Chapter 1

0D Analytical Modeling of Airplane Motions

The 0D modeling process tries to obtain variations as functions of time for all parameters of the motions of the plane.

The plane is considered here as a solid body moving freely through open space and therefore includes six degrees of freedom (DOF):

– three translational motions by three rectangular directions;

– three rotational motions classically described by Euler angles.

The plane is also under the influence of three external force systems which are:

– aerodynamic forces;

– propulsion forces;

– gravitational forces.

1.1. References: axis systems on use

In order to define the spatial motion of the airplane, we make use of two geometrical references.

1.1.1. *Galilean reference: R_0*

This geometrical reference has its origin center matched with the center of mass G of the airplane. The three principal rectangular axes are:

– Gx_0: horizontal, generally oriented to the West;

– Gy_0: horizontal, oriented to the North;

– Gz_0: vertically downward.

Gx_0, Gy_0 and Gz_0 form a direct rectangular reference.

NOTE.– Gz_0 is directed downward, due to the natural tendency of the airplane to descend when left to the effects of gravity.

This Galilean reference is in accordance with Newton’s first principle which makes use of the absolute components of the accelerations to be equal to the components of external forces.

1.1.2. *Airplane reference: R_B (body) also called “linked reference”*

This geometrical reference also has its center matched with G, the center of mass of the plane, but is physically linked to the airframe. Its three principal axes are: GX, GY and GZ.

GX, GY, GZ are preferably the principal axes of inertia of the plane and (GXYZ) is direct.

GXZ is the plane of symmetry of the airplane, with the exception of a few particular airplanes with asymmetric engine setups (Blohm and Voss, for instance; see Figure 1.1).

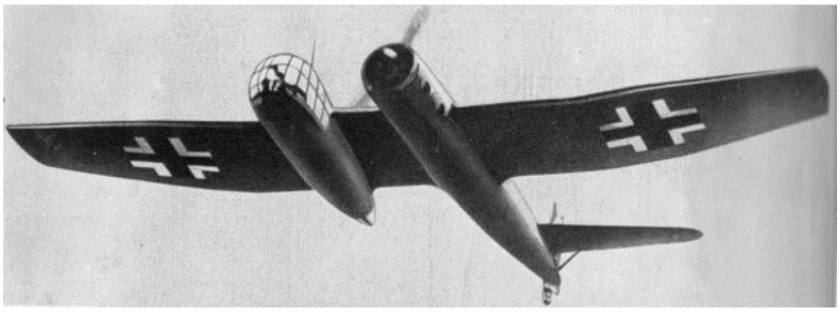

Figure 1.1. *Blohm and Voss BV 141*

(GXYZ), also called $R_{B,}$ is the preferred reference for use with torque equations due to the fact that the inertias remain constant.

We can move from the Galilean reference to the body reference by making three Eulerian rotations, which are:

– Ψ (Psi): Yaw angle;

– Θ (Theta): Pitch angle;

– Φ (Phi): Roll angle.

a) Yaw rotation (ψ)

This first Euler rotation is made around the Gz_0 axis.

$$\Psi = \text{Yaw angle}$$

The associated angular velocity is: $\dot{\psi} . Z_0$

$$(Gx_0\, y_0\, z_0) \underset{(\psi)}{\rightarrow} (Gx'\, y'\, z_0)$$

The relationship between the cosine directors are:

$$x' = \cos\psi . x_0 + \sin\psi . y_0$$

$$y' = -\sin\psi . x_0 + \cos\psi . y_0 \qquad [1.1]$$

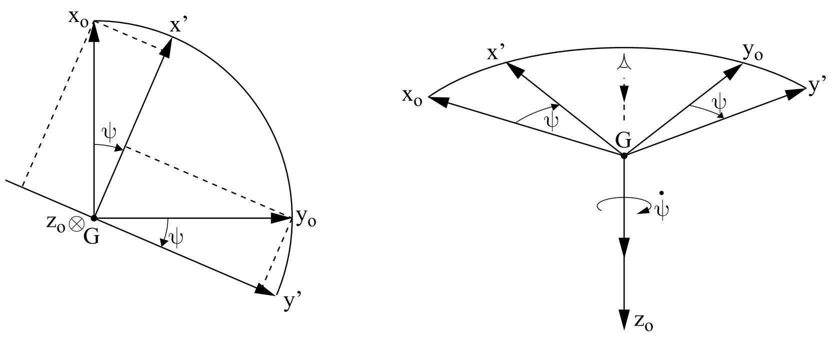

Figure 1.2. *First Euler rotation ψ around the Gz$_0$ axis*

b) Pitch rotation (θ)

This second rotation is made around the Gy' axis.

Θ = Pitch angle

The associated angular velocity is: $\dot{\theta}$. y

$$(Gx'y'z_0) \xrightarrow{(\theta)} (GXy'z')$$

The relationships between the cosine directors are:

$$X = \cos\theta .x' - \sin\theta . z_0$$

$$z' = \sin\theta . x' + \cos\theta . z_0 \qquad [1.2]$$

And as obtained before:

$$z_0 = \cos\theta . z' - \sin\theta . X$$

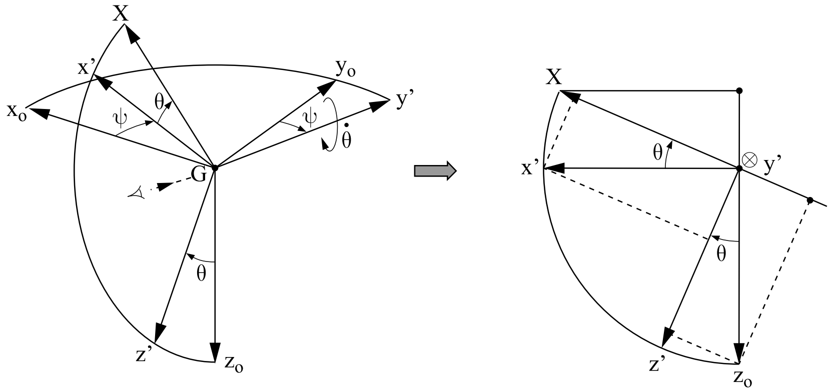

Figure 1.3. *Second Euler rotation θ around the Gy' axis*

c) Roll rotation (φ)

The third rotation (φ) is made around the GX axis.

Φ is the roll angle.

The associated angular velocity is: $\dot{\varphi}$ .X

$$(GXy'z') \xrightarrow{(\varphi)} (GXYZ)$$

The relationships between cosine directors are:

$$Y = \cos\varphi \,.y' + \sin\varphi \,.\, z'$$

$$Z = -\sin\varphi \,.\, y' + \cos\varphi \,.\, z' \qquad [1.3]$$

therefore:

$$z' = \sin\varphi \,.Y + \cos\varphi .Z$$

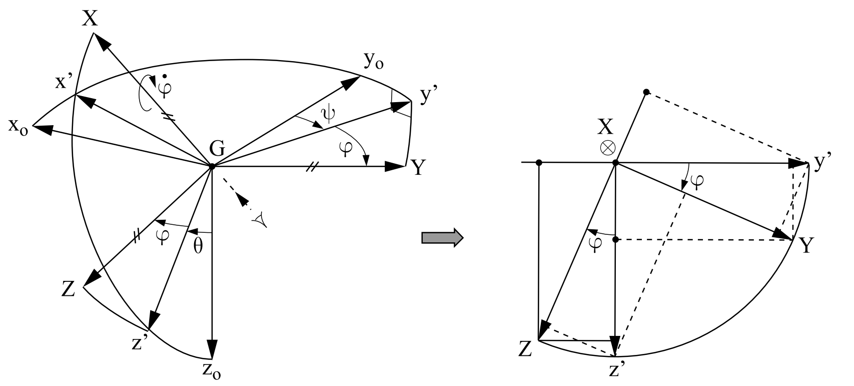

Figure 1.4. *Third Euler rotation φ around the GX axis*

1.1.3. *Resultant angular velocity*

The resultant angular velocity can be expressed as:

$$\vec{\Omega} = \dot{\psi} \cdot \vec{z}_0 + \dot{\theta}\vec{y} + \dot{\varphi}\vec{x} \qquad [1.4]$$

These rotations are made around three axes which do not form a rectangle.

We can now express the components of $\vec{\Omega}$ by the reference R_B linked to the airplane:

$$\vec{\Omega} = \dot{\psi} . \left(\cos\theta . \vec{z'} - \sin\theta . \vec{X} \right) + \dot{\theta} . \left(\cos\varphi . \vec{Y} - \sin\varphi . \vec{Z} \right) + \dot{\varphi} . \vec{X} .$$

therefore:

$$\vec{\Omega} = \dot{\psi} . \left[\cos\theta . \left(\sin\varphi . \vec{Y} + \cos\varphi . \vec{Z} \right) - \sin\theta . \vec{X} \right] + \cos\varphi . \dot{\theta} . \vec{Y} - \sin\varphi . \dot{\theta} . \vec{Z} + \dot{\varphi} . \vec{X} .$$

$$\vec{\Omega} = \left(\dot{\varphi} - \sin\theta . \dot{\psi} \right) . \vec{X} + \left(\cos\theta . \sin\varphi . \dot{\psi} + \cos\varphi . \dot{\theta} \right) . \vec{Y} + \left(\cos\theta . \cos\varphi . \dot{\psi} - \sin\varphi . \dot{\theta} \right) . \vec{Z}$$

Now expressed by R_B (GXYZ) reference:

$$\Omega/\left(GXYZ\right) = \begin{bmatrix} p = \dot{\varphi} - \sin\theta\ .\dot{\psi} \\ q = \cos\theta\ .\sin\varphi\ .\dot{\psi} + \cos\varphi\ .\dot{\theta} \\ r = \cos\theta\ .\cos\varphi\ .\dot{\psi} - \sin\varphi.\dot{\theta} \end{bmatrix} \quad [1.5]$$

NOTE.– p, q and r are the components of the vector resultant angular velocity expressed by the airplane body reference R_B. They can be measured on a real airplane by angular velocity sensors, commonly called “gyrometers”.

The Euler parameters: ψ, θ, φ are actually more difficult to obtain.

This point will be covered in more detail hereafter.

Equations [1.5] can be written under matricial form:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\sin\theta \\ \cos\varphi & 0 & \cos\theta.\sin\varphi \\ -\sin\varphi & 0 & \cos\theta.\cos\varphi \end{bmatrix} * \begin{bmatrix} \theta \\ \varphi \\ \psi \end{bmatrix} \quad [1.6]$$

It is then possible to solve this matricial equation in order to extract the derivatives of Euler angles which figure in the column vector $[\theta \ \ \varphi \ \ \psi\ ]^T$ of [1.6].

The characteristic determinant of this equation is called Δ:

$$\Delta = \begin{vmatrix} 0 & 1 & -\sin\theta \\ \cos\varphi & 0 & \cos\theta.\sin\varphi \\ -\sin\varphi & 0 & \cos\theta.\cos\varphi \end{vmatrix} \quad [1.7]$$

$$\Delta = -(\cos^2\varphi.\cos\theta + \sin^2\varphi.\cos\theta) = -\cos\theta.$$

The minor $N_{\dot\theta}$ associated with $\dot\theta$ is:

$$N_{\dot\theta} = \begin{vmatrix} p & 1 & -\sin\theta \\ q & 0 & \cos\theta.\sin\varphi \\ r & 0 & \cos\theta.\cos\varphi \end{vmatrix}$$

$$N_{\dot\theta} = -(\cos\theta.\cos\varphi.q - \cos\theta.\sin\varphi.r)$$

$$\text{Then: } \dot\theta = N_\theta / \Delta = \cos\varphi.\, q - \sin\varphi\, .r \qquad [1.8]$$

In the same way, the minor $N_{\dot\varphi}$ associated with $\dot\varphi$ is written as:

$$N_{\dot\varphi} = \begin{vmatrix} 0 & p & -\sin\theta \\ \cos\varphi & q & \cos\theta.\sin\varphi \\ -\sin\varphi & r & \cos\theta.\cos\varphi \end{vmatrix}$$

$$N_{\dot\varphi} = -p.(\cos^2\varphi\, .\cos\theta + \sin^2\varphi.\cos\theta) - \sin\theta.(\cos\varphi.r + \sin\varphi.q)$$

$$N_{\dot\varphi} = -p.\cos\theta - \sin\theta.(\cos\varphi\, .r + \sin\varphi.q)$$

$$\text{Then: } \dot\varphi = N_{\dot\varphi} /\Delta = p + tg\theta.(\cos\varphi.r + \sin\varphi.q) \qquad [1.9]$$

The last minor N_ψ associated with $\dot\psi$ is:

$$N_{\dot\psi} = \begin{vmatrix} 0 & 1 & p \\ \cos\varphi & 0 & q \\ -\sin\varphi & 0 & r \end{vmatrix}$$

$$N_{\dot\psi} = -(\cos\varphi.r + \sin\varphi.q)$$

$$\text{Then: } \dot\psi = N_{\dot\psi} /\Delta = (\cos\varphi /\cos\theta).r + (\sin\varphi /\cos\theta).q \qquad [1.10]$$

See the *Euler block* as shown in Figure 1.5.

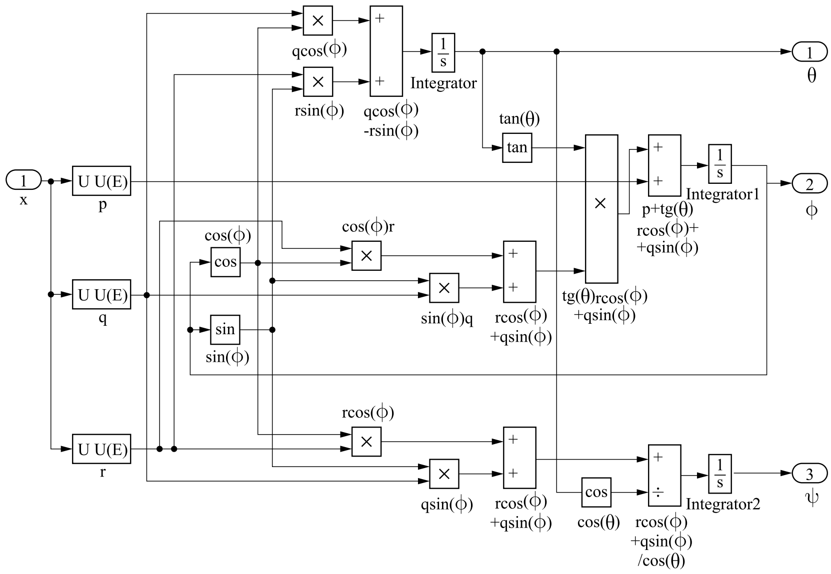

Figure 1.5. *Euler block*

This MATLAB/SIMULINK operator converts the linked parameters (p, q, r) to Euler angles:

(Theta, Phi, Psi)

1.2. Equations of motion of the airplane

The equations of motion translate the two fundamental principles of solid mechanical bodies:

1) The “quantity of acceleration” of the solid body in translation is equal to the resultant vector of all external forces (Newton’s principles).

NOTE.– The quantity of acceleration is exactly the opposite value of the inertial force.

As a result, we obtain three equations for equilibrium forces.

2) The "dynamic momentum" of the solid body is equal to the resultant torque of the external forces.

As a result, we obtain three equations for torque equilibrium.

1.2.1. *Expression of Newton's principle*

Two vectors define the motion of the solid body as functions of time:

– $\vec{V}$ (Uvw) and $\vec{\Omega}$ (p q r) at any time.

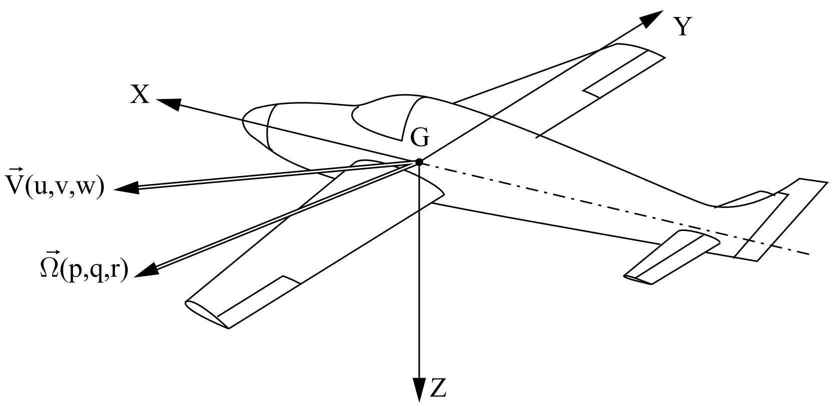

Figure 1.6.

Let us call the expression of the quantity of acceleration of the solid body Q_a

$$\overrightarrow{Q_a} = m.(\overrightarrow{dV} / dt) / R_0 \quad [1.11]$$

In this formula:

m = mass of the airplane (considered as a solid body).

$\overrightarrow{dV}$ /dt = time derivative of the velocity vector by a Galilean reference (velocity).

Due to the fact that $\vec{V}$ and $\vec{\Omega}$ are expressed by the body reference, which is mobile, we can state that:

$$(\overrightarrow{dV} / dt) / R_0 = (\overrightarrow{dV} / dt) / R_B + (\vec{\Omega} \wedge \vec{V}) / R_B \quad [1.12]$$

This formula states that the absolute velocity of the solid ($\overrightarrow{dV}$/dt)/R_0 can be expressed by two components which are written by the body reference (theorem of differentiation by mobile reference).

Written under matricial form:

$$(dV/dt)/R_0 = \begin{bmatrix} U \\ v \\ w \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \wedge \begin{bmatrix} U \\ v \\ w \end{bmatrix} = \begin{bmatrix} U + qw - rv \\ v + rU - pw \\ w + pv - qU \end{bmatrix}$$

So, Newton's first principle can be written as:

$$\begin{cases} m . \left(U + qw - rv \right) = F_x \\ m . \left(v + rU - pw \right) = F_Y \\ m . \left(w + pv - q U \right) = F_Z \end{cases} \quad [1.13]$$

1.2.2. *Expression of the dynamic momentum*

At first we can assume that GX, GY and GZ are matched with the principal axis of inertia, so the matrix of inertia of the plane is diagonal:

$$[I] = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

If the references (G XYZ) and (G xyz) are not confused, the matrix of inertia is complete:

$$[I] = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}$$

In the case where the matrix of inertia is diagonal, we design the kinetic momentum, sometimes called angular momentum, of the plane by H. This is expressed by R_B:

$$\vec{H}/_{RB} = \begin{bmatrix} A\,.\,p \\ B\,.\,q \\ C\,.\,r \end{bmatrix}_{R_B} \quad \text{So: } \delta\vec{H}/\delta t_{/RB} = \begin{bmatrix} A\,.\,\dot{p} \\ B\,.\,\dot{q} \\ C\,.\,\dot{r} \end{bmatrix}_{R_B} \quad \text{And: } \vec{\Omega} \wedge \vec{H}/_{RB} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \wedge \begin{bmatrix} A\,.\,p \\ B\,.\,q \\ C\,.\,r \end{bmatrix}_{RB} = \begin{bmatrix} (C\text{-}B)q.r \\ (A\text{-}C)r.p \\ (B\text{-}A)p.q \end{bmatrix}$$

The dynamic momentum of the plane, which is the time derivative of the kinetic momentum by the Galilean reference, can be written as:

$$\delta\vec{H}/\delta t_{/R0} = \delta\vec{H}/\delta t_{/RB} + \vec{\Omega} \wedge \vec{H}/_{RB}$$

We notice that ($\delta H/\delta t_{/R0}$) can be expressed by its components in R_B.

The expressions of external torques by the body reference are also expressed by R_B:

$$\Sigma M_{ext} \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix}_{R_B}$$

When making use of R_B referencee, the equilibrium equations for torque motions can be written as:

$$\begin{cases} A.\dot{p}+(C-B).q.r = M_X = \text{sum of roll torques} \\ B.\dot{q}+(A-C).r.p = M_Y = \text{sum of pitch torques} \\ C.\dot{r}+(B-A).p.q = M_Z = \text{sum of yaw torques} \end{cases} \quad [1.14]$$

The final equations defining the motions of the airplane are:

– Equilibrium between the components of the quantity of acceleration and the external forces [1.13].

– Equilibrium between the components of the dynamical momentum and the external torques [1.14].

– Values of the components of the resultant angular velocity [1.5].

The usual equations are shared into two groups:

– Longitudinal (symmetrical) group:

$$\begin{cases} m\left(\dot{U}+q.w-r.v\right)=F_X \\ m.\left(\dot{w}+p.v-q\,U\right)=F_Z \\ B.\dot{q}+\left(A-C\right).r.p=M_Y \\ q=\cos\theta.\sin\varphi.\dot{\psi}+\cos\varphi.\dot{\theta} \end{cases} \quad [1.15]$$

– Transversal (asymmetrical) group:

$$\begin{aligned} & m.\left(\dot{v}+r.U-p.w\right)=F_Y \\ & A.\dot{p}+\left(C-B\right).q.r=M_X \\ & C.\dot{r}+\left(B-A\right).p.q=M_z \\ & p=\dot{\varphi}-\sin\theta.\dot{\psi} \\ & r=\cos\theta.\cos\varphi.\dot{\psi}-\sin\varphi.\dot{\theta} \end{aligned} \quad [1.16]$$

These equations are described by MATLAB/SIMULINK as body and Euler blocks.

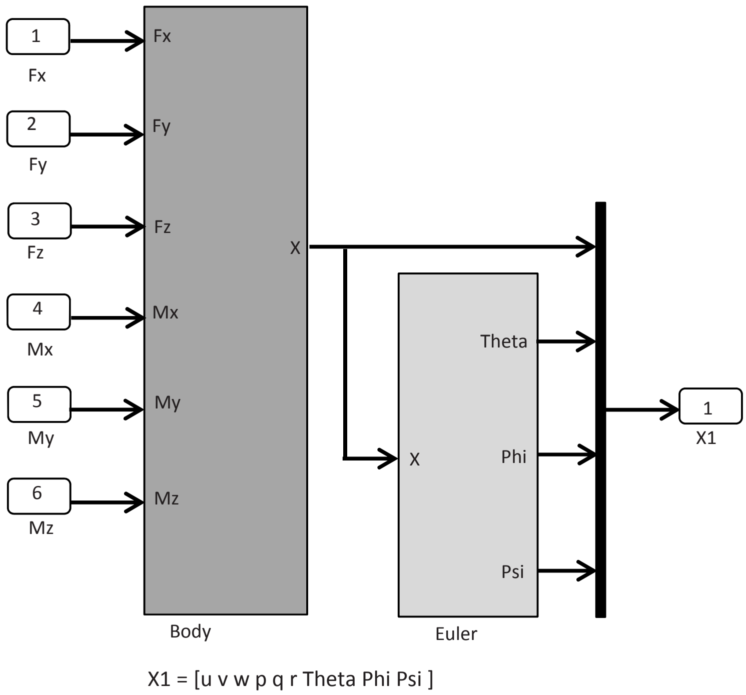

Figure 1.7. *Body and Euler blocks*

1.3. Description of external forces and torques

External forces and torques exerted on the airplane have three origins, all described by the body reference (G XYZ).

1.3.1. *Aerodynamic forces and torques*

For an airplane, these elements are preponderant. Their components are:

– F_{Xa} = aerodynamic drag;

– F_{Ya} = side lift;

– F_{Za} = vertical lift;

– M_{Xa} = aerodynamic roll torque;

– M_{Ya} = aerodynamic pitch torque;

– M_{Za} = aerodynamic yaw torque.

These components depend on the angular positioning of the velocity vector by the body reference, which is:

$$\vec{V}/_{RB} = (U, v, w)^T/_{RB}$$

The angles which position the velocity vector by the body reference are as follows.

Attack angle: α

This is the angle between the projection of the velocity vector on the plane of symmetry (GXZ) and the GX axis.

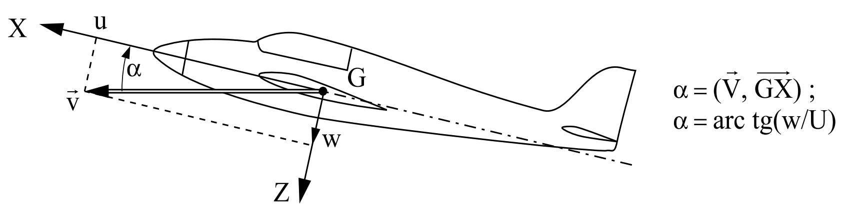

Figure 1.8. *Attack angle α*

Sideslip angle: β

This is the angle between the projection of the velocity vector on the (GXY) plane and the plane of symmetry of the airplane.

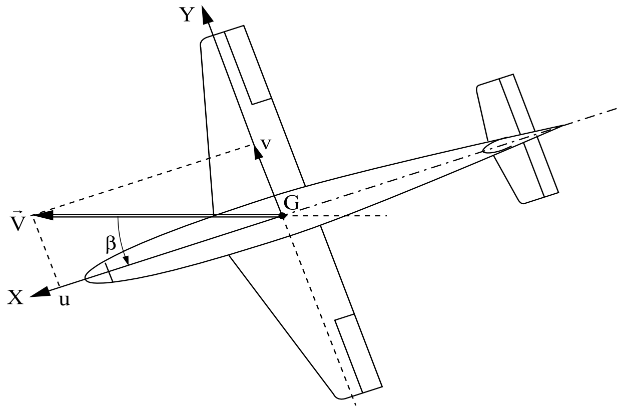

Figure 1.9. *Attack angle β*

So:

$$\beta = (V, GX)$$

$$\beta = \text{Arctg}(-v/U) \qquad [1.17]$$

NOTE.– In Figure 1.9, the velocity vector $\beta < 0$ is on the right side of the airplane.

The modulus of the velocity vector is expressed as:

$$|V| = (U^2 + v^2 + w^2)^{1/2}.$$

The SIMULINK scheme to compute V, alpha and beta can be displayed as in Figure 1.10.

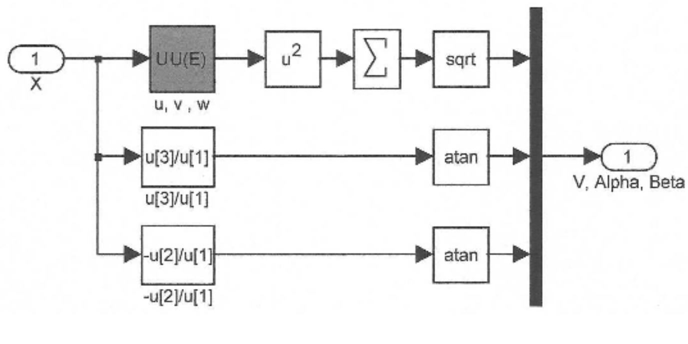

Figure 1.10. *SIMULINK scheme to compute V, alpha and beta*

1.3.2. *Sign rules*

The sign rules for forces and torques respect the same rules as those adopted for the axis of the referencee (GXYZ).

For angles and torques:

– Positive: from X toward Y, from Y toward Z, and from Z toward X.

With regards to the displacement commands:

– Roll command displacement: δ_l ;

– $\delta_l < 0$ for an aileron stick displacement to the left side.

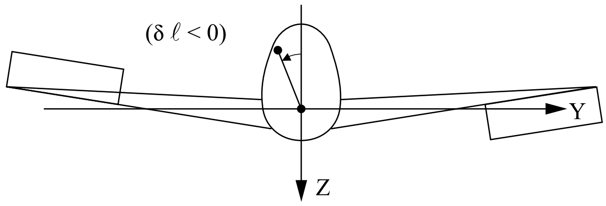

Figure 1.11. *Aileron stick displacement to the left side, where the left aileron goes up and the right aileron goes down. (Rear view)*

Pitch control surface displacement: δ_m

$\boldsymbol{\delta_m > 0}$ for an elevator stick displacement to the front, in this case, the elevator control surface goes down (from Z toward X, see Figure 1.12).

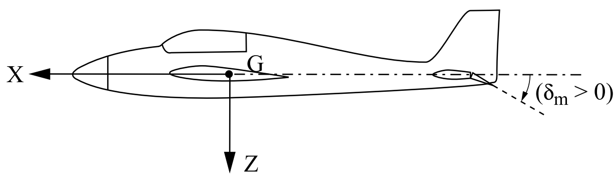

Figure 1.12. *Elevator stick displacement to the front, where the elevator control surface goes down*

Yaw control surface displacement: δ_n

$\boldsymbol{\delta_n > 0}$ for a left displacement of the yaw control surface (see Figure 1.13).

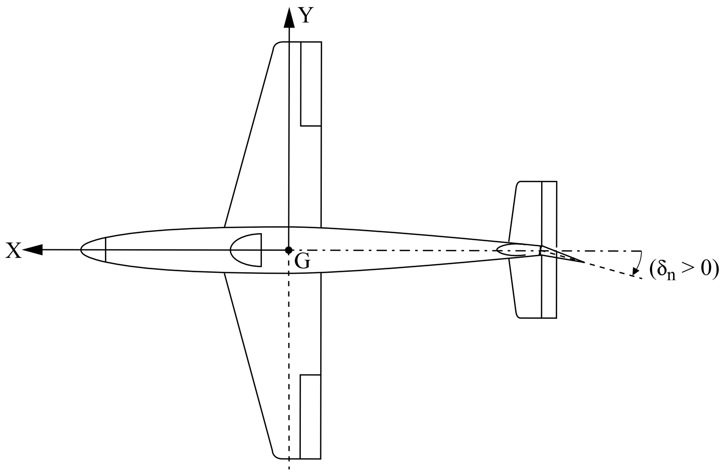

Figure 1.13. *Left displacement of the yaw control surface*

1.4. Description of aerodynamic coefficients

The coefficients of forces and torques depend on the following parameters:

– α: attack angle;

– β: sideslip angle;

– p: roll angular velocity;

– q: pitch angular velocity;

– r: yaw angular velocity;

– δ_l: roll control surface angular displacement;

– δ_m: pitch control surface angular displacement;

– δ_n: yaw control surface angular displacement.

1.4.1. *Drag coefficient: C_x*

The Bernouilli principle states that:

$$F_X = - q_0 .S . C_x \quad \text{where} \quad q_0 = ½ .\rho .V^2 \text{ (kinetic pressure)} \qquad [1.18]$$

F_X is represented by the Lilienthal polar (C_z = function (C_X)) with respect to the body axis.

C_Z and C_X both depend on the attack angle.

NOTE.– $C_X > 0$ for $F_X < 0$ (see Figure 1.14).

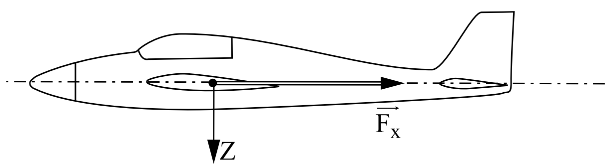

Figure 1.14. *Drag force is generally negative*

1.4.2. *Side lift coefficient: C_Y*

The Side Lift depends on two parameters:

– the slip angle of the fuselage: $C_{Y\beta}$;

– the angular displacement of the yaw control surface: $C_{Y\delta n}$.

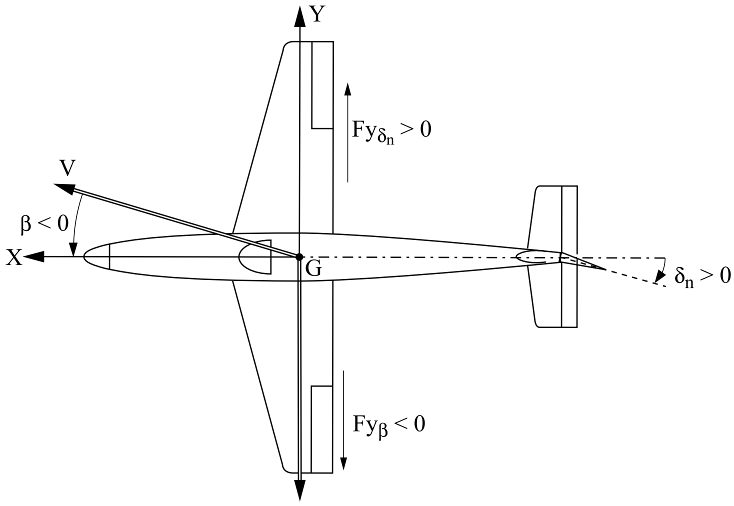

Figure 1.15. *Dependance of Side Lift on β and δ_n*

$$F_{Y\beta} = q_0.S.\,(\Delta C_Y/\Delta\beta).\beta = q_0.S.\,C_{Y\beta}.\beta \quad [1.19]$$

For:

$\beta < 0 \rightarrow F_{Y\beta} < 0$ as shown in Figure 1.15.

And also:

$$F_{Y\delta n} = q_0\,.\,S\,C_{Y\delta n}.\,\delta_n$$

For:

$\delta_n > 0 \rightarrow F_{Y\delta n} > 0$ as shown in Figure 1.15.

1.4.3. *Vertical lift due to attack angle: $C_{Z\alpha}$*

The angle between the velocity vector and the neutral axis of the fuselage Gx is designed by α (which can sometimes be the principal axis of inertia).

For: $\alpha > 0 \rightarrow F_{Z\alpha} < 0$ (as shown in Figure 1.16)

So: $F_{Z\alpha} = - q_0.S.(\Delta C_Z / \Delta\alpha).\alpha = - q_0. S .C_{Z\alpha}. \alpha$. [1.20]

$C_{Z\alpha} = \Delta C_Z / \Delta \alpha$ is commonly called the "aerodynamic stiffness of the plane".

Vertical lift due to elevator control surface angular displacement

The elevator angular displacement is called δm.

For $\delta_m < 0 \rightarrow F_{Z\delta m} > 0$ as shown in the following figure.

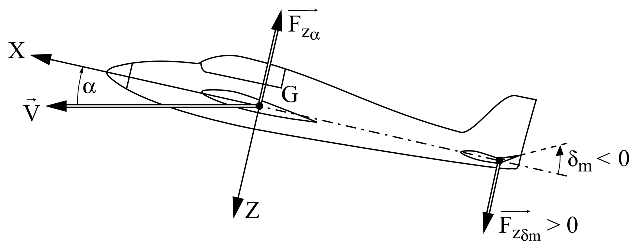

Figure 1.16. *Vertical lift due to elevator control surface angular displacement*

$$F_{Z\delta m} = -q_0 . S . (\Delta C_Z / \Delta\delta_m).\delta_m = -q_0 . S . C_{Z\delta m} .\delta_m \quad [1.21]$$

$C_{Z\delta m}$ is commonly called "elevator efficiency".

1.4.4. *Lift due to pitch angular velocity: C_{Zq}*

There is a modification to the attack angle of the pitch stabilizer; this is due, simultaneously, to pitch angular velocity and translational velocity V:

$\delta_{\alpha e} = \text{Arctg} (w_e / V)$ (see Figure 1.17)

$w_e = l . q \rightarrow \delta_{\alpha e} = \text{Arctg} (l.q/ V) \sim l.q /V$

The lift variation is:

$\Delta\ F_{Ze} = q_0 \ .\ S_e \ .(C_{ze})\ \delta_{\alpha e} = q_0 \ .\ S_e \ .(C_{ze})\ 1\ .\ q\ /\ V = q_0 \ .S_e .$
$C_{ze} .1 / V = C_{zq} \ .\ q\ .$

For the complete plane:

$$C_{Zq} = C_{Ze}\ .\ 1\ /V$$

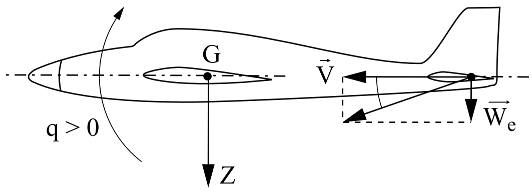

Figure 1.17. *Modification of the elevator attack angle due to angular pitch velocity*

This coefficient creates a coupling between the pitch angular velocity and the Vertical Lift.

1.4.5. *Roll coefficients (due to β, δ$_l$, p)*

1.4.5.1. *Roll due to slip angle (commonly named "dihedral effect"):* $C_{L\beta}$

The dihedral effect creates a torque, in an aerodynamic way, which allows the plane to bank to the inner side of the turn. This effect constitutes the principal difference between planes and cars. Cars tend to bank to the outer side of the turn – a phenomenon which is neither comfortable nor safe for the passengers inside.

A good plane, therefore, naturally banks to the inner side of the turn.

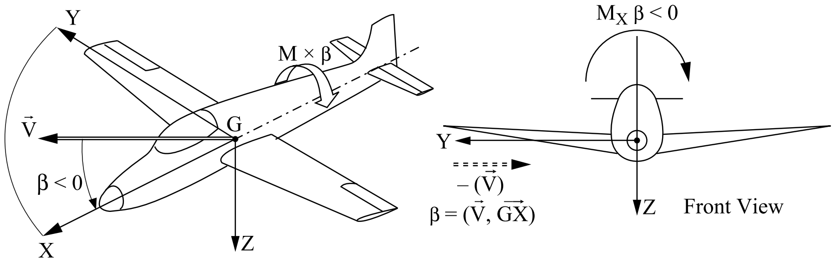

Figure 1.18. *Roll torque M_X due to side slip angle β*

The wings' lateral tips curve upwards so that the relative wind hits the lower face at the external side, creating a roll torque which banks the plane inside the turn.

So, for:

$$\beta < 0 \quad \rightarrow \; M_{X\beta} \; = \; q_0.\, S\, .\, 1\, .(\Delta C_L/\Delta\beta\,).\beta \; = \; q_0.S\, .1\, .C_{L\beta}.\beta \; < 0$$

[1.22]

1.4.5.2. *Roll due to ailerons' angular displacement $C_{L\delta l}$*

The lateral motion of the control stick acts on the ailerons' angular displacements.

When the control stick moves to the left, the left aileron goes up and the right aileron goes down, creating a control torque which enables the plane to make a left roll.

So for:

$$\delta L < \; 0 \; \rightarrow M_{X\delta L} = \; q_0.\, S\, .\, 1\, .(\, \Delta CL\, /\, \Delta\delta L\,).\delta L = \; q_0 SlC_{L\delta l}.\delta l \rightarrow(\, \Delta Cl\, /\, \Delta\delta l = C_{L\delta l}) < 0$$

[1.23]

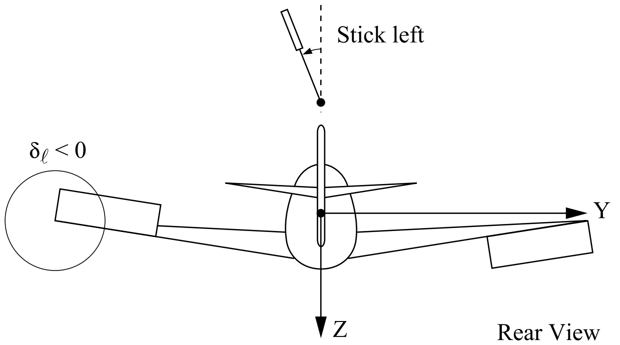

Figure 1.19. *The control stick moves to the left, the left aileron goes up and the right aileron goes down, the plane to make a left roll*

1.4.5.3. *Roll due to roll angular velocity (C_{Lp}): damping coefficient*

For a positive roll angular velocity:

$$p > 0 \quad \rightarrow \quad M_{Xp} = - \tfrac{1}{2}.\rho.V.S.\ l^2.(\Delta C_L/\Delta p\ ).p = - \tfrac{1}{2}.\rho.V.S.l^2.C_{Lp}.p \qquad [1.24]$$

This torque, which is proportional to an angular velocity, becomes a damping torque, with a stabilizing tendency.

Its property is to be increased with the aspect ratio of the wing (and horizontal stabilizer).

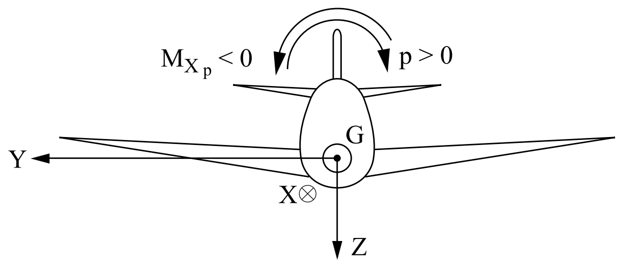

Figure 1.20. *The damping torque* M_{X_p} *is on the opposite side of p*

1.4.6. *Pitch coefficients (due to α, δ_m, q , static curvature)*

1.4.6.1. *Pitch due to attack angle: Cmα (aerodynamic pitch stiffness)*

This is the pitch stabilizing effect of the horizontal fin.

For:

$$\alpha > 0 \rightarrow M_{Y\alpha} = -q_0.S.l.(\Delta C_m/\Delta\alpha).\alpha = -q_0.S.l.C_{m\alpha}.\alpha < 0 \quad [1.25]$$

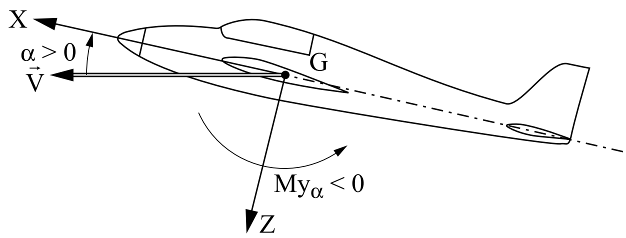

Figure 1.21. *The horizontal fin creates a torque on the opposite side of α*

1.4.6.2. *Pitch due to angular elevator deflexion: $C_{m\delta m}$ (elevator efficiency)*

For a given elevator deflexion δ_m:

$$\delta_m > 0 \rightarrow M_{Y\delta m} = -q_0.S.l.(\Delta C_m/\Delta\delta_m).\delta_m = -q_0.S.l.C_{m\delta m} < 0$$

[1.26]

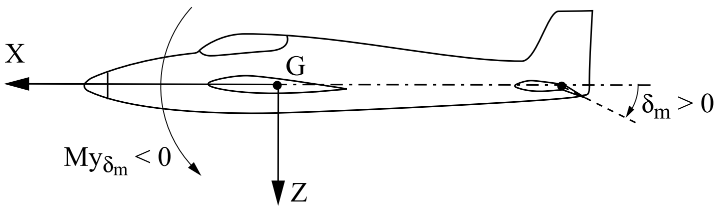

Figure 1.22. *For an elevator control surface motion going down, the pitch torque is negative*

1.4.6.3. *Pitch due to pitch angular velocity: C_{mq} (pitch damping)*

For a given pitch angular velocity q (positive for instance), the reaction torque is negative.

$$q > 0 \;\rightarrow\; M_{Yq} = -\,½.\rho\,.V\,.S.\,l^2.(\Delta C_m/\Delta q).q \;=\; -\,½.\rho.V.S.l^2.C_{mq}.q \;<\; 0 \qquad [1.27]$$

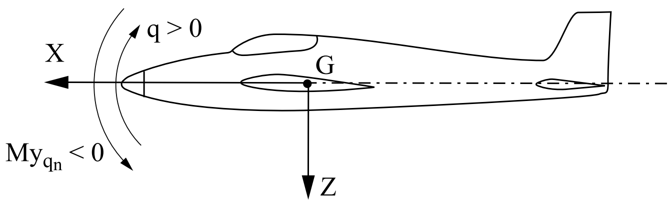

Figure 1.23. *Pitch torque and angular velocity are opposite*

1.4.6.4. *Pitch due to profile section curvature: C_{m0}*

For a non-symmetrical profile section (a CLARK Y for instance) there is a static momentum which is positive:

$$M_{y0} \;=\; q_0.S\,.l.\,C_{m0} \;\; > 0\,.$$

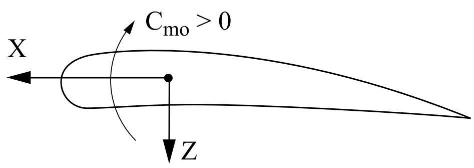

Figure 1.24. *For a "normal" profile section, the static momentum is positive*

This momentum C_{m0} tends to cause an upward pitch motion for the plane.

The wing alone, with such a section, is not self-stabilizing, and the built up attack angle increases continuously until the stall attack angle is reached and the plane crashes.

1.4.7. *Yaw coefficients (due to β, δ_n, r)*

1.4.7.1. *Yaw due to slip angle: $C_{n\beta}$ (directional stability)*

The principal cause of this effect is due to the vertical fin, and sometimes due to the fuselage if there is a large surface at the rear of the plane.

Its principal effect is to align the longitudinal GX axis with the velocity vector.

$$\text{For } \beta < 0 \rightarrow M_{Z\beta} = -q_0.S.l.(\Delta C_n/\Delta\beta).\beta = -q_0.S.l.C_{n\beta}.\beta > 0 \quad [1.28]$$

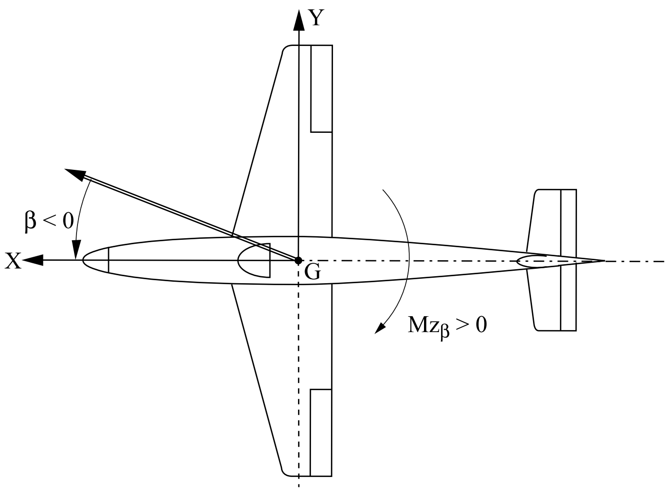

Figure 1.25. *The directional stability aligns GX with the velocity vector*

1.4.7.2. *Yaw due to directional control surface angular motion: $C_{n\delta n}$ (directional efficiency)*

For:

$$\delta n > 0 \rightarrow MZ\delta n = - q0.\ S.l.(\Delta Cn/\Delta\delta n).\ \delta n < 0. \quad [1.29]$$

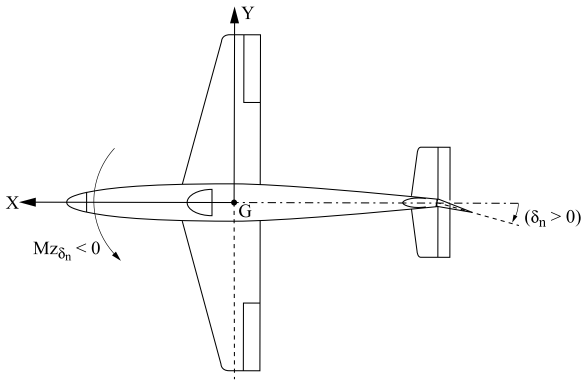

Figure 1.26. *Rudder control surface motion creates yaw torque*

1.4.7.3. *Yaw due to yaw angular velocity: C_{nr} (yaw damping)*

This damping torque is partially due to the directional fin and partially due to the rear area of the fuselage.

Insufficient yaw damping leads to bad behavior called "snaking", where the airplane progresses like a snake.

$$\text{For: } r > 0 \rightarrow M_{Zr} = -\tfrac{1}{2}.V.S.l^2.(\Delta C_n/\Delta r).r = -\tfrac{1}{2}.V.S.\,l^2.C_{nr}.r < 0 \quad [1.30]$$

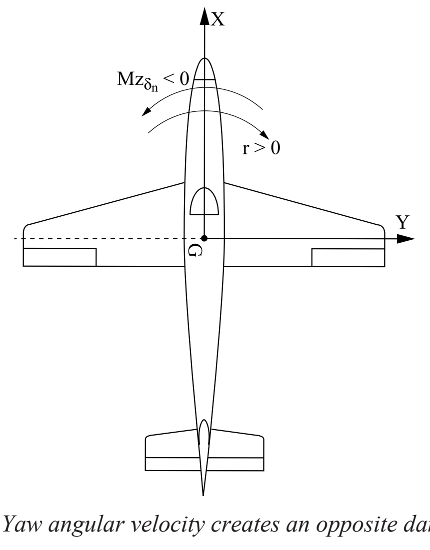

Figure 1.27. *Yaw angular velocity creates an opposite damping torque*

All of these formulae are condensed in the "AERO" block of the software, DYNAVION.

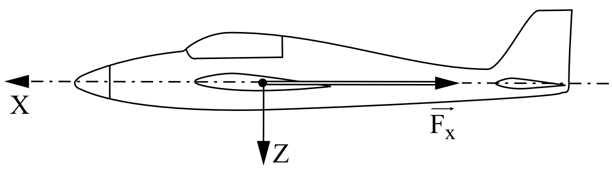

Figure 1.28. *Evaluation of aerodynamic coefficients as functions of the airplane's configurations: drag coefficient CX*

We consider here the Lilienthal reference attached to the airplane.

The drag coefficient C_X belongs on:

– the fuselage form and frontal section;

– the wing area and its profile;

– the horizontal and vertical fins.

All of these data are usually measured in the wind tunnel, but wing and stabilizer drag can be compiled by standard documents [ABB 59].

They are proper to each airplane, the number of them (18) is condensed by values and signs in the following SIMULINK Modulus called AERO.

The sign conventions for displacements and rotations, forces and torques respect the conventions in use in the reference (GXYZ). Mainly:

– For displacements and forces: positive like GX, GY, GZ.

– For angular displacements and torques:

- positive for X to Y;
- positive for Y to Z;
- positive for Z to X.

The values of these coefficients are recorded inside the “gains” block (Figure 1.29), they are all positive.

The signs of these coefficients are materialized inside the summing blocks.

The AERO block requires the following as inputs:

– U: longitudinal velocity (by GX axis) of the airplane;

– Beta: sideslip angle;

– Delta_n: angular deflection of the rudder control surface;

– Alpha: attack angle;

– Delta_m: angular deflection of the elevator control surface;

– Delta_l: angular deflection of the ailerons;

– p: roll angular velocity;

– q: pitch angular velocity;

– r: yaw angular velocity;

– C_{m0}: static pitch coefficient of the wing section.

This AERO block provides as outputs:

– the three aerodynamic forces (by the Lilienthal reference): FXA, FYA, FZA;

– the three aerodynamic torques (by the same reference): MXA, MYA, MZA.

These six components are the external impulses which make the plane move.

The organization of the AERO block is detailed, as shown by the following SIMULINK diagram (Figure 1.29).

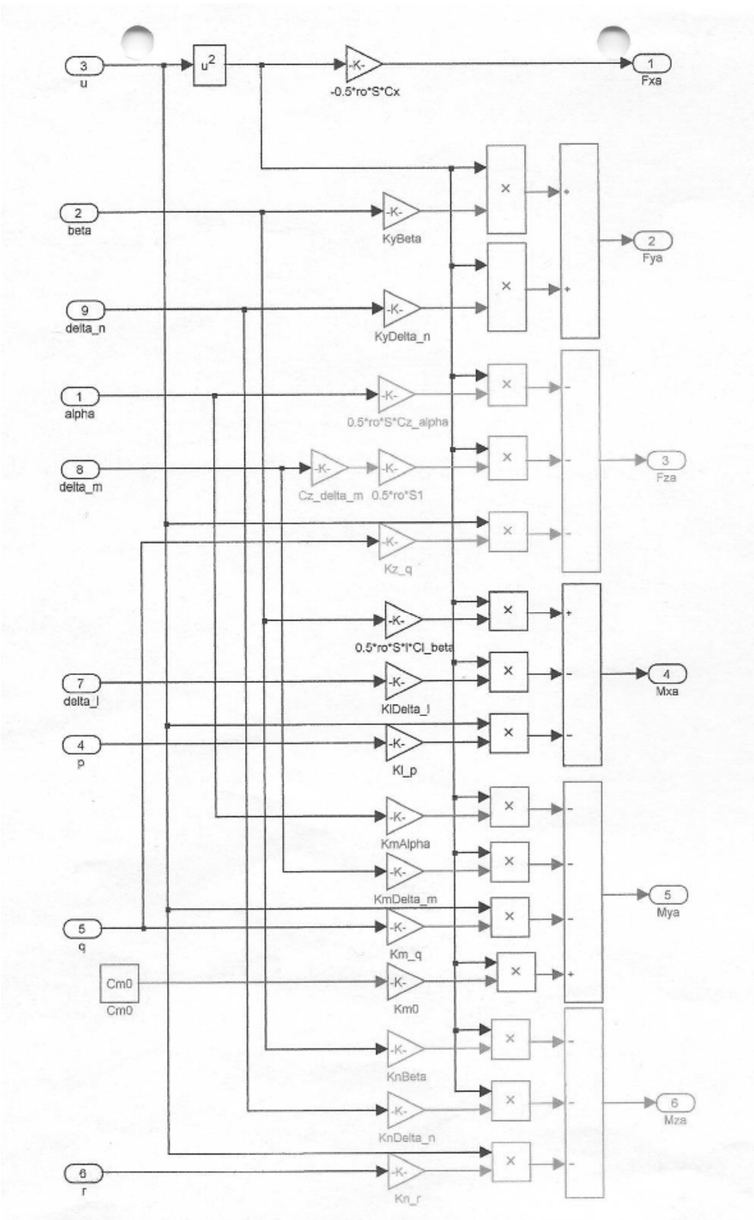

Figure 1.29. *SIMULINK diagram. All the necessary inputs are displayed on the left side, and the six outputs are displayed on the right side*

1.5. Aerodynamic data of a supersonic airliner for valuation of the software

The following data[1] for the French/British airliner Concorde in the landing configuration are provided here as validation elements for the software resulting from this document:

$C_{y\beta} = 0.67$

$C_{y\delta n} = 0.01$

$C_{Z\alpha} = -3.1$

$C_{Z\delta m} = -0.7$

$C_{Zq} = 0$

$C_{l\beta} = 0.1$

$C_{l\delta l} = 0.02$

$C_{lp} = -0.056$

$C_{m\alpha} = -0.062$

$C_{m\delta m} = -0.02$

$C_{m0} = 0$ (pitch torque balanced wing)

$C_{mq} = -0.2$

$C_{n\beta} = -0.153$

$C_{n\delta n} = -0.01$

$C_{nr} = -0.125$

Control surface deviations:

$\delta_l = 0$ (no ailerons motion)

$\delta_m = -0.0395$ (pitch control surface up)

$\delta_n = 0$ (no rudder motion)

1 Data from Sud-Aviation of 1963.

The coefficient of lift (C_z) and drag (C_x) are not linear functions of the attack angle α, and they are expressed during this subsonic phase by the following formulae:

$$C_Z = a_0 + a_1.\alpha + a_2.\alpha^2 + C_{Z\delta m}.\delta_m$$

$$C_x = b_0 + b_1.C_z + b_2.C_Z^2 \qquad [1.31]$$

1.6. Horizontal flight as an initial condition

The horizontal flight is taken as the initial condition prior to any perturbation. As a result, the velocity vector of the center of mass of the airplane is located in the horizontal plane (Gx_0y_0):

– α_0 is the attack angle;

– θ_0 is the pitch angle;

– $\alpha_0 = \theta_0 = (Gx_0, GX)$ in the present situation.

The pitch control surface is activated by two successive motions:

– δ_{mT} = trim deflection (permanent order);

– δ_{mC} = command pitch order.

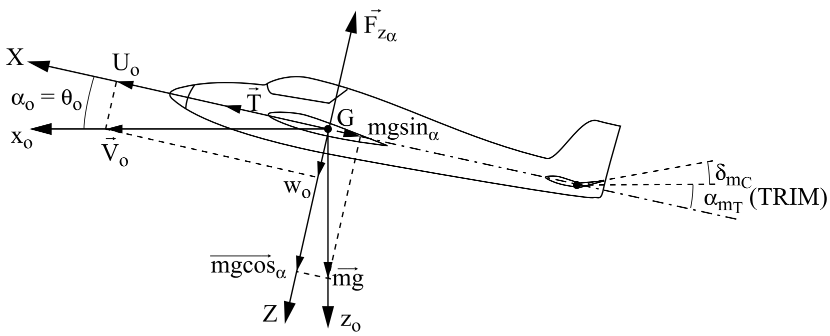

Figure 1.30.

To solve this problem, we have to consider the following equations:

$$m.g.\sin\alpha \ + q.\ S.C_x \ = \ T \ \rightarrow \ \text{forces balance by GX}$$

$$m.g.\cos\alpha \ = \ q.S.C_z \ \rightarrow \ \text{forces balance by GZ} \qquad [1.32]$$

$$M_{Y\alpha} = \ q.S.l.(-C_{m\alpha}.\alpha - C_{m\delta m}.\delta_{mT}) \ \rightarrow \text{torques balance around GY}$$

For subsonic flight, the Lilienthal polar (by GXZ) can be expressed as two functions of the attack angle α:

$$C_Z \ = \ a_0 + a_1.\alpha + a_2.\alpha^2 \quad \text{(wing and fuselage)}$$

$$C_Z \ = \ a_0 + a_1.\alpha + a_2\alpha^2 + C_{Z\delta m}.\delta_m \quad \text{(complete airplane)} \qquad [1.33]$$

By the numerical example for validation, we have to consider the following equations: [2]

$$C_Z \ = \ -0{,}0269 + 2{,}15.\alpha + 3{,}46.\alpha^2 - 0{,}7.\delta_m$$

$$C_X \ = 0{,}0256 - 0{,}061.C_Z + 0{,}0556.C_Z^2 \qquad [1.34]$$

Attack angle α_0 at the horizontal flight

We consider the equation about the force balance by GZ:

$$mg.\cos\alpha_0 \ = \ q.S.C_{Z0} \ = (a_0 + a_1.\alpha_0 + a_2.\alpha_0^2 + C_{Z\delta m}.\delta_m).q.S \qquad [1.35]$$

By this balance condition, there is a reciprocal condition between α and δ_m due to the torque balance around GY:

$$q.S.l.(-C_{m\alpha}\ .\alpha_0 - C_m.\delta_m.\delta_{mT}\ ) = 0\ ; \rightarrow \ C_{m\delta m}.\delta_{mT} \ = -C_{m\alpha}.\alpha_0$$

As a result:

$$\delta_{mT} \ = -(C_{m\alpha}/C_{m\delta m}).\alpha_0 \qquad [1.36]$$

2 Sud-Aviation and ONERA data, 1963.

Equation [1.36] is the static elevator angular deviation used to obtain the static attack angle α_0 at the horizontal velocity V.

The complete lift coefficient of the airplane becomes:

$$C_{Z0} = a_0 + a_1.\alpha_0 - C_{Z\delta m}.(-C_{m\alpha}/C_{m\delta m}).\alpha_0 + a_2.\alpha_0^2$$

The total lift coefficient becomes:

$$C_{Z0} = a_0 + (a_1 +(C_{m\alpha} . C_{Z\delta m}/C_{m\delta m})).\alpha_0 + a_2.\alpha_0^2 \quad [1.37]$$

As for the total drag coefficient:

$$C_{X0} = b_0 + b_1.C_{Z0} + b_2.C_{Z0}^2 \quad [1.38]$$

We also obtain the expression of the necessary thrust:

$$T_0 = m.g.\sin\alpha_0 + q.S.C_{X0} \quad [1.39]$$

The attack angle α_0 required for horizontal flight can be obtained by a graphical method, this attack angle is the value of α at the intersection that is displayed in equation [1.40] and Figure 1.31.

$$C_{Z0} = m.g.\cos\alpha_0 /(q. S) = a_0 +(a_1 + (C_{m\alpha}.C_{Z\delta m}/C_{m\delta m}).\alpha_0 + a_2.\alpha_0^2 \quad [1.40]$$

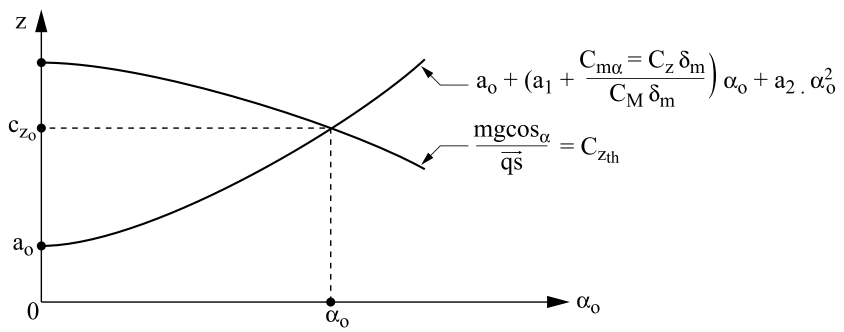

Figure 1.31.

The calculation of the successive data of the horizontal flight follows this schedule:

V_0

$\alpha_0 = \theta_0$

C_{Z0}

C_{X0}

$U_0 = V_0 . \cos\alpha_0$

$w_0 = V_0 . \sin\alpha_0$

$q = 0.5 . \rho . U_0^2$

$T_0 = m.g.\sin\alpha_0 + q.S.C_{X0}$

T_0 is the thrust-force necessary in order for the plane to sustain the velocity V_0.

1.7. Effect of gravitational forces

As the airplane moves through the atmosphere it is submitted to a uniform gravitational field. The resulting effect of this field on the airplane is a unique vertical force applied in a downward direction to the center of the airplane's mass.

We will now evaluate the components of this force by the airplane reference (GXYZ).

We consider the pitch angle (θ) and the roll angle (φ) which position the airplane reference in relation to the Galilean reference ($Gx_0y_0z_0$).

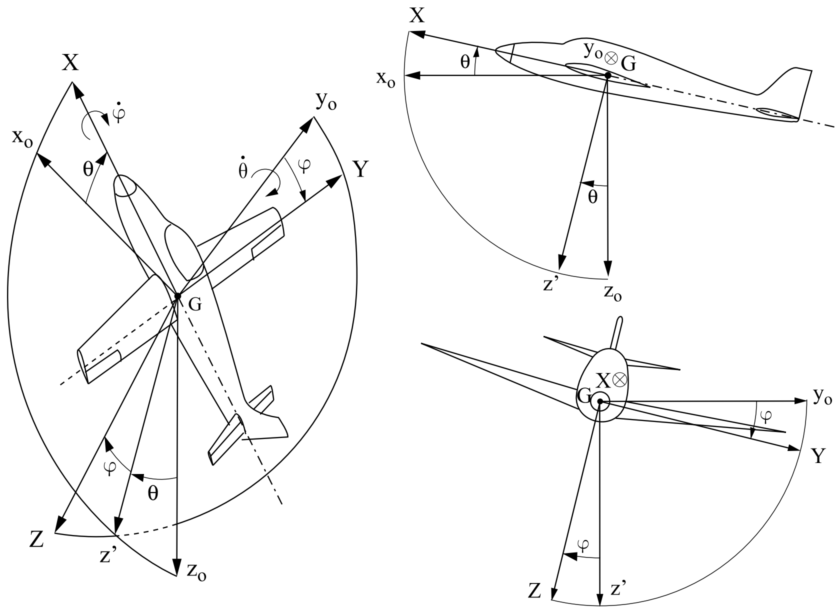

Figure 1.32. *Angular positioning by the horizontal plane Gx_0y_0*

We thus obtain the following equations:

Pitch rotation (θ)

$X = \cos\theta \, . \, x_0 - \sin\theta \, . \, z_0$

$z' = \sin\theta \, . \, x_0 + \cos\theta \, . \, z_0$

$z_0 = \cos\theta \, . \, z' - \sin\theta \, . \, X$ [1.41]

Roll rotation (φ)

$Y = \cos\varphi \, . \, y_0 + \sin\varphi \, . \, z'$

$Z = -\sin\varphi \, . \, y_0 + \cos\varphi \, . z'$

$z' = \cos\varphi \, . \, z + \sin\varphi \, . \, Y$ [1.42]

So, by successive substitutions, we can express X, Y, Z by x_0, y_0, z_0 as follows:

$X = \cos\theta . x_0 - \sin\theta . z_0$

$Y = \cos\varphi . y_0 + \sin\varphi. (\sin\theta . x_0 + \cos\theta .z_0) \rightarrow Y = \sin\theta . \sin\varphi . x_0 + \cos\varphi . y_0 + \cos\theta.\sin\varphi. z_0$

$Z = -\sin\varphi .y_0 + \cos\varphi. (\sin\theta .x_0 + \cos\theta . z_0) \rightarrow Z = \sin\theta. \cos\varphi . x_0 - \sin\varphi.y_0 + \cos\theta . \cos\varphi . z_0$

So, written under the matricial form:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta.\sin\varphi & \cos\varphi & \cos\theta.\sin\varphi \\ \sin\theta.\cos\varphi & -\sin\varphi & \cos\theta.\cos\varphi \end{bmatrix} * \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad [1.43]$$

The gravitational force is linearly confused with z_0 and directed downwards.

$$\overrightarrow{F_G} = m.g. \overrightarrow{z_0}$$

or $$\overrightarrow{F_G} / R_0 = \begin{bmatrix} 0 \\ 0 \\ m.g. \end{bmatrix}$$

So, the components of F_G expressed via the body reference are:

$$\begin{bmatrix} F_{XG} \\ F_{YG} \\ F_{ZG} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta.\sin\varphi & \cos\varphi & \cos\theta.\sin\varphi \\ \sin\theta.\cos\varphi & -\sin\varphi & \cos\theta.\cos\varphi \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \\ m.g \end{bmatrix} \quad [1.44a]$$

After development, the components of the vector gravitational force expressed by the body reference are:

$$F_G = \begin{bmatrix} F_{XG} & = & -m.g.\sin\theta \\ F_{YG} & = & m.g.\cos\theta.\sin\varphi \\ F_{ZG} & = & m.g.\cos\theta.\cos\varphi \end{bmatrix} \quad [1.44b]$$

See the *External forces block* (Figure 2.12).

So we were able to define the forces coming from three different origins:

– aerodynamic;

– thrust;

– gravity.

These forces are combined together and act as external forces, which appear as second members of the equations of the airplane's motion.

See the *Forces and Torques blockset* (Figure 2.11).

1.8. Calculation of the trajectory of the airplane in open space

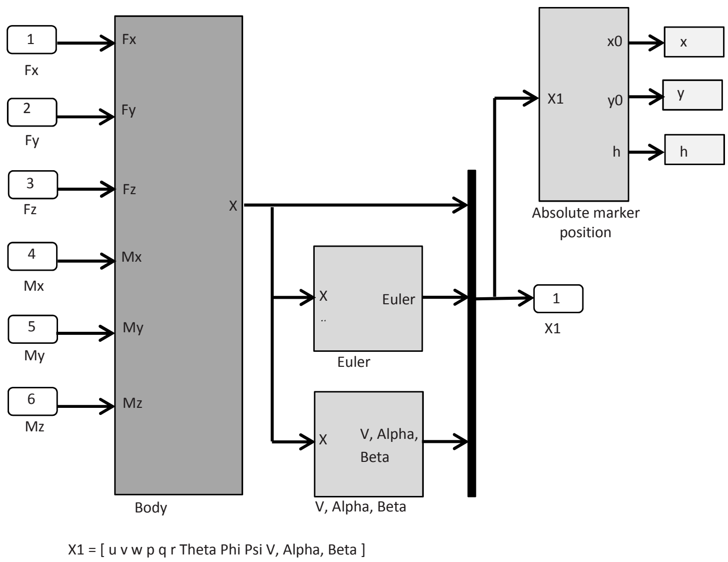

Figure 1.33.

The motion equations of the airplane form a "state vector" written as:

$$| X | \quad = \quad | \, U \; v \; w \; p \; q \; r \, |^T$$

This state vector includes the components of the velocity vector (U, v, w) and also those of the motion around the center of mass (p, q, r).

We start with the components of V in the airplane reference:

$$V = U.X + v.Y + w.Z$$

We now have to convert these components into those of the Galilean reference.

Once integration is made, we obtain the coordinates of G by the Galilean reference which represent the trajectory of the airplane.

This operation is acceptable due to the fact that (GXYZ) is Galilean.

Now we have to take into account the three Eulerian rotations which allow the passage from ($Gx_0y_0z_0$) to (GXYZ) which are: Ψ, θ, φ.

Yaw rotation Ψ

$$(Gx_0\, y_0\, z_0) \underset{(\Psi)}{\rightarrow} (Gx'\, y'\, z_0)$$

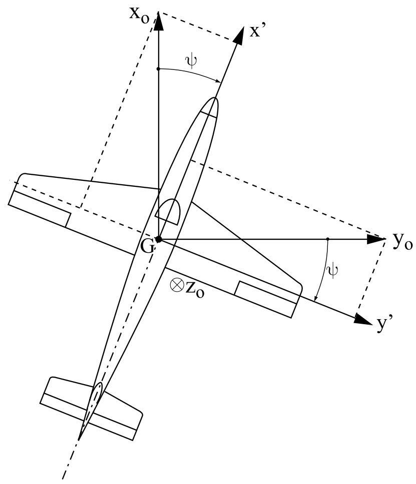

Figure 1.34. *First Euler rotation ψ*

$$x' = \cos\Psi . x_0 + \sin\Psi . y_0$$

$$y' = -\sin\Psi . x_0 + \cos\Psi . y_0$$

Under matricial form:

$$\begin{bmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{bmatrix} * \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

We call Δ the characteristic determinant of this equation.

$$\Delta = \begin{bmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{bmatrix} = \cos^2\Psi + \sin^2\Psi = 1$$

To solve this equation and obtain x_0 and y_0 we need to form:

$$N_{X0} = \begin{bmatrix} x' & \sin\Psi \\ y' & \cos\Psi \end{bmatrix} = \cos\Psi . x' - \sin\Psi . y'$$

$$x_0 = N_{X0} / \Delta = 1/\Delta . (\cos\Psi . x' - \sin\Psi . y') = \cos\Psi . x' - \sin\Psi . y'$$

[1.45]

$$y_0 = N_{Y0} / \Delta = 1/\Delta . (\cos\Psi . y' + \sin\Psi . x') = \cos\Psi . y' + \sin\Psi . x'$$

Pitch rotation (θ)

$$(G\ x'\ y'\ z_0) \underset{(\theta)}{\rightarrow} (G\ X\ y'\ z')$$

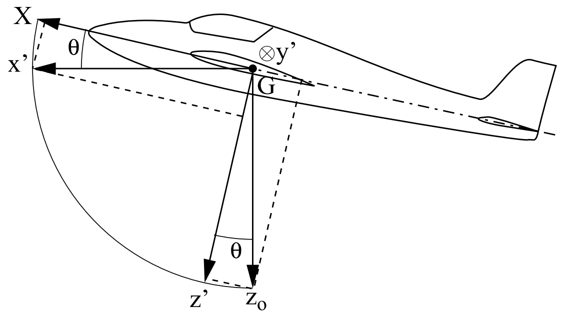

Figure 1.35. *Second Euler rotation θ*

$$X = \cos\theta .x' - \sin\theta .z_0 = \cos\theta .(\cos\Psi .x_0 + \sin\Psi .y_0) - \sin\theta .z_0$$

$$X = \cos\theta .\cos\Psi .x_0 + \cos\theta .\sin\Psi .y_0 - \sin\theta .z_0 \qquad [1.46]$$

Also, as shown in Figure 1.34:

$$z' = \sin\theta .x' + \cos\theta .z_0 ;$$

Roll rotation (φ)

$$(G\ X\ y'\ z') \xrightarrow{(\varphi)} (G\ X\ Y\ Z\ )$$

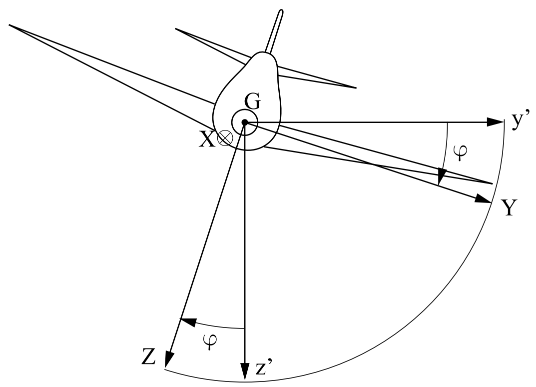

Figure 1.36. *Third Euler rotation φ*

$Y = \cos\varphi.y' + \sin\varphi.z' = \cos\varphi.(\sin\Psi.x_0+\cos\Psi.y_0)+\sin\varphi.[\sin\theta.(\cos\Psi.x_0+\sin\Psi.y_0)+\cos\theta.z_0]$

The components of Y grouped by x_0, y_0, z_0 are:

$$Y = (\sin\varphi.\sin\theta.\cos\Psi - \cos\varphi.\sin\Psi).x_0 + (\cos\varphi.\cos\Psi +\sin\varphi.\sin\theta.\sin\Psi)\ y_0 +\sin\varphi.\cos\theta.z_0 \quad [1.47]$$

As shown in Figure 1.36:

$Z = -\sin\varphi.y' + \cos\varphi.z'$

$Z = -\sin\varphi.(-\sin\Psi.x_0 +\cos\Psi.y_0) +\cos\varphi.(\sin\theta.x'+\cos\theta.z_{0)})$

$Z = -\sin\varphi.(-\sin\Psi.x_0+\cos\Psi.y_0)+\cos\varphi.[\sin\theta.(\cos\Psi.x_0+\sin\Psi.y_0)+\cos\theta.z_0]$

$$Z = (\sin\varphi.\sin\Psi+\cos\varphi.\sin\theta.\cos\Psi).x_0 +(\cos\varphi.\sin\theta.\sin\Psi - \sin\varphi.\cos\Psi).y_0+\cos\varphi.\cos\theta.z_0 \quad [1.48]$$

We already have the components of the velocity vector expressed in the body reference which are: U, v, w:

$V = U.X +v.Y +w.Z$

Now the expression of V in the Galilean reference will be:

$V = U.\cos\theta.\cos\Psi.x_0+ U.\cos\theta.\sin\Psi.y_0-U.\sin\theta.z_0$

$+v.(\sin\varphi.\sin\theta.\cos\Psi-\cos\varphi.\sin\Psi).x_0 + v.(\cos\varphi.\cos\Psi$

$+\sin\varphi.\sin\theta.\sin\Psi).y_0+v.\sin\varphi.\cos\theta.z_0$

$+w.(\sin\varphi.\sin\Psi+\cos\varphi.\sin\theta.\cos\Psi).x_0+w.(\cos\varphi.\sin\theta.\sin\Psi -\sin\varphi.\cos\Psi).y_0+w.\cos\varphi.\cos\theta.z_0$

Finally, the components of the velocity vector in the Galilean reference are:

$V/x_0 = U.\cos\theta.\cos\Psi + v.(\sin\varphi.\sin\theta.\cos\Psi \cos\varphi.\sin\Psi) + w.(\sin\varphi.\sin\Psi + \cos\varphi.\sin\theta.\cos\Psi)$

$V/y_0 = U.\cos\theta.\sin\Psi + v.(\cos\varphi.\cos\Psi + \sin\varphi.\sin\theta.\sin\Psi) + w.(\cos\varphi.\sin\theta.\sin\Psi - \sin\varphi.\cos\Psi)$

$V/z_0 = -U.\sin\theta + v.\sin\varphi.\cos\theta + w.\cos\varphi.\cos\theta$ [1.49]

The motions of the airplane inside the Galilean reference are obtained by integrating the precedent expressions.

For practical reasons, in order to consider the height of the airplane as a positive upright we use:

$H = -z_0$

So, we merely consider the “altitude” H of the airplane as the value obtained by the integration of dH/dt which is:

$dH/dt = \dot{H} = U.\sin\theta - v.\sin\varphi.\cos\theta - w.\cos\varphi.\cos\theta$ [1.50]

The following SIMULINK blockset condenses all the necessary operations to obtain the trajectory of the airplane.

The inputs of this blockset is the State Vector

$[X\ 1] = [U\ v\ w\ p\ q\ r]^T$.

The outputs are: x_0 , y_0 , h .

Details of the calculations are given in Figure 2.10, inside the block named:

– “Absolute marker position” → (inside the Galilean reference).

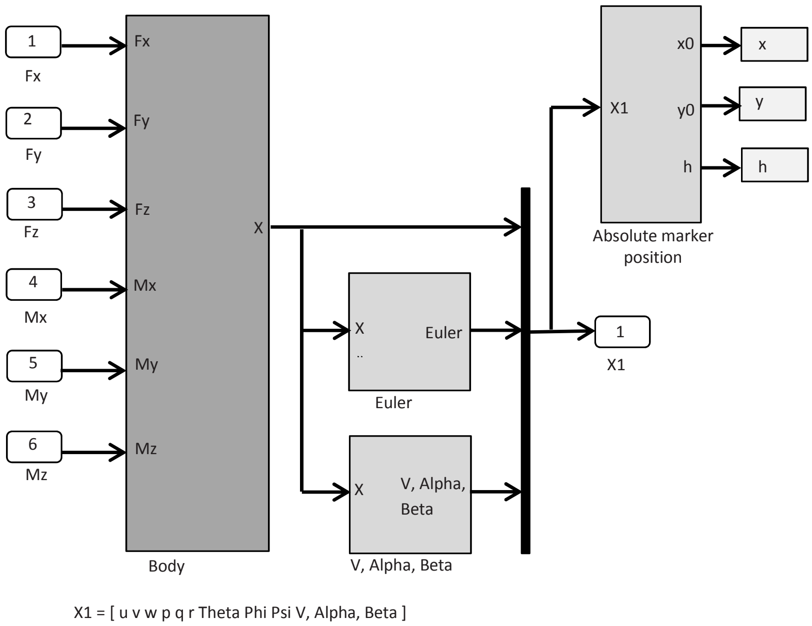

Figure 1.37.

This block represents the overall calculation of the motions of the airplane in application of Newton's principle:

– the input is the state vector [X_1];

– the output represents the placement of the airplane in the open space:

- x_0 , y_0 , h .

The details of this computing process are shown in Figure 1.38.

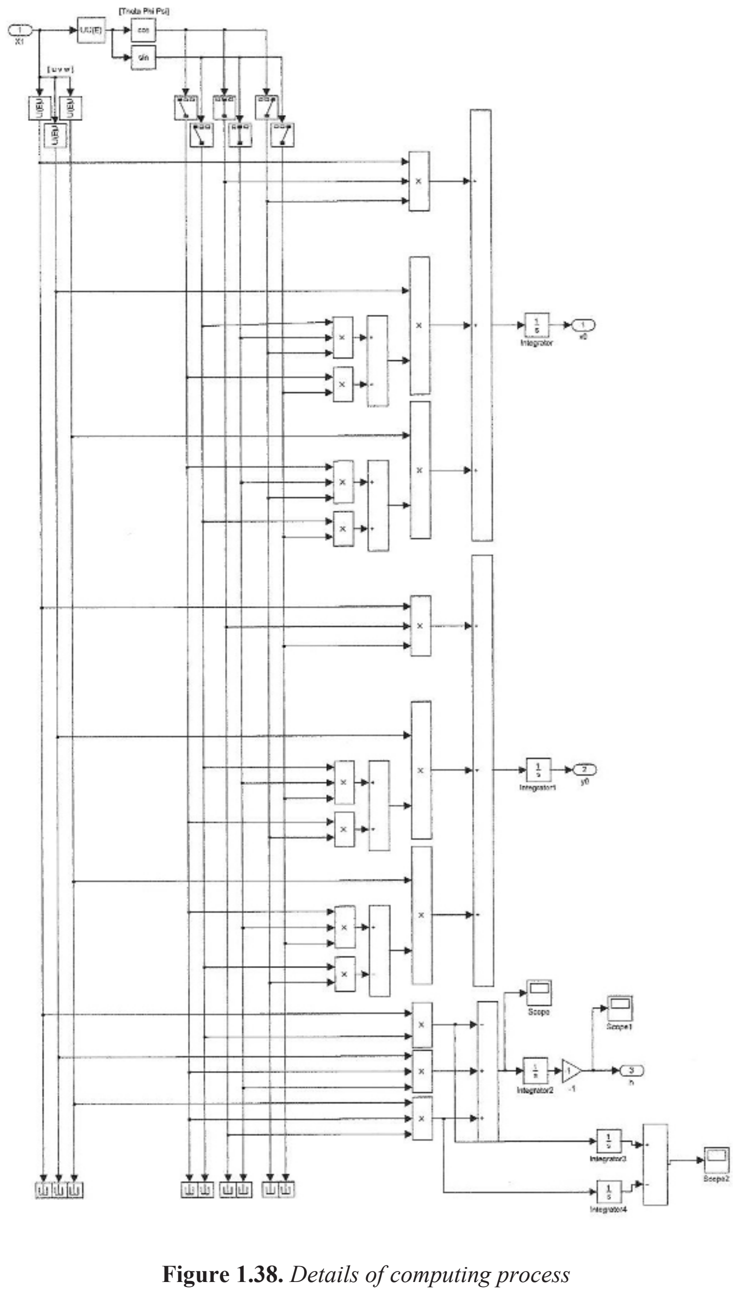

Figure 1.38. *Details of computing process*

1.9. Validation by comparison with ONERA Concorde data

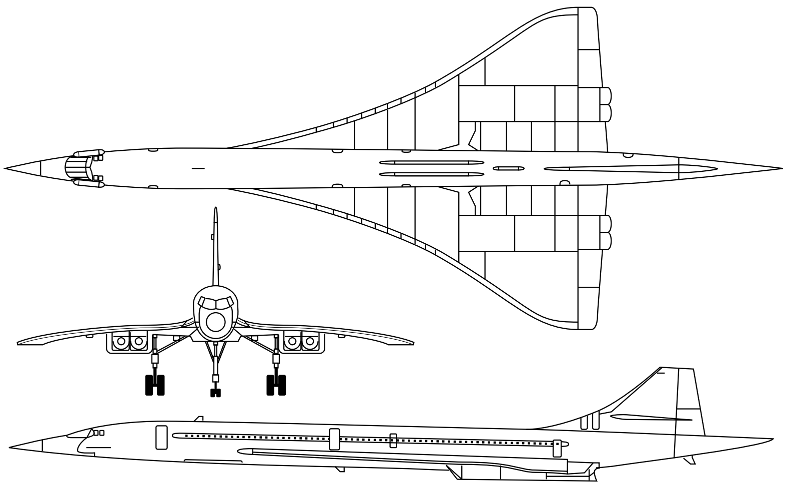

Figure 1.39.

In the early 1960s, there was a common project between Sud-Aviation and ONERA on a supersonic airliner called Concorde.

ONERA was in charge of the feasibility of this project on domains like aerodynamics, flying qualities and structural vibrations.

One of these projects was a version with a total mass of 85,000 kg, flying at Mach 2.2 at the altitude of 16,600 meters.

The numerical data, shown here, deal with the longitudinal motions of the airplane, including the two general modes (vertical and pitch), and three primary symmetrical vibration modes.

These vibration modes have their frequencies placed at:

– 2.59 Hz, 3.22 Hz and 5.10 Hz

The coupling influence between these modes and the general modes does not negatively impact the flying qualities of the airliner – one of the principal conclusions of this survey.

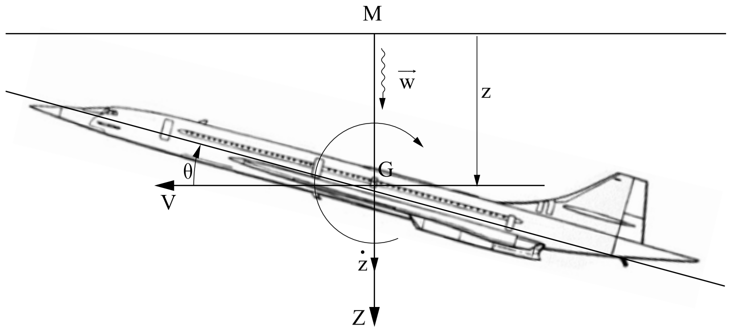

Figure 1.40. *Lateral figure of Concorde, showing altitude z and Pitch angle θ*

The numerical results show little deviation between ONERA methods and those used in this book.

Figures 1.41 and 1.42 display the responses of the airplane after a step input of 0.01 radians (about 0.6 degrees) of the elevator control surface (elevons), first for ONERA, and then for DYNAVION.

The figures show:

– The response of the airplane as pitch angle (θ) as a function of time.

– The response of the airplane as attitude angle (α) as a function of time.

For a step input of the elevator command, the pitch angle response displays an integration mode as a function of time (linear increase mode), while the pitch angle mode displays an almost constant response without integration.

Before the application of the elevator step input the airplane moves freely at an attitude angle $\alpha_{0,}$ which is the attitude angle obtained after the very slow angular motion, called "phugoid", is stabilized.

When the elevator step input is then applied (at t= t_P), the attitude response superimposes the attitude angle as $(\alpha - \alpha_0)$.

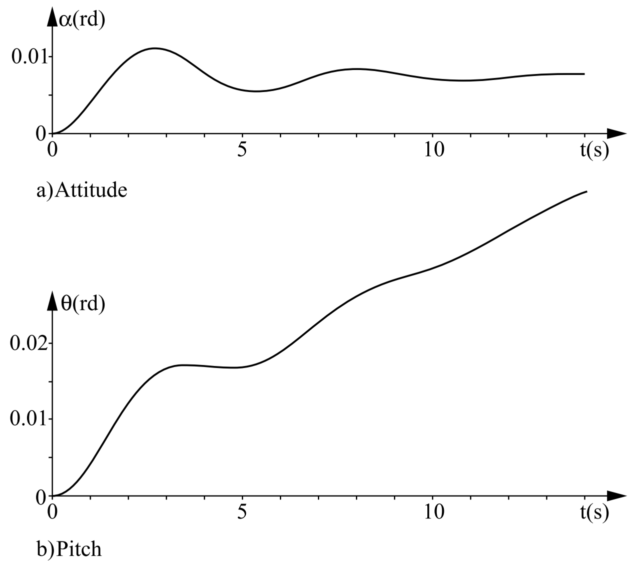

Figure 1.41. *ONERA results (A.T. PHAM report / 1963)*

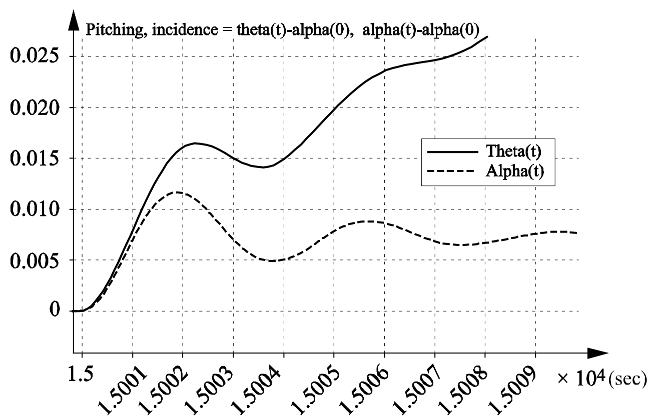

Figure 1.42. *DYNAVION results*

This comparison valuates DYNAVION software as a research tool for airplane dynamics.

The data of this plane are condensed in the file named “Données Concorde Mach 2.2 m” shown in Figure 1.43.

```
28/02/13 17:40  D:\Papa\Recherche Dynamique\ATPDynA...\DonneesConcordeMach2_2.m   1 of 2

% DonneesConcordeMach2_2.m
% Fichier données concorde en supersonic
% E. Grunn
% 20/06/06

% Constantes
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    g = 9.81;
    ro = 0.1506; % 16600 mètre

% Masse Inerties Géométrics
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    m = 8.5*10^4;
    A = 6.54*10^5;
    B = 8.15*10^6;
    C = 7.27*10^6;
    S = 310;
    l = 25.73;

% Coefficients aérodynamiques
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Dérivées aérodynamiques
    Cx=.018;                    % Lecture courbe abbaque

    Cy_beta = 0.67;       % INUTILE ETUDE LONGT
    Cy_delta_n = 0.01;       % INUTILE ETUDE LONGI

    Cz_alpha = 653/S;
    Cz_delta_m = 51.72/S;
    Cz_q = 1576/S/l;

    Cl_beta = 0.1;       % INUTILE ETUDE LONGI
    Cl_delta_l = 0.02;       % INUTILE ETUDE LONGI
    Cl_p = 0.056;       % INUTILE ETUDE LONGI

    Cm_alpha = 644/S/l;
    Cm_delta_m = 488/S/l;
    Cm_q = 30324/S/l^2;
    Cm0 = 0;

    Cn_beta = 0.153;       % INUTILE ETUDE LONGI
    Cn_delta_n = 0.01;       % INUTILE ETUDE LONGI
    Cn_r = 0.125;       % INUTILE ETUDE LONGT


% Calcul des gains aéro
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    KyBeta=0.5*ro*S*Cy_beta;
    KyDelta_n=0.5*ro*S*Cy_delta_n;

    KzAlpha=0.5*ro*S*Cz_alpha;
    KzDelta_m=0.5*ro*S*Cz_delta_m;
    Kz_q=0.5*ro*S*l*Cz_q;

    KlBeta=0.5*ro*S*l*Cl_beta
```

```
28/02/13 17:40  D:\Papa\Recherche Dynamique\ATPDynA...\DonneesConcordeMach2_2.m   2 of 2

KlDelta_l=0.5*ro*S*l*Cl_delta_l;
Kl_p=0.5*ro*S*l^2*Cl_p;

KmAlpha=0.5*ro*S*l*Cm_alpha;
KmDelta_m=0.5*ro*S*l*Cm_delta_m;
Km_q=0.5*ro*S*l^2*Cm_q;
Km0=0.5*ro*S*l;

KnBeta=0.5*ro*S*l*Cn_beta;
KnDelta_n=0.5*ro*S*l*Cn_delta_n;
Kn_r=0.5*ro*S*l^2*Cn_r;
```

Figure 1.43. *Listing of DYNAVION for Concorde data*

1.10. Definitions of aerodynamic coefficients and derivatives

1.10.1. *Aerodynamic coefficients*

We have to consider six aerodynamic coefficients, three for forces and three for torques, they are:

– total lift coefficients (wing, fuselage, elevator);

– drag coefficients;

– side lift coefficients;

– pitch coefficient;

– yaw coefficient;

– roll coefficient (generally this coefficient is neglected).

1.10.2. *Total lift coefficient*

We consider the wing and horizontal stabilizer separately as slender lifting surfaces. Only the fuselage is considered as a lifting body.

Lifting surfaces have lift coefficients which depend mainly on the attack angle and aspect ratios, as shown below in Figure 1.44, extracted from *Theory of Wing Sections* [ABB 59].

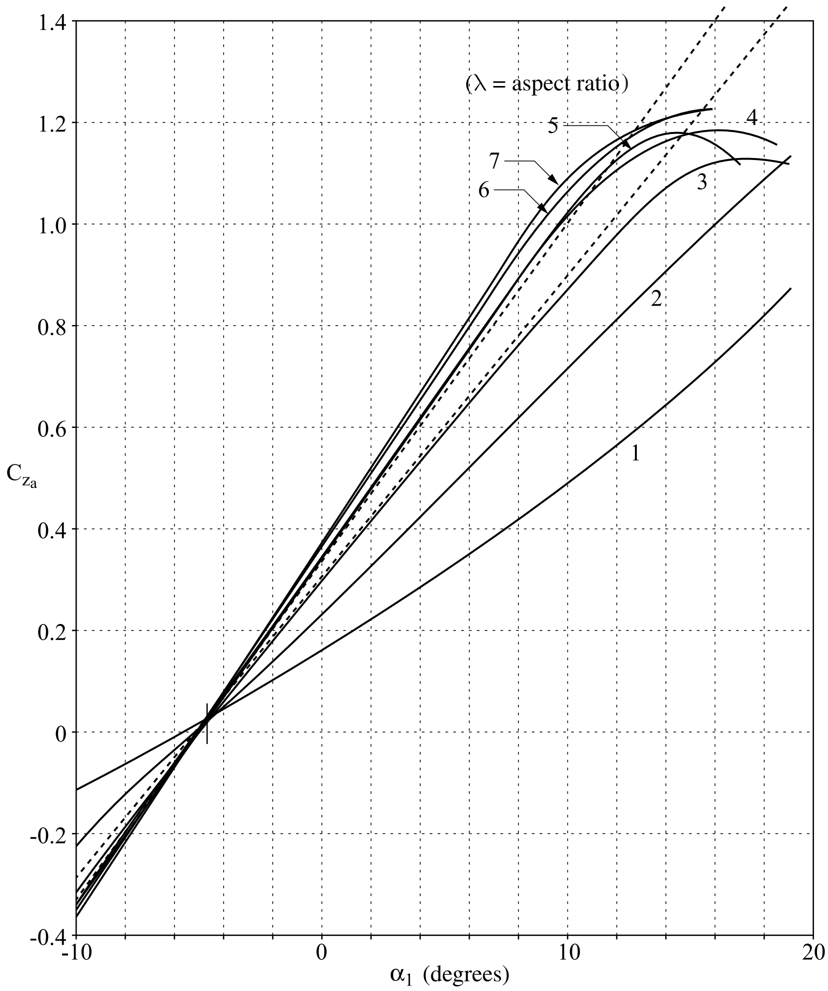

Figure 1.44.

This diagram shows the variation of the lift coefficient as a function of attack angle α and of aspect ratio λ.

To evaluate the total lift coefficient of the airplane, we have to consider Figure 1.45.

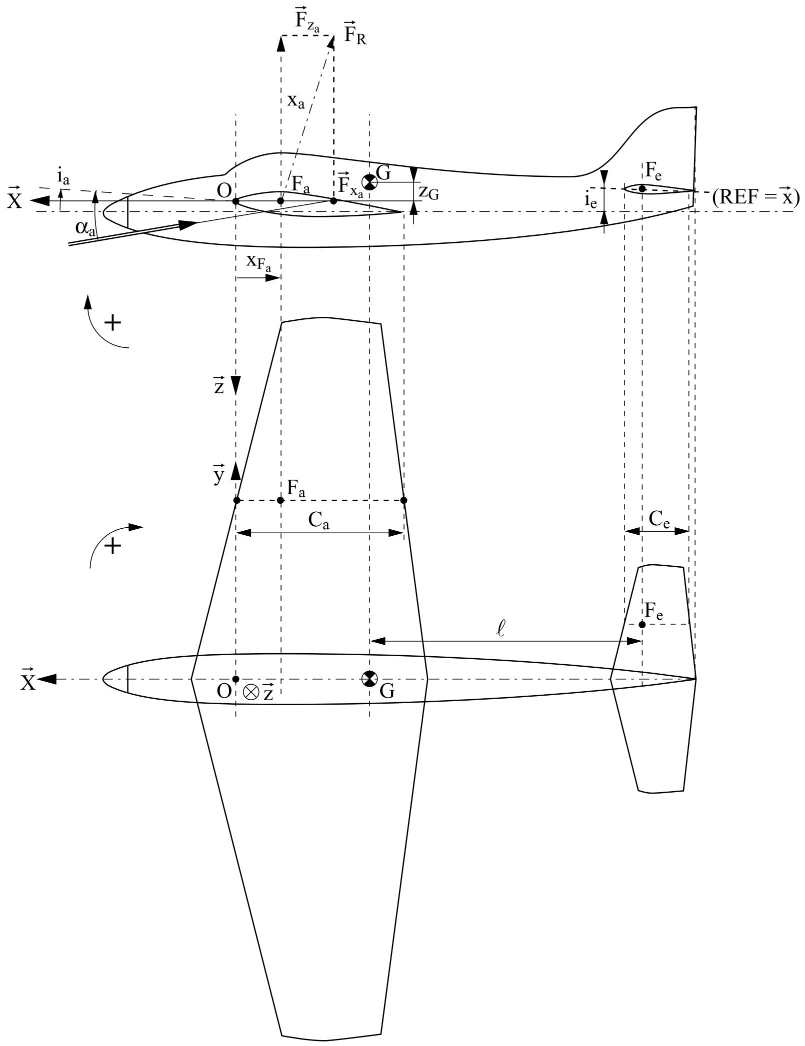

Figure 1.45. *Geometrics of the wing and the horizontal stabilizer*

In this figure, we consider O to be the center of coordinates at the leading edge of the middle wing section.

The rectangular body reference is connected to the plane as usual (OX to the front, OY to the right, OZ on the underside).

Aerodynamic forces are expressed by Lilienthal data in this reference.

Lift wing force F_{Za} is applied on F_a which is the focal point of each wing. The lift center of the total wing is situated at the same longitudinal coordinate, but at the centerline of the plane.

By similar considerations we can define the position of F_e which is the focal point of the horizontal stabilizer.

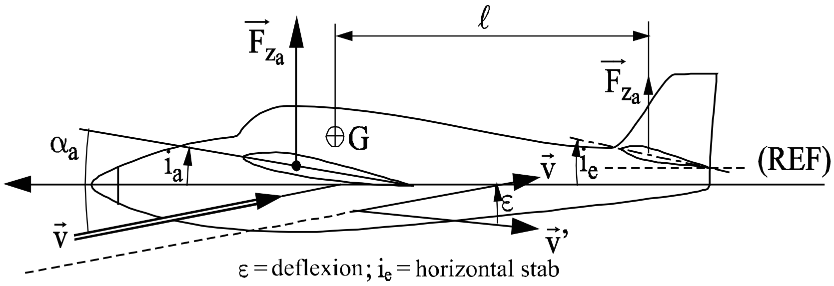

Figure 1.46. *Figure showing angular deviations of wing and stabilizer*

– i_a defines the angular deviation of the wing, relative to the OX axis;

– i_e the angular deviation of the horizontal stabilizer to the OX axis;

– α_a the attack angle of the wing;

– α_e the attack angle of the horizontal stabilizer, as shown on the precedent figure.

The total lift is:

$$F_z = F_{za} + F_{ze}$$

$$½.\rho . V_a^2.S.C_z = ½ . \rho .V_a^2. S. C_{za} + ½.V_e^2.S .C_{ze} \quad [1.51]$$

Dividing the two members by $½ .\rho. V_a^2. S$:

$$C_Z = C_{Za} + (½.\rho .V_e^2.S_e)/(½ .\rho .V_a^2.S) .C_{Ze}$$

We can call this:

$\mu = (½..\rho.V_e^2) / ½.\rho.V_a^2 = \text{Efficiency} \quad (0.8 < \mu < 1.2)$

$C_Z = C_{Za} + \mu .(S_e / S). C_{Ze} \; ; \quad (S_e /S \sim 0.25)$ [1.52]

(Dimensionless)

The total lift coefficient is equal to the lift coefficient of the wing, plus a percentage of the lift coefficient of the horizontal stabilizer.

Differentiating from α:

$\Delta C_Z /\Delta\alpha = \Delta C_{Za}/\Delta\alpha + (\mu.S_e/S) .(\Delta C_{Ze}/\Delta\alpha_e.)\Delta\alpha_e/\Delta\alpha$

So for the complete airplane:

$C_{Z\alpha} = C_{Z\alpha a} + \mu . S_e / S . \Delta\alpha_e/\Delta\alpha . C_{Z\alpha e}$ [1.53]

$C_{Z\alpha a}$ and $C_{Z\alpha e}$ both depend on the aspect ratios of the wing and the stabilizer.

The ratio of attack angles between the stabilizer and the wing is about 0.5 due to deflection law.

Now we have the complete description of the airplane's lift capability.

1.10.3. *Drag characteristics: (dimensionless)*

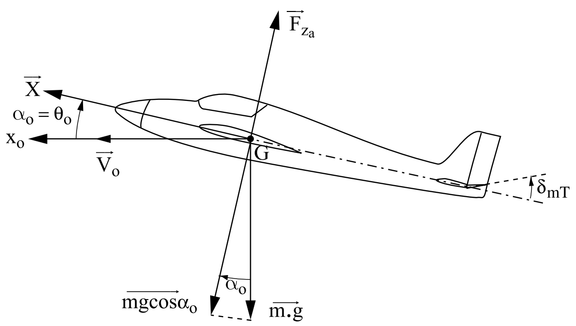

Figure 1.47. *An elevation time δ_{mT} is necessary to maintain an attack angle α_0*

We will consider the horizontal static flight condition at the velocity V_0 as a basis.

The kinetic pressure $\overline{q}$ is:

$$\overline{q} = ½ .\rho .V_0^2$$

Considering the force balance by the reference (GXYZ), essentially by GZ:

$$m.g.\cos\alpha_0 = \overline{q} . S . C_{Z0} \rightarrow C_{Z0}$$

This is the lift coefficient necessary for the plane to sustain this velocity.

For a given plane, C_{Z0} is a function of the attack angle α (drag due to lift) and elevator angular displacement (trim) δ_{mT}.

So: $\overline{q} .S .C_{Z0} = \overline{q} . S .(a_0 + a_1. \alpha_0 + a_2. \alpha_0^2 + C_{Z\delta m}.\delta_{mT})$

δ_{mT} is the necessary angular trim displacement to obtain the attack angle α_0, taking into account the pitch balance equation:

$$\overline{q} .S . l .(- C_{m\alpha}.\alpha_0 - C_{m\delta m}.\delta_{mT}) = 0 \ . \rightarrow \quad \delta_{mT} = - (C_{m\alpha} / C_{m\delta m}). \alpha_0 \quad [1.54]$$

$$m.g.\cos\alpha_0 / \overline{q} .S = a_0 + (a_1 + (C_{m\alpha}.C_{Z\delta m}/C_{m\delta m}).\alpha_0 + a_2.\alpha_0^2$$

We can solve this equation graphically to obtain α_0.

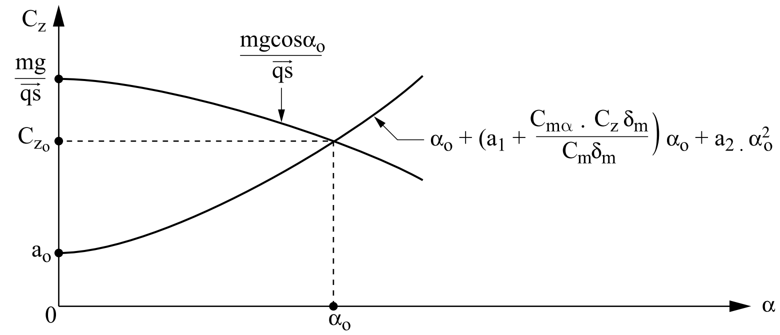

Figure 1.48. *Graphical solution to obtain α_0*

Once α_0 is obtained, we can compute C_{Z0}, then C_{X0}.

C_{X0} is the total drag due to the shape of the airplane (principally the fuselage) and to the lift (induced drag).

The drag of the plane's body shape can only be measured in a wind tunnel, due to the difficulties associated with 3D flow simulations.

Figure 1.49. *On this example (X-24 Prototype), we notice that the airflow near the plane is predominantly 3D*

Lilienthal polar

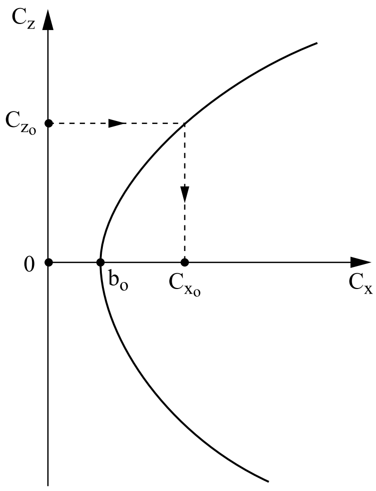

Figure 1.50. *The Lilienthal polar plot CZ as a function of CX by the body reference*

1.10.4. *Side lift coefficient: C_Y (dimensionless)*

The side lift capability of an airplane principally depends on rudder surface and fuselage shape.

These two properties both play a role in the side lift and yaw-centering characteristics of the plane.

If the fuselage has a substantial surface at the front, the side lift effect is predominant, for a given sideslip angle, the plane has more capability to make a “knife-edge” flight.

If the rudder is predominant, the yaw torque effect is more important than the side-force effect and the plane will have more tendency to go with the wind. This means that it will have some difficulty in sustaining a knife-edge flight, while its directional stability is convenient.

Figure 1.51. *From this point of view, a good plane manages a correct mix between directional stability and knife-edge capability*

1.10.5. *Roll coefficients*

Roll efficiency: $C_{L\delta l}$

Roll efficiency is defined as the ratio of roll torque on a given aileron's control surface angular motion.

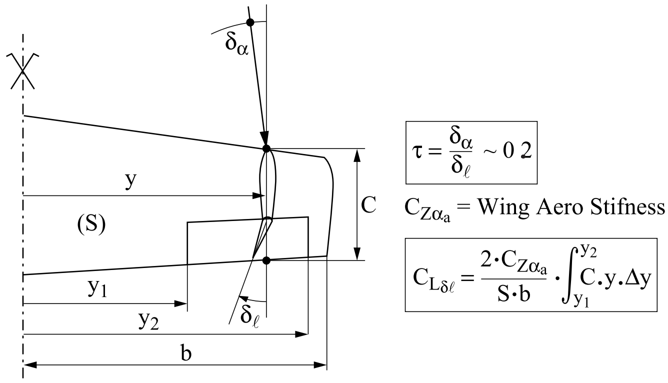

Figure 1.52. *The figure above shows the half-wing planform*

b = half wingspan

c = current width (wing chord)

y_1 = initial lateral coordinate of the aileron

y_2 = extreme coordinate of the aileron

An angular motion δ_1 of the aileron control surface is equivalent to a change in attack angle of δ_α. Experimental measurements show that:

$$\delta_\alpha / \delta_l \sim 0.2$$

If $C_{Z\alpha a}$ is the lift coefficient of the wing surface (which depends mainly on the wing section), the expression of aileron efficiency is:

$$CL\delta l = (2.CZ\alpha a / S.b).\int C.y.dy . \qquad [1.54]$$

Roll damping coefficient

When the plane makes a roll motion, it induces an orthogonal and linearly increasing field of velocity on the whole wingspan, which, combined with the velocity V of the plane, creates changes to the attack angle alongside the two half wingspans, and thus, also creates changes to the aerodynamic torque, with an intensity proportional to the roll angular velocity of the plane.

This damping torque stabilizes the roll motion of the plane by decreasing oscillations.

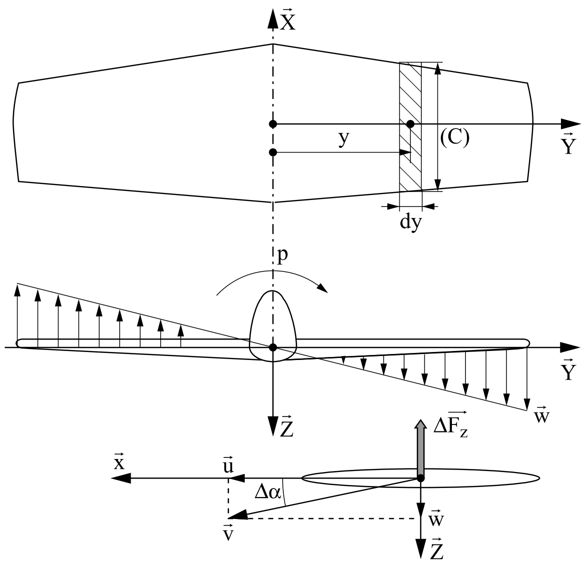

Figure 1.53. *The change of attack angle along the side of the wingspan due to roll angular velocity*

The total aerodynamic stiffness of the wing is known as $C_{Z\alpha}$. It depends on the wing aspect ratio and the wing airfoil section.

The elementary surface is: c(y).dy. This surface is placed at y from the centerline of the plane.

Due to the roll angular velocity p, the local angle of attack is: (p .y / V).

The elementary force is:

$$C_{Z\alpha}.(\, c(y).dy\,/S)\,.(p.y/V\,).$$

The elementary torque is:

$$C_{Z\alpha}.(\, c(y).dy\,/S)\,.\,(\,p.y^2\,/V).$$

The total roll torque due to the roll angular velocity of the plane is then:

$$L = 2\,.\,p\,.\int_0^b (C_{Z\alpha}/(S.V)).c(y).dy \;\; ; \quad \text{(b = half wingspan) (roll torque)}$$

We define as:

$$C_{Lp} = L/p = 2.(C_{Z\alpha}/(S.V)).\int_0^b c(y).y^2.dy$$

We can also make this work for the horizontal stabilizer, so that the total roll damping coefficient of the plane is:

$$\underbrace{C_{Lp} = 2\,.\,\left(\, C_{Z\alpha w}\,/\left(S.V\right)\right).\int_0^b c\left(y\right).y^2.dy}_{\text{(Wing)}} + \underbrace{2\,.\,\left(C_{Zes}\,/\left(S.V\right)\right).\int_0^b c\left(y\right).y^2.dy}_{\text{(Stabilizer)}} \;;\left(m.N\,/\,rd.s^{-1}\right) \qquad [1.56]$$

We notice that this roll damping coefficient takes into account the square of the wingspan. This property shows us that gliders with high-aspect ratio wings are presumably very "lazy" in roll motion.

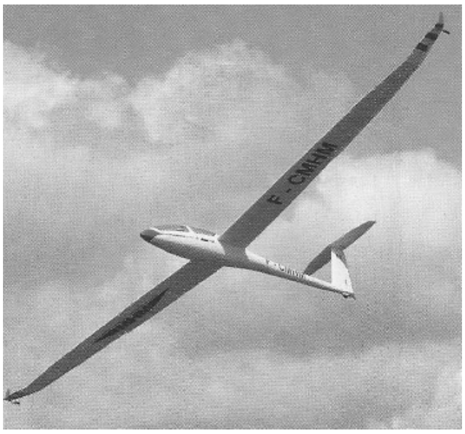

Figure 1.54. *A glider with high-aspect ratio wings*

1.10.6. *Pitch coefficients*

We distinguish two types of pitch coefficients: pitch stiffness and pitch damping.

These two functions are principally provided by the horizontal stabilizer (or elevator); part of this function can be assumed by special wing airfoils (self-stabilizing airfoils).

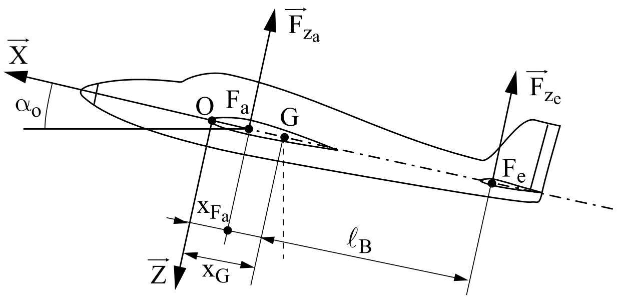

Figure 1.55. *O is the center of the body reference*

We have to consider three points:

– G the center of mass of the plane, placed at OG.

– F_a is the focal point of the wing located at the quarter of the mean wing chord, at GF_a.

– F_e is the focal point of the horizontal stabilizer, located at the quarter of the mean stabilizer chord.

The distance between G and F_e is called the horizontal lever = l_B.

In normal flight conditions, the plane stabilizes itself at an attack angle equal to α_0.

(See section 1.6. – Horizontal flight as an initial condition).

Any disturbance causes the plane to move slightly, but it naturally returns to the attack angle α_0.

Pitch stiffness

For a given disturbance $\Delta\alpha$ (> 0 for instance) and $\bar{q} = ½.\rho.V^2$ (dynamic pressure).

The wing generates an aerodynamic force which is: $-\bar{q}.S.C_{Z\alpha}.\Delta\alpha$ (this force is negative here).

Then an aerodynamic torque which is:

$$M_{Y\alpha} = \bar{q}.S.l.C_{M\alpha}.\Delta\alpha = GF_a.q.S.C_{Z\alpha}.\Delta\alpha$$

where: l = reference length = mean wing chord.

GF_a is the distance between the center of mass and the focal point of the wing.

We can define: $C_{M\alpha} = M_{Y\alpha} / (\bar{q}.S.l) = (GF_a / l).C_{Z\alpha}.\Delta\alpha$.

The horizontal stabilizer generates:

$$M_{Ye} = \bar{q}.S.l.C_{Me} = -GF_e.\bar{q}.S.C_{Ze}.\Delta\alpha .$$

The total aerodynamic torque is:

$$M_Y = M_{Ya}+M_{Ye}=\bar{q}.S.(GF_a.C_{Za}-GF_e.C_{Ze}).\Delta\alpha =(l_a.C_{Za}-l_e.C_{Ze}).\bar{q}.S.\Delta\alpha .$$

The total pitch coefficient will be written as:

$$C_M = M_y / \bar{q}.S.l = ((l_a.C_{Za} - l_e.C_{Ze})/l).\Delta\alpha .$$

The total aerodynamic stiffness is then:

$$C_{M\alpha} = d\,C_M / d\,\Delta\alpha = (l_a.C_{Za} - l_e.C_{Ze})/ l .$$

Pitch damping

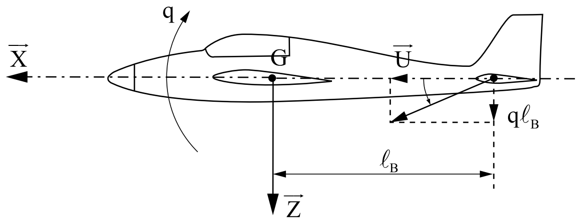

Figure 1.56. *Change of attack angle at the stabilizer due to* $\vec{V}$ *and* q

For a given pitch angular velocity q, there is a vertical velocity $q.l_B$ at the level of the horizontal stabilizer, which gives an attack angle (q.lB/U), combined with the velocity U (translation of the plane), at this place.

Thus a supplementary lift at the horizontal stabilizer:

$$\Delta F_{Ze} = ½.\rho . V^2 .(q.l_B / U). C_{Z\alpha e}$$

Where $C_{Z \alpha e} = \Delta C_{Ze} /\Delta \alpha_e$ = slope of variation of C_{Ze} as function of α_e

Then a pitch torque:

$$M_Y = ½.\rho .V^2 .(q.l_B^2 /V) .C_{Z\alpha e} . = ½ . \rho.V.q.l_B^2.C_{Z\alpha e} = ½ .\rho.V. C_{Zq} .q$$

So the coefficient of pitch torque as a function of angular velocity q can be written as:

$$C_{Zq} = C_{Z\alpha e}.l_B^2/V \qquad [1.57]$$

The pitch damping depends, of course, on the efficiency of the horizontal stabilizer ($C_{Z\alpha e}$) and also on the square of the horizontal stabilizer lever l_B.

Good pitch damping requires sufficient horizontal stabilizer length.

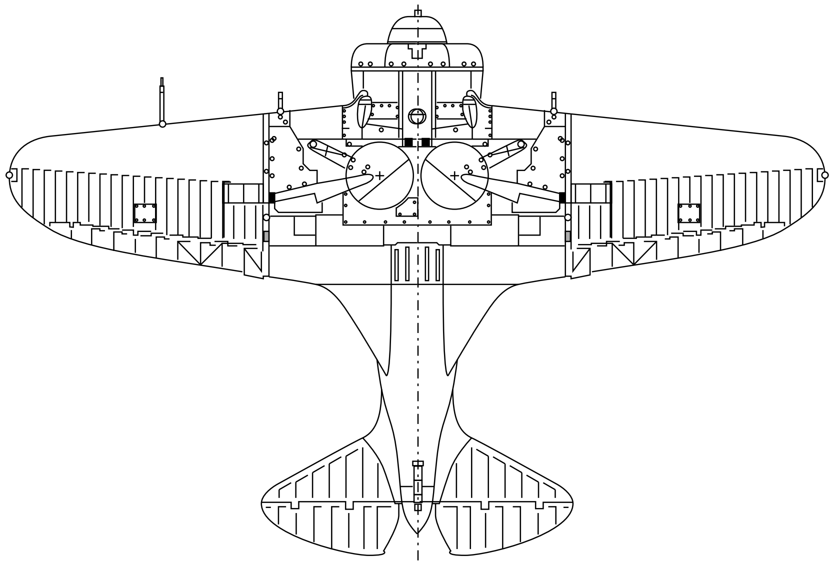

Figure 1.57. *The POLIKARPOV I-16 "RATA" with a very short pitch lever*

This drawing shows a Russian WW2 fighter, the POLIKARPOV I-16 "RATA".

This plane was not very successful as a fighter, probably due to its very short pitch lever, leading to a critical control.

Russia lost many of these planes during the 1943–1944 phase of the Russian campaign.

German fighters were better equipped with their MESSERSCHMITT – BF 109 which were fitted with a more convenient (longer) pitch lever.

Despite the fact that Russian pilots were numerous, the balance of victories was clearly in favor of the Germans.

Figure 1.58 shows another concept, more slender, especially in longitudinal view, called the MESSERSCHMITT – BF 109.

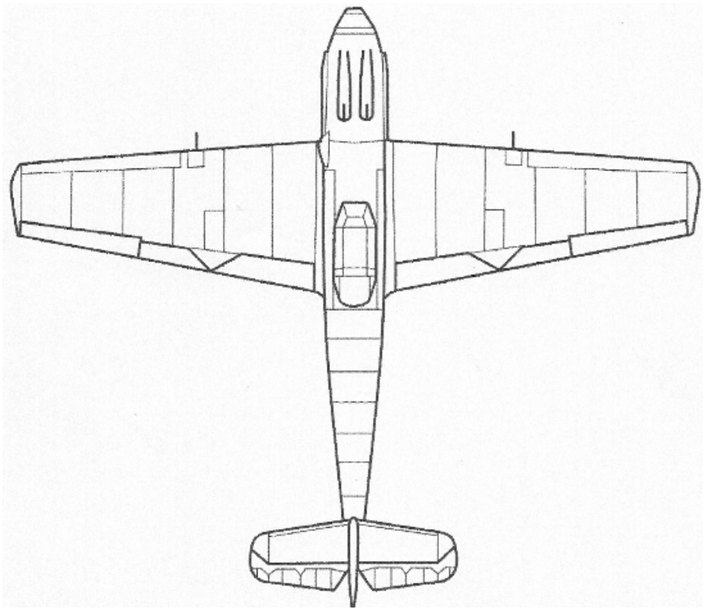

Figure 1.58. *The MESSERSCHMITT – BF 109*

1.10.7. *Yaw coefficients*

We distinguish two types of yaw coefficients: yaw stiffness and yaw damping.

These two functions are insured principally by the rudder, and partly by the lateral fuselage surface.

As shown in Figure 1.59:

– the rudder area is S_R;

– the lever of the rudder is l_R, distance between the center of mass and the focal point of the rudder.

The two inputs for yaw motions are:

– slip angle β;

– yaw angular velocity r.

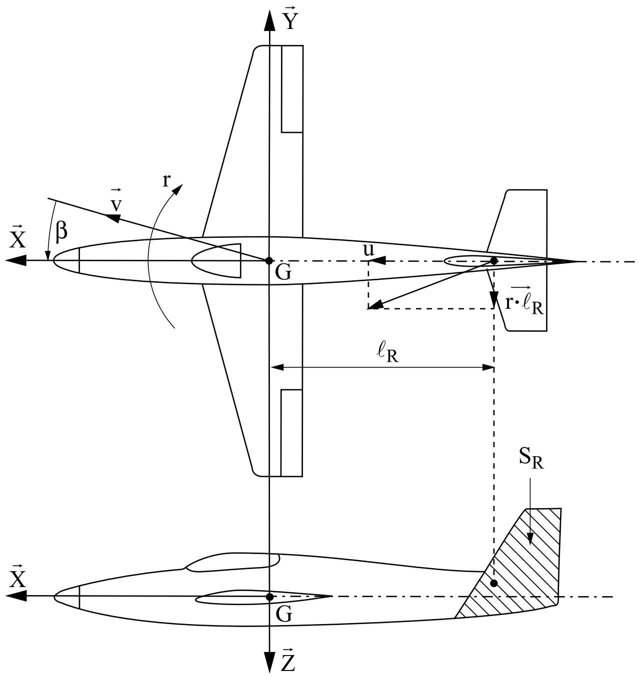

Figure 1.59. *Parameters of yaw stiffness and yaw damping*

Yaw stiffness

For a given slip angle β (negative on the figure), there will be a side force F_R at the level of the rudder:

$$F_R = ½.\rho .V^2 .S_R.C_{Y\beta R}.\beta . \quad (F_R \text{ is negative on Figure 1.59})$$

Where $C_{Y\beta R}$ is the aerodynamic stiffness of the rudder.

Thus a yaw torque which is:

$$M_{ZR} = l_R. F_R = ½.\rho.V^2.S_R.l_R.C_{Y\beta R}.\beta \quad [1.58]$$

Yaw damping

For a given yaw angular velocity r, there will be a lateral velocity $r.l_R$ at the level of the rudder (exactly at the focal point of the rudder).

Thus, a modification of ($r.l_R/V$) slip angle in the same place.

Thus, creation of a lateral force equal to:

$$F_R = ½.\rho.V^2.S_R .C_{Y\beta R}.(r.l_R/V) .$$

Thus, a yaw torque (related to yaw velocity) which is:

$$M_{ZR} = l_R.F_R = ½ .\rho .V .S_R .l_R^2. C_{Y\beta R}. \beta ; \quad [1.59]$$

We notice that the yaw damping is proportional to the square of the yaw lever, and this property is important for the plane's yaw oscillations in turbulent atmospheres ("snaking").