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A Brief History of Naval Hydrodynamics

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Humankind did not await the equations governing fluid mechanics before sailing. From the earliest times, humans learnt how to make the water carry them through experience, how to use wind or the oar to propel themselves and how to use the rudder to steer (Figure 1.1).

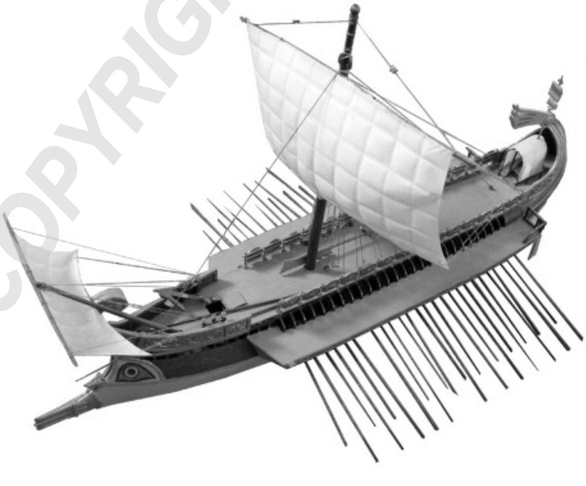

Figure 1.1. *Model of a Roman trireme (source: Wikimedia Commons). For a color version of this figure, see www.iste.co.uk/sigrist/fluidstructure.zip*

Fluid–Structure Interaction,
coordinated by Jean-François SIGRIST and Cédric LEBLOND.

Several centuries before our time, shipbuilding and navigation techniques had already achieved a certain level of maturity based on purely empirical knowledge, which continued without major innovation until the 17th century.

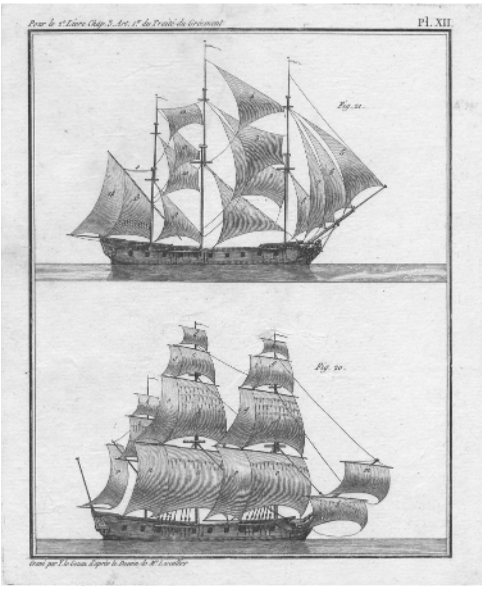

Figure 1.2. *Extract from* Traité pratique du gréement des vaisseaux et autres bâtiments de mer*, Daniel Lescallier, Paris, 1791 (source: personal collection)*

With the discovery of differential calculation, flow theory appeared, while an original experimental method was drawn up using tests on reduced models. This last method prevailed until the last quarter of the 20th century as the only precise and reliable scientific representation of the phenomena linked to shipping hydrodynamics. From the beginning of the 1960s, the development of algorithms for the geometric representation and numerical analysis of partial differential equations, underpinned by the exponential power of computers, gradually made it possible to solve the complex equations involved in hydrodynamics. Today, it has resulted in the “virtual test basin”.

1.1. The emergence of a new science

From the 4th century BCE, Aristotle attempted to provide philosophical definitions of movement and liquids, but it is to Archimedes, a century later, that we owe the first scientific manuscripts on fluid mechanics. His famous principle on the equilibrium of floating bodies still forms the basis of naval architecture. We also owe to him the invention of the screw, the rotation of which in fluid makes it possible to imprint an axial movement on this fluid. It was only 2,000 years later, in 1768, that the French mathematician Paucton gave a mathematical description of this, from which he deduced a theory of propeller propulsion.

Archimedes' principle and the concept of pressure linked to it was analyzed by the mathematician Flamand Stevin, to whom we owe the term hydrostatics. Then, in the middle of the 17th century, Blaise Pascal provided the expression of pressure within a fluid at rest, known as Pascal's law. Two physicists, one Dutch, Christiaan Huygens, and the other French, Edme Mariotte, members of the entirely new *Académie des sciences* (Figure 1.3), sought to determine the law of a projectile's resistance in a fluid and determined experimentally that this resistance varies as the square of the velocity of the body.

Figure 1.3. *Colbert presents the members of the* Académie royale des sciences *created in 1667 to Louis XIV, Henri Testelin (1616–1695), oil on canvas (1666), and château de Versailles (detail). In the center, Huygens and Mariotte (source: RMN-Grand Palais, château de Versailles/Gérard Blot). For a color version of this figure, see www.iste.co.uk/sigrist/fluidstructure.zip*

In *Principia Mathematica*, published in 1687, Newton demonstrated this result using a particular – erroneous! – model of fluid in the form of collisions of minuscule solid spheres that strike the body. This model did, however, enjoy great popularity and was used for nearly a century to find the form of minimal resistance. In his *Traité du navire*, published in 1746, Pierre Bouguer described a method for calculating collisions based on approximation of the hull using small, flat surface elements (Figure 1.4).

However, a scientific revolution began in 1684 with the publication of the *Nova methodus pro maximis et minimis* by the German Gottfried Wilhelm von Leibniz, in which there appeared briefly the rudiments of differential and integral calculus. This new theory took several generations to be fully developed. The first to familiarize themselves with this mathematical advance and complete it was the Bernoullis, a line of Swiss scholars. They deduced from this a variational principle in mechanics, the foundation of modern numerical analysis in mechanics, which Lagrange generalized under the name of the principle of virtual power.

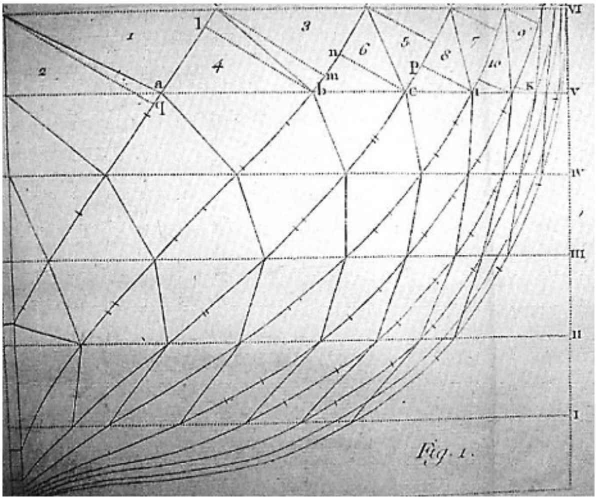

Figure 1.4. *Representation of a hull using plane elements (source: Bouguer (1746))*

The last quarter of the 17th century was marked by a flurry of scientific activity that was fueled by, among other factors, the growing needs of shipbuilding. Commercial rivalry and the defense of colonial possessions in the Americas, Antilles and east Indies, and the clash of French, English, Dutch and Spanish ambitions led to an increase in the number of ships, their size and their weaponry.

In the "artisanal" industry, carried out by carpenters whose knowledge was empirical and inherited, ships became bigger through homothety and extrapolations, and the result was often disappointing: on August 10, 1628, the *Vasa*, the pride of the king of Sweden, capsized in good weather in the Bay of Stockholm, two hours after its launch.

Colbert and his son Seignelay, while still reforming the organization of the arsenals (weaponry), committed themselves to perfecting and harmonizing practices in ship design and ship building and undertook the first scientific work on the subject. In 1681, in an attempt to test new mathematical theories, they ordered the construction – under the supervision of admirals Anne-Hilarion de Tourville and Abraham Duquesne – of two small frigates, with the aim of carrying out a test comparing them on the Grand Canal at Versailles. One was of "classical" shape, and the other united all the advances and new concepts of the era. Unfortunately these models, made at different scales in the absence of laws known as laws of similitude, could not give significant results.

Static stability, the reduction of running resistance and the position of the point of sail were subjects that the best scientific minds of the time tackled, encouraged by every European Academy of Science (Paris, Berlin, Saint Petersburg). Science became a strategic tool in the 18th century. Three publications marked a decisive stage in the development of hydrodynamics, the name of which was given specifically by one such publication: *Hydrodynamica* by Daniel Bernoulli, which appeared in 1738 (Figure 1.5).

In this book, the author established flow equations in ducts and formulated his famous principle of conservation, known today as Bernoulli's equation (or theorem). The second book, which certainly had the most significant and lasting impact on naval architecture, is the *Traité du navire* by Pierre Bouguer, which appeared in 1746. This book, written in French, was widely published and served as the basis for teaching shipping engineers (who replaced the old marine master carpenters) in the school in Paris created for this purpose by Henri-Louis Duhamel du Monceau[1] in 1741. In his book, Pierre Bouguer established the principle of the stable equilibrium of floating hulls and gave the formula for calculating the metacenter.

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[1] The school, then called the *Petite école du Louvre* gave lessons in a room at the palace, neighboring the *salle des séances* of the Academy of Sciences. Like Duhamel du Monceau, inspector general *de la Marine*, Pierre Bouguer, hydrographer to the king, and Borda, several teachers in this school became members of the *Académie des sciences* and the *Académie de marine* (created in 1752).

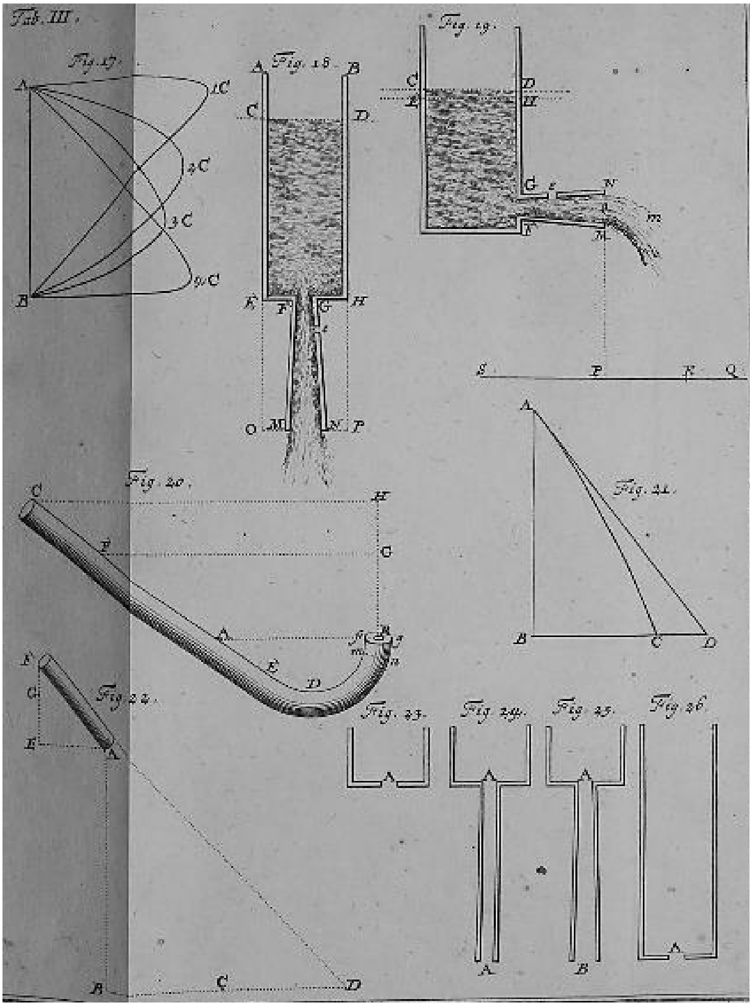

Figure 1.5. *Extract from* Hydrodynamica *by Daniel Bernoulli (source: Bernoulli (1738))*

Finally, Leonhard Euler published *Scientia Navalis* in 1749; a challenging, highly theoretical book in Latin, which consequently had little impact. However, the role of Euler in the development of the theory of fluid dynamics went well beyond the book. Originally from Basle, like the Bernoulli family with whom he had frequent exchanges, Euler was interested in the theory of fluids very early on. A member of the Academy of Sciences at both Saint Petersburg and Berlin, Euler took the application of infinitesimal calculation further, to fluid flows. In 1755, he proposed a continuous field model (pressure, velocity) described by equations with partial derivatives: Euler equations for "perfect fluids", an expression he introduced to distinguish them from "semi-fluids" endowed with cohesion; he defined streamlines, along which he generalized the Bernoulli equation. His entire work on fluid and acoustic mechanics was published in French in his *Principes généraux du mouvement des fluides* in 1757.

With Bernoulli and Euler, Jean Le Rond d'Alembert was one of the main artisans of the process that developed the theoretical foundations of the science of the motion of fluid. In his first writings on hydrodynamics, which appeared between 1744 and 1752, d'Alembert followed the same path as Bernoulli but extended his flow model from one dimension to two dimensions and introduced the laws of conservation that would be used by Euler in his equations. In his *Essai d'une nouvelle théorie sur la résistance des fluides*, d'Alembert showed the limits of the theory with his famous "paradox" and launched a debate that continued and grew until the end of the century. The scientific community was then divided between hydrodynamicists who, with Lagrange, strove to perfect the theory, and hydraulicists, who leaned more toward an experimental and practical approach. The debates were virulent and the enmity fierce. D'Alembert sought analytical solutions of flat movement using a method that prefigured the study in complex variables, and in 1781 in his *Mémoire sur la théorie du mouvement des fluides*, Lagrange introduced the major concept of potential flow for incompressible, irrotational perfect fluids. In 1788, he established the "Lagrangian" formulation of equations, an alternative to that of Euler, with his *Mécanique analytique* (Lagrange 1989).

Jean-Charles de Borda, a mariner and mathematician successor to Duhamel du Monceau at the head of the Engineering School[2], favored experiments over models and theory and engaged in virulent criticism, based on personal antagonism, of Bernoulli's and d'Alembert's models. The *abbé* Charles Bossut, a pupil of d'Alembert and professor of mathematics at the Engineering School, was distinguished by the many experiments he carried out. Faced with the quarrels that tore apart the *Académie des sciences*, Turgot, general controller of the kingdom's finances, created a "Chair of Hydrodynamics of the Louvre", funded by the king, which he gave to Bossut, and in 1775 tasked him, along with d'Alembert and Condorcet with "examining methods for perfecting navigation within the kingdom".

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[2] The shipbuilding engineers were established as a *Corps* (a guild or body) in 1765. They took the name *d'ingénieurs du Génie maritime* (maritime engineers) in 1795. The *Petite école* became the *l'École d'application du Génie maritime* and its pupils were already recruited to the *École polytechnique* (French technical school).

Their primary objective was to examine the problem of the resistance of fluids and "consult empirical evidence either to check elements of known theories, or to procure data that could serve as a basis for new solutions". The experiments, carried out according to an extremely rigorous procedure, made it possible to overturn the laws of resistance stated by Newton in his *Principia*.

1.2. Perfecting the theory

1.2.1. *Fluids, viscosity and turbulence*

At the beginning of the 19th century, mechanics was the prerogative of mathematical engineers in France, such as Dupin, a maritime engineer, or Navier, an engineer of *Ponts et Chaussée* (Bridges and Causeways).

In his *Mémoire à l'Académie sur les lois du mouvement des fluids* of 1822, Henri Navier introduced the notion of viscosity and the hypothesis of the adherence of fluid to the walls over which it flows. Cauchy, Poisson and Saint-Venant pursued his idea, and a British physicist, George Gabriel Stokes, in his *Report on Recent Researches on Hydrodynamics* presented to the British Association for the Advancement of Science in 1846, established the definitive form of equations for the flow of incompressible fluid under the name Navier–Stokes equations. Stokes, Poiseuille, Hagen and Couette focused on finding solutions to the Navier–Stokes equations. These solutions hypothesized flow in parallel layers, which would be called "laminar flows".

In 1883, an Irish professor at the University of Manchester, Osborne Reynolds, experimentally demonstrated the transition of a laminar flow into a turbulent flow by varying the speed within a tube. From this, he deduced the dimensionless ratio between viscous effects and inertia effects that governs this transition, the Reynolds number, as well as the averaged turbulent form of Navier–Stokes equations, termed Reynolds-averaged Navier–Stokes (RANS) equations.

In 1904, it was a German professor in Hanover, Ludwig Prandtl, who formulated a double-scale model to reformulate Navier–Stokes equations in the vicinity of a wall and who introduced the concept of the boundary layer. Boundary layer equations make it possible to match a perfect fluid solution, located far from the wall, to an "interior" solution, in the vicinity of the wall, where viscous effects are concentrated. This method prefigured a general mathematical approach that was theorized from the 1950s, the matched asymptotic expansion method.

Unfortunately, in their averaged Reynolds form, Navier–Stokes equations are not closed, that is, they are incomplete, and an additional equation, called a turbulence model, must be added to them. The development of turbulence models remains one of the most active domains of fundamental research in fluid mechanics today. The Boussinesq hypothesis, formulated in 1877, presupposes proportionality between turbulent constraints (Reynolds stress) and volume expansion rates and makes it possible to introduce the notion of turbulent viscosity. It remains the basis for most current turbulence models.

The 20th century was marked by a considerable number of publications on turbulence, among which we find the greatest names in mathematics and physics: Poincaré, Prandtl, von Kármán and Heisenberg (McDonough 2007). Statistical models started from the hypothesis formulated by Taylor in the 1930s, according to which turbulence is a random phenomenon and can therefore be described by statistical instruments. In 1941, the Russian mathematician Kolmogorov provided an energy "cascade" model for vortices of different characteristic lengths, down to the smallest, named the "Kolmogorov scale" where the energy of turbulence is dissipated. This description is now the basis of numerical solution methods, including large eddy simulation (LES).

Deterministic models consider, following Poincaré, that turbulence can be represented by Navier–Stokes equations, hence deterministically, as soon as we know how to solve them, up to a scale similar to the Kolmogorov scale. This approach leads to direct methods (direct numerical simulation (DNS)) for solving Navier–Stokes equations. These methods are very demanding in terms of computational resources, and today they are still at an initial stage.

Resulting from chaos theory, "structural models" consider the solutions of Navier–Stokes equations to be dynamic systems and lead to the modeling of laminar, transitional and turbulent flows around the notion of attractors, the dimension of which is a function of the Reynolds number. They use the formalism of Hamiltonian mechanics, another representation, along with Eulerian and Lagrangian, of mechanical equations.

As these complex theories developed, many measurement campaigns sought to determine the laws of variation of the average friction of fluid on a surface (usually a flat surface) depending on the Reynolds number. Among the many empirical laws deduced from it, a particular law has been recommended since 1957 for the needs of naval hydrodynamics (law ITTC 57).

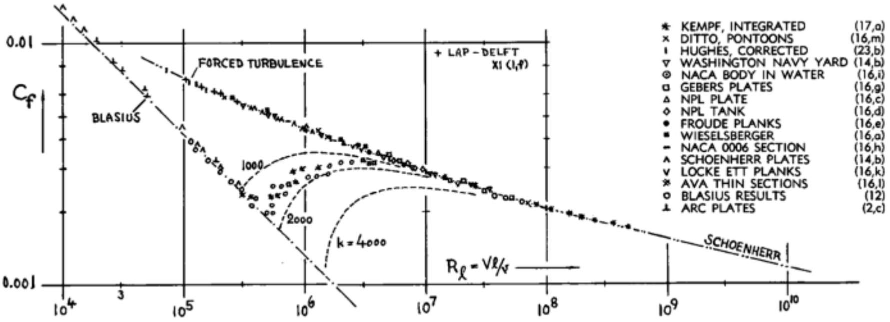


Figure 1.6. *Friction coefficient on a flat surface depending on the Reynolds number (source: Hoerner (1965))*

The relatively recent interest in sounds and vibrations generated by flow around vehicles in the air or in water led to a focus on the coherence, not only temporal but also spatial, of turbulent flow in contact with a wall. This led to the development of frequency and wavenumber spectra. The Corcos model was complemented by many works, especially in the region of low numbers of waves, called an "acoustic" zone.

1.2.2. *Potential theories*

Interest in viscosity and friction did not, however, preclude continuation of the work initiated by Lagrange, Laplace and Poisson on the properties of some mathematical functions, in particular solutions to the equation of the velocity potential of incompressible perfect fluid, the Laplace equation.

In 1828, a British mathematician, George Green, established potential theory and determined the expression of elementary potential functions applicable, in particular, to electromagnetism. He established an essential theory of integration called Green's theorem. Green's mathematical advances on electromagnetism stimulated work on field theory developed over the century by Gauss, William Thomson and Maxwell. In the second half of the 20th century, they were essential to the numerical solution of potential flows.

In 1870, a Scottish physicist, John William Macquorn Rankine, published *Shipbuilding, Theoretical and Practical.* In it, he developed a theory of closed streamlines induced by pairs of sources and wells which led to the definition of ovoid shapes.

Pursuing d'Alembert's idea on plane flows and the properties of the current function, known under the name "Cauchy–Riemann equations", Augustin Cauchy was another engineer of *Ponts et Chaussées*, who in the 1810s established the theory of complex functions. The complete theory of analytical functions was proposed by Bernhard Riemann at Göttingen in 1851. In fluid mechanics, complex potential theory played a major role in the study of two-dimensional flows, especially in the development of Kutta's theory for lifting profiles, at the start of the 20th century.

In the 1930s, John von Neumann, a Hungarian mathematician who emigrated to the United States, developed the bases of functional analysis and operator theory. He gave his name to the type of wall condition on an obstacle in perfect fluid, the Neumann condition, as well as the mathematical expression of problems including this condition. We therefore refer to "Neumann–Stokes" or "Neumann–Kelvin" problems for the study of free surface flow around a moving hull. Functional analysis paves the way for numerical analysis.

1.2.3. *Waves*

Since Leonardo da Vinci, the nature of waves at sea fascinated mathematicians and physicists; a number of scientists, from Newton to Lagrange, have attempted to describe it. The latter established, in his work already cited, that at shallow depths, the speed of a surface wave is equal to the square root of the product of the acceleration of gravity and the water's height. However, it was for Franz von Gerstner, an Austrian physicist, to formulate, in 1804, the hypothesis of circular trajectories of fluid particles in water.

In 1815–1816, Cauchy and Siméon Denis Poisson, another pupil of Lagrange and Laplace, provided solutions to the linearized problem of waves at infinite depth using a decomposition into a Fourier series for the first time. In 1845, the English mathematician George Bidell Airy provided a linear description of sinusoidal waves, called "Airy waves". It was extended in 1847 by Stokes to higher order solutions obtained by development in a Taylor series of the potential of velocities close to the free surface. Stokes's drift revealed a boundary condition on a wave breaking in shallow waters.

In 1863, Rankine gave an exact, rotational solution of Euler equations, the trochoidal wave. Other solutions, including Stokes's higher order developments, were proposed over the years. First, Boussinesq, then Korteveg and de Vries in

1895 introduced a new system of equations applicable at very shallow depths and gave an exact solution to them, known as a cnoidal wave.

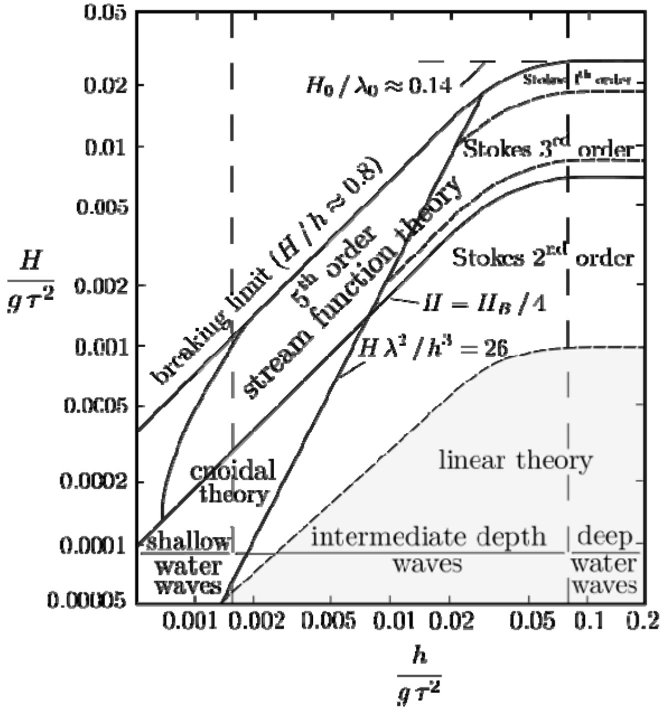
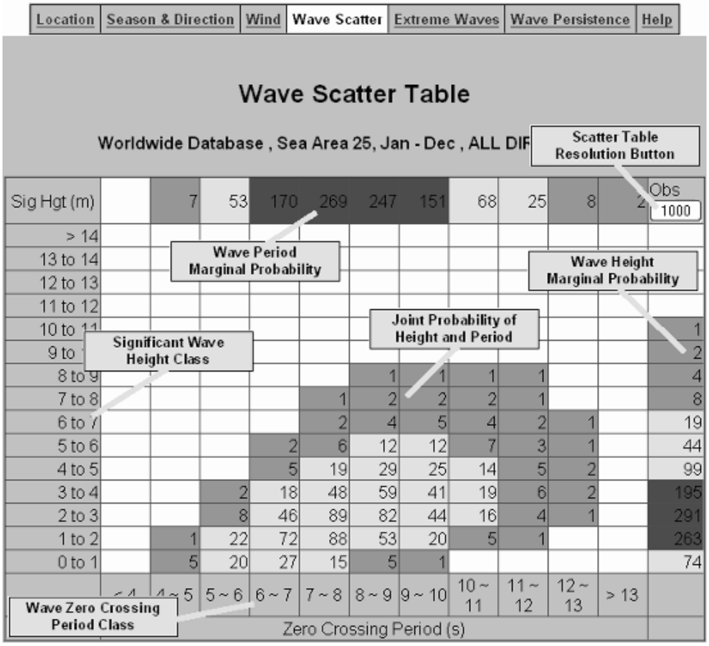


Figure 1.7. *The validity of different wave models (source: Le Mehauté (1976)). For a color version of this figure, see www.iste.co.uk/sigrist/fluidstructure.zip*

In 1887, William Thomson, who became Lord Kelvin in 1892, gave a general expression of an irregular wave in the form of a Fourier integral. With the German Hermann von Helmholtz, he studied the stability of a water–air wave interface. However, real advances for naval applications came after the Second World War through the application of statistical and probabilistic models to the results of observations made at sea.

Willard Pierson, then Gerhard Neumann, professors at the University of New York at the beginning of the 1950s, started from the hypothesis that real waves can be represented as the superposition of a large quantity of sinusoidal waves of infinitesimal size and introduced description by energy spectrum. Many spectrum models, adapted to different ocean or meteorological environments, were then established (Pierson-Moskovitz, Longuet-Higgins, Darbyshire, Breitschneider, JONSWAP, ITTC). These families of spectra are generally described by one or two parameters linked to wind strength.



Wave Scatter Table

Worldwide Database , Sea Area 25, Jan - Dec , ALL DIF

Sig Hgt (m)		7	53	170	269	247	151	68	25	8	2	Obs 1000
> 14												
13 to 14												
12 to 13												
11 to 12												
10 to 1												1
9 to												2
8 to 9						1	1	1	1			4
7 to 8					1	2	2	2	1			8
6 to 7					2	4	5	4	2	1		19
5 to 6				2	6	12	12	7	3	1		44
4 to 5				5	19	29	25	14	5	2		99
3 to 4			2	18	48	59	41	19	6	2		195
2 to 3			8	46	89	82	44	16	4	1		291
1 to 2		1	22	72	88	53	20	5	1			263
0 to 1		5	20	27	15	5	1					74
	< 4	4 ~ 5	5 ~ 6	6 ~ 7	7 ~ 8	8 ~ 9	9 ~ 10	10 ~ 11	11 ~ 12	12 ~ 13	> 13	
	Zero Crossing Period (s)											

Figure 1.8. *Wave distribution diagram model*[3]*. For a color version of this figure, see www.iste.co.uk/sigrist/fluidstructure.zip*

An empirical scale measuring wind strength was established in 1805 by the English hydrographer Francis Beaufort and adopted in 1830 by the Royal Navy for the needs of sailing ships. In 1920, after steamships had replaced sail ships, the Navy, and later the International Maritime Organization, adopted an equivalent scale, called the “Douglas scale”, to define the state of the sea from a visual estimate of significant wave height. However, it was only after the Second World War that there was interest in the statistical analysis of many measurements at sea and that probabilistic wave theory was formed.

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[3] Source: http://www.globalwavestatisticsonline.com/Help/wave_scatter.htm.

1.3. Ship theory

Throughout the 19th century and during the first half of the 20th century, an applied science called "ship theory" developed. According to a definition accepted at the time, "by ship theory, we understand all studies on the mechanics of a non-deformable geometric solid immersed in a fluid or floating on its surface, an abstraction made from the very structure of the body" (Doyere 1927). Indeed, ship theory covers the definition of the geometrical shapes of ships and related calculations, the study of static, its dynamic on a calm sea and on waves, and finally the description of the function of different types of wind and mechanical thrusters, which today we call naval hydrodynamics. The description of the framework and the processes for implementing materials form the domain of shipbuilding. These two disciplines form the two branches, theoretical and practical, of naval architecture (Bovis 2009, 2016).

1.3.1. *Stability*

Charles Dupin's work on analytical geometry led at the start of the 19th century to the theory of the geometrical representation of hulls and to analytical calculations of surfaces, volumes, centers of buoyancy and metacenters for the purposes of stability calculations. Since then, calculations of metacentric height, at all angles of incline, and stability curves have become systematic in shipbuilding projects. The increase in the weight of ships, led at the end of the century by the development of artillery and armor plating, created difficulties in providing ships with a sufficient reserve of stability; this was shown dramatically, for example, in the wreck of the English frigate *HMS Captain* in 1869 (Figure 1.9) or the French series of *cuirassés chavirables* (capsizing battleships)[4].

Quite quickly, there appeared to be a need to impose design criteria to ensure a minimum level of stability. The first criterion, called a load line, was made compulsory around 1900. In the wake of work by Finn Jaakko Rahola, different initial stability criteria were developed from the 1930s by states and classification societies. The wreck of the *Titanic* in 1912 nevertheless demonstrated the risks linked to waterways and led to a focus on the conditions of stability after damage. From 1968, stability was the subject of international regulation, established and regularly amended by the International Maritime Organization (IMO).

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[4] Six months after its completion, the *HMS Captain* overturned on a rough sea in the dark, with more than 500 victims. Bouvet gave a dramatic example of a *cuirassé chavirable* (capsizing ship): in 1915, one of them overturned and sank in less than a minute after hitting a mine in the Dardanelles, with 658 dead.

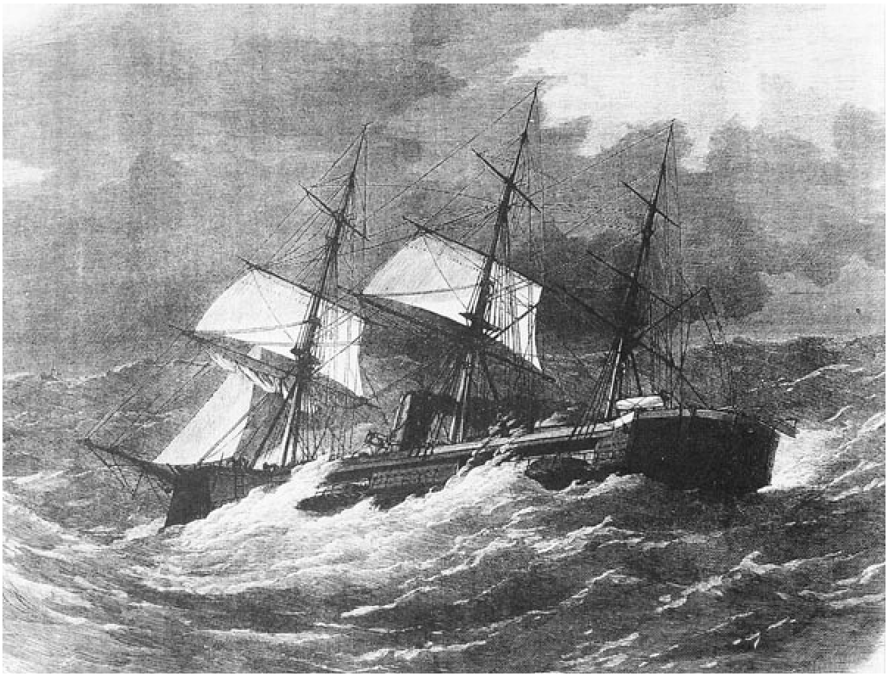

Figure 1.9. *The sinking of HMS Captain, William Frederick Mitchell (1845–1914), Illustrated London News, volume LVII, 24 September 1870 (source: Wikimedia Commons)*

1.3.2. *Resistance to forward motion*

Newton’s theory on collision has been recognized as imprecise, and the d’Alembert paradox has demonstrated the limit of solutions in perfect fluid. An approach involving resistance to forward motion therefore developed at the beginning of the 19th century using an empirical approach. The proportionality of the resistance to the square of the speed and the surface of the mid-ship section, widely accepted since Bossut’s tests, was called into question by numerous measurements of towing at sea, measurements facilitated by the advent of mechanical steam propulsion. In 1842, Frédéric Reech demonstrated that the proportionality coefficient is only constant, for similar hulls, if their speeds are proportional to the square roots of their dimensions.

In 1857, Siméon Bourgeois, a sailor and engineer, also established that a ship’s running resistance is the sum of two components, one linked to friction and varying with the square of the speed and another varying only with the speed. He demonstrated the presence of a minimum in the variation curve of the resistance with speed. Like the British architect Isambard Brunel, William Froude, himself an engineer, followed the making of the revolutionary ships, the *Great Western* and the *Great Eastern*, and noted their deficiencies. Charged by Admiralty to study optimal hull shapes, he convinced them to build the first test tank at Torquay at the start of the 1870s. He established the bases for the experimental study of running resistance defined the Froude number and formulated the Froude hypothesis. By predicting the running resistance of the corvette *HMS Greyhound* to within 10%, Froude convinced the naval community of the benefit of testing using models.

The path opened up by William Froude and pursued by his son Robert Edmund were extended rapidly with the construction of test tanks in several countries at the turn of the 20th century. In 1932, the International Conference of Ship Tank Superintendents was created, today the International Towing Tank Conference (ITTC), which brings together more than 100 centers. The ITTC sought to harmonize test procedures on models and methods of model-real-world extrapolation. Towing and self-propulsion tests and the prediction of a ship's propulsive power were the subject of the procedure called ITTC 78.

The advent of tests on models allowed systematic tests on the shapes of similar hulls to be carried out, leading to the development of standard series, such as the Taylor series, established in 1910, Wigley hull shapes, the 60 series or, for submarines, the 58 series, still called teardrop hulls. Similarly, a series of propellers with different geometries were defined, and their hydrodynamic performances were deduced in the form of families of curves (Troost, AU and Gawn series). These series and their test results were widespread and served for a long time as a reference for naval architects for choosing dimensions and predicting performance.

In 1887, Lord Kelvin established the characteristics of waves in the wake of a moving object near a free surface and, in particular, demonstrated that the Kelvin angle is a universal constant.

Afterward, the characteristics of wakes were demonstrated using different mathematical approaches.

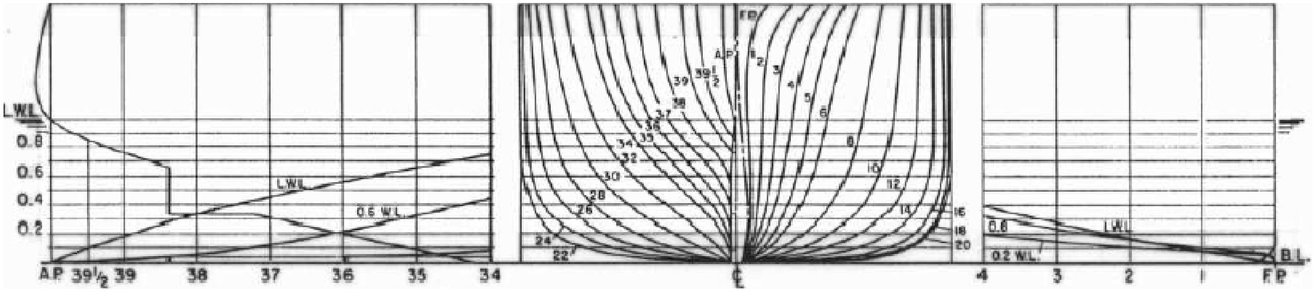

Figure 1.10. *Series 60 (source: F.H. Todd, Methodical experiments with models of single-screw merchant ships, Report 1712, David Taylor Model Basin, 1963)*

The first theoretical model of the wave resistance of a slender hull, known as "slender body approximation", was produced in 1898 by an Australian mathematician, John Henry Michell, in a remarkable publication (Figure 1.11). It was exceptional in its rarity – Michell published only 20 articles in his career – but also in its simplicity and finally because of the precision of its results. Twenty years later, the same method for removing boundary conditions was applied to linear airfoil theory.

$$R = \frac{4\rho g^2}{\pi U^2} \int_1^\infty (I^2 + J^2) \frac{\lambda^2 d\lambda}{\sqrt{\lambda^2 - 1}}$$

$$I = \int\int_H \eta_x(x, z) e^{\lambda^2 g z / U^2} \cos(\lambda g x / U^2) dx dz$$

Figure 1.11. *Michell's Formula (1898): J is identical to I with the sine function replacing the cosine function. $\eta_x(x,z)$ defines the shape of the hull either side of the plane $y = 0$, and the hull is assumed thin, that is, $\eta_x \ll 1$*

Unfortunately, Michell's work was ignored until his results were found in 1923 by Thomas Havelock, an English mathematician. Havelock introduced the potential of flow induced by a singular disruption in the vicinity of the linearized free surface and thus obtained the Green function of the Neumann–Kelvin problem, known as the Havelock formula (Figure 1.12). He established different methods of calculating wave resistance. By adding together the effect of sources distributed on the hull's longitudinal median plane, he found Michell's formula. He calculated the characteristics of the Kelvin wake and established a formula for calculating wave resistance from the energy radiated in the wake, far from the hull.

$$\begin{aligned} \phi = \frac{m}{r_1} - \frac{m}{r_2} - \frac{4gm}{c^2} \int_0^{\pi/2} e^{-k_0(f-z)\sec^2\theta} \sin(k_0 sec\,\theta) \times \\ \times \cos(k_0 y \sin\theta \sec^2\theta) \sec^2\theta\, d\theta - \frac{4gm}{\pi c^2} \\ P \int_0^{\pi/2} \sec^2\theta d\theta \int_0^\infty \frac{e^{-k(f-z)} \cos(kx\cos\theta)\cos(ky\sin\theta)}{k - k_0\sec^2\theta} dk \end{aligned}$$

Figure 1.12. *Havelock's Formula (1934). Potential of flow from a source m moving at speed Vo in the vicinity of a free surface, with $k_o = g/v_o^2$*

A detailed mathematical description of a hull's wave field was given by a Russian mathematician, Nicolai Kochin, in 1951. The Kochin function, in various approximate forms, was used by many authors to establish wave resistance models accessible through analytical calculation.

The Havelock source method is only practically usable in simple cases (slender hulls of infinite depth). In 1930, W.C.S. Wigley altered Havelock's calculations and obtained results far removed from practical experiments despite a correction for the viscosity effect introduced in 1938.

Guilloton introduced an approximate method in 1939, applying the Michell formula over a series of elementary volumes forming a hull of any shape. He developed his model over several publications until the 1970s, and many researchers followed his path, seeking either to demonstrate its mathematical foundations or to improve them. However, the analytical approach and the tricks of solving and correcting it were giving way to numerical solution models.

1.3.3. *Roll, pitch and seakeeping*

The study of a ship's roll on a calm sea was approached very early on by Euler, Jean Bernoulli and Bouguer. In 1757, in a thesis awarded by the *Académie des sciences* and which would become a reference work for over a century, Daniel Bernoulli approached the subject of roll on waves. However, this model, which was used as late as 1859 by Dupuy de Lôme for calculating the compensation torque on hulls, was marred by the shortcomings of the wave model used.

The first theory considering the orbital movement of waves resulted from Froude's work in 1861. This still imperfect theory was the subject of a great many polemics as well as additional work by Froude himself, by Rankine and indeed by the great English naval architect John Scott Russell. A major subject for shipbuilding, roll on waves engaged the greatest engineers until the turn of the century: Frédéric Reech, director of the *École du Génie maritime* (Naval Engineering School), Émile Bertin, director of shipbuilding in France, Sir William White, *Chief Naval Constructor* in Great Britain, and Alexei Kriloff, professor at the Saint Petersburg Naval Academy. Froude and Bertin developed specific instruments for measuring roll.

The ship was considered a forced linear oscillator, and many publications based on systematic measurements in tanks and at sea aimed to specify the differential equation of the roll (or of the pitch) on regular waves to independently evaluate the hydrodynamic coefficients and the influence of the forward speed to improve the knowledge and calculation of added masses and added damping coefficients. The excitation forces of the oscillator due to the effect of waves were calculated using the Froude–Krylov hypothesis, which consists of taking into account the wave's pressure field on the surface of the hull in the absence of the physical hull.

At the same time, and particularly between the two world wars, mechanical devices called stabilization devices were developed to reduce the amplitude of the ship's oscillations: gyroscopes, acting moving weights, passive and active anti-rolling tanks and rudders.

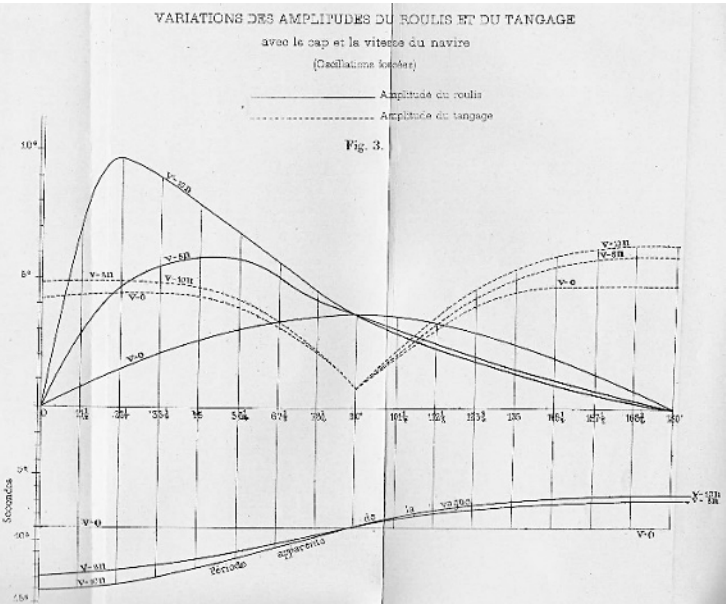

Figure 1.13. *Oscillations of a ship on a rough sea (source: Bindel (1989))*

In 1954, a researcher at the Stevens Institute of Technology, B.V. Korvin-Kroukovsky, introduced the notion of coupling between pitching and heaving movements. Consideration of the six degrees of freedom of the ship's movement led to the formation of a system of six differential equations, and experimental techniques were developed to identify the different hydrodynamic coefficients of these equations. In 1957, Korvin–Kroukovsky proposed a method for calculating coefficients using strips. Applicable to slender hulls, the "strip method", or "strip theory", consists of successively calculating the hydrodynamic stress on a two-dimensional strip perpendicular to the ship's axis. This makes it possible, moreover, to estimate the shearing and bending forces applied to the ship's hull girder. The Korvin–Kroukosvky model came to fruition with the development of the first digital calculators, and with additions over the years, it has been the reference model for several decades.

The linear oscillator model was finally extended to the case of a ship on irregular waves. In 1953, Manley St Denis from *David Taylor Model Basin* and Willard Pierson introduced the notion of a transfer function, making it possible to transform the wave spectrum into movement spectra. The transfer functions linked to each degree of freedom were called response amplitude operators (RAO). RAOs were determined either through an experimental path, on a model or on the sea, or theoretically using the strip method.

1.3.4. *Propeller and cavitation*

Applications of Bernoulli and Paucton’s theoretical work on helical thrusters were not imposed for several decades. The first use of this type of propeller appeared on the precursors of submarines, Bushnell’s *Tortue* in 1776 and Fulton’s *Nautilus* in 1804, in both cases with the help of man power.

With the development of steam in the first decades of the 19th century, new modes of propulsion came into force. The propeller and paddle wheels competed for nearly 50 years. Several patents for propellers were placed during the first half of the nineteenth century, notably by the French Frédéric Sauvage in 1832 and Francis Petit Smith in England in 1836. It was only from 1845 and the comparison in Britain of two corvettes, the *HMS Rattler* provided with a propeller and the *HMS Alecto* provided with paddle wheels, that the propeller gradually came to hold sway on ships.

However, its calculation remained rudimentary, based on the idea that the propeller cuts into the water like a cork screw in cork. Empirical relationships were sought, using several experiments at sea, as much in France as in England, between the speed and size of the ship (surface of the mid-ship), the power of the machine and the diameter and geometric pitch of the propeller. It was not until 1865 that Rankine established an operating model describing the exchange of momentum, which was developed in 1873 by R.E. Froude and known as an actuator disk. This model was refined again by Joukovski in 1918.

In 1873, Joseph-Émile Joëssel, a maritime engineer at the Marine institute at Indret, near Nantes in France, carried out experiments to determine the pressure and the center of pressure on a slender plane at incidence and determined a quadratic empirical law relating to the sine of the angle of incidence to consider the finite wingspan.

A new theory, the blade element theory, Figure 1.14, was proposed by W. Froude in 1878.

This theory was developed by Stefan Drzewiecki in 1892 (Drzewiecki 1892). Drzewiecki assumed that each cylindrical blade section behaves as a section of infinite wing at incidence. The angle of incidence is deduced from the difference between the geometric pitch of the section and the hydrodynamic pitch defined by the forward speeds and rotation of the section. This angle still had to be corrected by the relative slip of the propeller relative to the speed of the ship. The characteristics of each section had to be determined experimentally. In 1915, Albert Betz, a German aerodynamics specialist, suggested using actuator disk theory to theoretically determine the speed of slip and thus the local incidence.

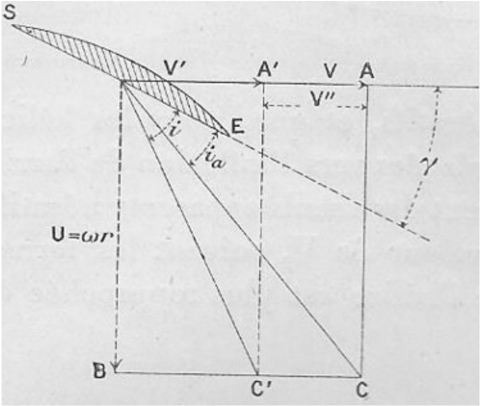

Figure 1.14. *Blade element theory (source: Pollard and Dubedout (1894))*

The main defect of the blade element theory was that it did not consider the effects of finite span, that is, the interactions between blade sections. It was not until the work of Kutta and Joukovski, but especially the lifting line theory proposed by Ludwig Prandtl in 1918 and applied by Betz to the propeller the following year, that there appeared a new theory called vortex theory. This theory became the basis for methods for designing propellers[5] and is still used today. Vortex theory is based on representing blades using a double system of linked or free vortices, the first on an axis parallel to the blade's guide and linked to the lift through circulation according to Joukovski's law, the others drawn in the wake of the blade and representing the variations in lift from one section of the blade to another.

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[5] It is usual to use the term "inverse calculation" to refer to the mathematical method allowing, from the performance sought and from the constraints given, the definition of the geometrical shape of the most suitable blades. The name "direct calculation" is given to the model allowing the calculation of performance in open water or behind the hull (thrust, torque, mechanical stresses, cavitation, etc.) of a propeller of given geometry.

Many hydrodynamicists devoted themselves, between the two wars, to perfecting this theory with the aim of attaining ever greater speeds. Thanks to the work of Émile Barrillon and Roger Brard, successive directors of the towing tank in Paris, France achieved several records, such as that of the destroyer *Le Terrible*, which exceeded 45 nodes in 1935; or, after several attempts, the winning of the blue ribbon crossing the Atlantic on the liner *Normandie*. The helical vortex model was achieved in 1955 by Hermann Lerbs, director of the hull tank at Hamburg.

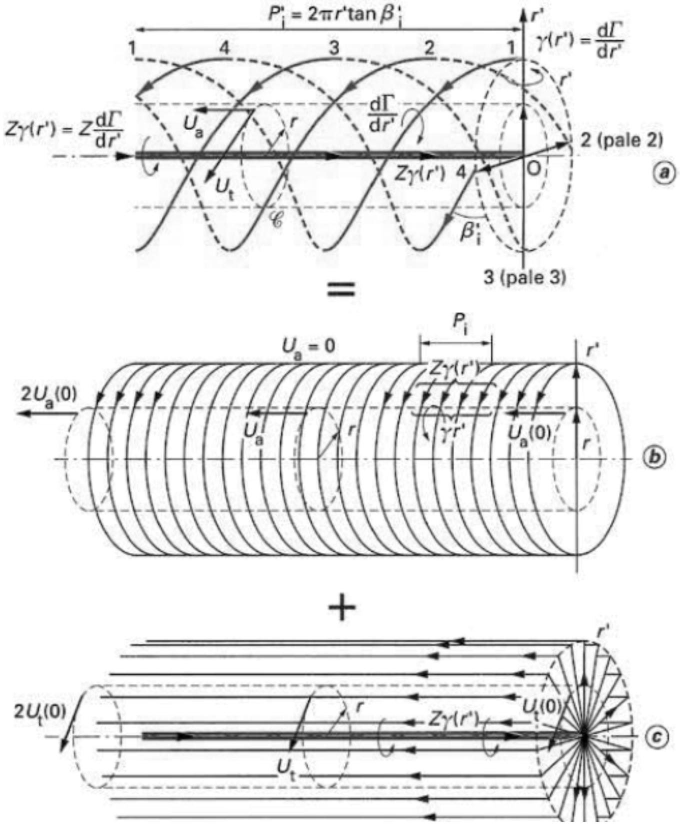

Figure 1.15. *H. Lerbs' vortex model (1955) (source: Aucher (1996)). For a color version of this figure, see www.iste.co.uk/sigrist/fluidstructure.zip*

After the Second World War, the inverse calculation method was sufficiently developed for design purposes, and scientists addressed direct calculation by developing new models, called airfoil models, which were adapted rapidly to solution using nascent numerical methods.

With the rapid increase in line shaft power, a new phenomenon, propeller racing, appeared and degraded ships' propulsive performance. While in France, Jacques-Augustin Normand attributed the phenomenon to a "break in the water column", the English engineer Charles Parsons named it "cavitation" and built the first cavitation tunnel. The phenomenon was studied theoretically by Lord Rayleigh, who in 1917 established the equation for the implosion of an empty cavity within a fluid. This equation was completed in 1949 by Milton Plesset to establish the equation known as the Rayleigh–Plesset equation. Until 1970, many authors sought to complete and to define it through various physical effects and various boundary conditions, especially in the vicinity of a solid wall such as a propeller blade.

The methods for theoretically predicting cavitation remained very limited, despite a vast range of publications and models. Additionally, numerous cavitation tunnels, of very substantial size, were built across the world from the 1930s until the last quarter of the 20th century with the construction of the *Grand tunnel hydrodynamique* in France or the Large Cavitation Channel in the United States. Tests were conducted in these with the intention of evaluating thruster performance, as well as studies that had a fundamental influence on many parameters (microbubbles, Reynolds number, roughness) on cavitation and its effect (noise, vibrations, erosion).

1.4. The numerical revolution

From the 1970s, in parallel with the refinement of the boundary layer theory, a new approach started to be used to study flow around a hull (a problem called diffraction-radiation to recall the two response terms of a hull on waves) with solution of the Neumann–Kelvin problem, that is, the solution of the Laplace equation of perfect incompressible fluids with the Neumann condition on the exact geometry of the hull and a linearized free surface condition (with or without the ship's speed). The solution, for each incident wave frequency, relies on the integral representation of the velocity potential, in which the solution is formulated in terms of the integrals of the potentials of fundamental singularities (source and doublets) on the surfaces bounding the fluid domain. Because of this, it is also called the singularity method. To limit the integral to a finite domain, that of the hull's surface, we combine the fundamental singularities (Rankine sources) with other analytical functions to develop Green's functions that satisfy all the problem's boundary conditions (free surface, bottom, walls) beyond the surface of the body. The expression of these Green's functions is even more difficult to establish, and the boundary conditions are more complex. The numerical solution is achieved after discretizing the hull into finite elements surface. This is what is called the boundary elements method (BEM), introduced in 1964 by John Hess and A.M.O. Smith of the *Douglas Aircraft Company* (Figure 1.16).

Originally developed without a free surface, it was applied to floating platforms for which the slender body hypothesis could not be applied, and commercial calculation codes appeared from the end of the 1970s.

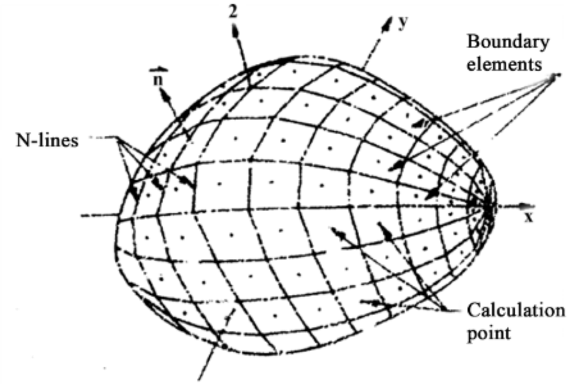

Figure 1.16. *Boundary elements method (source: Hess and Smith (1964))*

Application of the BEM method nevertheless proved very difficult in the case of ships because of their speed. Determination of Green’s function in this case proved to be very complex. A number of publications, including those by Pierre Guével’s team at the *École centrale de Nantes*, were intended for this aim. The integral representation also, in essence, contains disruptive solution terms, called irregular frequencies, which must be eliminated. A formulation using finite differences applied to a volume mesh of the fluid (the finite element method) was developed in parallel by Kwan Jung Bai at the University of Seoul. This formulation in fact combines a representation using finite elements in the vicinity of the hull (localized finite element method) and then in integral form beyond a cutoff surface. The finite element method prefigured calculations for solving RANS equations.

At the beginning of the 1960s, the vortex theory of screw propellers was used, notably by Justin Kerwin at the Massachusetts Institute of Technology, to develop methods called “lifting surface methods” with, unlike the lifting line, a pressure distribution and vortices along a chord section (this is the vortex lattice method). This method presupposes that the wingtip vortices from the wing are dispersed in the wake and are concentrated at the tip of the wing after a transition distance. This method opened the way for particle methods introduced in the 1980s.

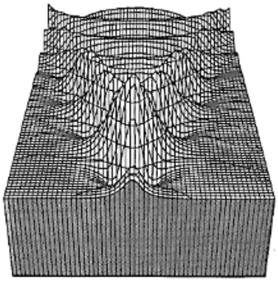

Figure 1.17. *Wave field calculation using localized finite elements (source: E. Masson, ENSTA, 1990)*

Another approach, the very general boundary element method, was applied in 1970 by T.S. Luu at the University of Orsay for slender blade profiles (Figure 1.18) and then extended by Hess in 1972 to thick airfoils. This modeling allows for much more exact consideration of the leading-edge flow and thus lends itself to extending the model to boundary layer calculations with layer separation and, *finally*, cavitation. This method takes further advantage of the mathematical equivalence between doublets, used in classical integral representation, and localized vortices, used to translate wing lift, an equivalence known as Hess identity.

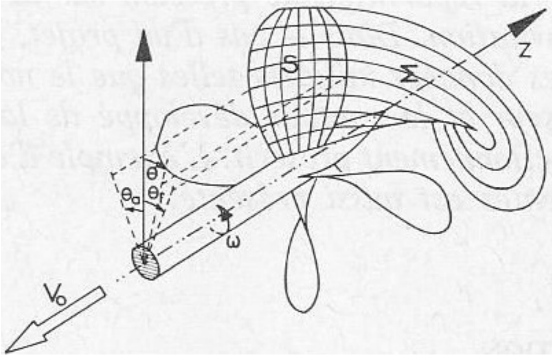

Figure 1.18. *Singularities method (source: T.S. Luu, ATMA, 1970)*

The last revolution – at least on the scale of this concise overview of a science that has been constantly reinvented for three centuries – arrived with the last decade of the 20th century, which saw the appearance of the first, and soon numerous numerical solution codes for Navier–Stokes equations, computational fluid dynamics (CFD).

Increasing computer power made it possible to tackle these calculations, which were very demanding in terms of computational resources. In fact, they required a considerable number of basic volumes and finite elements, for which automatic definition methods (meshes) were defined: structured meshes, unstructured meshes, adaptative meshes, moving meshes, etc. Specific processing is needed to consider the free surface and it gives rise to the volume of fluid method (VOF). Other models appeared in the 2000s aside from RANS equation models: Lagrangian particle methods of the SPH type (smoothed particle hydrodynamics) or vortex simulation LES-type methods. The power of these models, whichever they are, and their application domains will continue to increase in the coming years.

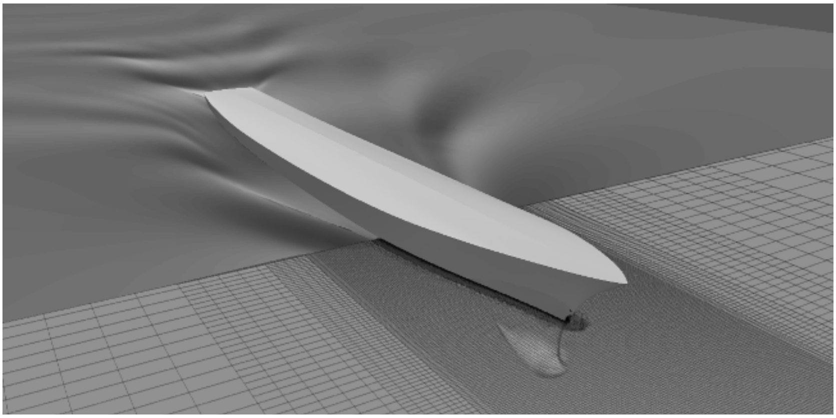

Figure 1.19. *Complex meshing around a hull and the free surface (source: Sirehna/Naval Group). For a color version of this figure, see www.iste.co.uk/sigrist/fluidstructure.zip*

This brief history of naval hydrodynamics stops at the dawn of the 20th century with the emergence of methods for numerically solving Navier–Stokes equations. These methods are now developing so fast that a historical overview is quite clearly impossible. It is the subject of so much work, so many publications and applications, in the domain of research as much as in business-related services, that it is impossible to credit any one team with significant advances.

The chapters that follow will broadly explore the state of the art of the different methods that are competing today within the "numerical towing tank" and their advantages as well as their drawbacks. Fluid–structure interaction is the first stage in studying the associated phenomena, in this case hydrodynamics and the local dynamics of structures in response to and in contribution to flows.

“Hydro-vibro-acoustics” is still a new stage. The massive use of complex composite materials, with the aim of developing biomimetic, adaptive, multifunctional naval structures, has incited the pursuit of the construction of multiphysic models with the integration of new phenomena such as electromagnetism.

Computer power will be available to support this pathway and numerical models will be needed: this is what the authors of this book – brought together by Jean-François Sigrist and Cédric Leblond – are working on.

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