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Use of Value-at-Risk (VaR) Techniques for Solvency II, Basel II and III

In recent years, banks and insurance companies have been subjected to more and more regulations in order to increase transparency of their risk management and to minimize the risk of bankruptcy. On the one hand, Basel II (in 2004) and Basel III in (2010), which are, respectively, the second and third of the Basel Accords, are international recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision. On the other hand, the Solvency II Directive is a European Directive that codifies and harmonizes the European insurance regulation. Solvency II is currently scheduled to come into effect on January 1, 2014. These two standards aim at measuring the solvability of banks or insurance companies in time intervals (one day, 10 days, one year, etc.). To do so, both standards use the value at risk (VaR) concept. This chapter discusses the importance of the VaR in these international standards and the restrictive assumptions under which this concept is used.

1.1. Basic notions of VaR

1.1.1. *Definition*

Formally, VaR measures the worst expected loss over a given horizon under normal market conditions at a given confidence level. For example, a daily VaR of $30 million at a 99% confidence level means that there is only one chance in 100, under normal market conditions, for a loss greater than $30 million to occur. Usual time horizons are one day, one month or one year. Usual confidence levels are 95%, 99% or 99.5%.

VaR at level α, with $\propto \in]0,1[$, is mathematically defined by the quantile of the random variable X under the probability distribution P, as shown in the following formula:

$$\mathrm{VaR}_{\propto}(X) = \inf\{x \in \Re / P[X \leq x] \geq \propto\} \qquad [1.1]$$

VaR is based on three key parameters: the probability distribution of losses, the time horizon and the confidence level.

The probability distribution of losses depends on the underlying portfolio (e.g. assets held by a bank or an insurance company, or the claims of a portfolio of insured people). This probability distribution of losses is generally difficult to estimate at best, in particular when the underlying portfolio is heterogeneous (e.g. if the portfolio is composed of many different types of asset).

Choosing a consistent time horizon depends on four major criteria. First, the time horizon chosen must match the holding period of the underlying portfolio to assess. This is the reason there is a major difference between banks and insurance companies. Whereas the time horizon is short term for banks (a few days), it is long term for insurance companies (the duration of the liabilities is often close to eight years). Second, portfolio composition should remain unchanged on the time horizon. This is verified in the context of Solvency II and Basel II–III because the portfolios are valued in runoff. Third, the time horizon chosen should be consistent with the degree of risk aversion of the bank or insurance company. Fourth, the time horizon chosen must be the same across the several institutions that we would like to compare. This implies that supervisors set a time horizon common to all banks or all insurance companies in order to compare them.

The confidence level depends on three major criteria. First, the confidence level obviously depends on the risk aversion of the bank or insurance company. Second, as for the time horizon, the confidence level must be the same across the several institutions that we would like to compare. Third, for reliable risk management, defining a relatively low threshold is better so as to obtain observations of overtaking and to check the robustness of the calculation.

1.1.2. *Calculation methods*

In this section, we not only give a comprehensive analysis of VaR, but also briefly discuss the different methods for its calculation, which are fully discussed in the subsequent chapters both in the fields of Gaussian and non-Gaussian finance. Although VaR is a very intuitive concept, its measurement is a challenging statistical problem. Three methods are available for the calculation of VaR: the historical, the parametric and the Monte Carlo methods. All these three methods have their own strengths and weaknesses. The problem is that the results each of these methods yield can be very different from each other.

The *historical method* simply consists of using the percentiles of the actual historical returns to calculate the VaR; therefore, any anomalies (such as fat-tails, skewness and kurtosis) and non-normality are included. This method is easy and relatively fast to implement and is suitable for all types of underlying portfolios. However, it implies that the future is assumed to behave like the past, which is obviously unrealistic. In other words, this method is based entirely on the assumption of stationarity of the loss distribution, which is not verified in reality.

In the *parametric method*, distribution of gains/losses is assumed to follow a parametric distribution (often a normal or lognormal distribution). Historical data (observations of gains/losses of the past) are used to better adjust the parameters of the chosen distribution. Once the distribution is correctly parameterized, the quantile can easily be calculated. The main advantages of this approach are its relatively simple structure and the speed of calculations. If the institution using the parametric approach is trading only regular, linear instruments, then the level of accuracy obtained is reasonably good. However, results become unreliable when the portfolios include significant numbers of nonlinear instruments. Moreover, this method is also based on the assumption of stationarity of the loss distribution, which is not verified in reality, particularly in times of crisis. In addition, while the historical method accepts distributions for what they were, the analytical method, in contrast, assumes and imposes a normal distribution for all exposures. This requires large approximations and is rarely accurate.

While other methods are based exclusively on past elements, the *Monte Carlo method* is intended to simulate the future evolution of risk factors. A probability distribution is defined for each risk factor and parameters of each distribution are estimated on the basis of the past of these risk factors. Then, a large number of gains/losses are simulated based on the distributions obtained for risk factors. A histogram of the gains/losses is then plotted to determine the quantile. Monte Carlo analysis is the most powerful method to calculate VaR because it can account for a wide range of risks. This approach can provide a much greater range of outcomes than historical simulation, and it is much more flexible than the other approaches. Unlike the analytical method, the distribution need not be normal and can contain fat-tails. In addition, this method is suitable for all types of underlying portfolios. The biggest drawback of this method is that implementing simulation models is very computationally intensive. Therefore, it is the most expensive and time-consuming method and tends to make it unsuitable for large, complex portfolios.

Figure 1.1 depicts the three methods, their advantages and limitations.

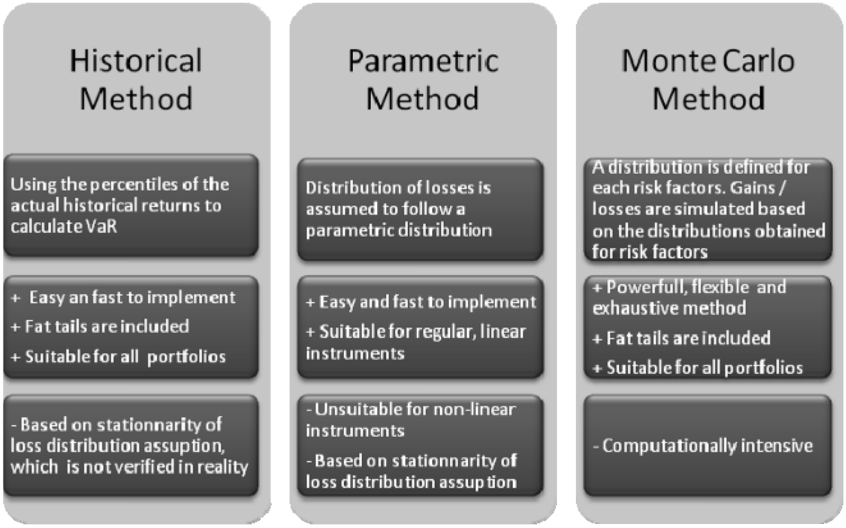

Figure 1.1. *Comparison of the three methods of calculation of VaR*

1.1.3. *Advantages and limits*

VaR is a risk measurement tool that has advantages and limits. On the one hand, VaR can be seen as a cornerstone in risk management because it is an easy-to-understand method for calculating exposure to risks in single digits. It provides a unified framework for a meaningful, easy to interpret, aggregate measure of risks. For example, a VaR at 99.95% over one year corresponds to one bankruptcy every 200 years. A VaR at 99% over 10 days corresponds to one bankruptcy every 1,000 days. Moreover, VaR uniformly treats different types of risk. For example, the insurance company can calculate two VaRs (one on credit risk and the other on interest rate risk) and can easily compare the results if the level of confidence and the time horizon are the same.

Concerning the limits, VaR is, in practice, often computed under the assumption that the distribution is normal; hence, estimates of tail probabilities can be obtained by estimating the mean and variance of the distribution. Nevertheless, this is often inaccurate. VaR offers little guidance in exploring tail events and is difficult to estimate. This limitation is especially profound for products with an asymmetrical risk profile, such as options and mortgages. In fact, VaR does not capture the subtleties of the probability distribution, especially for tail probabilities. For example, two different probability distributions can give an identical VaR. The two graphs below illustrate this point: in both cases, VaR at 84% is equal to 50, while the probability distribution is different. However, the second distribution hides a greater risk than the first distribution. When the loss exceeds the VaR, there is a high probability that this loss is much greater with the second distribution (approximately 200). In this example, a risk indicator, such as conditional VaR (CVaR) (detailed in section 1.2.2), would be more appropriate.

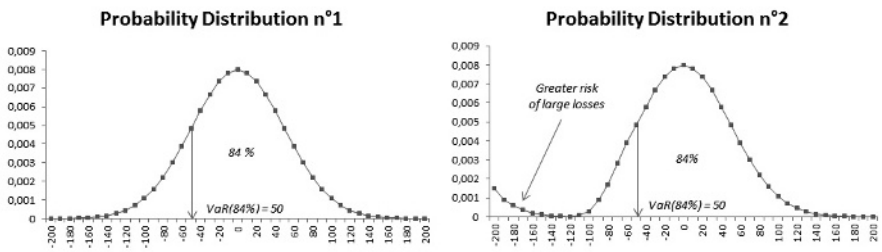

Figure 1.2. *Comparison of two probability distributions on VaR*

Moreover, assumptions used for volatilities and correlations, upon which the VaR calculation is highly dependent, can break down during periods of market stress. Indeed, securities that seem uncorrelated during normal times may become extremely highly correlated during market crises.

Furthermore, VaR is not intended to anticipate a crisis because it is only really secure when the underlying distribution is stationary. This assumption is generally not observed in reality; so, to correct this, a stressed VaR, which uses data from 2008 (during the crisis) to be calibrated, was added in a revision of Basel II.

VaR does not capture differences in the liquidity of portfolio assets. Indeed, in the VaR calculation, portfolio assets can be liquidated or covered regardless of their size, with no impact on the market and whether or not there is a crisis. To correct this underestimation of risk, it is possible to calculate a VaR adjusted to liquidity.

Last, but not least, VaR does not take into account wider risk factors such as policy and regulation.

Figure 1.3 shows the advantages and limitations of VaR.

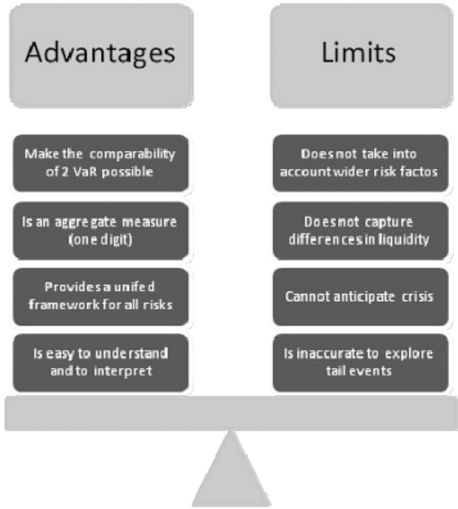

Figure 1.3. *Advantages and limitations of VaR*

1.2. The use of VaR for insurance companies

Solvency II is based, as are the Basel agreements, on a three-pillar approach:

– Pillar I consists of quantitative requirements (e.g. the amount of capital an insurer should hold).

– Pillar II sets out requirements for the governance and risk management of insurers as well as for the effective supervision of insurers.

– Pillar III focuses on disclosure and transparency requirements.

On the one hand, insurance companies use the VaR in the framework of Pillar I of Solvency II in order to calculate the regulatory capital. In this case, the confidence level and the time horizon are defined by the regulator and are common to all insurance companies. This allows for consistency and comparability between insurers.

On the other hand, insurance companies calculate their risk profile in the framework of Pillar II of Solvency II. To define their own risk and solvency assessment (ORSA), they must choose a tool to measure risk (VaR, Tail VaR (TVaR), etc.), a confidence level and a time horizon to better quantify their risk profile.

1.2.1. *Regulatory approach*

The Solvency II Directive incites insurers to take into account all the risks they face and to hold sufficient capital or equities with respect to these risks. More precisely, Solvency II introduces new solvency requirements, which are more risk-sensitive and sophisticated than in the past. In the framework of Solvency II, business is considered in runoff. The capital requirement is determined using a two-level approach, which corresponds to VaR with different levels of confidence on a one-year horizon. The first level is the minimum capital requirement (MCR), which is the capital representing the threshold that triggers ultimate supervisory measures in the event that it is breached. The second level is the solvency capital requirement (SCR), which can be calculated using the standard formula, a full internal model or a partial internal model. These two levels of capital requirements are described in the following sections.

1.2.1.1. *Calculation of the SCR*

1.2.1.1.1. SCR calculation using the standard formula

Solvency II explicitly states that SCR calculation is based on a VaR of equity over a one-year horizon and with a confidence level of 99.5%. The exact definition of SCR in the QIS5 specifications is the following:

> SCR.1.9. The SCR should correspond to the Value at Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period. The parameters and assumptions used for the calculation of the SCR reflect this calibration objective.

QIS5 Technical Specifications, 5 July 2010

To calculate the SCR, insurance companies must use the risk cartography of Solvency II, which includes risk modules (Market, Health, Default, Life, Non-Life, Intangible) and risk submodules (Mortality, Longevity, Disability/Morbidity, Lapse, Expense, Revision and Catastrophe (CAT) for the life module). This calibration is maintained at the module level by stress tests or closed formulas:

> SCR.1.10. To ensure that the different modules of the standard formula are calibrated in a consistent manner, this calibration objective applies to each individual risk module.

QIS5 Technical Specifications, 5 July 2010

The following figure shows the Solvency II risk cartography established for the Quantitative Impact Study no 5 (QIS5).

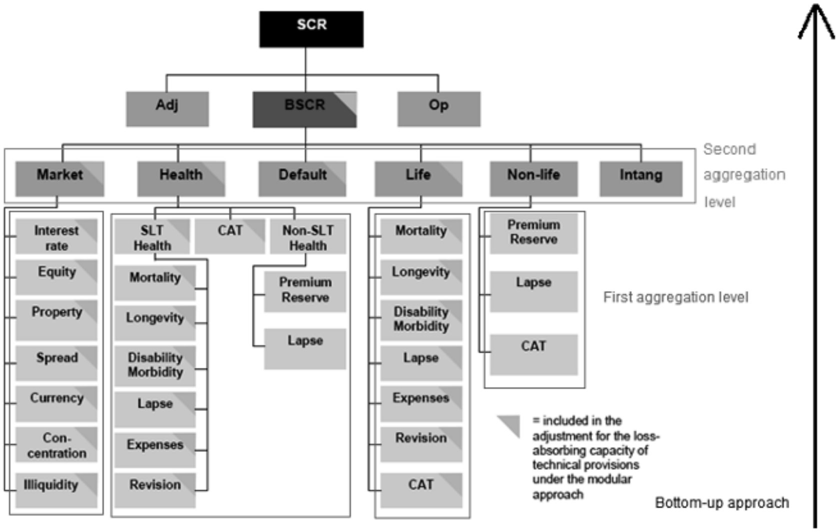

Figure 1.4. *QIS 5 Risk cartography (Source: QIS 5 Technical Specifications)*

In fact, capital charges are determined using a bottom-up approach. First, the capital requirements are evaluated for each risk submodule. Second, the capital requirements of submodules are aggregated using a prescribed correlation matrix to obtain the capital requirement of risk modules. Third, another aggregation using a prescribed correlation matrix is performed between the capital requirements of risk modules to give the basic SCR. Finally, a separate loading for operational risk is added to obtain the company SCR.

To illustrate this, let us consider the submodule Mortality included in the module SCR Life. The capital requirement for this submodule is defined as the result of the following mortality scenario:

$$\text{Life}_{\text{mort}} = \frac{\Delta NAV}{\text{mortshock}} \qquad [1.2]$$

where ΔNAV is the change in the net value of assets minus liabilities.

Mortshock (which means mortality shock) is a stress test corresponding to a permanent 15% increase in mortality rates for each age and each policy where the payment of benefits (either lump sum or multiple payments) is contingent on mortality risk.

The calibration made by the Solvency II Directive states that the coefficient of 15% mortality shock is equivalent to the calibration of a VaR at 99.5% over one year. This methodology is the same for the other sub-modules.

Unfortunately, the calibration method of shocks in the standard formula is not explicit. Moreover, given the chosen time horizon and confidence level, it seems impossible to perform backtesting. It is, therefore, difficult to accurately assess the robustness of this method. In addition, the choice of correlation coefficients implies a notion of subadditivity because an aggregation of several modules will give a result less than the sum of separate modules. This method may be subject to criticism. Finally, the choice of time horizon may be challenged because the average duration of insurance contracts is close to eight years.

1.2.1.1.2. SCR calculation using a full internal model

The goal of an internal model is to create a stochastic model with structures and relationships that best depict the company's business. The risks incorporated into an internal model are likely to be the same as those included in the standard model. The definition of the SCR remains the same as it corresponds to a VaR of equity at 99.5% over a one-year horizon. The calculation of the SCR using an internal model is based on the distribution of equities in a year. To calculate the SCR, two steps are needed:

– to project assets and liabilities under the historical probability in order to evaluate the quantile 99.5% of the Net Asset Value at time $t = 1$;

– to discount at time $t = 0$, the value of the quantile 99.5% in order to calculate the amount of capital, which invested at a risk-free rate will enable the insurance company to avoid bankruptcy at time $t = 1$ in 99.5% of cases.

Internal models enable the insurance company to quantify risk and determine the capital requirement on the basis of the company's specific risk. Nevertheless, they require expertise and resources for model building, calibration, validation, interpretation and communication. Finally, they require supervisory approval.

1.2.1.1.3. SCR calculation using a partial internal model

The partial model option allows companies to replace some components of the standard formula SCR with results from a full internal model. These values are then combined with the standard model results for the remaining risks and are correlated using either the standard model or company-derived correlation factors.

1.2.1.2. *Calculation of MCR*

The definition of MCR in the QIS5 specifications is as follows:

> The linear function used to calculate the Minimum Capital Requirement shall be calibrated to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 85% over a one-year period.

Article 129, Directive 2009/138/CE of the European Parliament and the Council, 25 November 2009

Thus, the calculation of the MCR is also based on a VaR of equity over a one-year horizon, but at a different level of confidence. Moreover, the MCR has an absolute threshold, which depends on the nature of the business. This threshold should be between 25% and 45% of the SCR (whatever the method of calculating it). More details on this threshold are given in the Implementing Measures on Solvency II (IML2):

> EIOPA (European Insurance and Occupational Pension Authority) – formerly CEIOPS (Committee of European Insurance and Occupational Pension Supervisors) – calibrated the MCR linear formula relative to the SCR standard formula. The life linear formula was fitted to a benchmark percentage (35%) of the SCR standard formula; whereas the non-life calibration was built on the standard deviation parameters used in the premium and reserve risk sub-module of the SCR standard formula.
>
> Admittedly, the relationship between the 85% and 99.5% confidence levels cannot be described by a fixed percentage across all probability distributions. EIOPA, (formerly CEIOPS) however, considers that the 35% ratio – which corresponds to the middle of the 25%–45% corridor – is broadly consistent with the range of distribution assumptions used in the SCR standard formula.

EIOPA (formerly CEIOPS) Advice for Level 2 Implementing Measures on Solvency II, 8 April 2010

The MCR is calculated in three steps:

– The linear MCR is the sum of the MCR Life and the MCR Non-Life, which are, in both cases, linear formulas. The MCR Life depends on technical provisions and capital at risk. The MCR Non-Life depends on technical provisions and premiums over the last 12 months.

– The combined MCR compares the linear MCR with 25% and 45% of the SCR.

– The final MCR is the maximum between the combined MCR and a threshold fixed by the regulator.

From the above calculation method, it is clear that the MCR is obviously dependent on the SCR. The underlying assumption is that 35% of the SCR corresponds to an 85% VaR over one year, which can be criticized.

Figure 1.5 shows the two levels of capital requirements in the Solvency II framework.

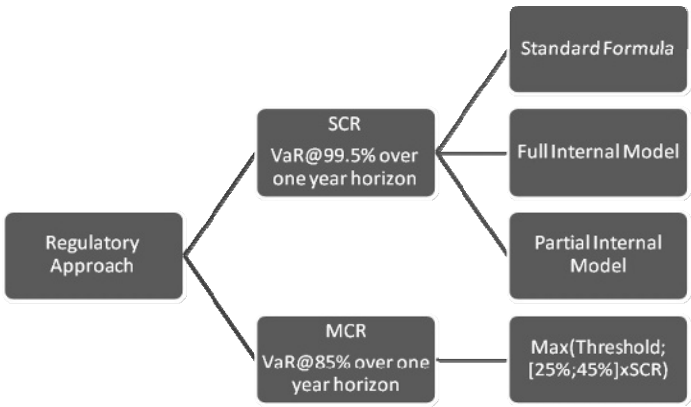

Figure 1.5. *The two levels of capital requirements in the Solvency II framework*

1.2.2. *Risk profile approach*

While the regulatory approach is useful to compare insurance companies to each other, it does not necessarily fit the insurance company's objectives. In fact, the regulatory approach can be seen as limited from the company's viewpoint. For example, in the calculation of the SCR:

– The valuation is performed using a risk-neutral approach; therefore, only the average of the simulations has sense, and there is no visibility on the extreme scenarios.

– The projection horizon is one year, which can lead to inadequate protection for policy holders whose insurance policies have a longer period. For insurers, this horizon is not compatible with the need to manage contracts and risks on the medium and long term. It would be better to have a more long-term horizon, which would be compatible with the management horizon of the insurance company and the horizon of strategic processes (such as trade policies, reinsurance policies, and asset and liability management strategies).

– Cash-flow projections are performed considering a runoff portfolio. Business plans are thus not taken into account, and this prospective approach can provide useful information to the insurance company.

– The scope of risks is limited to the Solvency II mapping, whereas other risks may be important for the insurance company (e.g. strategic, business or reputation risks).

Therefore, it is necessary to define a management tool that provides an alternative assessment of the solvency relative to Pillar I.

To do so, insurance companies are encouraged in the Solvency II Directive to implement their ORSA. The following extract of the Directive gives more details:

> As part of its risk-management system every insurance undertaking and reinsurance undertaking shall conduct its own risk and solvency assessment. That assessment shall include at least the following:
>
> a) the overall solvency needs taking into account the specific risk profile, approved risk tolerance limits and the business strategy of the undertaking;
>
> b) the compliance, on a continuous basis, with the capital requirements, and with the requirements regarding technical provisions;
>
> c) the significance with which the risk profile of the undertaking concerned deviates from the assumptions underlying the Solvency Capital Requirement as laid down in Article 101(3), calculated with the standard formula or with its partial or full internal model.

DIRECTIVE 2009/138/EC of the European Parliament
and the Council, 25 November 2009

Therefore, to better assess its Risk Profile, the insurance company has to define the risk cartography that corresponds at best to reality and to choose the risk metric that corresponds at best to its problematic.

The choice of the risk metric is a key element in the ORSA. The insurance company has to choose:

– an indicator (such as a result, a value or a solvency indicator);

– a time horizon, which is compatible with its management horizon;

– a risk measurement tool;

– a level of confidence.

For example, an insurance company can choose to calculate a VaR over a five-year horizon with a confidence level of 95% based on the SCR (as a solvency indicator).

Therefore, the VaR is not the only possible choice the insurance company can make. Other risk measurement tools exist, such as TVaR, Conditional Tail Expectation (CTE), CVaR and Expected Shortfall (ES), which are described in detail in Chapter 2.

The main advantage of VaR is that it is the simplest risk measurement tool. Its limitation is that this simplicity can hide other risks (such as fat-tails and liquidity risk), which can be better assessed by other risk measurement tools (e.g. TVaR, CTE and CVaR). Therefore, the risk manager can use one or several risk measurement tools in the framework of risk profile calculation, knowing the limitations of each of these tools.

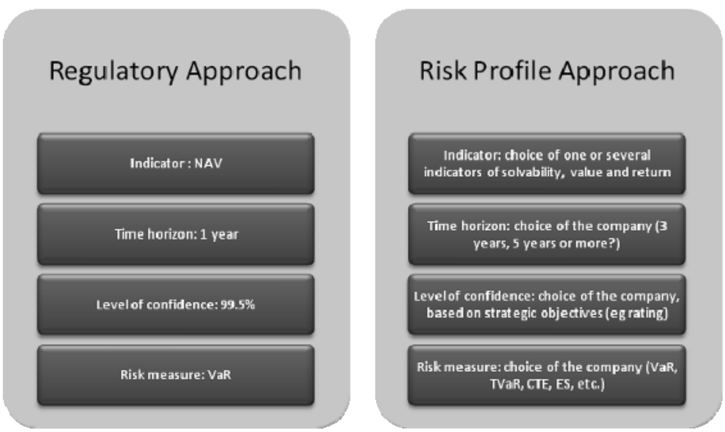

Figure 1.6. *Comparison between regulatory and risk profile approaches*

1.3. The use of VaR for banks

VaR has been used by banks as a key indicator for many years. Basel II and III agreements use this risk measurement tool, and section 1.3.1 explains the process.

1.3.1. *Basel II*

Basel II, such as Solvency II, is based on a three-pillar approach. The VaR is used in Pillar I, which deals with MCRs.

In the framework of Basel II, banks must meet the following ratio to determine their capital requirement:

$$\text{McDonough ratio} = \frac{\text{Regulatory capital}}{\text{Credit risk} + \text{Operational risk} + \text{Market risk}} \geq 80\% \qquad [1.3]$$

To measure their different risks (credit, operational and market risks), banks have two options: apply the standard method or use an internal model. This structure is similar to Solvency II.

On the one hand, the standard method involves a system of ratings of the assets in order to determine the level of risk. The ratings are based on two aspects: the probability of default (PD) and the rate of loss given default (LGD). If the ratings for both components (PD and LGD) are performed by outside agencies (regulatory, rating agencies), this indicates that the bank uses the standard method. If the bank calculates its own PD, the method is called internal ratings-based (IRB) foundation approach. If the bank calculates its PD and LGD, the method is called an IRB advanced approach.

On the other hand, the bank can use internal models to assess its risk. In this case, the VaR can be used according to the requirements specified in the Basel II standards. The main quantitative restrictions for the calculation of required capital with the VaR are the following:

– The VaR should be calculated on one day (in order to make the average VaR over several days).

– The confidence level used is set to 99%.

– The holding period, which is the time of holding the asset or entire portfolio for which a VaR is calculated and the time horizon of the VaR, is set to 10 days. If the holding period is less than 10 days, methods can be used by providing the necessary justification to obtain the result on 10 days detention.

– In the case of using historical data to determine the VaR, the minimum period of observation shall be one year. For example, the calculation of the VaR on 10 days must be based on observations of 365 working days. This enables the bank to obtain 36 values to construct the empirical distribution of gains/losses on 10 days.

– Banks must calculate a stressed VaR. This corresponds to the calculation of VaR calibrated on historical data for one year when a crisis occurs. Typically, many banks use the period 2007/2008 as a reference to simulate a crisis. This particular point was added in the revisions of 2009. It was incorporated following the crisis of 2007/2008 to overcome one of the drawbacks of VaR. It is a well-known fact that the VaR only makes sense if the distribution is stationary, which is rarely true in reality. Without involving a “stressed VaR,” the “classic” VaR can never take into account the occurrence of a crisis.

The formula to calculate the required capital is the following:

$$c = \max\{VaR_{t-1}; m_c \cdot \mathrm{VaR}_{\mathrm{ava}}\} + \max\{s\mathrm{VaR}_{t-1}; m_s \cdot s\mathrm{VaR}_{\mathrm{ava}}\} \quad [1.4]$$

where:

– VaR_{t-1} is the previous day’s VaR.

– $\mathrm{VaR}_{\mathrm{avg}}$ is the average of the 60 previous VaRs.

– m_c and m_s are coefficients, which are determined by the regulatory and equal at least to three. It can be incremented to one according to the results of the backtesting.

– $s\mathrm{VaR}_{t-1}$ is the last stressed VaR obtained.

– $s\mathrm{VaR}_{\mathrm{avg}}$ is the average of the 60 previous stressed VaRs.

1.3.2. *Basel III*

Basel II has shown its limitations in 2007 with the subprime crisis. With Basel III, the Basel Committee wished to learn from this systemic crisis by strengthening many aspects of the regulation; particularly two points: the strengthening of equity and the stakeout of the liquidity.

Concerning the strengthening of equity, the aim is to redefine the different layers of capital, their level of quality and quantity. Moreover, to address the crisis of 2007, the committee introduced a strengthening of the measure of market risk with the use of a stressed VaR already mentioned in the revisions of Basel II in 2009. Indeed, the crisis of 2007 has revealed an underestimation of risk measured by VaR models in periods of high volatility. A measure of the stressed VaR is required to address the procyclicality of VaR.

Concerning the stakeout of the liquidity, the aim of this measure is to ensure that banks have an adequate level of liquidity to deal with a sudden liquidity shock.

Last, but not least, the VaR calculation under Basel III is exactly the same as mentioned in Basel II.

1.4. Conclusion

This chapter has highlighted the fact that VaR has become a key indicator for risk management not only for banks, but also for insurance companies. The calculation of both regulatory equities is based on a VaR, whose confidence level and time horizon depend on the characteristics of the underlying business. On the one hand, regulatory capital is based on a VaR over one year at 99.5% for insurance companies; on the other hand, it is a VaR over one day at 99% for banks. Whatever the calibration, calculation methods are similar between banks and insurance companies. The next chapter describes the classical methods of VaR.