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Objections and Rebuttals of Current Laws

1.1. Discussion on the general concepts

1.1.1. *Choosing a three-dimensional space*

As we perceive it, space is three dimensional. To define the position $\boldsymbol{x}$ of a point in this space, classical Newtonian or relativistic mechanics introduced the notion of a Galilean or inertial frame of reference within which any point particle not submitted to a force moves following a translational motion with uniform velocity. This frame of reference is defined by a coordinate system denoted by $\mathbb{R}^3(x, y, z)$ in three dimensions spanned by the three orthonormal unit vectors $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$. The theory of relativity introduces another space dimension based on the product between celerity of light and time $(c_0 t)$; the whole then forms what is called space-time.

The concept discussed now relates to the dimension with which a physical phenomenon can be interpreted. Despite the three-dimensional space being an accepted notion, the dimension of a fixed physical phenomenon can be the subject of further reflection. For example, the propagation of a compression wave has, in essence, a single direction; this phenomenon is extended to any type of longitudinal wave, swell, acoustic wave propagation or light. The velocity of the wavefront is called longitudinal celerity, whose symbol c_l is different for each of the mentioned phenomena. This characteristic is strictly independent of the velocity of transport of the material medium or particle; it is an intrinsic property of the medium, the sea, the air or the vacuum. When the source of acoustic waves emits in all directions of the space, it is referred to as isotropic, but the phenomenon basically remains in a single direction in the space. When two plane waves enter the same area of the space, they form two-dimensional interference fringes. The same phenomenon is observed for shock waves, for example, in the wake of nozzles to form fixed structures called Mach disks. In all the examples, the phenomenon of longitudinal wave propagation has a single direction; it does not need to be modeled in a three-dimensional space.

Extending this principle to other dimensions of space is a cause and effect approach; the reason for this is that the velocity in the direction of the wave cannot exceed the celerity of the medium.

The second type of wave is transverse, and it is defined by the celerity c_t; these waves are generated by a motion orthogonal to the direction of propagation. They are also associated with only one direction, that of orthogonal propagation to the plane of motion that gives rise to them. They are said to be polarizable because propagation in a fixed direction can be generated by any directional movement in the orthogonal plane under consideration. They can be observed in the ground during seismic phenomena, in elastic solids or in space in the form of gravitational waves. Although these waves are generated in a two-dimensional plane, the propagation is still in a single spatial dimension. Thereby, independently of the waves being longitudinal or transverse, only the common direction of these waves must be considered to model them.

Why is such emphasis given to the dimension of three-dimensional space when the phenomena of propagation have only one direction? It is the very foundation behind modeling the phenomena of physics that is addressed through this question. The fact that the frame of reference $\mathbb{R}^3(x, y, z)$ of classical and relativistic mechanics is no longer employed leads to a decisive change in the way physics is understood. The consequences of such a choice are considerable because the frame of reference $\mathbb{R}^3(x, y, z)$ is attached to the very existence of the concept of continuous medium, to that of differentiation at a single point, to integration and, in general, to mathematical analysis.

1.1.2. *Galileo's law of free-falling bodies*

The free fall of a body under the influence of the Earth's gravity, which dates back to the 17th century, is a law devised by Galileo. It expresses the equivalence between mass associated with gravity, called "gravitational mass", and inertia related to mass or "inertial mass."

1.1.3. *Uniform rotation, a Galilean motion*

In classical mechanics, and namely the theory of relativity, constant-velocity rotation is an accelerated motion. The principle of inertia or Newton's first law states that if a body is in rectilinear translational motion at a constant velocity V_0 it continues to follow it as long as no force is applied to it. This uniform translational motion leads to the notion of inertial frame of reference. A frame of reference that is not in rectilinear and uniform translational motion with respect to a Galilean frame of reference is a non-inertial frame of reference. Therefore, all accelerated motion corresponds to non-inertial reference frames. Most of the time, the motion of rotating bodies is considered to be accelerated because they are subject to centrifugal acceleration. In fact, the Galilean and inertial frames of reference refer to the same concept, namely, that of the principle of inertia and Newton's first law. The definition of the Galilean frame of reference is very restrictive because it only addresses the motion of uniform translation; it is written as:

$$\rho\,\gamma \equiv \rho\,\frac{d\mathbf{V}}{dt} = 0 \qquad [1.1]$$

For the acceleration to be zero, the motion has to be steady, $\partial \mathbf{V}/dt = 0$, and the sum of the inertia terms also zero. In continuum mechanics, the inertial term is written equivalently in different manners:

$$\rho\,\mathbf{V}\cdot\nabla\mathbf{V} = \rho\,\nabla\left(\frac{|\mathbf{V}|^2}{2}\right) - \rho\,\mathbf{V}\times\nabla\times\mathbf{V} = \nabla\cdot(\rho\,\mathbf{V}\otimes\mathbf{V}) - \rho\,\mathbf{V}\,\nabla\cdot\mathbf{V} \qquad [1.2]$$

where $\boldsymbol{\mathcal{L}} = -\mathbf{V}\times\nabla\times\mathbf{V}$ is the Lamb vector. In the case of a uniform motion with velocity $\mathbf{V}_0$ on a straight trajectory, it is easy to show that the inertial term is equally zero, and the condition [1.1] is indeed verified.

Let us examine why a rotational motion at a constant velocity induces a fictitious force in the Navier–Stokes equation. Given the angular velocity $\boldsymbol{\Omega}$, the local velocity is thus equal to $\mathbf{V}_{rot} = \boldsymbol{\Omega}\times\mathbf{r}$ and only a rotation about the Oz axis is taken into consideration here such that $\boldsymbol{\omega} = \boldsymbol{\Omega}\cdot\mathbf{e}_z$. In cylindrical coordinates, the only component of the non-zero Navier–Stokes equation is the one following coordinate r. Since mechanical equilibrium is not certain within the context of an inertial frame of reference, the acceleration is expressed in a rotating frame of reference where the centrifugal acceleration, $-\boldsymbol{\Omega}\times\boldsymbol{\Omega}\times\mathbf{r}$, enables us to write:

$$\rho\,\boldsymbol{\omega}^2\,r\,\mathbf{e}_r - 2\,\rho\,\boldsymbol{\omega}^2\,r\,\mathbf{e}_r = -\rho\,\boldsymbol{\omega}^2\,r\,\mathbf{e}_r \qquad [1.3]$$

where the two terms on the left-hand side of relation [1.3] represent inertia in the context of a Galilean frame of reference and the term on the right-hand side corresponds to a fictitious centrifugal force. All other terms, especially viscous terms, are a priori zero.

The necessary presence of this fictitious centrifugal force to establish mechanical equilibrium is to be attributed to the formulation of the equations of mechanics in a global Cartesian frame of reference. To establish that a vector equation or a vector is zero, all three of its components must be zero simultaneously. We should first observe that the equilibrium translated by relation [1.3] is borne only by terms following $\mathbf{e}_r$.

The first term is the gradient of $|\mathbf{V}|^2/2$ and the term of the second member is also the gradient of a centrifugal potential $\boldsymbol{\omega}^2 \; r^2/2$. Therefrom, the Lamb vector $\boldsymbol{\mathcal{L}} = -\mathbf{V} \times \nabla \times \mathbf{V}$ can only be a gradient of a function of r identical to the other two terms. This perspective of the mechanical equilibrium of a uniform rotational motion is questionable. Indeed, no constraint applies in the orthoradial direction $\mathbf{e}_\theta$, while basic common sense makes us believe that an acceleration following θ contributes to ensuring mechanical equilibrium.

The origin of the fictitious forces within the Navier–Stokes equation is due to the form of the perceived acceleration that reveals three fictitious accelerations: (i) the centrifugal acceleration, (ii) the Coriolis acceleration and (iii) the Euler acceleration corresponding to that of the rotating coordinate system. Nonetheless, the problem of a uniform rotational motion must not be assimilated to the problem of changing the frame of reference where a velocity field is added to the local celerity of the material medium in order to obtain solutions more easily. Nor should the dynamic actions to be carried out to obtain a fixed motion be mistaken for the motion itself that emerges from a purely kinematic understanding. There is no legitimate reason to consider that uniform rotational motion, with zero divergence and constant curl, is a non-inertial problem. A particle on its circular path carries on its motion without involving other accelerations or forces that are specifically related to a change in direction or velocity.

In fact, the assumptions that have shaped modern mechanics are the basis of choices that sometimes prove to be questionable and maintain the status quo about how space and time should be perceived [POI 17]. This is in particular the notion of a global reference frame in a three-dimensional space denoted by $\mathbb{R}(x, y, z)$. Newton and Leibniz introduced differential and integral calculus, which are perfectly justified concepts in mathematics, as is analysis itself. In physics, laws can deviate from the basic concepts of mathematics; this is the case for vector addition, which does not apply to the velocities with orders of magnitude of the celerity of light in vacuum c_0. The transposition of local laws of physics to objects far from the Universe is not necessarily ensured by a change in global reference frame.

The point of view developed further on consists of considering that the equation of motion filters out some uniform motions including constant velocity rotation, namely, that it is invariant by a rotation defined by the velocity field $\mathbf{\Omega} \times \mathbf{r}$; it would be an extension to Galileo's principle of inertia, according to which a rotational motion at a constant velocity continues indefinitely if it is not subjected to any external action. In fact, it is necessary that the rotational motion $\mathbf{V}_{rot}$ satisfies the condition $d\mathbf{V}_{rot}/dt = 0$. This is not the case with the equations of mechanics whose terms of inertia create an artifact, that is, an inertia compensated by a fictitious centrifugal force.

1.1.4. *Galileo and Lorentz invariants*

Sometimes presented as contradictory, they are in fact disjointed and complementary. Let us first consider Galilean transformation. A frame of reference $\mathbb{R}'$ moves at a constant velocity $\mathbf{V}_0$ with respect to a reference frame $\mathbb{R}$. The spatial coordinate in the direction of the wave is represented by $\boldsymbol{r}(t)$ in the latter reference frame and by $\boldsymbol{r}'(t)$ in the reference frame $\mathbb{R}'$. To shift from $\mathbb{R}$ to $\mathbb{R}'$, the following transformation should be applied:

$$\begin{cases} \boldsymbol{r}'(t) = \boldsymbol{r}(t) - \mathbf{V}_0\, t & [1.4a] \\ t' = t & [1.4b] \end{cases}$$

If a law of physics is invariant as a result of this transformation, it will be said that it follows Galileo's invariance. By applying Newton's law in the reference frame $\mathbb{R}'$, it is easy to show that:

$$\mathbf{F}'(t) = m\,\boldsymbol{\gamma}'(t) = m\,\boldsymbol{\gamma}(t) = \mathbf{F}(t) \qquad [1.5]$$

The law of the initial frame of reference is valid in any frame of reference. This interpretation of the Galileo transformation is relative to a change of reference frame, which is used to predict motion within the reference frame $\mathbb{R}'$ by considering that the laws are the same in any Galilean or inertial reference frame. In fact, it should be noted that invariance can be implicitly achieved with the corresponding law $\mathbf{F}(\mathbf{V}_0) = 0$ regardless of the form of the latter.

The Lorentz transformation [POI 05] is presented as an extension of Galileo's transformation to relativistic motions. One of its forms is written as:

$$\begin{cases} \boldsymbol{r}' = \gamma\ (\boldsymbol{r} - \mathbf{V}\, t) & [1.6a] \\ t' = \gamma\ \left(t - \dfrac{\mathbf{V}\,\boldsymbol{r}}{c^2}\right) & [1.6b] \end{cases}$$

where γ is the Lorentz factor $\gamma = 1/\sqrt{1 - \mathbf{V}^2/c^2}$. It should be noted that the Lorentz transformation affects only the component corresponding to the direction of the wave, while the other space components satisfy the Galileo transformation. Although this transformation [1.6] allows the Galileo transformation to be found [1.5] when $\mathbf{V} \to 0$, they are not compatible; the factor γ that can be seen in front of the parentheses causes phenomena such as length contraction and duration dilation to appear. These constraints are in fact ad hoc abstractions that enable the behavior of the velocity of a particle to tend toward the celerity of the medium on a straight trajectory. The very form of the Lorentz factor excludes any possibility of a velocity greater than the celerity of the medium.

Galileo's invariance or principle of inertia proves to be incomplete, because it refers to the motion of uniform translation only. Rotational motion is considered to be accelerated in both Newtonian mechanics and the theory of relativity. Although Galileo and Lorentz transformations should be compatible without the abstractions of length contraction and time dilation being ratified, they will have to be intrinsically verified by the laws of physics; this is the case for Maxwell's equations or the wave equation. An extension of Galileo's invariance to uniform rotation motion is therefore essential.

1.1.5. *Distinction between velocity and celerity*

Velocity is a relative quantity independent of the medium that possesses an orientation and a direction, whereas celerity is a positive scalar quantity strictly associated with the medium under consideration. The notion of wave–particle duality introduced by Einstein [EIN 05] and de Broglie [BRO 95] at quantum scales can be transposed to larger scales considering that the particle has a velocity (or momentum) and the wave is associated with a celerity.

In fluid mechanics, Hugoniot [HUG 89] introduced the Mach number, the ratio $M = V/c$, in the definition of his theorem established from Euler's equation, the conservation of mass and the definition of the celerity of gas, $c^2 = dp/d\rho$:

$$\frac{dV}{V}\left(1 - \frac{V^2}{c^2}\right) + \frac{dS}{S} = 0 \qquad [1.7]$$

in which S is the duct cross-section considered in the one-dimensional model of ideal fluid flow.

The use of the Mach number in fluid mechanics is justified by qualifying the nature of the flow, namely, subsonic for $M < 1$ or supersonic if $M > 1$. Through this expression [1.7], it links the space curvature to the evolution of velocity, for example, in a Laval nozzle. When the convergent is followed by a constant cross-section channel, Hugoniot's theorem leads to a limit velocity that cannot be greater than the local celerity of the medium, this is the phenomenon of choking flow. However, although velocity and celerity are expressed with the same units, we should bear in mind the different natures of these quantities; comparing a relative quantity to another intrinsic to the medium can be challenging.

1.1.6. *Space–time or space and time?*

The notion of space–time, introduced in the 17th century by Lerond-d'Alembert, was recovered by the theory of special relativity and then developed in general relativity by Einstein. Until then, Newtonian mechanics considered space and time to be disjointed concepts. In special relativity, space–time is constructed from the classical spatial coordinates, (x, y, z), and a spatial coordinate defined by the product of the celerity of light by a time, $d = c_0\, t$. Space–time is then characterized by a quadruplet $(c_0\, t, x, y, z)$. One of the consequences affects the meaning of times and lengths, which then depend on the chosen reference frame. Within the context of the theory of relativity of the 19th and the 20th century where the emergence of these concepts is linked to light and gravitation, space–time is a way of describing the phenomena observed from a very legitimate perspective of unification.

In the theory of relativity, lengths are no longer fixed but depend on the time lapse dt being considered; using the distance $c_0\, dt$ as a length, the metric is written as:

$$ds^2 = dx^2 + dy^2 + dz^2 - c_0^2\, dt^2 \qquad [1.8]$$

where ds is the relativistic Minkowski length. The Minkowski space–time metric is a relativistic invariant. As a matter of fact, since $c_0\, dt = \gamma\,(c_0\, dt' + V dx'/c)$, $dx = \gamma\,(dx' + V dt')$, $dy = dy'$ and $dz = dz'$, then the relativistic distance:

$$ds^2 = dx^2 + dy^2 + dz^2 - c_0^2 dt^2 = dx'^2 + dy'^2 + dz'^2 - c_0^2 dt'^2 \qquad [1.9]$$

remains unchanged during the transformation between the two inertial coordinate systems. Two different events can thus be separated by a zero distance.

If we write $L_0 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$ the classical mechanics distance, the relativistic distance is equal to $L = \gamma\, L_0$, that is, a length contraction that depends on the velocity compared to the celerity of light V/c_0. For small values of this ratio, we find again the definition of the classic length. Correlatively, relativity induces a relativistic time such that $\tau = \tau_0/\gamma$, where τ_0 is time at rest. Relativistic time, or the time lapse between two events, undergoes dilation as the velocity increases toward the celerity of light. This time becomes infinite when $V = c_0$ so that a particle cannot reach, and of course exceed, the celerity of light in a vacuum. Length contractions and time dilation are ad hoc hypotheses in accordance with the other concepts of relativity and in particular with the Lorentz transformation.

Objections can be raised about this concept of space–time, even though it is no longer debated within the scientific community. Is it possible to arrive at the same conclusions and results by exploring other potential avenues in order to come back to the Newtonian notions of space and time?

First of all, the space traveled by an object whose velocity is equal to V at the end of a time t in the direction of the velocity is indeed equal to $\boldsymbol{l} = V\, t$; it is the distance traveled by a photon whose velocity is assumed to be equal to the celerity of light $V = c_0$. In the general case of a particle whose velocity is arbitrary, the distance traveled remains equal to $\boldsymbol{l} = V t$. What is then the significance of the product ct when the velocity of the wave is an intrinsic property of the medium and not a velocity? This notion of length built on celerity does not correspond to the underlying notion of displacement $U = V\, t$. Moreover, in what way is the celerity of light c_0 an absolute reference? Swell propagation or sound or seismic wave propagation have nothing in common with the celerity of light; these propagate at celerities depending on the medium considered, namely, gas, liquid or solid. In the theory of relativity, the celerity is always equal to c_0, the celerity of light in vacuum. This notion of space–time inhibits any unification of the laws of physics.

Going back to the concepts of disjointed space and time is not impossible. In this case, the lengths are fixed and time flows regularly and does not depend on velocity. The concepts of the theory of relativity initially based on thought experiments on clock and time are abandoned. Space and time no longer depend on the frame of reference, which must also be an abandoned concept. The undisputed results of special and general relativity must then be naturally found by the new laws of physics.

1.2. Objections to the equations of classical mechanics

1.2.1. *Equations of mechanics*

The equations of classical mechanics all have equivalent structures originating in the fundamental law of mechanics, Newton's second law, where the product of mass by acceleration is equal to the sum of forces. This is the case for Euler's equations for non-viscous compressible flows, the Stokes equation for creeping viscous flows, Navier–Stokes equations for viscous flows, the Navier–Lamé equation for solid elastic media, Darcy's law describing flows in porous media, etc. The most emblematic of these is probably the Navier–Stokes equation for fluid flows, which gathers together all the terms that can be found in the others. The equations of fluid mechanics are presented under multiple formulations that depend on the problem at hand; they are often associated with laws of conservation of energy in any form, enthalpy, internal energy, entropy, etc. Let us consider one of the most classic forms, omitting the complementary laws:

$$\begin{cases} \rho \left(\dfrac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \nabla \left(\lambda \, \nabla \cdot \mathbf{V} \right) + \nabla \cdot \big(\mu \left(\nabla \mathbf{V} + \nabla^t \mathbf{V} \right) \big) & [1.10a] \\ \dfrac{d\rho}{dt} = -\rho \, \nabla \cdot \mathbf{V} & [1.10b] \end{cases}$$

where ρ is the density and λ and μ are the Lamé coefficients of the fluid. The adjoint law, the conservation of mass, is strictly necessary; the Navier–Stokes vector equation is actually not autonomous, including for incompressible flows. The divergence of the tensor of the strain rates $\mathbf{D}$ can be developed to include the velocity rotational. Since the Lamé coefficients are variable quantities with respect to other quantities, the divergence of the Cauchy stress tensor is written as:

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} = \nabla \cdot \left(-p \, \mathbf{I} + \lambda \, \nabla \cdot \mathbf{V} \, \mathbf{I} + 2 \, \mu \, \mathbf{D} \right) & [1.11a] \\ \nabla \cdot \boldsymbol{\sigma} = -\nabla \Big(p - \left(\lambda + 2 \, \mu \right) \nabla \cdot \mathbf{V} \Big) - \mu \, \nabla \times \nabla \times \mathbf{V} & [1.11b] \\ \qquad\qquad + \nabla \cdot \mathbf{V} \, \nabla \lambda + \nabla \mu \cdot \big(\nabla \mathbf{V} + \nabla^t \mathbf{V} \big) & \end{cases}$$

After a few rearrangements, the Navier-Stokes equation becomes:

$$\begin{cases} \rho \, \dfrac{d\mathbf{V}}{dt} = -\nabla \left(p - \left(\lambda + 2 \, \mu \right) \nabla \cdot \mathbf{V} \right) - \mu \, \nabla \times \nabla \times \mathbf{V} & [1.12a] \\ \qquad\qquad + \nabla \cdot \mathbf{V} \, \nabla \lambda + \nabla \mu \cdot \big(\nabla \mathbf{V} + \nabla^t \mathbf{V} \big) & \\ \dfrac{d\rho}{dt} = -\rho \, \nabla \cdot \mathbf{V} & [1.12b] \end{cases}$$

When the flow can be considered incompressible and with constant viscosities, that is $\nabla \cdot \mathbf{V} = 0$, $\mu = const$ and $\lambda = const$, equation [1.12] simplifies and becomes the frequently used classical form:

$$\begin{cases} \rho \left(\dfrac{\partial \mathbf{V}}{\partial t} + \dfrac{1}{2} \, \nabla \left(|\mathbf{V}|^2 \right) - \mathbf{V} \times \nabla \times \mathbf{V} \right) = -\nabla p - \nabla \times \left(\mu \, \nabla \times \mathbf{V} \right) & [1.13a] \\ \nabla \cdot \mathbf{V} = 0 & [1.13b] \end{cases}$$

The density ρ remains a variable property, for example, as a function of temperature and pressure. Regardless of the methodology adopted to solve this system of equations, density and pressure have to be linked by a law of state. Furthermore, the number of unknowns and equations leads to closing the system by a relationship between these quantities. The simplification of [1.12] to [1.13] assumes that the constraint $\nabla \cdot \mathbf{V} = 0$ is a priori satisfied, which is not the case; this is simply a desired condition. In fact, it is the group $(\lambda + 2\,\mu)\;\nabla \cdot \mathbf{V}$ that should be considered, because this term is of the order of magnitude of the pressure and the other terms of the equation; the larger the values $(\lambda + 2\,\mu)$, the smaller the divergence. The values of these two viscosities attributed to fluids, and in particular the Stokes assumption, do not intrinsically guarantee the incompressibility of the flow.

1.2.2. *Divergence and curl operators*

The application of divergence and curl operators on the Navier–Stokes vector equation has a particular significance, because it creates second-order terms, some of which have questionable physical meaning, especially in situations where density is variable; this case is not taken into consideration in the remainder of this section.

The material derivative of the Navier–Stokes equation is equal to $d\mathbf{V}/dt = \partial \mathbf{V}/\partial t$ $+\boldsymbol{\kappa}$, where $\boldsymbol{\kappa}$ represents inertia; in continuum mechanics, it is written indifferently as $\mathbf{V} \cdot \nabla \mathbf{V}$, $\nabla \cdot (\mathbf{V} \otimes \mathbf{V}) - \mathbf{V}\;\nabla \cdot \mathbf{V}$ or even $\nabla(|\mathbf{V}|^2/2) - \mathbf{V} \times \nabla \times \mathbf{V}$. The latter term is none other than the Lamb vector $\nabla \cdot \boldsymbol{\mathcal{L}} = -\mathbf{V} \times \boldsymbol{\omega}$ with $\boldsymbol{\omega} = \nabla \times \mathbf{V}$, the vorticity vector. Let us consider the first term of the latter form by setting $\phi_i = |\mathbf{V}|^2/2$, the inertial potential; its divergence is equal to $\nabla \cdot \nabla \phi_i = \nabla^2 \phi_i$ and its zero rotational, $\nabla \times \nabla \phi_i = 0$:

$$\begin{cases} \nabla \cdot \left(\dfrac{d\mathbf{V}}{dt} \right) \equiv \dfrac{\partial}{\partial t} \nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla \left(\nabla \cdot \mathbf{V} \right) + \nabla^2 \phi_i + \left(\nabla \cdot \mathbf{V} \right)^2 - 2\, I_2 & [1.14a] \\ \nabla \times \left(\dfrac{d\mathbf{V}}{dt} \right) \equiv \dfrac{\partial}{\partial t} \nabla \times \mathbf{V} + \mathbf{V} \cdot \nabla \left(\nabla \times \mathbf{V} \right) - \nabla \times \mathbf{V} \cdot \nabla \mathbf{V} & [1.14b] \end{cases}$$

where I_2 is the second invariant of the tensor $\nabla \mathbf{V}$ and the first two right-hand side terms of these two relations, respectively, represent the material derivative of the divergence and the velocity curl, $d(\nabla \cdot \mathbf{V})/dt$ and $d(\nabla \times \mathbf{V})/dt$. The divergence of the Lamb vector is also written as $\nabla \cdot \boldsymbol{\mathcal{L}} = \mathbf{V} \cdot \nabla \times \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \boldsymbol{\omega}$ and comprises the flexion terms and enstrophy. These two terms are interpreted as properties of turbulent flows [HAM 08].

The first relation of [1.14] can be simplified in the presence of an incompressible flow, $\nabla \cdot \mathbf{V} = 0$; there remains the scalar potential Laplacian, but also the second invariant of the tensor $\nabla \mathbf{V}$. The latter is expressed by an orthogonal plane to the unit vector $\mathbf{n}$ such as $\mathbf{n} = \mathbf{t} \times \mathbf{m}$, where $\mathbf{t}$ and $\mathbf{m}$ are the unit vectors of the considered plane; if u and v are the components of the velocity in this plane, the term I_2 is the exterior product $\nabla_s u \wedge \nabla_s v$, where ∇_s is the gradient operator on the surface under consideration. It is then possible to define a pseudo-vector $\boldsymbol{\mathcal{I}}$ whose component would be associated with each plane of normal $\mathbf{n}$. This invariant I_2 has no reason to be zero, even with zero divergence. This scalar is zero on the volume Ω, but not locally like other invariants of continuum mechanics [TES 13]. Therefore, it appears that the constraint $I_2 = 0$ becomes a compatibility condition to locally ensure that $\nabla \cdot \mathbf{V} = 0$ in the context of the search for a strong solution.

The application of the curl operator to the velocity material derivative is generally written as:

$$\nabla \times \left(\frac{d\mathbf{V}}{dt} \right) \equiv \frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{V} \cdot \nabla \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \nabla \mathbf{V} \qquad [1.15]$$

where $\boldsymbol{\omega} = \nabla \times \mathbf{V}$ is the vorticity vector. The first two terms of the second member represent its material derivative $d\boldsymbol{\omega}/dt$. In two spatial dimensions, $\mathbf{V}$ and $\boldsymbol{\omega}$ are orthogonal and their dot product is zero:

$$\nabla \times \left(\frac{d\mathbf{V}}{dt} \right) \equiv \frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{V} \cdot \nabla \boldsymbol{\omega} \qquad [1.16]$$

This difference between the curl of the two and three spatial dimensional material derivative can be seen as an artifact due to the vectorial formulation of the equations of mechanics. Indeed, the curl of the velocity $\nabla \times \mathbf{V}$ is a pseudo-vector or an axial vector that only has a meaning when it is assigned to a surface with normal $\mathbf{n}$. It is defined by the Stokes theorem and computed on the contour of the considered surface; the vector $\boldsymbol{\omega}$ does not have a meaning when it is defined at a point within the context of the notion of a continuous medium. For a non-viscous fluid, equation [1.15] corresponds to the conservation of vorticity.

1.2.3. *The Stokes relation, an erroneous assumption*

The Navier–Stokes equation in [1.10] for a Newtonian fluid shows the two Lamé coefficients, λ the compressive viscosity and μ the shear viscosity. To assign a value to λ within the stress tensor, Stokes proposed a hypothesis linking the two Lamé coefficients in a relation $\eta = \lambda + 2/3\ \mu$, where η is called the *bulk viscosity*. This value of the bulk viscosity is usually identified as zero, and in other cases, it is the compression viscosity that is set to zero; this value is obtained as a result of solving the Boltzmann equation for monoatomic gases at very low pressure. If $\eta = 0$, Stokes' assumption leads to the relation $3\ \lambda + 2\ \mu = 0$; since μ is a positive measurable quantity, it follows that λ can take negative values, which seems unacceptable if we give it the role of a viscosity. In fact, many authors have tried to measure the value of λ for dense gases or liquids, and the values obtained vary greatly. It turns out that this Stokes hypothesis is erroneous, even for monoatomic gases [GAD 95b, RAJ 13].

The Navier–Stokes equation [1.12] shows the group $(\lambda + 2\ \mu)$ associated with compression-related quantities such as pressure p and the velocity divergence $\nabla \cdot \mathbf{V}$. This grouping of the two Lamé coefficients defines a single property, compressibility, or rather its inverse multiplied by a time constant dt, $dt/\chi_T = (\lambda + 2\ \mu)$. This expression can be compared to the elastic coefficients of a solid where the compressive wave modulus is equal to $M = (\lambda + 2\ \mu)$, where λ and μ are the Lamé coefficients of the elastic solid. The equivalence between fluid and solid is expressed through the expression of displacement $\mathbf{U} = \mathbf{V}\, dt$. The isostatic modulus of elasticity $K = 1/\chi_T$ defines the grouping $(\lambda + 2/3\ \mu)$. Its perfectly measurable value for water is about $K = 2 \times 10^6$ Pa and, since its dynamic viscosity is equal to $\mu \approx 10^{-3}$, it is easy to find $\lambda \approx K = 2 \times 10^6$ Pa and to see that Stokes' law is wrong.

By construction, there can only be two independent coefficients to represent each of the different effects, the effects of compression by the compressibility coefficient χ_T and shear by the dynamic viscosity μ. The 81 coefficients of the elasticity tensor $\mathbf{C}$ are reduced to two coefficients for an isotropic fluid by considering a sequence of rotations and symmetries, $\mathbf{C}_{12} = \lambda$ and $\mathbf{C}_{44} = 2\ \mu$. The value of λ deduced from $\lambda + 2/3\,\mu$ introduced into equation [1.10] has absolutely no influence on the behavior of the Navier–Stokes equation; given the ratio of $\mu/K \approx 10^{-3}/10^6$, it represents a hypercompressible medium. This is the reason why the law of conservation of mass is adjoint thereto. It indirectly introduces the real compressibility of a medium, even for a gas where the ratio becomes $\mu/K \approx 10^{-5}/10^5$ for air under normal conditions. For an isentropic flow, the determining parameter is its celerity $c_s = \sqrt{\rho\, \chi_s}$, where χ_s is its isentropic compressibility. In conclusion, system [1.10] formed by the Navier–Stokes equation and its adjoint, the conservation of mass, does not require any value to be assigned to λ for both compressible and incompressible flows. It should be noted that if the value of the compression viscosity such as $\lambda = 1/\chi_T$ were adopted in the Navier–Stokes equation [1.10], resorting explicitly to mass conservation would become unnecessary. Indeed, the operator $\nabla\, (\lambda\, \nabla \cdot \mathbf{V})$ would immediately ensure the conservation of mass; more specifically, a very high value of λ would implicitly result in imposing $\nabla \cdot \mathbf{V} = 0$, a constraint that describes an incompressible flow.

1.2.4. *Non-relativistic equations*

Just like the other equations of classical mechanics, the Navier–Stokes equation is a priori not relativistic. Several attempts to make this equation satisfy the Lorentz invariance remain inconclusive for different reasons. First, the physical properties, density ρ, the viscosity coefficients λ and μ make the Lorentz transformation difficult to apply. The existence of nonlinearities of the terms of inertia seriously complicates this work. Finally, the conservation of mass outside the equation itself restricts the chances of simply obtaining an equation of relativistic motion that nevertheless retains its properties at velocities much lower than the celerity of light c_0.

As such, system [1.10] exhibits a hyperbolic character related to its ability to represent longitudinal waves of celerity c_l. It very correctly translates linear and nonlinear waves such as shock waves. The transposition between sound waves and light waves is not only formal; swell, acoustic waves and Hertzian waves share the same nature and the propagation properties naturally depend on the frequencies considered. One phenomenon makes it possible to understand the legitimacy of comparing acoustic waves and light waves: it is the limitation of the material velocity to the celerity of the medium on a rectilinear trajectory when the inertial curvature is zero, c_l for fluids and c_0 for light in vacuum. In a shock tube, the solution to Euler's equation is in the form $\boldsymbol{x} = \pm c_l t$, where $\boldsymbol{x}$ is the x-axis of the wavefront; the velocity cannot exceed the celerity of sound in the fluid. The conclusions drawn by Einstein from the experiment by Michelson and Morlay [MIC 87] on the non-existence of Aether at the end of the 19th century are that the "speed of light" is a value of material velocity that cannot be exceeded and that it is the same in any inertial frame of reference. These two perfectly established observations in fluid mechanics and special relativity make us question a single formalism, but the Navier–Stokes equation is not a wave equation, even if the nonlinearity due to the terms of inertia is disregarded.

Another issue is related to the viscous effects that have no equivalent in special relativity, where the propagation of gravity waves with a celerity equal to c_0 has the same nature as the propagation effects of polarizable transverse waves in solid mechanics. The viscous term $\mu \nabla^2 \mathbf{V}$ or its equivalent $-\nabla \times (\mu \nabla \times \mathbf{V})$ are therefore not the appropriate form to represent these transverse waves at a celerity c_t. The Navier–Stokes equation reflects the fact that, even if these waves exist, they are instantaneously dissipated; this instantaneous character is not admissible in physics, there is always a time constant, even a very small one, which ensures the transition for the attenuation of transverse waves in Newtonian viscous fluids. The diffusion of the transverse momentum implies its dissipation [CAL 24d]. From this point of view, the phenomena of longitudinal and transverse propagation in elastic solids which are governed by the Navier–Lamé equation are more compatible with the relativistic formalism. In fact, although fluid mechanics and solid mechanics are supposed to be brought together within the mechanics of continuous media, it is clear that the equations are still different.

So why is it important to look for an equation that is just as representative of viscous flows and light propagation? The Navier–Stokes equations have remained virtually unchanged for more than two centuries, even though very important discoveries have been made in physics during the same time period. Maxwell's equation is relativistic and the equation of fluid motion should be relativistic one day. Cosmology introduced the notion of cosmological fluid by integrating the expansion of the Universe into Euler's equations; this concept of expansion is still non-existent in the current equations of classical mechanics where Galileo's invariance becomes insufficient. Similarly, the equivalence between mass and energy of special relativity is a pillar of physics that is neglected. The current equations of mechanics are based on the conservation of mass, but also on that of energy duplicated by an equation of motion, which also expresses a conservation of energy. These equations are redundant and the number of variables used is also in excess. These findings deserve special attention and thorough examination to reduce the number of equations, variables and even the number of fundamental units in which they are expressed.