1

General Remarks

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1.1. Introduction

Any matter that is not at absolute zero emits radiation. This radiation is electromagnetic by nature. Electromagnetic radiation is characterized by its frequency or its wavelength as well as by the sizes of the electric and magnetic fields that it transports, or better, by the energy it transports. The entire wavelength or frequency spectrum can be used by radiation. Electromagnetic radiation has been modeled by Maxwell and thermal emission by Planck. We note that what we normally call light, more precisely visible light forms, is only a small part of a wavelength spectrum ranging from zero to infinity. Our concern here will be to decipher the radiative energy transmitted or received by material bodies, by linking it with radiations that can be identified by their wavelengths (or their frequencies). We will go over the definition of these terms. Moreover, the propagation of radiative energy involves a particular metrology called photometry. We should therefore introduce some common definitions from this domain.

1.2. Propagation of a sinusoidal electromagnetic wave

1.2.1. *Frequencies and wavelengths*

We will refer to some specialist work for a general study of wave propagations, as well as for the modeling of electromagnetic waves.

We will simply recall here the definitions of two importance parameters, the frequency ν and wavelength λ of a ray.

An electromagnetic radiation is manifested by the evolution in space of an electric field $E(x,t)$ and a magnetic field $B(x,t)$ paired, and so variable in time and space.

In the simplest case of a plane wave propagating along a rectilineal axis $0x$ (which results in the "light ray"), the space state is identical on any plane of abscissa x normal to this axis. The fields $E(x,t)$ and $B(x,t)$ are therefore expressed in the form of a function such that

$$f(x,t) = A\cos 2\pi\left(\nu t - \frac{x}{c}\right) = A\cos 2\pi\left(\frac{t}{T} - \frac{x}{c}\right) \qquad [1.1]$$

This formula, which can be notably obtained by solving the laws of electromagnetism (Maxwell's theory), is thus interpreted:

At a given point, f is a wave function, of amplitude A. ν is the frequency of the perturbation observed, and T is the time for a movement or period.

We can "follow" the propagation in space. We seek the evolution in time of the abscissa of a given state (particular value of f). f is therefore a function of time and space, $f(\vec{r},t)$.

This perturbation moves at celerity c (celerity since it does not involve moving matter, but a state of perturbation). During a period T, the wave moves from a distance $\lambda = c\,T$, called a wavelength.

For electromagnetic waves, this celerity varies depending on the medium crossed. It has the form $\frac{c}{n}$, where n is the medium's refraction index. The celerity of the "light", often denoted as c, is a universal constant $c = 299\,792\,458\, m.s^{-1} \approx 300\,000\, km.s^{-1}$. This is the maximum possible celerity for an electromagnetic wave.

Moreover, in a void, c is the same for all wavelengths. For material bodies, the refraction index varies with the wavelength (dispersion phenomenon).

The wavelength of a radiation therefore depends on the medium traversed, unlike the frequency.

A radiation can be formed by several frequencies. When there is only one, we speak of a monochromatic radiation.

We can therefore characterize a monochromatic radiation in two ways:

– by its frequency ν;

– by its wavelength λ in the void.

Instinctively, we could consider that the frequency is the most logical characteristic of a radiation; this frequency is independent of the medium. The celerity of the light varies with the medium; it is equal to $V = \frac{c}{n}$, where n is the refraction index, which leads to the wavelength being redefined as $\lambda = \frac{c}{n\nu}$.

In the end, the wavelength, that is, the distance crossed during a period, is merely an indirect characteristic.

Nevertheless, considering orders of size here calls for a different choice.

We will consider a visible light, red for example, of $\lambda = 700\,nm = 7.10^{-7}\ m.$ The radiation frequency is in the order of:

$$\nu = \frac{c}{\lambda}\lambda \sim \frac{3.10^8}{7.10^{-7}} = 4{,}3.10^{14}\ Hz \qquad [1.2]$$

We see that the orders of size of their wavelengths are far more comfortable to use than those of the frequencies.

In this book, we choose, like many other authors, to characterize radiations by their wavelengths λ in a vacuum. Note “in the vacuum”, as in practice, we systematically omit this specification for simplicity.

1.2.2. *Radiation spectrum*

The quantity of energy carried by the wave is given by the knowledge of the As.

A general ray carries several waves of different wavelengths, each allocated an amplitude $A(\lambda)$. ***The data from*** $A(\lambda)$ ***constitutes a spectrum***.

We will see that in practice, this parameter $A(\lambda)$ will be expressed implicitly from the luminance $L_\lambda(T)$. The distribution of energy depending on the wavelength will define a spectrum.

This spectrum may be continuous or discontinuous. In a continuous spectrum, an energy density is attributed to each wavelength. A continuous spectrum can span the entire range of wavelengths, from $\lambda = 0\ m$ to $\lambda \to \infty$, where a body can only emit in one or more bounded area wavelengths or $\lambda = 700\ nm$ to $\lambda = 2000\ nm$, for example.

A body can only emit (this is the case with atoms) for some wavelengths, or rather, a limited number of small areas around a wavelength $\lambda_1, \lambda_2, \lambda_3, \ldots.\lambda_n$, etc. We therefore speak of a radiation spectrum.

Electromagnetic waves have received denominations linked to their generating phenomenon, or to their use.

We can thus cite, following a range of decreasing wavelengths, that is, at increasing frequencies:

Radio frequency waves, from 3 mm to 33 km. Old radio stations at amplitude modulation were called by their wavelength. We distinguish long wave (LW), medium wave (MW) and short wave (SW) amateur radio domains expressed in wavelength. The dials for our receivers in “modern” radio, in frequency modulation, are graduated in MHz. We distinguish VHF (metrical waves, for example radio FM), UHF (centimetric waves, for example television) and SHF (millimetric waves, for example radar). We signal the case of specialist "microwaves" for heating (cooking, etc.), whose length is fixed at $\lambda = 12.2\ cm$ to avoid interference with radio.

The infrared domain from $700\,\text{nm}$ to $1\,\text{mm}$. We distinguish (ISO cutting, but there are others) near infrared ($0.78\,\mu\text{m} - 3\,\mu\text{m}$), mid infrared ($3\,\mu\text{m} - 50\,\mu\text{m}$) and far infrared ($50\,\mu\text{m} - 5\,\text{mm}$).

The visible light domain. The International Commission on Illumination defines the vision of the reference observer for a wavelength in a vacuum of $380\,\text{nm}$ to $780\,\text{nm}$.

We can see from this domain, defined from the performances of a particular sensor, the human eye is very narrow.

The ultraviolet domain, comprised between the visible and X-rays. Often called black (since it is not visible) light, the Sun is its main source. They are responsible, among others, for tanning, but also for sunburn and cancers. The atmospheric ozone layer partially protects us from it.

We distinguish:

$UV\ A$ ($400\,\text{nm} - 315\,\text{nm}$);

$UV\ B$ ($315\,\text{nm} - 280\,\text{nm}$);

$UV\ C$ ($280\,\text{nm} - 100\,\text{nm}$).

NOTE.– Although its wavelengths are higher than the visible spectrum, we use the term **infra**-red for radiations whose frequency is lower than that of visible light. In the same way, although its wavelengths are less than those of the visible spectrum, we use the term **ultra**-violet for radiations whose frequency is higher than visible frequency.

X-rays

They are generated mainly by the interaction of an electron beam and a metal plate (anti cathode). They are a vital element in medical imaging, and remain as so, even though other methods have emerged. Wavelengths vary from (10^{-8} m) to 10^{-12} m.

Gamma rays

Gamma rays of wavelengths lower than a picometer ($1\text{ pm} = 10^{-12}$ m).

NOTE.– The lowest wavelengths correspond to the highest frequencies, so do the photons with the most energy. This explains why for gamma ultraviolet rays, the harmful effects of the rays on human biology, in particular, increase.

We will see that in thermics, for temperature ranges encountered in practice, ray emissions involve visible and near infrared.

1.3. The concept of photometry

1.3.1. *Geometric parameters*

The light ray is a model used in geometrical optics.

From the perspective of optical physics, it represents a "carrier" of the wave vector $\vec{k}$ of norm:

$$k = \frac{2\pi}{\lambda} \qquad [1.3]$$

From the perspective of photometrics, light energy does not propagate along a line, but is contained in a volume. This volume is "bounded" by a cone formed of "rays". This cone of light is measured in terms of a solid angle.

Consider the intersection of a cone issuing from a source point with a unit radius sphere. Defined in this way from a source point, the solid angle Ω is the area of the fraction of the unit radius determined by the cone.

To show that the extent of the cone is being discussed, we do not use Ω in m^2 but in **steradians** *sr*.

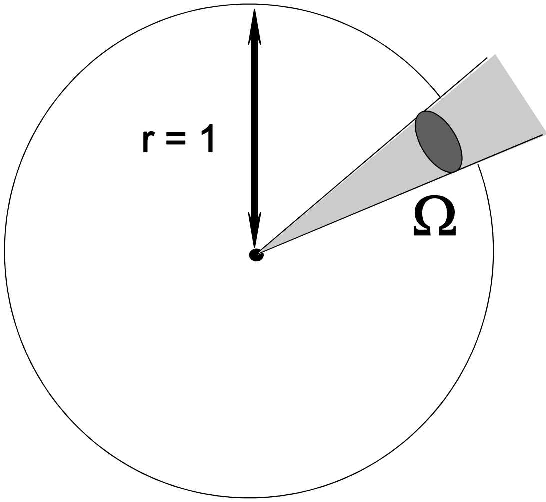

Figure 1.1. *Concept of the solid angle. For a color version of this figure, see www.iste.co.uk/ledoux/heat2.zip*

Information is often given on the solid angle from the area *S*, the intersection of the cone and a sphere of radius $R \neq 1$. The intersection of this cone with a sphere of radius $r = 1$ (with the area defining the solid angle, Ω) will be in homothety with S. For surfaces, this homothety will involve a relationship:

$$\frac{S}{\Omega} = \frac{R^2}{1} = R^2 \qquad [1.4]$$

For this reason, the small solid angle determined by a center *O*, a small spherical cap with area dS centered on *O*, will be:

$$d\Omega = \frac{dS}{R^2} \qquad [1.5]$$

This is particularly true for a cone with an infinitely small angle, which is determined from a surface dS centered in O' normal to the ray OO'. The solid angle is then expressed by:

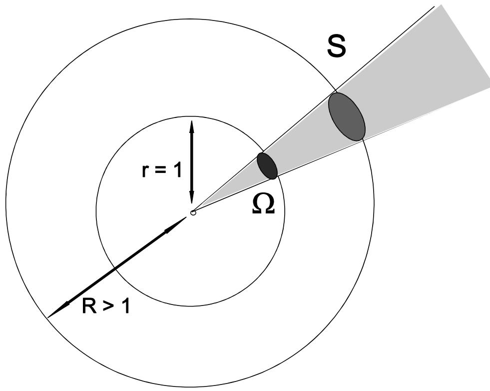

Figure 1.2. *Illustration of* $d\Omega = \frac{dS}{R^2}$*. For a color version of this figure, see www.iste.co.uk/ledoux/heat2.zip*

If the surface dS is not normal to OO', but has a normal that makes an angle θ with OO', the solid angle is calculated with the projection of dS on a normal plane at OO', which will then have an area of $dS\cos\theta$. The solid angle will then be expressed by:

$$d\Omega = \frac{dS\cos\theta}{R^2} \qquad [1.6]$$

Moreover, it shows that the solid angle defined by a half-angle cone at the top α is:

$$\Omega = 2\pi\left(1-\cos\alpha\right) \qquad [1.7]$$

which for a small angle gives:

$$d\Omega = 2\pi\sin\alpha\, d\alpha \qquad [1.8]$$

For the whole space, the solid angle becomes the surface of a sphere with radius $R = 1$:

$$\Omega = 4\pi R^2 = 4\pi\, sr \qquad [1.9]$$

The half-space is therefore equal to $2\pi\, sr$.

1.3.2. *Radiance*

The energy radiated by a surface dS in a direction that makes an angle θ with the normal to this surface element varies with:

– the absolute temperature T of the emitting surface;

– the wavelength interval $d\lambda$, or spectral range, one in which we express the emission.

Moreover, we return the transmission to a temperature T in a spectral interval $d\lambda$ to a normal surface to the axis of the basic solid angle $d\Omega$ considered. This surface has an area of $dS\cos\theta$.

The energy sent by a surface dS in a solid angle $d\Omega$, whose axis makes an angle θ with the normal to dS, is expressed in the form of a flow $d^3\Phi$ that is written as:

$$d^3\Phi = L_{T,\lambda}(\theta)\, dS \cos\theta\, d\lambda\, d\Omega \qquad [1.10]$$

This flow is differential three times, through dS, $d\lambda$ and $d\Omega$.

$L_{T,\lambda}(\theta)$ is the monochromatic radiance of the surface in the direction θ. This radiance is defined by the preceding expression.

NOTE.– The expression $L_{T,\lambda}(\theta)$ is common in books on radiation. It is clearly mathematically equivalent:

$$L_{T,\lambda}(\theta) \equiv L(T,\lambda,\theta) \qquad [1.11]$$

This particular writing highlights the fact that, in all instances, radiance is variable with T and λ, while it is not in all cases with θ (this will be the particular, important case expressed by Lambert's law). It should be noted that the angle θ is involved twice, in two different ways in the formula $[1.10]$:

– the θ figuring in $L_{T,\lambda}(\theta)$ marks the influence of the ray angle with the surface on electromagnetic emission (encrypted by the luminance);

– the θ figuring in $d\,S\cos\theta$ does not mark a variation of the emission with θ, but provides a geometric reference to a normal surface at the axis of the elementary solid angle $d\,\Omega$ considered.

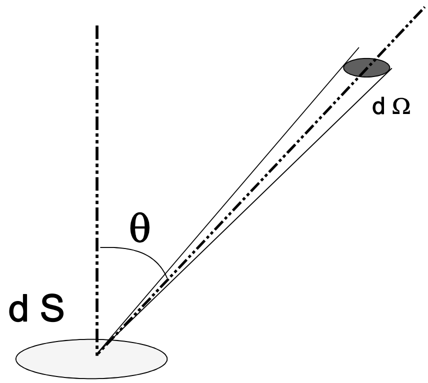

Figure 1.3. *The angle θ, For a color version of this figure, see www.iste.co.uk/ledoux/heat2.zip*

When we look at the energy emission of a surface over the whole spectrum of light, we integrate it over the wavelengths and have:

$$d^2\,\Phi = L_T\left(\theta\right) d\,S\cos\theta\;d\,\Omega \qquad [1.12]$$

with:

$$L_T\left(\theta\right) = \int_0^{\infty} L_{T,\lambda}\left(\theta\right) d\,\lambda \qquad [1.13]$$

This defines a total radiance.

We note that the flow $d^2\,\Phi$ in this instance is only twice differential.

NOTE.– Writing the radiance in $L_{T,\nu}\left(\theta\right)$

We do not use this writing here, but it can be found, especially in articles on radiation transfer. In this case, it is a good idea to know how to link them.

The flow expression $d^3\Phi$ can also be written in terms of a spectral frequency interval, *v*:

$$d^3\,\Phi = L_{T,\nu}\left(\theta\right) d\,S\cos\theta\, d\nu\, d\,\Omega \qquad [1.14]$$

The expression of $L_{T,\nu}\left(\theta\right)$ will then be different from that of $L_{T,\lambda}\left(\theta\right)$.

The relationship between these two radiance expressions is simple to find; noting that, in a set spectral interval, the energy mission should retain the same value, so:

$$\begin{aligned} d^3\,\Phi &= L_{T,\nu}\left(\theta\right) d\nu\, d\,S\cos\theta\; d\,\Omega = L_{T,\nu}\left(\theta\right) \\ d\nu &= L_{T,\lambda}\left(\theta\right)\, d\lambda\;\, d\,S\cos\theta\;\, d\,\Omega \end{aligned} \qquad [1.15]$$

We then identify:

$$L_{T,\nu}\left(\theta\right) d\nu = L_{T,\nu}\left(\theta\right)\, d\nu = L_{T,\lambda}\left(\theta\right)\, d\lambda \qquad [1.16]$$

Noting that:

$$\lambda = \frac{c}{\nu}\ , \qquad [1.17]$$

where c is the celerity of the light. We differentiate both terms of equality:

$$d\,\lambda = -\frac{c\,d\,\nu}{\nu^2} \qquad [1.18]$$

where

$$d\,\nu = -\frac{c\,d\,\nu}{\lambda^2} \qquad [1.19]$$

which is the relationship between $L_{T,\nu}\left(\theta\right)$ and $L_{T,\lambda}\left(\theta\right)$:

$$L_{T,\nu}\left(\theta\right)\, d\nu = -\frac{c\,d\,\nu}{\nu^2} L_{T,\lambda}\left(\theta\right) \qquad [1.20]$$

Or indeed

$$L_{T,\lambda}(\theta) = -\frac{c}{\lambda^2} L_{T,\nu}(\theta) \qquad [1.21]$$

1.3.3. *Bouguer–Lambert law*

We consider two infinitely small surfaces dS_1 and dS_2 facing one another, centered on two points O_1 and O_2. Their centers are distant by $r = O_1 O_2$; this parameter is large compared to area size scales. The normal to dS_1 makes an angle θ_1 with $O_1 O_2$. The normal to dS_2 makes an angle θ_2 with $O_1 O_2$.

dS_1 is the light-emitting surface, and dS_2 is the receiving surface. What is the flow received by?

Two solid angles will be used for the remainder of the explanation:

The solid angle $d\Omega,$ under which dS_2 is seen by O_1:

$$d\Omega_1 = \frac{dS_2 \cos\theta_2}{r^2} \qquad [1.22]$$

The solid angle $d\Omega_2$, under which dS_1 is seen by O_2:

$$d\Omega_2 = \frac{dS_1 \cos\theta_1}{r^2} \qquad [1.23]$$

The energy emitted by dS_1 and received by dS_2 is:

$$d\Phi_2 = L_T \cos\theta_1 \, d\Omega_1 \, dS_1 \qquad [1.24]$$

where L_T is the radiance of dS_1. This radiance is found elsewhere, defined by the preceding formula.

Replacing by its value, we find:

$$d\,\Phi_2 = L_T \cos\theta_1 \frac{d\,S_2 \cos\theta_2}{r^2} d\,S_1 \qquad [1.25]$$

which can be rewritten as:

$$d\,\Phi_2 = L_T\, d\,S_2 \cos\theta_2 \frac{d\,S_1 \cos\theta_1}{r^2} = L_T\, d\,S_2 \cos\theta_2\, d\,\Omega_2 \qquad [1.26]$$

We note that through this formula, if the surface was emitting the same luminance as before $d\,S_1$, L_{T1} the energy $d\,\Phi_2$ received by $d\,S_1$ would be equal to $d\,\Phi_1$. In fact:

$$d\,\Phi_1 = L_T\, d\,S_2 \cos\theta_2\, d\,\Omega_2 = L_T\, d\,S_2 \cos\theta_2 \frac{d\,S_1 \cos\theta_1}{r^2} = d\,\Phi_2 \qquad [1.27]$$

which forms Bouguer–Lambert's law.

The quantity $\frac{d\,S_1\, d\,S_2 \cos\theta_1 \cos\theta_2}{r^2}$ is often called the **optical range** determined by the surfaces $d\,S_1$ and $d\,S_2$.

NOTE.– This result is used particularly in establishing the Kirchhoff Law. In fact, two surfaces of the same emissivity, facing each other at the same temperature in an enclosure, are in thermodynamic equilibrium; they both send each other the same flow.

1.3.4. *Intensity*

When we have a body with small dimensions, we consider the flow $d\,\Phi$ that it emits over the whole surface, in a given direction. This energy flow $d\,\Phi$ is then defined in a small solid angle $d\,\Omega$ centered on the chosen direction. We then have to define monochromatic intensity $I\,(\theta,\lambda)$, by:

$$d^2\,\Phi = I\,(\theta,\lambda)\, d\,\Omega\, d\,\lambda = I_\lambda\,(\theta,)\, d\,\Omega\, d\,\lambda \qquad [1.28]$$

And a total intensity $I(\theta)$, by:

$$d\,\Phi = I(\theta)\,d\,\Omega \qquad [1.29]$$

The curve $I = I(\theta)$ is often represented in a polar diagram. The curve obtained is then called an intensity indicator. In practice, applied to a light source (an ampoule, for example), it will give, in particular, the energy sent by this ampoule in all directions.

1.3.5. *Lambert's law – a surface's emissivity*

1.3.5.1. *Lambert's law*

Many light sources obey this law, which will prove very practical in applications. Moreover, all emitting bodies considered in our examples satisfy it.

A light **source** meets Lambert's law when its radiance does not depend on the angle of incidence, that is, when it is identical in all directions.

We therefore have:

$$\frac{d\,L_{T,\lambda}(\theta)}{d\,\theta} = 0 \qquad [1.30]$$

Note that it can be written as:

$$d^3\,\Phi = L_{T,\lambda}\;d\,S\cos\theta\,d\lambda\,d\,\Omega \qquad [1.31]$$

The dependence of $L_{T,\lambda}$ on θ disappears, but following a remark already made above, the dependence on $\cos\theta$ remains.

1.3.5.2. *A surface's emissivity*

In practice, it is important to know the energy sent by a surface in all directions. We should therefore integrate radiance over the whole half-space, which is situated above the surface considered.

This notion is particularly important. We will make intensive use of it in the examples used in these chapters on radiation.

In practice, we will often have to establish balances on a surface. To do this, it is practical to express the energy sent by a surface in all directions. To do this, we will integrate the expression of the flow emitted over a half-space, which is a solid angle

$$\Omega = \int_{half\ space} d\,\Omega = \pi \qquad [1.32]$$

This leads us to the definition, important for the remainder of the book, of a monochromatic emittance, and a total emittance of a surface. This emittance is defined locally on an elementary surface $d\,S$.

We will thus have to define **monochromatic emittance** $M_{T,\lambda}$, the expression of the flow emitted in the whole half-space:

$$d^2\Phi_\lambda = \int_{half\ space} d^3\Phi = d\lambda \int_{half\ space} L_{T,\lambda}(\theta)\,d\,S\cos\theta\,d\,\Omega = M_{T,\lambda}\,d\,S\,d\,\lambda \qquad [1.33]$$

Thus,

$$M_{T,\lambda} = \int_{half\ space} L_{T,\lambda}(\theta)\,\cos\theta\,d\,\Omega \qquad [1.34]$$

In the same way, we will have to define **the total emittance** M_T, the expression of the flow emitted by $d\,S$ in the whole half-space:

$$d\Phi_\lambda = \int_{half\ space,\lambda} d^3,,\Phi = \int_{half\ space,\lambda} L_{T,\lambda}(\theta)\,d\,S\cos\theta\,d\lambda\,d\,\Omega = M_{T,\lambda}\,d\,S \qquad [1.35]$$

So,

$$M_T = \int_{half\ space} L_{T,\lambda}(\theta)\,\cos\theta\,d\,\Omega\,d\,\lambda \qquad [1.36]$$

We note that the integral in the whole half-space integrates the information on the dependence of the light with θ and $\cos\theta$.

We will not reproduce the resulting calculation here, which is purely geometrical by nature.

The result is particularly important, and we will use it implicitly or explicitly in the examples of a frequency application.

In fact, we find, for monochromatic emittance:

$$M_{T,\lambda} = \int_{half\ space} L_{T,\lambda}(\theta)\cos\theta\, d\Omega = \pi L_{T,\lambda}(\theta) \qquad [1.37]$$

and, integrated over all the wavelengths, we will find for the total emittance:

$$M_T = \int_{half\ space} L_{T,\lambda}(\theta)\cos\theta\, d\Omega\, d\lambda = \pi\, L_{T,\lambda}(\theta) \qquad [1.38]$$

This relationship, which results directly from the orthopty of the source, is also often called Lambert's law.

1.3.5.3. *Intensity indicator of a Lambertian source*

Another consequence of Lambert's law has implications for intensity. If we represent the intensity variation $I(\theta)$ with the incidence θ by vectors issuing from a single point, we trace what is called the intensity indicator. This type of diagram proves particularly useful in all problems involving light. In fact, it technically defines the source used.

For a Lambertian source, the intensity indicator is a circle.

NOTE.–

Lambert's name is linked to three different laws that should not be confused:

– Bouguer–Lambert's law on optical range;

– Lambert's law expressing the orthopty of a source and above all, linking emittance and radiance;

– Beer–Lambert's law, relating to the absorption by transparent or translucid bodies, which we will briefly present in Chapter 3.