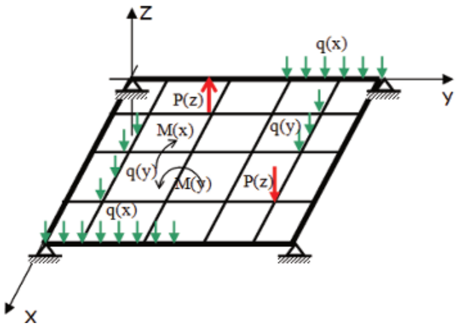


Figure 1.9. *Crossbeam structures*

1.5.5. *Arches*

Arches are structures with an inverted curvature in relation to cable structures (Figure 1.10). Arches are designed to bear loads with a long span and they must be rigid enough to maintain their shapes. The applied load generates a normal force, a shear force and a bending moment that are considered during the design phase. Arches are used in the construction of bridges, domes, sports halls, department stores, etc.

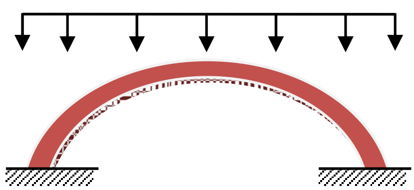


Figure 1.10. *A given arch*

1.6. Static indeterminacy of structures

Structures are grouped into two categories: (1) statically determinate structures and (2) statically indeterminate structures. For statically determinate structures, static equations are sufficient to determine the support reactions and consequently the internal actions: the bending moment, the shear force and the normal force.

In the opposite case, equilibrium equations are insufficient to determine the support reactions. This means that the number of unknowns of the given problem is much greater than that of the independent equations.

Every structure is in equilibrium under the effect of the applied loads when the result of forces and the sum of the moments on a point is zero. For plane structures, the three static equations must be fulfilled, namely:

$$\sum F_x = 0 \quad [1.1]$$

$$\sum F_y = 0 \quad [1.2]$$

$$\sum M_i = 0 \quad [1.3]$$

Therefore, the plane structure is called statically determinate if equations [1.1]–[1.3] are sufficient to determine the support reactions; otherwise it is called statically indeterminate. To analyze a statically indeterminate structure, it is necessary to add other independent equations so that the number of unknowns is equal to the degree of static indeterminacy of the problem. One thus defines the degree of static indeterminacy of a structure by the difference between the number of unknowns and that of the static equations.

1.6.1. *Trusses*

In the case of truss systems, the nature of the support designates the number of corresponding support reactions. However, in structural mechanics, a hinge is used with a vertical and horizontal reaction and a roller with a single vertical reaction (Figure 1.11).

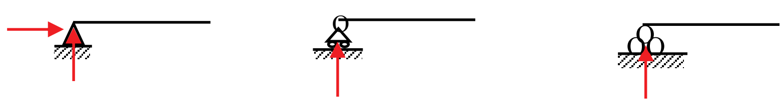


Figure 1.11. *Types of support and associated reactions*

In general, for a truss, which has b bars, n joints and r support reactions, the nature of the structure can be identified as following:

The structure is statically determinate externally if

$$b-(2n-3)=0 \quad [1.4]$$

It is statically indeterminate, "*f*", if

$$f=b-(2n-3)\succ 0 \quad [1.5]$$

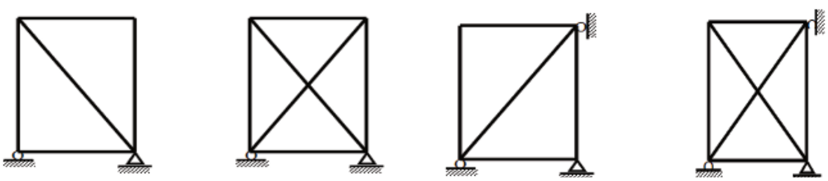
It is unstable when

$$f=b-(2n-3)\prec 0 \quad [1.6]$$

f is the degree of static indeterminacy of the system or structure.

EXAMPLE 1.1.–

Analyze the static indeterminacy of trusses (Figure 1.12).



(a) (b) (c) (d)

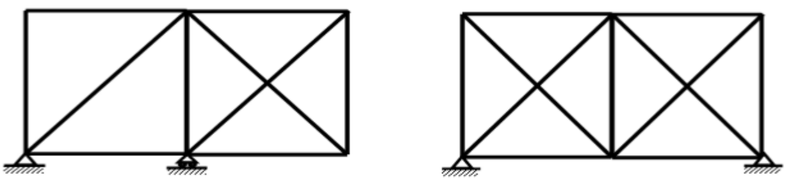
Figure 1.12. *Different structures of strusses*

System	(a)	(b)	(c)	(d)
b	5	6	5	6
$2n-3$	5	5	5	5
r	3	3	4	4
f	0	1	0	1

System	Internal analysis	External analysis
(a)	Statically determinate	Statically determinate
(b)	Once statically indeterminate	Statically determinate
(c)	Statically determinate	Once statically indeterminate
(d)	Once statically indeterminate	Once statically indeterminate

EXAMPLE 1.2.–

Analyze the static indeterminacy of trusses (Figure 1.13).



(I) (II)

Figure 1.13. *Given trusses*

– System (I)

$b = 10,\ 2n - 3 = 9$ and $r = 3$

The system is statically determinate externally and once statically indeterminate internally.

– System (II)

$b = 11, 2n - 3 = 9$ and $r = 4$

So, the system is once statically indeterminate externally and twice statically indeterminate internally.

1.6.2. *Beam and frames*

Aside from hinged and roller supports, beams and frames may contain recesses that are characterized by three independent actions (a horizontal reaction, a vertical reaction and a bending moment) (Figure 1.14).

Figure 1.14. *Reaction at fixed node*

In the same way, for a frame or a beam with n joints, b bars, r support reactions and k additional conditions (hinges), the degree of static indeterminacy of the system is given by the following formula:

$$f = (3b + r) - (3n + k) \quad [1.7]$$

Determine the degree of static indeterminacy of the frames (Figure 1.15).

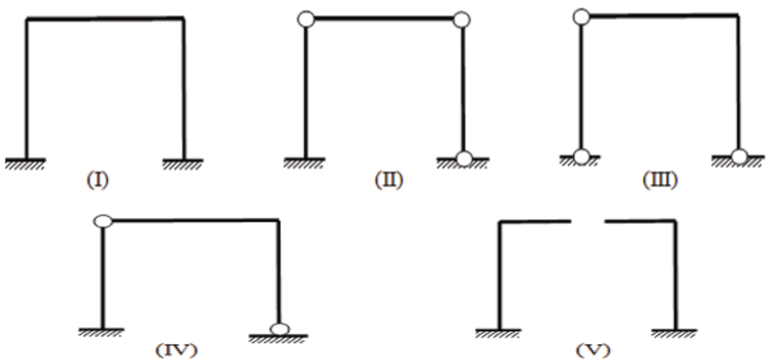


Figure 1.15. *Single span frames*

System	(I)	(II)	(III)	(IV)	(V)
b	3	3	3	3	3
r	6	5	4	4	6
n	4	4	4	4	4
k	0	2	1	1	3
f	3	0	0	0	0

EXAMPLE 1.3.–

Determine the degree of static indeterminacy of the frames (Figure 1.16).

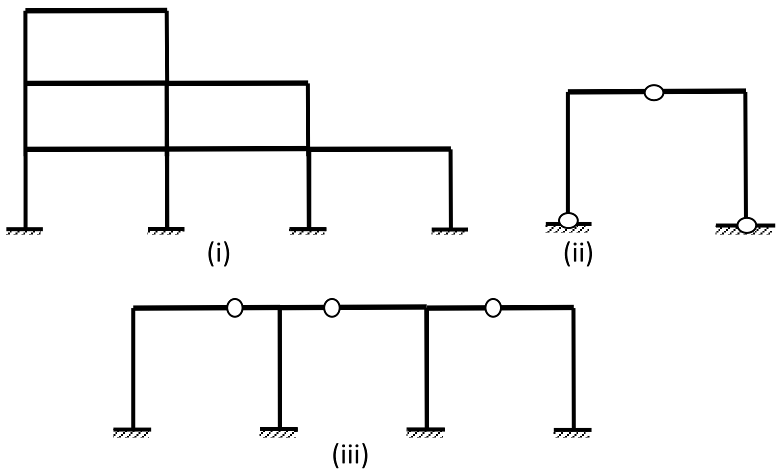


Figure 1.16. *Single and multiple span frames*

Applying equation [1.7], the degree of static indeterminacy of each system is:

– Structure (i)

$f = (3b + r) - (3n + k) = (3 \times 15 + 12) - (3 \times 13 + 0) = 18$, once statically indeterminate.

– Structure (ii)

$f = (3 \times 3 + 4) - (3 \times 4 + 1) = 0$, the system is statically determinate.

– Structure (iii)

$f = (3 \times 7 + 12) - (3 \times 8 + 3) = 6$, once statically indeterminate.

In the case of beams and when the vertical loads are perpendicular to the bar axes, the degree of static indeterminacy is written in the form

$$f = (2b + r) - (2n + k) \qquad [1.8]$$

EXAMPLE 1.4.–

Evaluate the degree of static indeterminacy of the beams given by Figure 1.17.

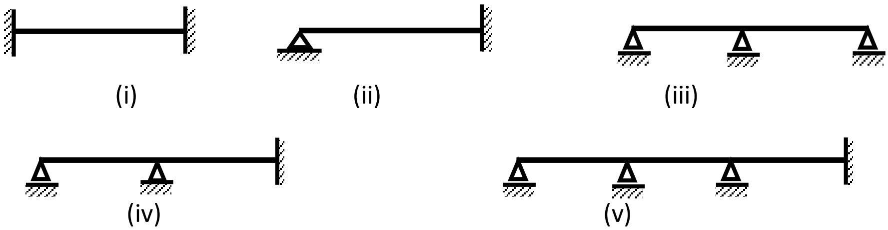


Figure 1.17. *Given beams*

– Beam (i): $f = (2b + r) - (2n + k) = (2 \times 1 + 4) - (2 \times 2 + 0) = 2$, once statically indeterminate.

– Beam (ii): $f = (2 \times 1 + 3) - (2 \times 2 + 0) = 1$, once statically indeterminate.

– Beam (iii): $f = (2 \times 2 + 3) - (2 \times 3 + 0) = 1$, once statically indeterminate.

– Beam (iv): $f = (2 \times 2 + 4) - (2 \times 3 + 0) = 2$, once statically indeterminate.

– Beam (v): $f = (2 \times 3 + 5) - (2 \times 4 + 0) = 3$, once statically indeterminate.

1.7. Sign convention

To establish the structural analysis methods presented in this work, we adopt *a priori* the following sign convention:

– normal force is positive when it causes traction. On the contrary, it is negative if it generates a compression;

– the bending moment is considered positive if its rotation is counterclockwise;

– the shear force is assumed to be positive if it is oriented downwards to the left side.

This sign convention is represented in Figure 1.18. The actions shown are considered positive.

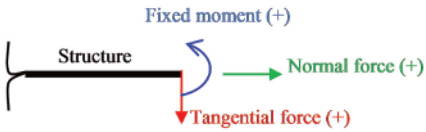


Figure 1.18. *Sign conventions of internal actions*

In the same context, the kinematic sign convention can be deduced based on the sign convention of the actions cited above (Figure 1.19).

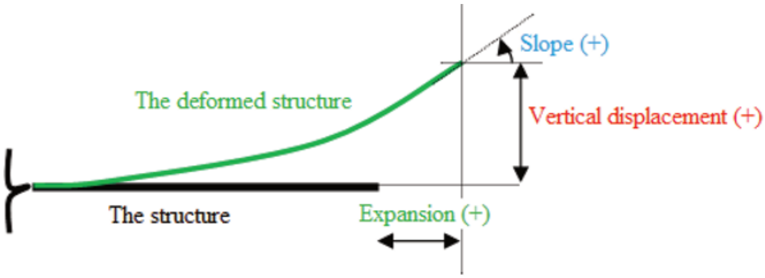


Figure 1.19. *Sign conventions of deflections*

1.8. Conclusion

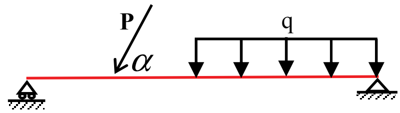
In this chapter, we presented the preliminary conceptual components of the structural analysis material. The teaching aims first of all to determine the internal actions at every point of a structure. This can be used for the design and dimensioning of the different elements of a structure.

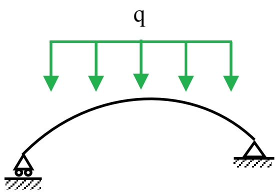
The distinction of the static indeterminacy of a structure plays a very important role in leading to the appropriate method of analysis. This is the distinction between the methods of analysis for statically determinate and statically indeterminate structures. The second part of the book is devoted to presenting methods of analysis for statically determinate structures. The teaching in this chapter makes it possible to differentiate between the nature of the structure and its application in the industrial sector. Finally, this chapter proposes a sign convention for internal actions and their corresponding kinematic consequences.

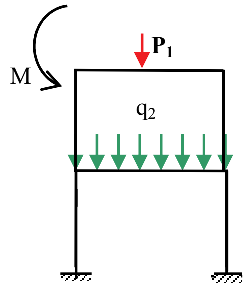
1.9. Problems

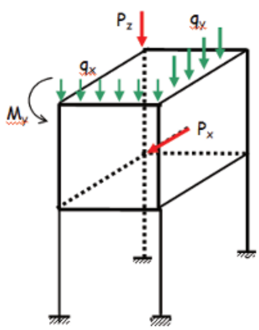
Exercise 1

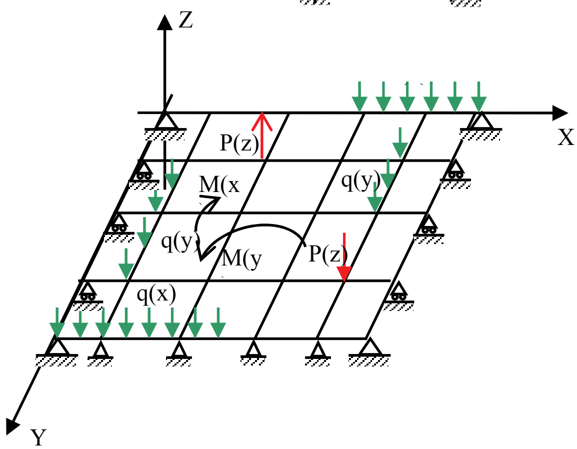
Specify the analysis conditions for each structure.

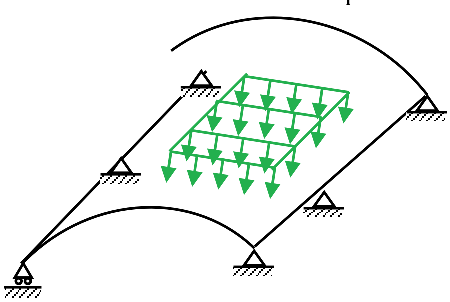


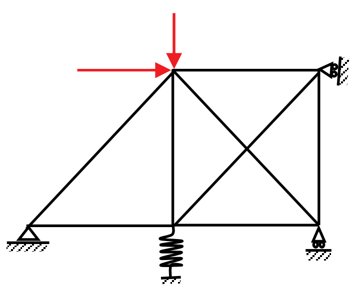






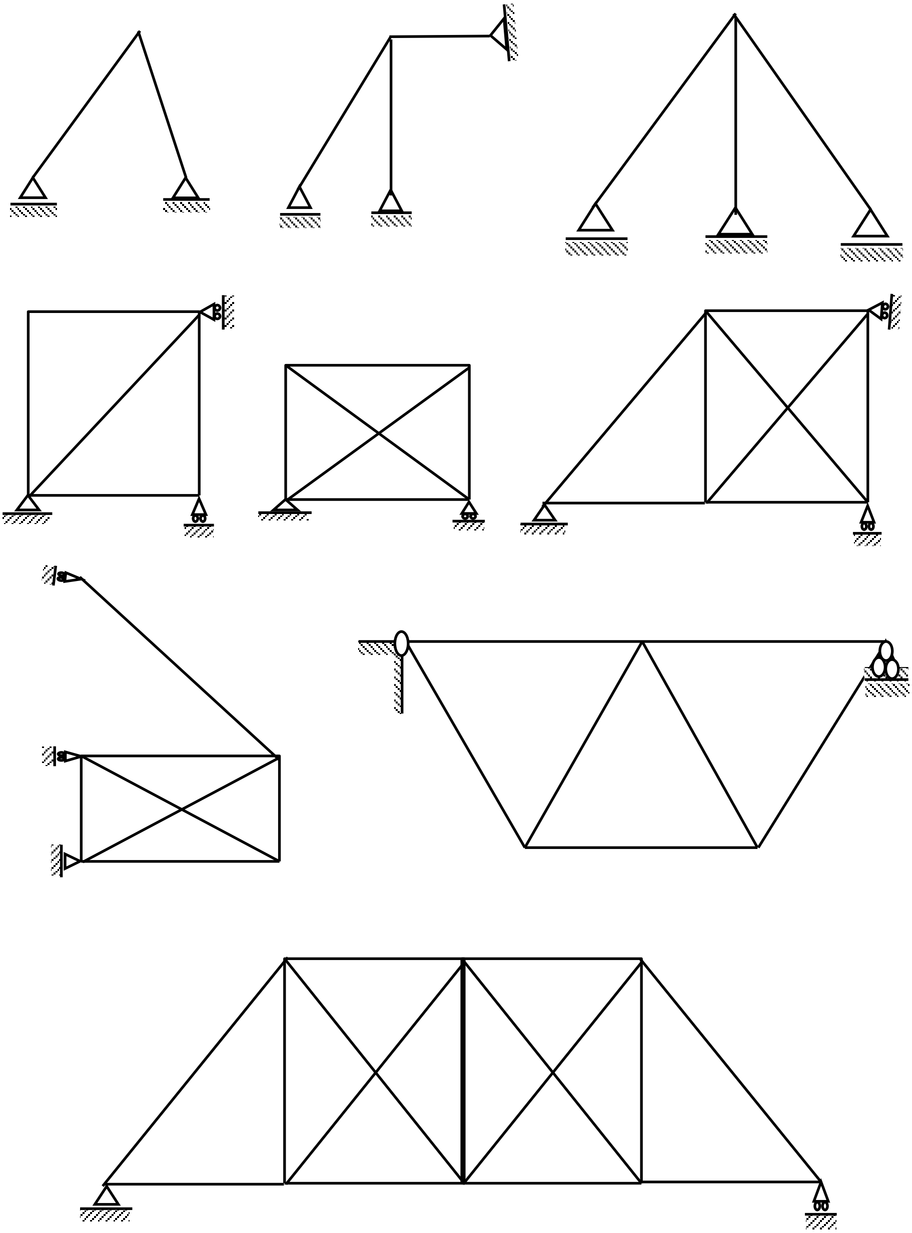






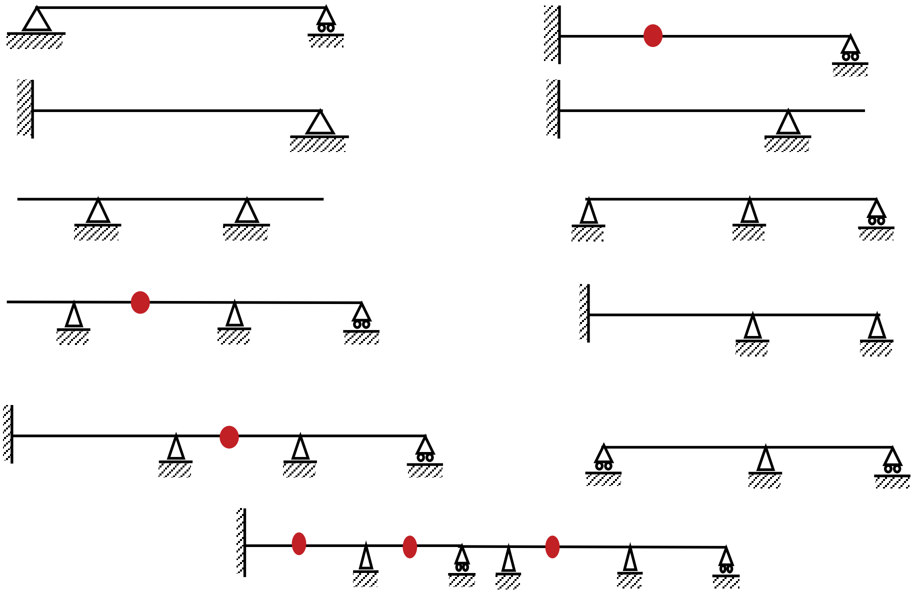
Exercise 2

Distinguish between statically determinate, statically indeterminate and unstable trusses.



Exercise 3

Differentiate the static indeterminacy of each beam.



● (designates a hinge)

Exercise 4

Determine the degree of static indeterminacy of the following frames: