PART 1

Network Variety and Modeling

1

Network Typology

1.1. Introduction

The rapid deployment of physical means of transport (air, sea, road) in the post-war period, followed quickly by the emergence of information technologies (Internet, telecommunications, GPS), made it possible to set up distributed organizations, or networks, which structure the economic and social life of the modern world. In their diversity, networks are characterized first by the functions and services they provide to the various economic (companies, consumers) and social actors (citizens, administrations) involved, and second by the technical solutions implemented to perform these functions and services.

1.1.1. *Network description levels*

In general, we can distinguish three levels of description for a given network: functional, organizational and technological.

The functional level recalls the purpose of the network (economic, commercial, social, etc.) and the value provided by it under different contexts (citizen, consumerist, professional, associative or private use). The point of view is therefore in terms of services rendered to users, in the form of the supply of physical goods (distribution networks for water, energy, foodstuffs and capital goods), intangible services (transport networks, care) or means of communication (IT, communication, and social networks). The issues here relate to the quality and availability of the service provided, and implicitly the associated socio-economic and environmental costs and impacts.

The organizational level describes the infrastructures and resources mobilized to offer the services expected by users. The term logistics, with indeterminate outlines and whose origin is attributed to the military field, is now widely used to designate the organizational aspects of a network. During design engineering issues concern the sizing of infrastructures, while network operation focuses on adapting capacities to demand.

The technological level relates to operational, physical and IT solutions, implemented to achieve the infrastructure, then operate the network.

In short, these three levels of detail (functional, organizational, technological) correspond to the respective concerns of the user, the service provider (operator) and the technology provider in the context of distributed systems, specific to networks.

In the diversity of networks, it is obviously not the technological description of network infrastructures that motivates this book, but rather the description of the organizations exploiting the infrastructures to offer services to users.

1.1.2. *Network, graph and flow*

The distributed nature of networks naturally leads to their architecture being represented by a directed graph. The vertices of the graph represent the nodes of the network, that is, the components (considered as such at the scale of representation chosen) of a distributed system. The arcs of the graph represent the links allowing the nodes of the network to interact by exchanging separable entities (material objects or information) or by continuous physical flows (energy, fluid). The use of flow graphs (flow networks, state machines, Petri nets, queuing networks, etc.) thus proves to be suitable both for the static structural representation of a network and the analysis of flows that reflect its dynamic behavior.

Moreover, the advantage of graphs is that they characteristically offer disaggregation/aggregation mechanisms for the multi-scale representation of a distributed organization. Indeed, any vertex representing a component of the network can, if necessary, be detailed by a graph offering an internal organizational view of the component. Conversely, a graph can be aggregated into a vertex offering an external synthetic view on part of the organization.

However, the semantics of the graph representing a network must be specified. We refer to a topological network when the vertices of the graph represent components located in distinct places – considered as such – and fixed in space, while the arcs describe the existing links between some of these places. Such a representation is suitable for the case of networks resulting from the interconnection of local organizations, such as transport and distribution networks, or IT networks. *A contrario*, we refer to a sociological network when the vertices of the graph designate animate beings as mobile components of the organization. The arcs thus represent the interactions between these beings. The distinction between a topological network and a sociological network is therefore based on the fixity, or conversely, the mobility of the network's components. The term "social networks" used today refers to the interaction of social groups, and by extension, IT services (Facebook, LinkedIn, etc.) intended for exchanges between members of social groups. The term digital social networks unambiguously refer to online web services supporting social networks. These digital social networks are based on an infrastructure associating topological networks (IT networks, cellular telephony network) that allow for broadcasts to users (equipped with mobile terminal mobile smart devices) of the social network.

1.1.3. *Shared or dedicated infrastructure*

The underlying infrastructure of a network can be shared with other networks, or be specific. We can in fact distinguish the following:

– General transport infrastructures (road, rail, air, sea) shared among a large number of operators for the transport of people and the distribution of goods. The flows of people or goods passing through these networks are made up of separable entities transported in vehicles or "containers" of varying size (cars, buses, trucks, trains, planes, boats, etc.).

– Dedicated infrastructures (electricity networks, municipal gas, water, telephone, IT and telecommunications networks), within which flows circulate through a physical effect.

1.1.4. *User inclusion*

Despite the diversity of networks, the modalities for including users in a network can be categorized into three situations (see Figure 1.1):

– The users themselves form, within vehicles, the circulating entities which pass through a topological network. This is the case with passenger transport networks.

– Users are the fixed terminations of a topological network. This is the case with distribution networks, such as fixed telephony and IT networks. In this case, the user is a consumer of services provided by the network.

– Users are the mobile components of a sociological network. This is the case with mobile communication networks and digital social networks. In this case, the user is both a generator and consumer of the information and content exchanged within the social network.

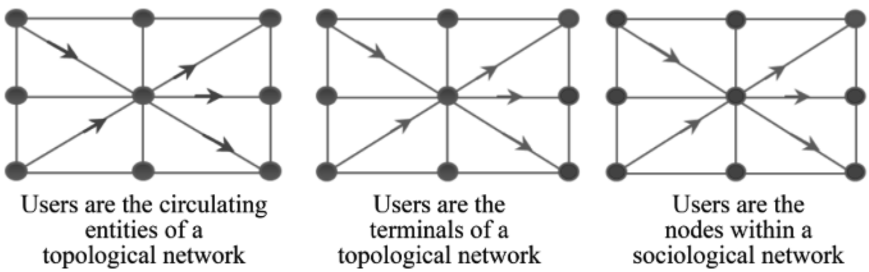

Figure 1.1. *Ways of including users in a network. For a color version of this figure, see www.iste.co.uk/bourrieres/networks.zip*

1.2. The principal networks

The diversity of networks can be analyzed by the nature of the services offered to users. These services consist of the supply of material goods, various services (transport, healthcare, etc.), energy, fluids, information and communications.

1.2.1. *(Human) transport networks*

Transport networks channel the mobility of people. Transport networks have a topological structure resulting from the interconnection of routes. In the case of road, air, rail and maritime networks, the connection points of the routes are, respectively, crossroads, airports, stations and ports. The vertices of the topological graph representing the network are, in an equivalent manner, at the same time the departure, passage and arrival points of users.

Individual or quasi-individual transport allows each user, as a simple pedestrian, cyclist or motorist, to travel in total autonomy using the networks offered by urban and rural road networks. The flows are then formed asynchronously according to the decision-making of individual users, who can go from any point of the network to any other point, under variable traffic conditions resulting from the behavior of other users.

A contrario, public transport networks (bus, train, plane, boat) offer travel services only between the stopping points (stations, stations, airports, ports) of the corresponding networks. These stopping points can also offer users a connection to other transport networks (from plane to train, from train to bus, etc.) according to the principles of intermodality and "intelligent transport" (Sumalee and Waiho 2018). Collective transport flows are formed synchronously by the execution of transport schedules (timetables).

1.2.2. *(Goods) distribution and collection networks*

Product distribution ensures the transfer of physical goods from production centers to distribution centers (hypermarkets, stores, etc.) or to private homes. The flows are made up of standardized containers (trucks, boxes, pallets, boxes, etc.) and share the same general transport infrastructure with the transport of people. The vertices of the topological graph representing a distribution network can be differentiated: we distinguish the places of production, the places of final delivery, and the intermediate places of warehousing and trans-docking. Various methods of operating the infrastructure exist. In a B2B (Business-to-Business) context, producers charter dedicated means of transport to deliver their customers directly. In a B2C (Business-to-Consumer) e-commerce context, many deliveries are intended for individual consumers or else to relay points. In this case, logistics platforms are used to consolidate shipments before transporting them to other logistics platforms where the goods will be unbundled for final delivery. Such a logistics organization jointly involves regional and local transport operators managing a fleet of low-capacity utility vehicles and interregional, international or intercontinental operators specializing in large volume freight (road, rail, air, sea). The distribution of goods in cities or urban logistics, subject to traffic congestion and increasing environmental requirements, is at the heart of the Smart City concept (Taniguchi and Thomson 2018).

In addition, the distribution of liquid substances (hydrocarbons, chemicals), which uses, within specific containers, general transport infrastructure (road, rail, air, sea), can be considered as falling within the scope of product distribution.

We should also mention the waste collection networks (used products of industrial, domestic, hospital origin) which, in reverse logic to that of the distribution networks, channel the flows from consumption points to dedicated treatment/recycling sites.

1.2.3. *Dedicated distribution and collection networks (of fluids and energy)*

The distribution of municipal gas, water and electricity uses dedicated infrastructure. Transfers from one point of the network to another are provided by physical means (gas or fluid pressure, electrical voltage). Here again, the vertices of the topological graph representing the network designate points of production, intermediate storage and final delivery.

Since the 2000s, electricity distribution networks have undergone rapid change, linked to the emergence of technologies for the electrical conversion of renewable primary energies, particularly those of wind and solar origin. Highly centralized up until then, electricity production has a tendency to massive distribution, meaning that the renewable component of the energy mix is evolving *vis-à-vis* the fossil fuel and nuclear component. The result is a great complexity in the management of electricity networks, which requires digital intelligence and Smart Grid-type models (Tuballaa and Abundob 2016).

In this regard, we should also mention the sewerage networks collecting both the wastewater of domestic and industrial origin to treatment plants.

1.2.4. *IT networks*

IT networks, connecting computers and servers located in fixed places, are part of topological networks. Extended by cellular infrastructures connecting them to users' mobile terminals, they become digital social networks. Since the emergence of the Web in the early 1990s, these networks have reached planetary dimensions through the successive interconnection of networks of increasing size: LAN (Local Area Network), RAN (Regional Area Network) and WAN (Wide Area Network).

Using dedicated interoperable infrastructures (cable networks, optical fibers, telephony, wireless networks), the transmission of computer data is carried out in packets, likely to be grouped, stored and unbundled, according to transport logistics and as such present a strong analogy with the case of product distribution networks. Indeed, memory-databases store data awaiting processing by applications, similar to physical storage centers that keep physical goods for processing or consumption.

1.2.5. *Communication networks*

The increasing convergence of communication and IT networks makes their distinction less and less relevant. In reality, fixed or mobile telephony infrastructures, combined with wireless transmission infrastructures, have today become the vector for information transmissions. Although sharing a common infrastructure that combines both analog and digital technologies, communication and IT networks are nevertheless distinguished at the functional level: the former are finalized by the transmission of signals in real-time (voice, image, sound, radar, etc.), the latter being more intended for the distribution of storable data (multimedia files) with a potential view to deferred consumption. The resulting communication processes can thus be synchronous (telephone communication, chats) or asynchronous (SMS, publication of files on digital social networks).

Media communication (press, television, advertising) can be considered as operators of communication networks providing information (here also synchronous or asynchronous) intended for citizens, consumers and professionals.

1.2.6. *Social and digital social networks*

A social network corresponds to a set of relationships between actors, individuals or a collective (Lemieux 1999; Mercklé 2004). These networks can be made up of individuals, groups of people or organizations. This whole can be organized, or not, and the nature of the relations can be of a very diverse nature, and the links structuring this social network can be characterized as strong or weak (Granovetter 1973). Interactions can take place through physical contact, remote collaboration, any type of social meeting, as certain forms of verbal or written communication depending on the situation. Social interaction plays an important role in studying the spread of information, innovation, ideas and influence among its members. When an event occurs – for example, the use of cell phones among students, the adoption of a new information system within the company, or the rise of a political movement in an unstable society –, it can die out quickly or bring about major breakthroughs within a population. The process of networking allows us to understand the dynamics and spread of information in social networks. Diffusion studies are numerous since diffusion phenomena are discussed in several disciplines: computer science (computer viruses, dissemination of information in a social network) (Girvan 2002), biology (epidemics) (Cauchemez 2011), physics and so on.

Digital social networks result from the creation of spaces for sharing between individuals in a community, based on a private, associative or professional logic. Under the term of social network sites, digital social networks are defined in Boyd and Ellison (2007) as "web services that allow individuals (1) to build a public or semi-public profile in a small space, (2) to structure a list of users with whom they share a connection, and (3) to view and browse their list of connections and those established by others within the same system". These networks rely on operators, such as Facebook, Twitter, Instagram and LinkedIn, which provide appropriate services for the animation of communities or for sociability interactions, such as publications (newspaper articles, stories, etc.), data storage and discussion forums, with different types and functions supporting these interactions.

According to Proulx (2015), a sociological analysis of digital social networks leads to taking into account four dimensions: the architecture of the platform, the structure of the social interactions that take place there, the user experience of the platform, and the quality of the user's contribution to the digital world.

Digital social networks are therefore located at the intersection of various social interactions resulting between individuals, groups, and/or networks. They are multi-layered networks, which result from the stacking of various social interactions – materialized by publications and information exchange –, communication, computer and energy networks. If mobility is included in the panorama, in particular thanks to a tool such as the smartphone, the use of digital social networks is also operationalized in a transport network (car, public transport, etc.). Digital social networks are therefore an example of the hybridization of networks of a different nature, between the technical and the social.

1.3. Characterization and typology of networks

We summarize below the functional or organizational characteristics resulting from the review of the main networks made above and propose a typology on this basis.

1.3.1. *Key characteristics*

We highlight the following general characteristics:

– The relationship of users to the network: the user can be a mobile element within a fixed network (transport network, mobile telephony, etc.), constitute a fixed terminal of a distribution network, a mobile terminal of a fixed network (GSM network) or a structural component of a social network.

– The role of logistics operators: with the exception of individual transport, where the user directly uses the infrastructure (roads, road network), access to the service generally requires the intermediation of operators. In the case of collective passenger and freight transport, operators ensure the provision of vehicle fleets, as well as the logistics programming (schedules, timetables) characterizing the service offered. These logistics operators develop services for users and are generally separate from infrastructure maintenance operators.

– A more or less dedicated infrastructure: road infrastructure is shared by a variety of individual users, public transport as well as transport and freight operators. Rail, air and sea transport networks use specific port and airport infrastructures shared by a large number of freight and passenger transport operators. Fluid and energy distribution networks (distribution of water, gas, electricity) use infrastructures totally dedicated to the physical nature of the flow. Finally, the electronic infrastructures underlying communication and digital networks, which themselves underpin multimedia networks and digital social networks, are the subject of an ever-evolving range of standardization.

– The extent of the network: through the interconnection of local infrastructures, some networks benefit from extensive coverage at the regional, national, international or even intercontinental level. The global scale is characteristic of transport, digital and communications networks. Energy distribution networks are interconnected internationally, even intercontinentally, to allow for the export/import of electric power and gas. Conversely, water distribution and sewerage networks are deployed at the local level.

1.3.2. *Network integration*

The main networks have been listed above from a functional point of view, that is, by the nature of the services provided to users. In reality, the realization of complex functionalities may require combining different networks through horizontal and/or vertical integration.

The horizontal integration of networks involves interconnecting networks of the same nature, so as to extend their coverage. Transport networks, the physical distribution of goods, energy or gas, and also telephone networks, the Internet and digital social networks are the result of such integration. Achieving this implies a strategic desire on the part of the various operators concerned, and also requires the removal of various technical, commercial and accounting obstacles, or even legal ones, through the use of standards and stabilized technologies. The result is a large-scale service, characterized by a simplification of usages for the user, and also a necessity for interoperability (Daclin *et al.* 2016), to ensure that the infrastructure, operating processes and information systems function in concert for multiple operators. Horizontal integration of networks is booming in many areas, such as multimodal transport in metropolitan areas (Jeekel 2019), healthcare networks (McCutcheon 2019) and among virtual businesses.

The vertical integration of networks consists of developing high functionality networks from existing networks of lesser functionality, which means that the latter constitute the infrastructure of the former. In particular, this type of integration has led to the use of telephone and communication networks as infrastructures to develop IT networks, themselves structured in several functional layers (OSI model). Another remarkable example of functional construction has been the emergence of digital social networks, resulting from a new service that uses IT networks as an infrastructure.

1.3.3. *Typology*

On the basis of the previous observations, Tables 1.1 and 1.2 provide a typology of networks reviewed in section 1.2.

For convenience, we separate the case of networks with material flows (transport of people, goods, fluids and energy) from the case of networks with intangible flows (information and communications).

The tables read as follows: three main lines represent respectively, from bottom to top, considerations related to infrastructure, operators and users, that is, increasing functional levels that lead to the genesis of services offered by the network.

Three main columns represent the actors involved (users, operators or technology providers), the level of description of the network and the classification of the principal networks.

Each network is thus positioned in a repository determining the nature of the service, the type of user inclusion, the possible operators managing overseeing the infrastructure and the type of infrastructure supporting the service.

In the case of information flow networks, Table 1.2 positions IT networks, themselves based on wired and wireless communication networks, within a multi-layered infrastructure that allows operators to offer services to users through three main types of information (notably television), telephony and the exchange of digital multimedia files (electronic messaging, sound, video), as well as digital social networks.

<table>
<tr><th>Actors</th><th colspan="2">Description level</th><th colspan="4">Classification of networks</th></tr>
<tr><td rowspan="2">Users</td><td rowspan="2">Service</td><td>Nature of service</td><td>Individual transport</td><td>Collective transportation</td><td>Product distribution recycling</td><td>Fluid and energy distribution
Sanitation</td></tr>
<tr><td>User inclusion</td><td colspan="2">Mobile
Flow element</td><td>Fixed
Network termination</td><td>Fixed
Network termination</td></tr>
<tr><td>Logistics operators</td><td colspan="2">Organization</td><td>N/A</td><td>Transporters</td><td>Producers
Distributors
Carriers
Communities</td><td>Producers
Distributors
Communities</td></tr>
<tr><td>Providers and integrators of technology</td><td colspan="2">Infrastructure</td><td colspan="3">General transport infrastructure (road, rail, sea, air)</td><td>Dedicated distribution infrastructures (oil pipelines, gas, electricity, water, sanitation)</td></tr>
</table>

Table 1.1. *Typology of material flow networks. For a color version of this table, see www.iste.co.uk/bourrieres/networks.zip*

Actors	Description level		Classification of networks		
Users	Service	Nature of service	Information	Multimedia communication	Digital social networks
		User inclusion	Fixed Topological network node	Mobile/fixed Topological/sociological network node	Mobile Sociological network node
Logistics operators	Organization		Media and advertising agencies	Multimedia operators	Digital social network operators (Facebook, LinkedIn, etc.)
Providers and integrators of technology	Infrastructure		IT networks (TCP / IP, OSI protocols, etc.)		
			Communication infrastructure (cable, fiber, wireless, GSM)		

Table 1.2. *Typology of information flow networks. For a color version of this table, see www.iste.co.uk/bourrieres/networks.zip*

1.4. Engineering issues

The broad spectrum of networks means that there are different engineering methods as per the respective fields: Civil Engineering for road networks, port and airport facilities; Industrial Engineering for distribution networks; Electrical Engineering for electrical networks; Computer Engineering for information systems and network engineering and so on.

From the perspective of a system, a network views its life cycle as subject to the general principles of system engineering (INCOSE 2015) and the management of complex projects (ISO 21500 2012), which are now well-established and which transcend traditional areas of application.

On this basis, we can consider network engineering issues in terms of two axes (Table 1.3):

– the chronology of the design-build-operate/maintenance-evolution-dismantling life cycle specific to the systems;

– the functional service-organization-infrastructure decomposition outlined in section 1.3.

The term "life cycle" describes the stages of design, construction and then operation of a network until its dismantling.

V-lifecycle models guide engineering activities through a cascade of specifications going from the general design of the system to a detailed design of its components, followed, in reverse order, through the stages of implementation, validation, integration up to the obtainment of the overall system (Eigner *et al.* 2017).

Spiral life cycle models (Boehm 2014) and agility principles (Garrison Darrin and Devereux 2017) focus, beyond the initial implementation of a system and its consequent maintenance, on recurrent reengineering activities which carry out successive evolutions of the system, generally to improve its performance, until its eventual obsolescence and dismantling.

Life cycle

Functional levels		Conception	Realization	Exploitation Maintenance	Re-engineering	Dismantling
	Service	Performance analysis				Removal of the service
		Innovation services	Implementation	Maintaining quality of service	Service enhancement	
	Logistics organization	Performance analysis				Dismantling of partnerships
		Establishing partnerships Co-design Capacity sizing	Organization implementation	Resource planning	Capacity enhancement	
	Infrastructure	Performance analysis				Dismantling infrastructure
		Infrastructure studies	Infrastructure implementation	Operational maintenance of infrastructure	Technological improvement	

Table 1.3. *Network engineering issues. For a color version of this table, see www.iste.co.uk/bourrieres/networks.zip*

Beyond the general principles of systems engineering, the engineering issues specific to networks emphasize the constitution and evolution of the partnership between economic players on which the co-creation of value and the associated service is based and offered to network users (Gay 2014; Guan *et al.* 2017). Throughout the life cycle of a network, engineering activities may involve all functional levels, or only some of them. Indeed, due to their often very high costs, infrastructures are stable over a fairly long-term, but do not, however, preclude a strong service innovation dynamic. The most striking example of this today is the profusion of new services taking advantage of GPS infrastructure, IT networks or the Internet of Things (Huan 2018).

1.5. Performance indicators, evaluation, optimization

The pursuit for performance is at the heart of organizational engineering (Jagdev *et al.* 2004). The use of performance indicators aims to qualify, measure and validate the performance of systems in the initial design, operational and improvement phases.

We specify here the nature of the performance indicators specific to networks, and next recall the evaluation and optimization approaches that apply to them. Since the theoretical resolution of the optimization problem is the subject of specialized literature, we will only present here the principle, in order to allow the uninformed reader to follow the case studies presented at the end of this work.

1.5.1. *Performance indicators*

The performance of networks is assessed, as with any system, from many points of view: economic performance (cost and quality of service), environmental impacts, societal impacts, health impacts and so on.

The environmental and health impacts are mainly due to the network infrastructure: carbon emissions from a transport or distribution network, electromagnetic radiation from telecommunications networks, energy dissipation from electrical networks, the energy consumption of data centers and so on. To a lesser extent, the logistics organization operating the infrastructure can achieve environmental performance, for example, the optimization of delivery rounds within a distribution network makes it possible to reduce the carbon footprint of the transport activity.

Economic indicators predominate at the functional levels of service and logistics organization of networks to qualify, through quality-cost-time indicators, the relationship between users and operators. We distinguish the following:

– Indicators perceived by the user-consumer of the service, such as *price* and *quality of service*. Quality of service refers to a set of parameters determining access to the service: the response time to the request and the safety and confidentiality in terms of the use of the service are generally the main consideration. In the case of a transport network, waiting times, transport times and security are the main elements determining the quality of service. In the case of mobile telephone networks, the ubiquity of service, the time to obtain information and the quality of voice are such indicators. In the case of digital social networks, several indicators can be taken into account to qualify the quality of service: for example, the guarantees offered by the platforms in terms of data confidentiality, or the possibilities offered by the platforms to manage one’s digital identity (*personal branding*) or the e-reputation of organizations. Other indicators can be taken into account, in particular, with regard to the user experience (Chen *et al.* 2018; Rhiu and Yun 2018). For an electrical network, this will be the voltage, waveform and frequency, as well as the power available at the point of consumption.

– *Productivity* includes all the parameters conditioning the reduction of network operating costs by operators. Very generally, these are management indicators relating the cost of the resources mobilized to provide the service with the revenue generated by users, which, in turn, increases with the level of activity within the network. When it comes to a transport network, or a distribution network, the filling rate of vehicle fleets is a major indicator. In the case of a network of manufacturing companies, we consider the load rate of units and equipment for the production and transport of goods. In the case of digital social networks, this could be the load rates of computer servers, the payroll of operating staff and so on.

Evidently, the performances seen, respectively, by the user and the operator, strongly interact with each other, not only by the passing on of operating costs onto the price of the service, but also by the load rate of the network, which relates the overall volume of network activity to its necessarily limited capacity to meet the demand. The increase in the network load rate induces a degradation in the quality of service. In transport networks, increased traffic creates waiting times due to the poor flow of vehicles. In a computer network, the influx of data tends to saturate the servers and create waiting times felt by the users. In an electrical network, high consumption demand degrades the operating voltage and so on. In short, the quality of the individual service experienced by a user decreases with the number of users requesting network resources. Hence, the reengineering of networks very broadly aims to enhance the structural capacity flows, when the service quality offered to users is degraded by a collective demand on the network that has become too intense.

1.5.2. *Evaluation and optimization*

Performance evaluation is a process of measurement, interpretation (the processing and aggregation of data) and diagnosis, based on the values of predefined indicators. The evaluation process can be applied to a real system or to a simulated model of said system. Clearly, however, performance evaluation is not an end in itself, but only a prerequisite for the expert engineering decision that leads to the implementation or improvement of systems in accordance with performance targets.

In an engineering process, the question that arises is to determine, from a set of possible system implementations, the solutions that lead to the best performance. The so-called experimental design test method (Taguchi *et al.* 1989) may be a realistic approach in the presence of a limited number of parameters. In the case of complex networks, the numerical simulation of a representative model of the system is now a widespread practice. The difficulty is then to have, not only the simulator necessary for evaluating the performance of the system, but also resolutive methods to guide the exploration of the parameters and identify effective solutions. Hence the need to carry out simulation campaigns, which can be long and expensive.

The optimization of a system is a design or improvement process based on a formal model of the system, and jointly on the definition of performance targets. Mathematical performance optimization is applicable to systems that are simple enough to be modeled mathematically. The optimization problem is expressed by the search for an *extremum* (minimum or maximum) of an objective function whose variables are subject to the constraints that characterize the model of the system. In itself, the objective function expresses the main performance sought, which often takes the form of a compromise between antagonistic performances. We refer to linear programming (LP) when, at the same time, the objective function and the constraints are linear functions of the variables.

If the variables are integers, this is integer linear programming (ILP), for example:

$$Maximize\ J = x_1 + x_2$$

under the constraints $3x_1 + 2x_2 \leq 11$ $3x_1 + 4x_2 \leq 16$ $x_1, x_2 \in N$

gives $Max(x_1 + x_2) = 4$ which we obtain (Figure 1.2) for the multiple solution:

$$x_1, x_2 = 0,4$$

$$x_1, x_2 = 1,3$$

$$x_1, x_2 = 2,2$$

$$x_1, x_2 = 3,1$$

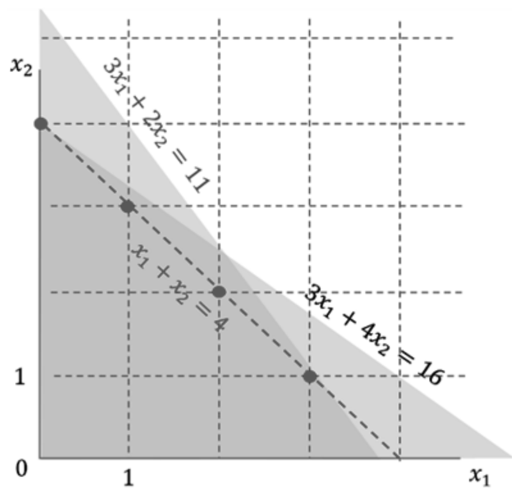

Figure 1.2. *Example of ILP. For a color version of this figure, see www.iste.co.uk/bourrieres/networks.zip*

This is quadratic programming (QP) when the objective function is a quadratic function of the variables, themselves subject to linear constraints. The quadratic formulation of the objective function facilitates the expression for optimal performance by the data of the targets $x_1^d, \dots, x_n^d$ towards which the variables must tend. For example, the following formula:

$$Minimize\ J = \alpha^2\left(x_1^d - x_1\right)^2 + \beta^2\left(x_2^d - x_2\right)^2$$

under the constraint $ax_1 + bx_2 \leq c$

states a two-variable $(x_1,\ x_2)$ optimization problem where the data x_1^d, x_2^d specifies the targets and where the coefficients α and β are used to weight the relative importance given to these targets. If the target point (x_1^d, x_2^d) is compatible with the constraint, we evidently have $Min\, J = 0$ with $(x_1, x_2) = (x_1^d, x_2^d)$ as a unique solution and the target is reached. Otherwise, the optimal solution $(x_1, x_2) = (\tilde{x}_1, \tilde{x}_2)$ is the one which corresponds to the point of tangency of the constraint with the ellipse of center (x_1^d, x_2^d) for which α and β are the semi-axes (Figure 1.3).

The use of optimization methods is facilitated by computerized solvers (CPLEX, GUROBI, XPRESS, MOSEK, etc.) implementing various exact and heuristic mathematical programming methods to set up decision support studios in engineering of systems.

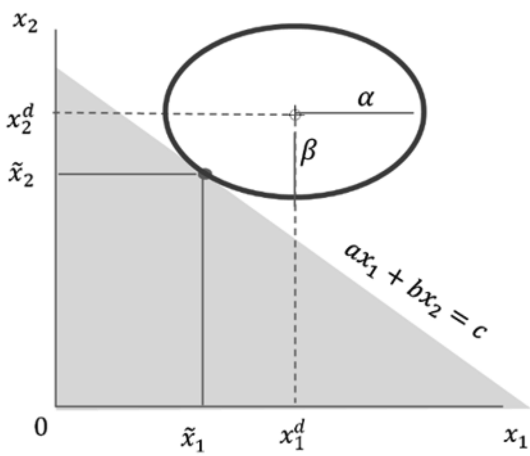

Figure 1.3. *Example of a quadratic program. For a color version of this figure, see www.iste.co.uk/bourrieres/networks.zip*

1.6. Conclusion

The multitude of networks that have developed over the past decades across a range of fields are the pillars which support the high level of services available to us today. A true paradigm of the modern world, the concept of networks takes advantage of the economic and/or social added value that can be expected from the intelligent interaction of resources and actors within a distributed but communicating organization.

In this chapter, we have endeavored to identify the invariants that make it possible to bring together, in their diversity, the main networks that surround us, so as to bring out both their common and discriminating characteristics.

This general overview on networks leads us to present, in Chapter 2, the knowledge that allows analysts to imagine a network, generally for design or improvement purposes. In doing so, we tackle the challenges inherent to modeling networks.