1

Electromagnetic Wave Scattering from Random Rough Surfaces: Basics

This chapter recalls the basic necessary concepts for dealing with electromagnetic wave scattering from random rough surfaces, by using integral equations. First, it recalls the notions of Maxwell equations, plane wave propagation, polarization, Snell-Descartes laws. Second, it gives a statistical description of the heights of random rough surfaces and defines the concept of electromagnetic roughness through the Rayleigh roughness parameter. Last, it introduces the integral equations describing the electromagnetic scattering, and the necessary Green functions, for both 2D and 3D problems, and defines the notion of a normalized radar cross section.

1.1. Introduction

In this book, the incident wave illuminating the surfaces will be considered as a plane wave. A wave can be called locally plane if it is located in the so-called *Fraunhofer zone*[1] of the transmitter source, or far-field zone of the source. This assumes that the source is far enough from the surface such that the incident wave may appear as a plane on a distance greater than any dimension of the surface [LYN 70a]. The media are assumed to be linear, homogeneous and isotropic (LHI), stationary and non-magnetic. The incident medium is perfectly dielectric[2], and can be assimilated to vacuum in general, although we will endeavor to write the equations in the general case of any lossless perfect dielectric medium.

1 The Fraunhofer zone or far-field zone corresponds to a distance R from the source which is greater than approximately $2D^2/\lambda$, where D is the greatest dimension of the source and λ is the transmitted electromagnetic wavelength.

2 A dielectric medium is called perfect if the considered dielectric medium does not have sources of load or current.

The problem of electromagnetic (EM) wave scattering from non-flat surfaces, called rough surfaces, has been studied for decades. In particular, let us quote the works of Lord Rayleigh [RAY 45, RAY 07], who was the first to give a rigorous definition of the EM roughness of a surface (characterized by the so-called Rayleigh roughness criterion, which will be detailed further). Among rough surfaces, two main categories may be distinguished: periodic surfaces (such as square surfaces, triangular surfaces, sawtooth surfaces and sinusoidal surfaces), which are deterministic, and random surfaces for which only some statistical features are known. This latter category is discussed in this book.

This chapter aims at introducing the main necessary concepts for understanding the tools used in the following chapters. In section 1.2, first, we will recall some generalities on EM waves and their propagation in LHI media. The case of dielectric media will be discussed in general, these media being potentially lossy dielectric[3]. Then, the interaction of these EM waves with a flat interface will be studied by detailing the reflection and transmission of a plane wave at a flat (perfectly conducting, lossless or lossy dielectric) interface of infinite length. In section 1.3, a description of random rough surfaces, with either spatial or spatiotemporal variations, will be given. However, we will focus here only on the cases where spatiotemporal varying surfaces are equivalent to spatial varying surfaces (ergodicity). An application in the maritime domain will be given. Also, the so-called Rayleigh roughness EM criterion will be described for making a distinction among a slightly rough, a moderately rough and a very rough surface. Finally, in section 1.4, the general problem of EM wave scattering from random rough surfaces will be presented, in order to calculate the EM power scattered by such surfaces. In the rough surface scattering community, this quantity is generally called *scattering coefficient* as a general describer. The more specific terms used in radar and optics will also be given.

1.2. Generalities

1.2.1. *Maxwell equations and boundary conditions*

In their local form, the Maxwell equations in dielectric media are given by [BOR 80]:

$$\operatorname{div} \boldsymbol{B} = 0, \tag{1.1}$$

$$\textbf{rot}\, \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1.2}$$

3 A dielectric medium is called lossy if the considered dielectric medium is free of charge, but not free of current. This is opposed to a lossless dielectric medium that is free of both charge and current.

$$\operatorname{div} \boldsymbol{D} = \rho, \qquad [1.3]$$

$$\mathbf{rot}\, \boldsymbol{H} = \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}. \qquad [1.4]$$

Usually, in Cartesian coordinates, the operator div is replaced by $\boldsymbol{\nabla} \cdot$ and the operator $\mathbf{rot}$ is replaced by $\boldsymbol{\nabla} \wedge$. The first two equations give the relationships of the fields' structure, and are valid irrespective of the medium. The last two equations depend on the considered medium. Here, $\boldsymbol{E}$ and $\boldsymbol{H}$ refer to the electric and magnetic field vectors, respectively, which compose the EM field. They are expressed in V/m and A/m, respectively. It is important to note that, throughout the book, the vectors will be denoted in bold, and the unitary vectors will be denoted in bold and with a hat. $\boldsymbol{D}$ and $\boldsymbol{B}$ refer to the electric displacement and the magnetic induction, respectively, and describe the action of the EM field on the matter. They are expressed in C/m^2 and Tesla, respectively. Finally, ρ and $\boldsymbol{j}$ refer to the densities of charge or current. They are expressed in C/m^3 and A/m^2, respectively. These quantities act as sources for the EM field. They check the charge conservation equation:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0\,.$$

For an LHI medium[4] (which is the case that we will always consider in the following), the quantities $\boldsymbol{D}$, $\boldsymbol{B}$ and $\boldsymbol{j}$ are related to $\boldsymbol{E}$ and $\boldsymbol{H}$ by the following constitutive relations:

$$\boldsymbol{D} = \epsilon\, \boldsymbol{E} = \epsilon_0 \epsilon_r\, \boldsymbol{E}, \qquad [1.5]$$

$$\boldsymbol{B} = \mu\, \boldsymbol{H} = \mu_0 \mu_r\, \boldsymbol{H}, \qquad [1.6]$$

$$\boldsymbol{j} = \sigma\, \boldsymbol{E}. \qquad [1.7]$$

where, ϵ, μ and σ are, respectively, the permittivity, the permeability and the conductivity of considered matter, with ϵ_0 and μ_0 as their constants in vacuum, which are equal to:

$$\epsilon_0 \simeq \frac{1}{36\pi \times 10^9} \text{ F/m}, \qquad [1.8]$$

$$\mu_0 \simeq 4\pi \times 10^{-7} \text{ H/m}. \qquad [1.9]$$

These two quantities check the relation:

$$\epsilon_0\, \mu_0\, c^2 = 1\,,$$

4 The linearity characterizes the fact that the quantities ϵ and μ are independent of the intensity of $\boldsymbol{E}$ and $\boldsymbol{H}$, the homogeneity that ϵ and μ do not depend on the considered space point and the isotropy that ϵ, μ and σ are scalar (i.e. they do not depend on any spatial direction).

with c as the celerity of light in vacuum. ϵ_r and μ_r are the relative electric permittivity and magnetic permeability, respectively: they are equal to 1 in vacuum. Let us recall that in the following, only non-magnetic media will be considered; consequently, the relative magnetic permeability $\mu_r = 1$. Moreover, propagation media will be assumed to be free of charge, $\rho = 0$, and most of the time free of current as well, $\boldsymbol{j} = \boldsymbol{0}$. A medium that is free of charge is then qualified as a dielectric medium; a distinction will be made between a dielectric medium free of current, which will be called *perfect dielectric medium* or *lossless dielectric medium*, and a dielectric medium not free of current, which will be called *lossy dielectric medium*.

1.2.1.1. *Boundary conditions*

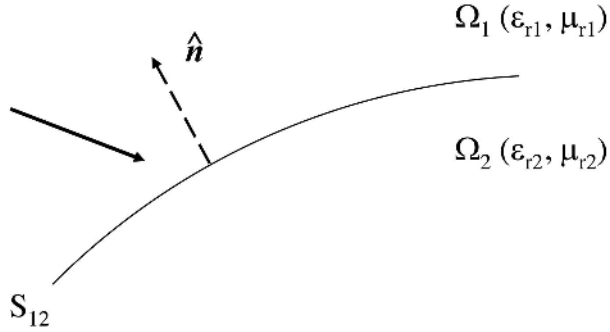

Figure 1.1. *Interface between two semi-infinite LHI media* Ω_1 *(incident medium) and* Ω_2

The Maxwell equations are applicable to infinite media, which does not reflect reality as every medium has boundaries. For practical applications of electromagnetics, it is essential to know how to deal with the problem of the boundary between two media of different EM properties. Let us assume that an arbitrary interface S_{12} separates two semi-infinite media (LHI) denoted by Ω_1 for the incident (upper) medium and Ω_2 for the transmission (lower) medium, respectively, and $\hat{\boldsymbol{n}}$ is a unitary vector that is orthogonal (normal) to the interface and oriented towards the incident (upper) medium Ω_1. The boundary conditions [KON 90, FAR 98, PÉR 01] may be written in the local form as follows:

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{B_2} - \boldsymbol{B_1}) = 0, \quad [1.10]$$

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{D_2} - \boldsymbol{D_1}) = \rho_s, \quad [1.11]$$

$$\hat{\boldsymbol{n}} \wedge (\boldsymbol{E_2} - \boldsymbol{E_1}) = \boldsymbol{0}, \quad [1.12]$$

$$\hat{\boldsymbol{n}} \wedge (\boldsymbol{H_2} - \boldsymbol{H_1}) = \boldsymbol{j_s} \wedge \hat{\boldsymbol{n}}, \quad [1.13]$$

where ρ_s and $\boldsymbol{j_s}$ represent the superficial (or surface) density of charge and the vector of superficial (or surface) density of current, respectively, which may exist at the boundary between the two media ($\rho_s = 0$ for dielectric media, $\rho_s = 0$ and $\boldsymbol{j_s} = \mathbf{0}$ for perfect dielectric media). Equations [1.10] and [1.12], called *continuity relations*, describe the continuity of the normal component of $\boldsymbol{B}$ and of the tangential component of $\boldsymbol{E}$ at the interface, respectively. The other two equations [1.11] and [1.13] describe the discontinuity of the normal component of $\boldsymbol{D}$ in the presence of superficial charges of density ρ_s and the discontinuity of the tangential component of $\boldsymbol{H}$ on a layer of current, respectively.

For the case where the lower medium is a perfectly conducting metal[5], the equations take the form:

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{H_1} = 0, \tag{1.14}$$

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{E_1} = -\rho_s/\epsilon_1, \tag{1.15}$$

$$\hat{\boldsymbol{n}} \wedge \boldsymbol{E_1} = \mathbf{0}, \tag{1.16}$$

$$\hat{\boldsymbol{n}} \wedge \boldsymbol{H_1} = -\boldsymbol{j_s} \wedge \hat{\boldsymbol{n}}. \tag{1.17}$$

Condition [1.16] is usually called the *Dirichlet boundary condition* and condition [1.17], in the absence of current, is usually called the *Neumann boundary condition*.

Using the same method, for the case when the two LHI media are perfect dielectric, the equations take the form:

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{H_2} - \boldsymbol{H_1}) = 0, \tag{1.18}$$

$$\hat{\boldsymbol{n}} \cdot (\boldsymbol{E_2} - \boldsymbol{E_1}) = 0, \tag{1.19}$$

$$\hat{\boldsymbol{n}} \wedge (\boldsymbol{E_2} - \boldsymbol{E_1}) = \mathbf{0}, \tag{1.20}$$

$$\hat{\boldsymbol{n}} \wedge (\boldsymbol{H_2} - \boldsymbol{H_1}) = \mathbf{0}. \tag{1.21}$$

1.2.2. *Propagation of a plane wave (Helmholtz equation and plane wave)*

The propagation equations of fields are obtained from the Maxwell equations by using the property $\mathbf{rot\ rot} = \mathbf{grad}\ \mathrm{div} - \boldsymbol{\nabla}^2$, where $\boldsymbol{\nabla}^2$ is the vector Laplacian[6]. Then, in a general way, we obtain:

$$\boldsymbol{\nabla}^2 \boldsymbol{E} - \epsilon\mu \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{1}{\epsilon} \mathbf{grad}\ \rho + \mu \frac{\partial \boldsymbol{j}}{\partial t}, \tag{1.22}$$

$$\boldsymbol{\nabla}^2 \boldsymbol{H} - \epsilon\mu \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = -\,\mathbf{rot}\,\boldsymbol{j}. \tag{1.23}$$

5 A perfectly conducting metal is characterized by a conductivity $\sigma \to \infty$.
6 In Cartesian coordinates, if we represent the scalar Laplacian by $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, the vector Laplacian of $\boldsymbol{A} = (A_x, A_y, A_z)$, $\boldsymbol{\nabla}^2 \boldsymbol{A}$, is defined by $\boldsymbol{\nabla}^2 \boldsymbol{A} = \Delta A_x \hat{\boldsymbol{x}} + \Delta A_y \hat{\boldsymbol{y}} + \Delta A_z \hat{\boldsymbol{z}}$.

For a perfect dielectric medium ($\rho = 0$, $\boldsymbol{j} = \mathbf{0}$), the equations reduce to:

$$\boldsymbol{\nabla}^2 \boldsymbol{E} - \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \mathbf{0}, \quad [1.24]$$

$$\boldsymbol{\nabla}^2 \boldsymbol{H} - \frac{1}{v^2}\frac{\partial^2 \boldsymbol{H}}{\partial t^2} = \mathbf{0}. \quad [1.25]$$

A wave equation of Alembert type is obtained, where $v = 1/\sqrt{\epsilon\mu}$ is the propagation speed of the wave. v can be written in the form $v = c/n$, where $n = \sqrt{\epsilon_r \mu_r}$ is the refractive index of the considered medium ($\mu_r = 1$ here) and c is the propagation speed in vacuum, defined by $c = 1/\sqrt{\epsilon_0\mu_0} \simeq 3 \times 10^8$ m/s.

In a general way, the solution of the propagation equation in a perfect dielectric medium for a plane progressive wave (PPW), which propagates in the direction $\hat{\boldsymbol{u}} = \boldsymbol{R}/\|\boldsymbol{R}\|$ at speed v, is written as [PÉR 04, BOR 80, FAR 98]:

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}_+ \left(t - \frac{\hat{\boldsymbol{u}} \cdot \boldsymbol{R}}{v}\right) + \boldsymbol{\Psi}_- \left(t + \frac{\hat{\boldsymbol{u}} \cdot \boldsymbol{R}}{v}\right), \quad [1.26]$$

where, by definition of the plane wave, the wave planes (or surfaces of the plane waves) are orthogonal to $\hat{\boldsymbol{u}}$, defined by the planes $\hat{\boldsymbol{u}} \cdot \boldsymbol{R} = C$, where C is a constant. The function $\boldsymbol{\Psi}_+$, sometimes called PPW+, is a PPW that propagates at speed v in the direction $+\boldsymbol{R}$. Likewise, $\boldsymbol{\Psi}_-$, sometimes called PPW−, is a progressive wave that propagates at speed v in the direction $-\boldsymbol{R}$. This wave function is checked by both $\boldsymbol{E}$ and $\boldsymbol{H}$, and it can be shown that:

$$\boldsymbol{H} = Z\, \hat{\boldsymbol{u}} \wedge \boldsymbol{E}, \quad [1.27]$$

where $Z = \sqrt{\epsilon/\mu} = Z_0\sqrt{\epsilon_r/\mu_r}$ is the *wave impedance* of the considered medium, with Z_0 the wave impedance of vacuum which is equal to $Z_0 = \sqrt{\epsilon_0/\mu_0} \simeq 120\pi\,\Omega$. Thus, $(\boldsymbol{E}, \boldsymbol{H}, \hat{\boldsymbol{u}})$ form a direct trihedral. The wave is then called transverse electromagnetic (TEM), because both vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ are orthogonal to the propagation direction given by $\hat{\boldsymbol{u}}$.

1.2.2.1. *Harmonic regime and harmonic plane progressive waves*

A *harmonic* plane progressive wave (HPPW) is a space–time function of real expression[7] [PÉR 04]:

$$\boldsymbol{\Psi}(\boldsymbol{R}, t) = A\cos\left[\omega\left(t - \frac{\hat{\boldsymbol{u}} \cdot \boldsymbol{R}}{v}\right) - \phi\right]\hat{\boldsymbol{\Psi}} = A\cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{R} - \phi)\,\hat{\boldsymbol{\Psi}}, \quad [1.28]$$

7 The generally retained solution of equation [1.26] for the HPPW is the PPW+, because most of the time, the chosen coordinate is such that the studied incident HPPW propagates away from the chosen origin point O.

where $\boldsymbol{k} = \omega/v\,\hat{\boldsymbol{u}}$ is the wave vector, ω is the pulsation in rad/s and ϕ is a constant phase term. In the following, we will consider the harmonic regime such that every EM quantity $\boldsymbol{G}$ is an HPPW of complex form:

$$\underline{\boldsymbol{\Psi}}(\boldsymbol{R}, t) = A \exp[\pm i(\omega t - \boldsymbol{k} \cdot \boldsymbol{R} - \phi)]\,\hat{\boldsymbol{\Psi}} = \underline{\psi}(\boldsymbol{R}) \exp(\pm i\omega t)\,\hat{\boldsymbol{\Psi}}, \qquad [1.29]$$

where $\underline{\psi}(\boldsymbol{R}) = A \exp[\mp i(\boldsymbol{k} \cdot \boldsymbol{R} + \phi)]$. Then, to simplify the notations, the complex fields will be represented by being underlined.

Depending on the sign convention $+$ or $-$ in $\exp[\pm i(\omega t - \boldsymbol{k} \cdot \boldsymbol{R})]$, the time derivative operator $\partial/\partial t$ is equivalent to a multiplication by $\pm i\omega$ and the space derivative operator $\boldsymbol{\nabla}\cdot$ is equivalent to a multiplication by $\mp i\boldsymbol{k}$. In the following, the retained convention is $\exp[-i(\omega t - \boldsymbol{k} \cdot \boldsymbol{R})]$[8]. Thus, the wave equation [1.24] of the electric field $\boldsymbol{E}(\boldsymbol{R}, t) = \boldsymbol{E}_0(\boldsymbol{R}) \exp(-i\omega t)$ in a free of charge and current medium becomes:

$$(\boldsymbol{\nabla}^2 + k^2)\,\boldsymbol{E} = \boldsymbol{0}, \qquad [1.30]$$

with $k^2 = \omega^2/v^2$ (dispersion relation), where k represents the wavenumber inside the considered perfect dielectric medium. This equation, which is called the *Helmholtz equation*, is also checked by the magnetic field $\boldsymbol{H}$.

By taking the superficial currents $\boldsymbol{j} = \sigma\boldsymbol{E}$ into account, the wavenumber $\underline{k}$ is expressed by the dispersion relation as:

$$\underline{k}^2 = \frac{\omega^2}{v^2}(1 + i\frac{\sigma}{\omega\epsilon}). \qquad [1.31]$$

In this case, the wavenumber $\underline{k}$ is complex and the wave is damped during its propagation inside the lossy medium. This wave is then called “pseudo-HPPW”.

1.2.3. *Incident wave at an interface: polarization*

Let us consider a plane EM wave propagating in a non-magnetic LHI medium. If the spatial frame is defined in Cartesian coordinates $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$, it is usually chosen (for the sake of simplicity) such that the wave propagates in the plane $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{z}})$ (see Figure 1.2). The polarization of an EM wave is defined by the properties of the incident electric field vector $\boldsymbol{E_i}$ of the wave in the given plane. When the wave interacts with an interface, the chosen plane is usually the *incidence plane*. The latter is defined by

8 However, note that this choice impacts the definition of the permittivities and refractive indices. With this choice of convention, they will have the form $\underline{a} = a' + ia''$ (with $a', a'' > 0$); otherwise, $\underline{a} = a' - ia''$.

the plane formed by the wave vector incident onto the surface $\hat{\boldsymbol{k}}_i$ and the normal to the surface $\hat{\boldsymbol{n}}$. In the case when the studied surface is flat, $\hat{\boldsymbol{n}} \in (\hat{\boldsymbol{x}}, \hat{\boldsymbol{z}})$ with constant direction whatever the surface point, the incidence plane $(\hat{\boldsymbol{k}}_i, \hat{\boldsymbol{n}})$ is identical to the plane $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{z}})$ as illustrated in Figure 1.2. In the case of a rough surface, the normal to the surface becomes a local normal that depends on the considered surface point. Considering an arbitrary rough surface for which the height ζ depends on the two horizontal parameters x and y, $\zeta(x, y)$, the normal does not belong to the plane $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{z}})$ *a priori*, then the incidence plane depends on the considered surface point. For better convenience, the polarization of the incident wave is defined relatively to the mean plane $(\hat{\boldsymbol{k}}_i, \hat{\boldsymbol{z}})$, as illustrated in Figure 1.3.

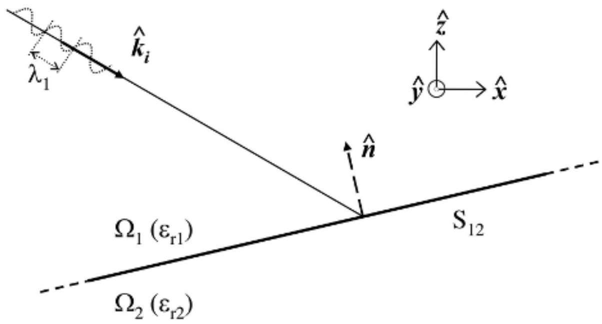

Figure 1.2. *Incident wave on an infinite flat surface: cut view in the incidence plane* $(\hat{\boldsymbol{k}}_i, \hat{\boldsymbol{n}})$

To study the polarization in the general case rigorously, it is necessary to consider an arbitrary elliptical polarization. However, by considering a Cartesian coordinate system and knowing that every polarization state of a wave can be represented by the combination of two linear horizontal and vertical components, we will study these two fundamental components.

A possible representation of the horizontal and vertical polarizations is given in Figure 1.3. Note that in the literature, various denominations of these polarizations are given: the horizontal (denoted by H) polarization is also called the transverse electric (denoted by TE) polarization or perpendicular (denoted by $\perp$ or s for *senkrecht*, which means perpendicular in German, in the optical domain) polarization. The vertical (denoted by V) polarization is also called the transverse magnetic (denoted by TM) polarization or parallel (denoted by $\|$ or p for *parallel* in the optical domain) polarization.

1.2.3.1. *Snell–Descartes laws and Fresnel coefficients*

Let us consider an HPPW of pulsation ω, which propagates in direction $\hat{\boldsymbol{k}}_i$ inside the medium Ω_1 of relative permittivity ϵ_{r1} onto the flat interface S_{12} that is assumed to be of infinite length. This wave is transformed into a reflected wave in direction

$\hat{\boldsymbol{k}}_{\boldsymbol{r}}$, and (possibly) a transmitted wave in direction $\hat{\boldsymbol{k}}_{\boldsymbol{t}}$. The continuity relations [1.18]–[1.21] at the interface imply that the pulsations of the three waves are identical (we talk about phase invariance), and that the planes of incidence $(\hat{\boldsymbol{k}}_{\boldsymbol{i}}, \hat{\boldsymbol{n}})$, of reflection $(\hat{\boldsymbol{k}}_{\boldsymbol{r}}, \hat{\boldsymbol{n}})$ and of transmission $(\hat{\boldsymbol{k}}_{\boldsymbol{t}}, \hat{\boldsymbol{n}})$ are equal. Thus, the first Snell–Descartes law states that for an incident ray, only one reflected ray exists and one refracted ray at the most exists, and that the planes of incidence, reflection and refraction are equal. Moreover, these continuity relations make it possible to establish the second Snell–Descartes law, for which the angles of reflection and transmission check the condition:

$$\theta_r = \pm\,\theta_i, \qquad [1.32]$$

$$\sqrt{\epsilon_{r2}}\sin\theta_t = \sqrt{\epsilon_{r1}}\sin\theta_i, \qquad [1.33]$$

where the angles are defined relatively to the normal to the surface, with:

$$\cos\theta_i = -\hat{\boldsymbol{k}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{n}}. \qquad [1.34]$$

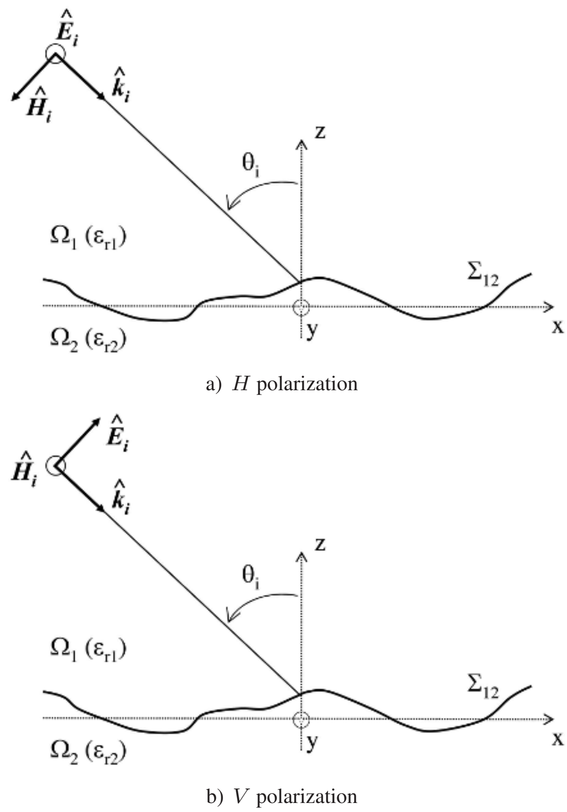

Figure 1.3. *Incident wave onto a random rough interface in horizontal (H) and vertical (V) polarizations: cut view in the mean incidence plane* $(\hat{\boldsymbol{k}}_{\boldsymbol{i}}, \hat{\boldsymbol{z}})$

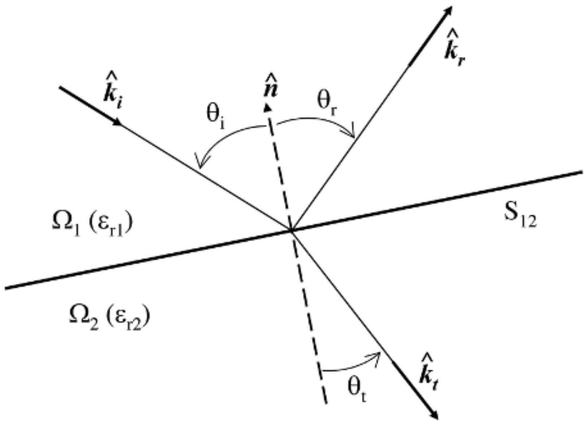

Figure 1.4. *Reflected and transmitted waves by a flat interface of infinite length ($\epsilon_{r1} < \epsilon_{r2}$ here)*

The reflection angle θ_r is equal to plus or minus θ_i, depending on whether the angles are oriented or not. In this paragraph, it is not necessary, but in the following, we will take oriented angles, at least for two-dimensional (2D) problems.

Likewise, from the boundary conditions for the electric field [1.19 and 1.20] and magnetic field [1.18 and 1.21] at the interface S_{12} between Ω_1 and Ω_2, the expressions of the so-called Fresnel reflection r_{12} and transmission t_{12} coefficients can be derived, in both horizontal (H) and vertical (V) polarizations. They are given by [COM 96]:

$$r_{12}^H(\theta_i) = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{\sqrt{\epsilon_{r1}} \cos\theta_i - \sqrt{\epsilon_{r2} - \epsilon_{r1} \sin^2\theta_i}}{\sqrt{\epsilon_{r1}} \cos\theta_i + \sqrt{\epsilon_{r2} - \epsilon_{r1} \sin^2\theta_i}}, \quad [1.35]$$

$$t_{12}^H(\theta_i) = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2\sqrt{\epsilon_{r1}} \cos\theta_i}{\sqrt{\epsilon_{r1}} \cos\theta_i + \sqrt{\epsilon_{r2} - \epsilon_{r1} \sin^2\theta_i}}, \quad [1.36]$$

in H polarization and:

$$r_{12}^V(\theta_i) = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} = -\frac{\epsilon_{r2} \cos\theta_i - \sqrt{\epsilon_{r1}}\sqrt{\epsilon_{r2} - \epsilon_{r1} \sin^2\theta_i}}{\epsilon_{r2} \cos\theta_i + \sqrt{\epsilon_{r1}}\sqrt{\epsilon_{r2} - \epsilon_{r1} \sin^2\theta_i}}, \quad [1.37]$$

$$t_{12}^V(\theta_i) = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} = \frac{2\sqrt{\epsilon_{r1}\epsilon_{r2}} \cos\theta_i}{\epsilon_{r2} \cos\theta_i + \sqrt{\epsilon_{r1}}\sqrt{\epsilon_{r2} - \epsilon_{r1} \sin^2\theta_i}}, \quad [1.38]$$

in V polarization, where θ_i is the local incidence angle defined by $\cos\theta_i = -\hat{\boldsymbol{k}}_{\boldsymbol{i}} \cdot \hat{\boldsymbol{n}}$. The transmission coefficient is expressed with respect to the reflection coefficient in H and V polarizations by:

$$t_{12}^H(\theta_i) = 1 + r_{12}^H(\theta_i), \qquad [1.39]$$

$$t_{12}^V(\theta_i) = \frac{n_1}{n_2} \left[1 - r_{12}^V(\theta_i)\right], \qquad [1.40]$$

respectively.

It should be noted that, often in the literature on rough surface scattering, a slightly different definition is given, as r_{12}^V is replaced by $-r_{12}^V$.

1.2.3.2. *Study of some particular cases*

For normal incidence ($\theta_i = 0$), equations [1.35]–[1.38] become:

$$r_{12}^H(0) = r_{12}^V(0) = \frac{n_1 - n_2}{n_1 + n_2}, \qquad [1.41]$$

$$t_{12}^H(0) = t_{12}^V(0) = \frac{2\,n_1}{n_1 + n_2}, \qquad [1.42]$$

and for a low-grazing incidence angle $\theta_i \to \pm\pi/2$, they become:

$$r_{12}^H(\pi/2) = -1, \qquad [1.43]$$

$$r_{12}^V(\pi/2) = +1, \qquad [1.44]$$

$$t_{12}^H(\pi/2) = t_{12}^V(\pi/2) = 0. \qquad [1.45]$$

In the case of a perfectly conducting lower medium ($\sigma \to \infty$ or $\epsilon_{r2} = i\infty$), we get $\forall\theta_i$:

$$r_{12}^H(\theta_i) = r_{12}^V(\theta_i) = -1, \qquad [1.46]$$

$$t_{12}^H(\theta_i) = t_{12}^V(\theta_i) = 0. \qquad [1.47]$$

Then, it is usually said that for the reflected wave, the field is reversed.

1.2.3.3. *Limit angle and Brewster angle*

If the incident wave goes from a less refractive to a more refractive medium ($n_1 < n_2$) and the incident wave is grazing $\theta_i \to \pi/2$, a *limit angle of transmission* θ_t^l appears, which is defined by:

$$\sin\theta_t^l = n_1/n_2. \qquad [1.48]$$

For an air–glass interface ($n_2 = 1.5$), $\theta_t^l \simeq 41.8°$. For an air–sea interface without losses ($n_2 = \sqrt{53}$), $\theta_t^l \simeq 7.9°$. Conversely, if $n_1 > n_2$, at the limit incidence angle $\sin\theta_i^l = n_2/n_1$, the angle of transmission is equal to $\pi/2$. Thus, beyond this incidence angle, there is no transmitted wave in the far field.

The reflection coefficient goes to 0 only in V polarization, for an incidence angle called the *Brewster incidence angle* θ_i^B, which is defined by:

$$\tan\theta_i^B = n_2/n_1. \qquad [1.49]$$

For an air–glass interface, $\theta_i^B \simeq 56.8°$. For an air–sea interface, $\theta_i^B \simeq 82.2°$.

1.3. Random rough surfaces: statistical description and electromagnetic roughness

In this section, the statistical description of random rough surfaces is presented, by using the height distribution and autocorrelation function. An application to sea surfaces is given. Finally, the concept of *EM* roughness of a rough interface is given through the Rayleigh roughness criterion.

1.3.1. *Statistical description of random rough surfaces*

Here, the description of a random rough surface with height variations ζ is given in detail. These variations are characterized by the height probability density function (PDF) and the height autocorrelation function (or the height spectrum).

1.3.1.1. *Surface height PDF*

The surface height PDF $p_h(\zeta)$ represents the statistical height distribution of the random rough surface. Three important pieces of information are contained in this PDF:

– its mean value: here, the mean surface height, ζ_0;

– its standard deviation: here, the surface height standard deviation, σ_h;

– the type of this density: Gaussian, Lorentzian, exponential, etc.

Most of the time, a rough surface is characterized by a Gaussian height PDF (see Figure 1.5):

$$p_h(\zeta) = \frac{1}{\sigma_h\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\zeta-\zeta_0}{\sigma_h}\right)^2\right]. \qquad [1.50]$$

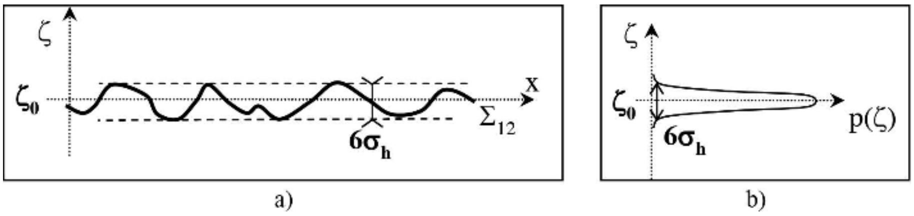

Figure 1.5. *One-dimensional (1D) random rough surface of Gaussian statistics (a) and its height distribution (b)*

Thus, for a Gaussian height PDF, 99.73% of the surface heights are contained between $\zeta_0 - 3\sigma_h$ and $\zeta_0 + 3\sigma_h$. The height PDF checks:

$$\langle 1 \rangle = \int_{-\infty}^{+\infty} p_h(\zeta)\, d\zeta = 1, \text{ and } \langle \zeta \rangle = \int_{-\infty}^{+\infty} \zeta\, p_h(\zeta)\, d\zeta = \zeta_0. \qquad [1.51]$$

The mean (average) height ζ_0 will be taken as 0 in general for the sake of simplicity. The statistical average over the heights $\langle \zeta \rangle$ is called *first-order statistical moment* (or mean value). The *centered second-order statistical moment* (or variance), $\langle (\zeta - \zeta_0)^2 \rangle = \langle \zeta^2 \rangle$ (for $\zeta_0 = 0$), corresponds here to the average over the square of the heights:

$$\langle \zeta^2 \rangle = \int_{-\infty}^{+\infty} \zeta^2\, p_h(\zeta)\, d\zeta = \sigma_h^2. \qquad [1.52]$$

$\sigma_h = \sqrt{\langle \zeta^2 \rangle}$ is the surface height standard deviation, which is also called root mean square (RMS) height.

1.3.1.2. *Surface (spatial) height autocorrelation function and height spectrum*

The (spatial) autocorrelation function between two surface points M_1 and M_2 represents the statistical correlation between these two points, with respect to their horizontal distance $\boldsymbol{r_d} = \boldsymbol{r_2} - \boldsymbol{r_1}$. It is maximum if $\boldsymbol{r_2} = \boldsymbol{r_1}$ (or $\boldsymbol{r_d} = \boldsymbol{0}$). Two important pieces of information are contained in this function:

– its correlation lengths along $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$, $L_{c,x}$ and $L_{c,y}$;

– its type: Gaussian, Lorentzian, exponential, etc.

It is defined by:

$$\begin{aligned} W_h(\boldsymbol{r_1}, \boldsymbol{r_2}) &= \langle \zeta(\boldsymbol{r_1})\, \zeta(\boldsymbol{r_2}) \rangle \\ &= \lim_{X,Y \to +\infty} \frac{1}{X\,Y} \int_{-X/2}^{+X/2} \int_{-Y/2}^{+Y/2} \zeta(\boldsymbol{r_1})\, \zeta(\boldsymbol{r_2})\, dx\, dy\,, \end{aligned} \qquad [1.53]$$

where (X, Y) are the surface lengths with respect to $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$, respectively. For a stationary[9] surface, $W_h(\boldsymbol{r_1}, \boldsymbol{r_2}) \equiv W_h(\boldsymbol{r_d}) = \langle \zeta(\boldsymbol{r_1})\, \zeta(\boldsymbol{r_1} + \boldsymbol{r_d}) \rangle$, with the property $W_h(\boldsymbol{r_d} = \boldsymbol{0}) = \sigma_h^2$. The autocorrelation coefficient $C(\boldsymbol{r_d})$ is equal to the autocorrelation function normalized by the RMS height (height standard deviation); it is written for a stationary surface as:

$$C_h(\boldsymbol{r_d}) = \frac{\langle \zeta(\boldsymbol{r_1})\, \zeta(\boldsymbol{r_1} + \boldsymbol{r_d}) \rangle}{{\sigma_h}^2}. \qquad [1.54]$$

$C_h(\boldsymbol{r_d}) = 1$ when $\boldsymbol{r_d} = 0$. The correlation length L_c is a characteristic value of the autocorrelation function, which determines the so-called *scale* of roughness of the surface. Typically, it corresponds to the horizontal distance (x_d for $L_{c,x}$ or y_d for $L_{c,y}$) between two surface points for which the autocorrelation coefficient is equal to $1/e$ (see Figure 1.6).

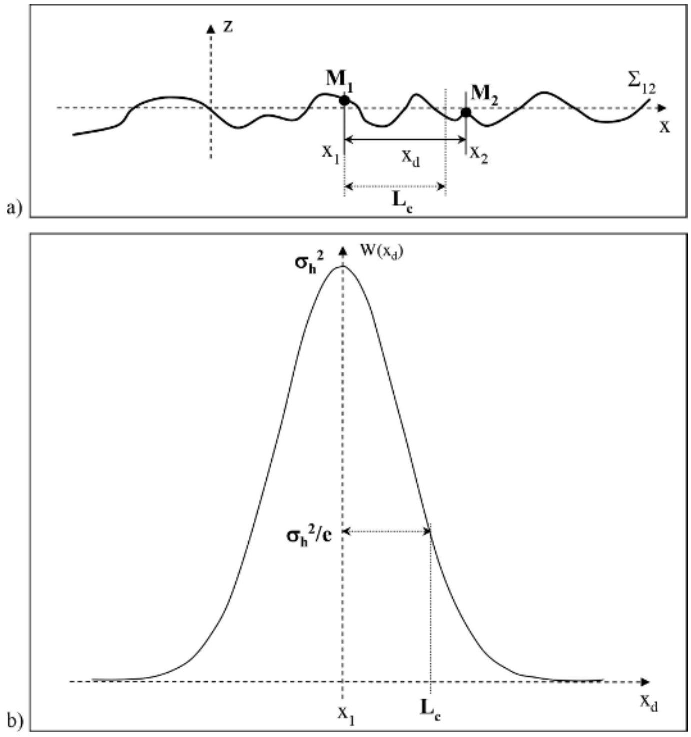

Figure 1.6. *2D random rough surface (a) and its height autocorrelation function (b), here taken as Gaussian*

9 In its usual definition, a stationary process is a stochastic process whose first moment and covariance do not change when shifted in time or space. As applied to surfaces, it means that the mean value and the autocorrelation function do not change with respect to space.

Finally, the height PDF and the height autocorrelation function make it possible to have a fair description of a random rough surface: indeed, for Gaussian statistics, surfaces with Gaussian height PDF and autocorrelation function have the property that all their statistical moments are related to the first two functions.

Usually, instead of using the height autocorrelation function, the *surface height spectrum*, which is the spatial Fourier transform of the autocorrelation function, is used. It is also often called the surface power spectral density function, and is defined for a stationary surface of infinite extent by the relation:

$$S_h(\boldsymbol{k}) = \mathrm{FT}[W_h(\boldsymbol{r_d})] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W_h(\boldsymbol{r_d}) \exp(-i\boldsymbol{k}\cdot\boldsymbol{r_d})\, d\boldsymbol{r_d}, \quad [1.55]$$

where $\boldsymbol{k}$ represents the spatial frequency per cycle vector or surface wave vector[10], which is homogeneous to rad/m. Likewise, the autocorrelation function can be defined from the spectrum by using an inverse Fourier transform as follows:

$$W_h(\boldsymbol{r_d}) = \mathrm{FT}^{-1}[S_h(\boldsymbol{k})] = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S_h(\boldsymbol{k}) \exp(+i\boldsymbol{k}\cdot\boldsymbol{r_d})\, d\boldsymbol{k}. \quad [1.56]$$

Typically, considered autocorrelation functions are Gaussian, Lorentzian or exponential. They are defined for 2D problems (also called 1D surfaces) as:

$$W_h(x_d) = \sigma_h^2\, \exp\left(-\frac{{x_d}^2}{L_c^2}\right), \quad [1.57]$$

$$W_h(x_d) = \frac{\sigma_h^2}{1 + {x_d}^2/L_c^2}, \quad [1.58]$$

$$W_h(x_d) = \sigma_h^2\, \exp\left(-\frac{|x_d|}{L_c}\right), \quad [1.59]$$

respectively. Their corresponding spectrum is then defined by:

$$S_h(k) = \sqrt{\pi}\,\sigma_h^2 L_c\, \exp\left(-\frac{L_c^2\,k^2}{4}\right), \quad [1.60]$$

$$S_h(k) = \pi\,\sigma_h^2 L_c\, \exp\left(-L_c\,|k|\right), \quad [1.61]$$

$$S_h(k) = \frac{2\,\sigma_h^2 L_c}{1 + L_c^2 k^2}, \quad [1.62]$$

10 It must not be confused with the electromagnetic wave vector.

respectively. It can be noted that the spectrum associated with a Gaussian autocorrelation function is also Gaussian (by Fourier transform), and that the spectrum of a Lorentzian autocorrelation is exponential and vice versa.

In addition to the RMS height σ_h and the correlation length L_c, other important statistical parameters can be useful to characterize a random rough surface. The first parameter is the surface RMS slope σ_s, which is defined by [OGI 91, SOU 01a, MAR 90]:

$$\sigma_s = \sqrt{\left\langle \left[\zeta'(x) - \langle \zeta'(x) \rangle\right]^2 \right\rangle} = \sqrt{\int_{-\infty}^{+\infty} \frac{dk}{2\pi}\, k^2 S(k)} = \sqrt{-W_h''(0)}. \qquad [1.63]$$

For a Gaussian PDF surface with Gaussian correlation [1.57], the RMS slope is related to the RMS height and the correlation length by the relation:

$$\sigma_s = \sqrt{2}\, \frac{\sigma_h}{L_c}. \qquad [1.64]$$

For a so-called 2D surface (3D problem), the same results are obtained by splitting along the $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ axes: $\sigma_{s,x}$ is expressed in terms of $L_{c,x}$, and $\sigma_{s,y}$ in terms of $L_{c,y}$.

The second commonly used parameter is the surface mean curvature radius R_c, which is defined for 1D surfaces (2D problems) as [OGI 91, SOU 01a, PAP 88]:

$$R_c = -\frac{\left[1 + \left\langle \zeta'(x)^2 \right\rangle\right]^{3/2}}{\langle \zeta''(x) \rangle}. \qquad [1.65]$$

For a Gaussian surface (i.e. Gaussian height PDF and Gaussian correlation [1.57]), under small slopes assumption, the mean curvature radius checks the asymptotic relation [PAP 88]:

$$R_c \simeq \frac{1}{2.76} \frac{L_c^2}{\sigma_h} \left(1 + \frac{3}{2} \frac{\sigma_h^2}{L_c^2}\right), \qquad [1.66]$$

which simplifies for RMS slope $\sigma_s \ll 1$ as:

$$R_c \approx 0.36\, \frac{L_c^2}{\sigma_h}. \qquad [1.67]$$

Sometimes, an additional parameter is used: the mean distance D_m between two successive peaks of the surface. It can be estimated by [MAR 90, FRE 97]:

$$D_m \simeq \pi \sqrt{\frac{\int_{-\infty}^{+\infty} dk\, k^2 S(k)}{\int_{-\infty}^{+\infty} dk\, k^4 S(k)}}. \quad [1.68]$$

Physically, it is expected that this distance D_m would be of the same order as the correlation length L_c. Indeed, for a Gaussian surface, this distance checks the condition:

$$D_m = \frac{\pi}{\sqrt{6}} L_c \simeq 1.28 L_c, \quad [1.69]$$

which is consistent with our qualitative physical prediction. Besides, it can be noted that (at least for a Gaussian correlation surface) the distance between two surface peaks is a bit greater than the correlation length.

1.3.1.3. *Other statistical tools*

In addition to the surface height PDF and/or autocorrelation function (or spectrum), in some cases other statistical tools that describe random rough surfaces may be used. Indeed, depending on the analytical models used to describe the EM scattering, an alternative statistical tool to the autocorrelation function (or its associated spectrum, like for the small perturbation method (SPM)) may be used. For instance, as studied further, the geometric optics (GO) approximation uses the slope PDF $p_s(\gamma)$. For a Gaussian process, it is defined for 1D surfaces (2D problems) by:

$$p_s(\gamma) = \frac{1}{\sigma_s \sqrt{2\pi}} \; \exp\left[-\frac{1}{2}\left(\frac{\gamma - \gamma_0}{\sigma_s}\right)^2\right], \quad [1.70]$$

where σ_s is RMS of the slopes $\gamma = \zeta'$ of the surface. In general, the mean value $\langle\gamma\rangle \equiv \gamma_0 = 0$. For 2D surfaces (3D problems), by assuming a correlated centered 2D Gaussian process, ($\langle\gamma_x\rangle \equiv \gamma_{0x} = 0$ and $\langle\gamma_y\rangle \equiv \gamma_{0y} = 0$), it is defined by:

$$p_s(\gamma_x, \gamma_y) = \frac{1}{2\pi\sqrt{|[C_2]|}} \; \exp\left[-\frac{(\sigma_{sy}^2 \gamma_x^2 + \sigma_{sx}^2 \gamma_y^2 + 2W_2 \gamma_x \gamma_y)}{2\,|[C_2]|}\right], \quad [1.71]$$

where $|[C_2]| = \sigma_{sx}^2 \sigma_{sy}^2 - {W_{2x}}^2 {W_{2y}}^2$ is the determinant of the slope covariance matrix $\{\gamma_x, \gamma_y\}$, with $W_{2x} = -\frac{\partial^2 W_h}{\partial x^2}$ and $W_{2y} = -\frac{\partial^2 W_h}{\partial y^2}$ as the surface slope autocorrelation functions and $(\sigma_{sx}, \sigma_{sy})$ as the RMS slopes of (γ_x, γ_y), respectively.

In the uncorrelated case, the slope PDF for an anisotropic Gaussian 2D process is written as:

$$p_s(\gamma_x, \gamma_y) = \frac{1}{2\pi\, \sigma_{sx}\sigma_{sy}} \; \exp\left(-\frac{\gamma_x^2}{2\,\sigma_{sx}^2} - \frac{\gamma_y^2}{2\,\sigma_{sy}^2}\right). \qquad [1.72]$$

Similarly to the height spectrum, the height characteristic function $\chi_h(q)$, which is equal to the statistical average over the complex exponential $\exp(iq\zeta)$, is sometimes used. For an even process, it is then equal to the Fourier transform of the height PDF as follows:

$$\chi_h(q) = \langle \exp(iq\zeta) \rangle = \int_{-\infty}^{+\infty} p_h(\zeta) \exp(iq\zeta)\, d\zeta. \qquad [1.73]$$

For a centered Gaussian process, it can be shown that:

$$\chi_h(q) = \exp\left(-\frac{1}{2}\, q^2 \sigma_h^2\right). \qquad [1.74]$$

For a stationary surface, the characteristic function of two surface points separated by a distance x_{12} is then given by:

$$\begin{aligned}\chi_h(q_1, q_2; x_{12}) &= \left\langle e^{i(q_1\zeta_1 + q_2\zeta_2)} \right\rangle \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p_h(\zeta_1, \zeta_2; x_{12}) e^{i(q_1\zeta_1 + q_2\zeta_2)}\, d\zeta_1 d\zeta_2, \qquad [1.75]\end{aligned}$$

where $p_h(\zeta_1, \zeta_2; x_{12})$ can be expressed for a Gaussian process by:

$$p_h(\zeta_1, \zeta_2; x_{12}) = \frac{1}{2\pi\sqrt{|[C_2]|}} \; \exp\left[-\frac{\sigma_h^2\zeta_1^2 + \sigma_h^2\zeta_2^2 - 2W_0(x_{12})\zeta_1\zeta_2}{2|[C_2]|}\right], \qquad [1.76]$$

where $|[C_2]| = \sigma_h^4 - {W_0}^2(x_{12})$ is the determinant of the height covariance matrix $\{\zeta_1, \zeta_2\}$, with $W_0 \equiv W_h$ as the height autocorrelation function. This function can be rewritten in the form:

$$p_h(\zeta_1, \zeta_2; x_{12}) = \frac{1}{2\pi\sqrt{|[C_2]|}} \; \exp\left(-\frac{1}{2} \boldsymbol{V_2^T} [C_2]^{-1} \boldsymbol{V_2}\right), \qquad [1.77]$$

where $\boldsymbol{V_2} = [\,\zeta_1\; \zeta_2\,]$. Thus, for a centered Gaussian process, the characteristic function can be written as:

$$\chi_h(q_1, q_2) = \exp\left\{-\frac{1}{2}\left[(q_1^2 + q_2^2)\sigma_h^2 + 2q_1q_2W_0(x_{12})\right]\right\}. \qquad [1.78]$$

This can be generalized to n random variables as follows:

$$\left\langle \exp\left[j\sum_{i=1}^{n} q_i\zeta_i\right]\right\rangle = \exp\left[-\frac{1}{2}\left\langle\left(\sum_{i=1}^{n} q_i\zeta_i\right)^2\right\rangle\right] = \exp\left[-\sum_{i=1}^{n}\sum_{j=1}^{n} q_i q_j \left\langle\zeta_i\zeta_j\right\rangle\right]. \quad [1.79]$$

1.3.2. *Specific case of sea surfaces*

It is important to note that for the case of a sea surface, the statistical description is more complex. This issue mainly concerns the description of the surface height spectrum, which is very different from a Gaussian, an exponential or a Lorentzian function. Then, it is necessary to study this specific case more closely. It is usually said that a sea surface has two main regimes of roughness: the capillary and gravity regimes (see Figure 1.7). The capillary regime corresponds to the so-called capillary waves, which are also called wavelets (or sometimes ripples). They are created by the action of a local wind. They characterize the so-called small-scale roughness of the sea surface, with correlation length L_{cs}. The gravity regime corresponds to the gravity waves, which are sometimes called swell. They characterize the so-called large-scale roughness of the sea surface, with correlation length L_{cl}. The RMS height and the correlation length of the gravity waves are significantly greater than those of the capillary waves, as illustrated in Figure 1.7.

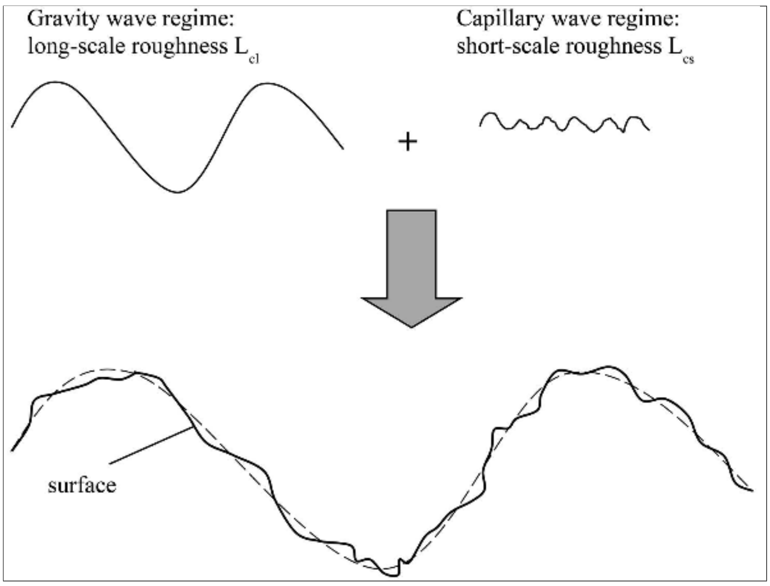

Figure 1.7. *Contribution of the two regimes of a sea surface: gravity and capillary waves*

The first works on height spectra of sea surfaces were mainly developed in the 1970s, but it was only in the 1980s and the 1990s that global spectra, which take both the gravity and the capillarity into account, were developed [BOU 99]. The following three spectra can be quoted: the spectra of Pierson, of Apel and of Elfouhaily *et al.* [ELF 97]. The latter, which was established in 1997, has been retrieved by experimental measurements [COX 54], contrary to the other two spectra. Indeed, it has been built on both experimental and theoretical bases that the previous two models did not consider. It represents a summary of the entirety of the work on this subject from the 1970s and has become a reference since then. The Elfouhaily *et al.* spectrum $S_E(k,\phi)$ is given by [ELF 97]:

$$S_E(k,\phi) = \frac{M(k)}{2\pi}\,[1 + \Delta(k)\cos 2\phi], \tag{1.80}$$

where $M(k)$ is the isotropic part of the spectrum and $\Delta(k)$ the anisotropic part, and ϕ the wind direction. $\phi = 0$ corresponds to the upwind direction, and $\phi = \pi$ to the downwind direction; $\phi = \pi/2$ and $\phi = 3\pi/2$ correspond to the two cross-wind directions.

From the Elfouhaily *et al.* spectrum, it is possible to retrieve the classical statistical parameters of the sea surface, due to the knowledge of the wind speed above the sea surface. For instance, the following relationships can be obtained [BOU 02a]:

$$L_c \simeq 3 \times 0.154\, {u_{10}}^{2.04}, \tag{1.81}$$

$$\sigma_h = \sqrt{\int_0^{+\infty} M(k)\, dk} \;\simeq 6.29 \times 10^{-3}\, {u_{10}}^{2.02}, \tag{1.82}$$

where u_{10} is the wind speed at 10 m above the sea surface (mean level), expressed in m/s. By construction, the Elfouhaily *et al.* spectrum is in agreement with the experimental Cox and Munk model [COX 54], which was established from airborne photographs. In this model, the global RMS slope σ_s is related to the RMS slope in the up-wind direction σ_{sx} and in the cross-wind direction σ_{sy} by $\sigma_s^2 = (\sigma_{sx}^2 + \sigma_{sy}^2)/2$, where:

$$\sigma_{sx}^2 = 3.16 \times 10^{-3}\, u_{12} \pm 4 \times 10^{-3}, \tag{1.83}$$

$$\sigma_{sy}^2 = 1.92 \times 10^{-3}\, u_{12} + 3 \times 10^{-3} \pm 2 \times 10^{-3}, \tag{1.84}$$

where u_{12} is the wind speed at 12.5 m above the sea surface. In [BOU 99] and [DÉC 04], a useful table giving the RMS slopes, the RMS height, the correlation length with respect to the Beaufort scale and to u_{10} can be found.

1.3.3. *Electromagnetic roughness and Rayleigh roughness criterion*

The first work on the scattering of waves from rough surfaces was made by Lord Rayleigh, who considered the problem of an HPPW propagating upon a sinusoidal surface at normal incidence [RAY 45]. This work led to the so-called Rayleigh roughness criterion, which makes it possible to establish the degree of EM roughness of a rough surface. It is used in practice in several simple models to describe the EM wave scattering from random rough surfaces. For instance, in ocean remote sensing, it is used in the Ament model [AME 53, FRE 06, FAB 06, PIN 07a] to calculate the grazing incidence forward (i.e. in the specular direction) radar propagation over sea surfaces; in optics to determine optical constants of films [YIN 96, YIN 97] and other applications [OHL 95, AZI 99, POR 00, CHO 06, XIO 06, MAU 07, REM 09], or in indoor propagation; in ray-tracing-based wave propagation models that take the wall roughness into account by introducing a power attenuation parameter [BOI 87, LAN 96, DID 03, JRA 06, COC 07].

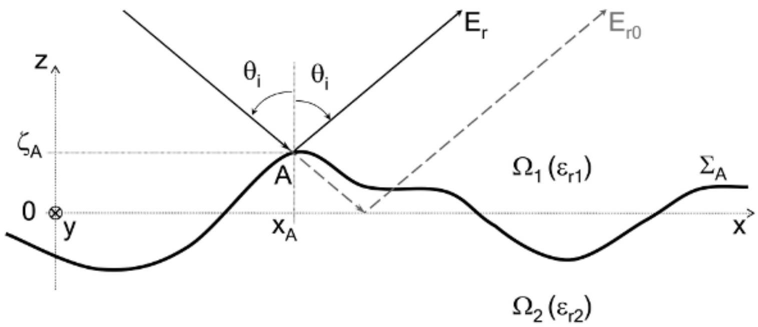

Figure 1.8. *Electromagnetic roughness (in reflection) of a random rough surface: phase variations of the reflected wave owing to the surface roughness*

The roughness (from an *EM* point of view) of a surface depends – obviously – on its height variations, but it is also related to the incident wavelength. Indeed, the *EM* roughness of a surface is related to the phase variations $\delta\phi_r$ of the wave reflected by the surface, owing to the surface height variations. It is obtained under the Kirchhoff-tangent plane approximation[11], which is valid for large surface curvature radii and gentle slopes. Let us consider an incident HPPW inside a medium Ω_1 of wavenumber k_1 on a rough surface with angle θ_i (see Figure 1.8). For the case of a random rough surface considered here, the total reflected field E_r results from the contribution of all reflected fields from the random heights of the rough surface. Then, to quantify the EM surface roughness, it is the phase *variation* $\delta\phi_r$ of the reflected field around its

11 (together with a further approximation, like the so-called scalar Kirchhoff approximation, or the method of stationary phase)

mean value (which corresponds to the phase of the mean plane surface) that must be considered. For the case of a rough surface (see Figure 1.8), the phase variation $\delta\phi_r$ is given by the relation:

$$\delta\phi_r = 2k_1\delta\zeta_A \cos\theta_i, \quad [1.85]$$

where $\delta\zeta_A = \zeta_A - \langle\zeta_A\rangle$ is the height variation, and θ_i is the incidence angle. $\langle\zeta_A\rangle$ is the mean value of the rough surface heights (with $\langle\cdots\rangle$ representing the statistical average), which is equal to 0 here in Figure 1.8.

If the phase variation is negligible, $\delta\phi \ll \pi$, for all positions of these points on the surface, then all the waves scattered (reflected) by the random rough surface are nearly in phase and will consequently interfere constructively. The surface is then considered as slightly or very slightly rough: it may be assimilated to a flat surface. On the contrary, if the phase variation checks $\delta\phi \sim \pi$, these rays interfere destructively. The contribution of the energy scattered in this specular direction is then weak, and the surface is then considered as rough.

The *Rayleigh roughness criterion* [OGI 91, TSA 00] assumes the following condition: if $\delta\phi < \pi/2$, the waves interfere constructively. Consequently, the surface can be considered as very slightly rough or even *flat* if $\delta\phi \ll \pi/2$. Conversely, if $\delta\phi > \pi/2$, the waves interfere destructively, and the surface can be considered as *rough*. To apply this local approach to the whole surface, it is necessary to consider a mean phenomenon, which implies quantifying this phenomenon by a statistical average on $\delta\phi$. The mean value of the surface heights being taken as zero, $\langle\zeta_A\rangle = 0$, the Rayleigh roughness parameter is quantified by the variance of the phase variation $\sigma^2_{\delta\phi}$. Knowing that $\langle\zeta_A^2\rangle = \sigma_h^2$ and $\langle\delta\phi\rangle = 0$, it is defined by:

$$\sigma^2_{\delta\phi} = \left\langle(\delta\phi)^2\right\rangle = \left\langle\left(2k_1\delta\zeta_A\cos\theta_i\right)^2\right\rangle = 4k_1^2\,\sigma_h^2\,\cos^2\theta_i. \quad [1.86]$$

The Rayleigh roughness parameter is then defined from the RMS value $\sigma_{\delta\phi}$. Its definition varies by a factor (coefficient) of 2, depending on the authors; here we take:

$$R_a = k_1\sigma_h\cos\theta_i, \quad [1.87]$$

which corresponds to $R_a = \sigma_{\delta\phi}/2$. The Rayleigh roughness criterion is then:

$$R_a < \pi/4, \quad [1.88]$$

which corresponds to $\sigma_h\cos\theta_i < \lambda/8$.

Thus, the EM roughness is not a phenomenon that is intrinsic to the surface: it depends on the incident wavelength $\lambda_1 \equiv \lambda$. It is the ratio σ_ζ/λ that determines the degree of roughness of a surface, for a given incidence angle. Besides, the influence of the term $\cos\theta_i$ is nearly always neglected. Nonetheless, it is not negligible when the incidence angle becomes grazing, $\theta_i \rightarrow 90°$: this implies that a surface can be considered as rough for moderate incidence angles and becomes only slightly rough for grazing angles.

If we look more closely at this roughness criterion, we can see that equation [1.87] can be rewritten in the form:

$$\sigma_h < \frac{1}{8}\frac{\lambda_1}{\cos\theta_i} = \frac{1}{8}\lambda_{app}, \qquad [1.89]$$

where $\lambda_{app} = \lambda_1/\cos\theta_i$ can be defined as an apparent wavelength along the normal to the mean surface [CRO 84] (by analogy with the apparent wavelength in a metallic waveguide).

From a more quantitative point of view, the field reflected by the surface $E_r(\boldsymbol{R})$ can be written under some assumptions[12] in the specular direction in the form:

$$E_r(\boldsymbol{R}) = E_i(\boldsymbol{R_S})\, r_{12}\, \exp[i\boldsymbol{k_r}\cdot(\boldsymbol{R}-\boldsymbol{R_S})]\, \exp(i\delta\phi), \qquad [1.90]$$

where $\boldsymbol{R_S}$ is a point of the average plane of the rough surface, that is for $z = 0$. It can be rewritten in the form:

$$E_r(\boldsymbol{R}) = E_r^{flat}(\boldsymbol{R})\, \exp(i\delta\phi), \qquad [1.91]$$

where $E_r^{flat}(\boldsymbol{R})$ corresponds to the field reflected by a flat surface. The Rayleigh roughness criterion quantifies the attenuation of the mean field scattered by the random rough surface. Here, the mean scattered field can be written as:

$$\big\langle E_r(\boldsymbol{R})\big\rangle = E_r^{flat}(\boldsymbol{R})\,\big\langle \exp(i\delta\phi)\big\rangle, \qquad [1.92]$$

with, for a surface with Gaussian statistics,

$$\Big\langle \exp(i\delta\phi)\Big\rangle = \exp\big(-\langle\delta\phi^2\rangle/2\big) = \exp\big(-2\,{R_a}^2\big)\,. \qquad [1.93]$$

More precisely, as we are interested in the power (or intensity) reflected by the random rough surface, the coherent power (intensity) attenuation due to the surface roughness is equal to $\exp(-g)$, where g is given by:

$$g = 4\,{R_a}^2. \qquad [1.94]$$

12 The basic necessary assumption is that the Kirchhoff-tangent plane approximation must be valid. An additional condition is also necessary; which, assuming negligible surfaces slopes are enough, typically corresponds to the scalar Kirchhoff approximation. Alternatively, the method of stationary phase can be applied to lead to the same attenuation in the specular direction.

However, note that for a non-Gaussian stationary surface, the attenuation term takes a different expression. Thus, the criterion $R_a = \pi/4$ corresponds to an attenuation of the scattered power of $\exp(-\pi^2/4) \simeq 0.085 \simeq -11$ dB, which is not negligible at all: the surface roughness is already significant. That is why, in order to qualify a random rough surface as *slightly rough*, a more restrictive criterion is sometimes given [SOU 01a, ULA 82]:

$$R_a < \pi/16, \qquad [1.95]$$

which corresponds to $\sigma_h \cos\theta_i < \lambda/32$. Then, the attenuation of the coherent scattered intensity is equal to $\exp(-\pi^2/64) \simeq 0.86 \simeq -0.7$ dB. This condition corresponds for normal incidence to the criterion called the Fraunhofer criterion:

$$\sigma_h/\lambda < 0.03. \qquad [1.96]$$

Conversely, for a surface to qualify as *very rough* compared to the wavelength, the coherent power is very strongly attenuated; then, it is generally negligible (compared to the incoherent power). Yet, the criterion $R_a = \pi/4$ corresponds to a power attenuation of $\exp(-\pi^2/4) \simeq 0.085 \simeq -11$ dB, which may not generally be a sufficient attenuation for the coherent power to be negligible. That is why, for a surface to qualify as *very rough*, the following more restrictive criterion can be used:

$$R_a > \pi/2, \qquad [1.97]$$

which corresponds to $\sigma_h \cos\theta_i > \lambda/4$. The attenuation of the coherent power (intensity) is then equal to $\exp(-\pi^2) \simeq 5.17 \times 10^{-5} \simeq -43$ dB: this attenuation is very significant this time, and the coherent power may be neglected (as compared to the incoherent power) *a priori*. Then, for moderate incidence angles, this corresponds to the following criterion on the surface RMS height:

$$\sigma_h/\lambda > 1/4. \qquad [1.98]$$

Another aspect of the surface roughness may be added here: the *scale* of the surface roughness. As pointed out earlier for sea surfaces, in the literature, the terms small-scale and large-scale roughnesses are commonly used. This roughness scale is typically characterized by the correlation length (denoted by L_c) of the surface. L_c is the horizontal distance that separates two surface points at which their autocorrelation falls down to $1/e$ of its maximum (at least for Gaussian and Lorentzian correlations). Then, it corresponds to the value for which the autocorrelation coefficient is equal to $1/e$. Note that this roughness scale is typically quantified by L_c in comparison to the incident wavelength λ.

1.3.3.1. *Rayleigh roughness criterion in transmission*

To our knowledge, the Rayleigh roughness criterion has always been defined only in the case when the wave is scattered by the rough surface into the incident medium, until recently [PIN 07a, PIN 07b, PIN 10]. Then, a Rayleigh roughness parameter in reflection and a *Rayleigh roughness criterion in reflection* were defined. Nevertheless, often the study focuses on a wave scattered in transmission through a (perfectly dielectric) rough surface. Thus, it is of interest to know in this specific case when a rough surface can be qualified as flat or rough. The same way is used to derive this criterion in transmission. The phase variation $\delta\phi_t$ of the ray transmitted through a surface point $A(x_A, \delta\zeta_A)$ in the direction of specular transmission θ_t (see Figure 1.9) is given by:

$$\Delta\phi_t = k_0 \, \delta\zeta \, (n_1 \cos\theta_i - n_2 \cos\theta_t), \tag{1.99}$$

where k_0 is the wavenumber inside vacuum, and $n_1 = \sqrt{\epsilon_{r1}}$ and $n_2 = \sqrt{\epsilon_{r2}}$ are the indices in media Ω_1 and Ω_2, respectively, checking the transmission Snell–Descartes law $n_1 \sin\theta_i = n_2 \sin\theta_t$ for the specular transmission. Using the same method as for the reflection, let us define the transmission Rayleigh criterion corresponding to the criterion on the phase variation $\delta\phi_t < \pi/2$. Then, the *transmission Rayleigh roughness parameter* $R_{a,t}$ is defined by:

$$R_{a,t} = k_0 \sigma_h \frac{|n_1 \cos\theta_i - n_2 \cos\theta_t|}{2}, \tag{1.100}$$

with the transmission Rayleigh criterion:

$$R_{a,t} < \pi/4. \tag{1.101}$$

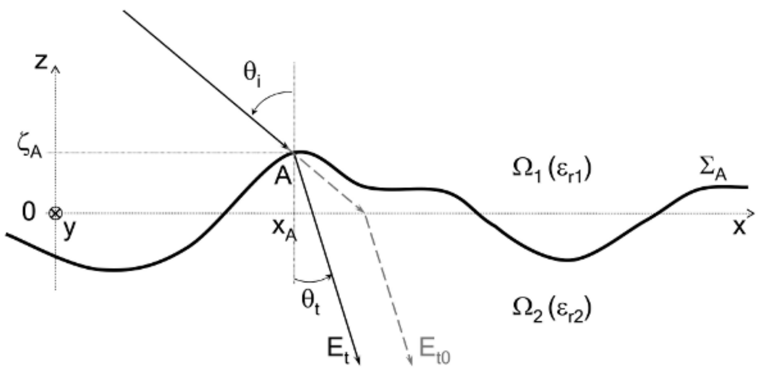

Figure 1.9. *Roughness in transmission of a random surface: phase variations of the transmitted wave owing to the surface roughness*

Note that the transmission Rayleigh roughness parameter is in general (for similar refractive indices of the two media) lower than that in reflection. The consequence of this result is important, as a random rough surface may be considered as very rough electromagnetically when the reflected wave is considered, whereas it may be considered as moderately rough or slightly rough electromagnetically when the transmitted wave is considered. Then, it is necessary to take the appropriate Rayleigh roughness parameter, depending on the studied configuration.

1.3.3.2. *Comparison between the Rayleigh roughness parameters in reflection and transmission*

A comparison between the Rayleigh roughness parameters in reflection and transmission makes it possible to compare the EM roughness between the cases of the reflected wave and of the transmitted wave. The Rayleigh parameter in reflection is given by equation [1.87] and the Rayleigh parameter in transmission by equation [1.100] where θ_t is related to θ_i by the Snell–Descartes law $n_1 \sin\theta_i = n_2 \sin\theta_t$. Figure 1.10 plots the normalized Rayleigh parameter, that is, for $k_0\sigma_h = 1$, for $\epsilon_{r1} = 1$ and for different values of ϵ_{r2} with respect to the incidence angle θ_i.

As a general remark, it can be seen that the Rayleigh roughness parameter in reflection decreases for the increasing incidence angle θ_i; by contrast, the parameter in transmission increases for increasing θ_i. Moreover, the latter increases for increasing ϵ_{r2} (for $\epsilon_{r1} = 1$). It can be noted that for values of relative permittivities ϵ_{r2} close to 1, for low incidence angles, the transmission Rayleigh parameter is lower than the reflection parameter; on the contrary, it is higher for higher incidence angles. Thus, it can easily be shown that the incidence angle for which the Rayleigh roughness parameters in reflection and transmission are equal is given, for $\epsilon_{r2} \geq \epsilon_{r1}$, by:

$$\theta_i^{rug} = \arccos\left(\sqrt{\frac{\epsilon_{r2} - \epsilon_{r1}}{8\,\epsilon_{r1}}}\right), \qquad [1.102]$$

if $\epsilon_{r2} \leq 9\epsilon_{r1}$. For $\epsilon_{r1} = 1$ in Figure 1.10, this gives $\theta_i^{rug} \simeq 69.3°$ for $\epsilon_{r2} = 2$ and $\theta_i^{rug} \simeq 52.3°$ for $\epsilon_{r2} = 4$. Here, $\epsilon_{r1} = 1$, so for relative permittivities $\epsilon_{r2} > 9$, the Rayleigh roughness parameter in transmission is always greater than that in reflection, as can be seen in Figure 1.10 for $\epsilon_{r2} = 53$. Thus, for $\epsilon_{r2} = 9$, the equality holds only for normal incidence, $\theta_i = 0°$.

In conclusion, for $\epsilon_{r2} > 9\epsilon_{r1}$, the Rayleigh roughness parameter in transmission is always greater than that in reflection. On the other hand, for lower permittivity values of ϵ_{r2}, it is greater only from a given incidence angle θ_i^{rug}, which is given by equation [1.102]. Then, for relative permittivities ϵ_{r2} close to 1 (for $\epsilon_{r1} = 1$) and moderate incidence angles, the Rayleigh roughness parameter in transmission is lower than that in reflection: in this case, the surface is rougher electromagnetically in reflection than in transmission.

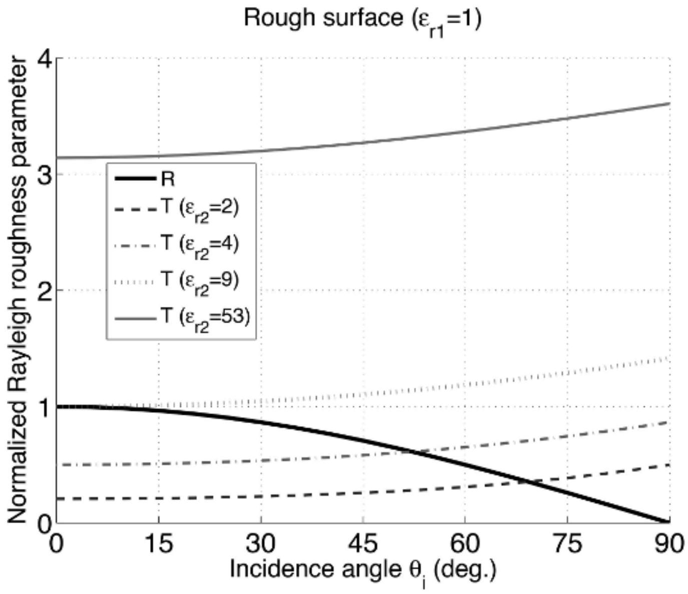

Figure 1.10. *Comparison of the normalized Rayleigh roughness parameters for $\epsilon_{r1} = 1$ and for different values of ϵ_{r2} with respect to the incidence angle θ_i*

1.3.3.3. *Specular and diffuse scattering in reflection and transmission: coherent and incoherent scattering*

For a surface of infinite extent, the fields scattered in reflection $\boldsymbol{E_r}(\boldsymbol{R})$ and transmission $\boldsymbol{E_t}(\boldsymbol{R})$ by a random rough surface can be split up into an average component and a fluctuating component as [SOU 01a, TSA 01b, CAR 03a]:

$$\boldsymbol{E_r}(\boldsymbol{R}) = \langle \boldsymbol{E_r}(\boldsymbol{R}) \rangle + \delta \boldsymbol{E_r}(\boldsymbol{R}), \quad [1.103]$$

$$\boldsymbol{E_t}(\boldsymbol{R}) = \langle \boldsymbol{E_t}(\boldsymbol{R}) \rangle + \delta \boldsymbol{E_t}(\boldsymbol{R}), \quad [1.104]$$

with

$$\langle \delta \boldsymbol{E_r}(\boldsymbol{R}) \rangle = \langle \delta \boldsymbol{E_t}(\boldsymbol{R}) \rangle = 0, \quad [1.105]$$

where $\langle ... \rangle$ represents the statistical average and δ represents the field variations. Then, the total intensity scattered by the surface may be expressed as the sum of the coherent and incoherent intensities as:

$$\langle |\boldsymbol{E_r}(\boldsymbol{R})|^2 \rangle = |\langle \boldsymbol{E_r}(\boldsymbol{R}) \rangle|^2 + \langle |\delta \boldsymbol{E_r}(\boldsymbol{R})|^2 \rangle, \quad [1.106]$$

$$\langle |\boldsymbol{E_t}(\boldsymbol{R})|^2 \rangle = |\langle \boldsymbol{E_t}(\boldsymbol{R}) \rangle|^2 + \langle |\delta \boldsymbol{E_t}(\boldsymbol{R})|^2 \rangle. \quad [1.107]$$

The term $|\langle \boldsymbol{E_{r,t}}(\boldsymbol{R})\rangle|^2$ represents the coherent intensity, owing to its well-defined phase relationship with the incident wave. It corresponds to the specular reflection or transmission of a flat surface (but its amplitude is potentially attenuated). The term

$$\langle|\delta \boldsymbol{E_{r,t}}(\boldsymbol{R})|^2\rangle = \langle|\boldsymbol{E_{r,t}}(\boldsymbol{R})|^2\rangle - |\langle \boldsymbol{E_{r,t}}(\boldsymbol{R})\rangle|^2 \qquad [1.108]$$

represents the incoherent intensity, owing to its angular spreading and its low correlation with the incident wave. It corresponds to the so-called diffuse reflection or transmission. Then, when the surface is flat, the coherent term is maximum and the incoherent term is zero, as all the incident energy is reflected or transmitted into the specular direction. When the EM roughness of the surface increases, the coherent term is attenuated, and the incoherent term increases. For a surface that is qualified (through the Rayleigh criterion) as (very) slightly rough, the coherent term is dominant, whereas for a surface that is qualified as moderately rough, the incoherent term is dominant. Finally, for a very rough surface, the coherent term may be neglected.

1.4. Scattering of electromagnetic waves from rough surfaces: basics

1.4.1. *Presentation of the problem (2D/3D)*

We are interested, in general, in a 3D problem and potentially study the 2D problem first, which is equivalent to a rough surface that is invariant along one direction. The chosen frame is an orthonormal Cartesian frame $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$, with an arbitrary origin O on the mean plane $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}})$ of the random rough surface.

Let $\boldsymbol{E_i}(\boldsymbol{R})$ be a plane EM wave, propagating inside the upper incident medium Ω_1 toward a rough interface Σ_{12} separating the lower medium Ω_2. The two media are assumed to be LHI, stationary, non-magnetic and of relative permittivities ϵ_{r1} and ϵ_{r2}, respectively. The separating interface Σ_{12} is of infinite extent. It is described by its height variations $z = \zeta(x, y)$. A point of space, denoted by $\boldsymbol{R}$, is expressed in the Cartesian frame by $\boldsymbol{R} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}} \equiv \boldsymbol{r} + z\hat{\boldsymbol{z}}$, and a point of the rough surface, denoted by $\boldsymbol{R_A}$, is expressed by $\boldsymbol{R_A} = x_A\hat{\boldsymbol{x}} + y_A\hat{\boldsymbol{y}} + \zeta_A\hat{\boldsymbol{z}} \equiv \boldsymbol{r_A} + \zeta_A\hat{\boldsymbol{z}}$.

The incident wave propagates along the direction $\hat{\boldsymbol{K}}_{\boldsymbol{i}} = (k_{ix}, k_{iy}, k_{iz})/|k_1| = (\hat{k}_{ix}, \hat{k}_{iy}, \hat{k}_{iz})$, having an (elevation) angle θ_i with respect to the vertical axis and an (azimuth) angle ϕ_i with respect to the axis $\hat{\boldsymbol{x}}$ in the plane $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}})$. The incident wave on the rough surface at the point A is written as:

$$\boldsymbol{E_i}(\boldsymbol{R_A}) = E_0 \exp(ik_1 \hat{\boldsymbol{K}}_{\boldsymbol{i}} \cdot \boldsymbol{R_A})\, \hat{\boldsymbol{e}}_{\boldsymbol{i}}, \qquad [1.109]$$

where $\hat{\boldsymbol{e}}_{\boldsymbol{i}}$ is the vector of polarization of $\boldsymbol{E_i}$. The total field on the dielectric surface in the incident medium Ω_1 is then the sum of the incident field and the field scattered (in reflection) inside Ω_1:

$$\boldsymbol{E}(\boldsymbol{R_A}) = \boldsymbol{E_i}(\boldsymbol{R_A}) + \boldsymbol{E_r}(\boldsymbol{R_A}), \qquad [1.110]$$

where $\boldsymbol{E_r}(\boldsymbol{R_A})$ is the scattered field in reflection in the direction $\hat{\boldsymbol{K}}_{\boldsymbol{r}}$, with angles (θ_r, ϕ_r) in a spherical frame. The total field on the dielectric surface in the medium of transmission Ω_2 is equal to the scattered field in transmission $\boldsymbol{E_t}(\boldsymbol{R_A})$ in the direction $\hat{\boldsymbol{K}}_{\boldsymbol{t}}$, with angles (θ_t, ϕ_t) in a spherical frame. The unknowns of the problem are then the scattered fields in reflection $\boldsymbol{E_r}(\boldsymbol{R})$ and transmission $\boldsymbol{E_t}(\boldsymbol{R})$ by the rough surface at any point of the space $\boldsymbol{R}$.

To express the normalized wave vectors $\hat{\boldsymbol{K}}$ as well as the polarization basis of the electric and magnetic fields $(\hat{\boldsymbol{v}}, \hat{\boldsymbol{h}}, \hat{\boldsymbol{K}})$ in the frame $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$, we can choose whether to orientate the angles or not. For a 3D problem, if we choose not to orientate the angles, by taking $\theta_{i,r,t} \in [0; \pi/2]$ and $\phi_{i,r,t} \in [0; 2\pi]$, we obtain in the basis $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$, [TSA 01b]:

$$\left\{\begin{array}{l} \hat{\boldsymbol{v}}_{\boldsymbol{i}} = (\, -\cos\theta_i \cos\phi_i, -\cos\theta_i \sin\phi_i, -\sin\theta_i \,) \\ \hat{\boldsymbol{h}}_{\boldsymbol{i}} = (\quad -\sin\phi_i \quad , \quad +\cos\phi_i \quad , \quad 0 \quad) \\ \hat{\boldsymbol{K}}_{\boldsymbol{i}} = (\, +\sin\theta_i \cos\phi_i, +\sin\theta_i \sin\phi_i, -\cos\theta_i \,) \end{array}\right. , \qquad [1.111]$$

$$\left\{\begin{array}{l} \hat{\boldsymbol{v}}_{\boldsymbol{r}} = (\, +\cos\theta_r \cos\phi_r, +\cos\theta_r \sin\phi_r, -\sin\theta_r \,) \\ \hat{\boldsymbol{h}}_{\boldsymbol{r}} = (\quad -\sin\phi_r \quad , \quad +\cos\phi_r \quad , \quad 0 \quad) \\ \hat{\boldsymbol{K}}_{\boldsymbol{r}} = (\, +\sin\theta_r \cos\phi_r, +\sin\theta_r \sin\phi_r, +\cos\theta_r \,) \end{array}\right. , \qquad [1.112]$$

$$\left\{\begin{array}{l} \hat{\boldsymbol{v}}_{\boldsymbol{t}} = (\, -\cos\theta_t \cos\phi_t, -\cos\theta_t \sin\phi_t, -\sin\theta_t \,) \\ \hat{\boldsymbol{h}}_{\boldsymbol{t}} = (\quad -\sin\phi_t \quad , \quad +\cos\phi_t \quad , \quad 0 \quad) \\ \hat{\boldsymbol{K}}_{\boldsymbol{t}} = (\, +\sin\theta_t \cos\phi_t, +\sin\theta_t \sin\phi_t, -\cos\theta_t \,) \end{array}\right. . \qquad [1.113]$$

The incident wave and the wave scattered in reflection check the Helmholtz equation [1.30] inside the upper medium Ω_1, and the wave scattered in transmission checks the Helmholtz equation inside the lower medium Ω_2:

$$(\boldsymbol{\nabla}^2 + k_\alpha^2)\boldsymbol{E} = \boldsymbol{0}, \qquad [1.114]$$

where $\alpha = 1$ in the upper medium and $\alpha = 2$ in the lower medium. The waves scattered in reflection and transmission on the rough interface Σ_{12} are related to the incident wave by the boundary conditions that are expressed in a general way by equations [1.10]–[1.13]. Finally, in order to fully describe the problem, it is necessary to have a condition of radiation at infinity, that is to say $\boldsymbol{R} \rightarrow +\infty$ [DÉC 04, SOM 54]. This condition is checked by the so-called Green functions that will be detailed hereafter.

1.4.2. *Huygens' principle and extinction theorem*

The Huygens principle [SOM 54, PÉR 04, FAR 93] is a fundamental principle of the undulatory theory of light. Its statement is as follows [FAR 93]: *Each point of a wave surface*[13] S_0 *reached by the light at an instant* t_0 *may be considered as a secondary source which transmits spherical wavelets. At the instant* $t > t_0$*, the wave surface* S *is the envelope of the wave surfaces transmitted by the secondary sources emanating from* S_0. This intuitive principle, which makes it possible to retrieve the laws of geometrical optics, was completed by the postulate of Fresnel in 1818: *Each point* M *of a surface* S *reached by the light may be considered as a secondary source which transmits a spherical wave whose amplitude and phase are those of the incident wave at the point* M. That is why this principle is sometimes called the principle of Huygens–Fresnel. It can be demonstrated from the propagation equations and the second theorem of Green.

1.4.2.1. *Expressions in the scalar case*

In the scalar case (2D or 3D), from the appropriate Green function (see equation [1.128] or [1.129] hereafter), inside the medium Ω_1 it can be shown that:

$$\left.\begin{array}{l}\boldsymbol{R} \in \Omega_1,\ E_1(\boldsymbol{R}) \\ \boldsymbol{R} \in \Omega_2, \quad 0\end{array}\right\} = E_i(\boldsymbol{R}) + \int_{\Sigma_A} d\Sigma_A \left(E_1(\boldsymbol{R_A}) \frac{\partial G_1(\boldsymbol{R_A}, \boldsymbol{R})}{\partial N_A} - G_1(\boldsymbol{R_A}, \boldsymbol{R}) \frac{\partial E_1(\boldsymbol{R_A})}{\partial N_A} \right), \quad [1.115]$$

where $E_1(\boldsymbol{R_A}) = E_i(\boldsymbol{R_A}) + E_r(\boldsymbol{R_A})$ is the total field on the surface Σ_A inside the incident medium, with $E_r(\boldsymbol{R_A})$ the field scattered in reflection. $\hat{\boldsymbol{N}}_{\boldsymbol{A}}$ is the normal to the surface Σ_A at the point A considered, oriented upward, that is to say, in our convention, toward the direction of positive z (see Figure 1.11). In the 2D and 3D cases, it is expressed by:

$$\hat{\boldsymbol{N}}_{\boldsymbol{A}} = \frac{-\gamma_{Ax}\hat{\boldsymbol{x}} + \hat{\boldsymbol{z}}}{\sqrt{1+\gamma_{Ax}^2}} \text{ (2D case)}, \quad [1.116]$$

$$\hat{\boldsymbol{N}}_{\boldsymbol{A}} = \frac{-\gamma_{Ax}\hat{\boldsymbol{x}} - \gamma_{Ay}\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}}{\sqrt{1+\gamma_{Ax}^2+\gamma_{Ay}^2}} \text{ (3D case)}, \quad [1.117]$$

13 A wave surface is the ensemble of points of equal light perturbation. If the wave surface is a plane, the wave is called plane (it is then called plane wave); if this surface is spherical, the wave is called spherical.

respectively, where $\gamma_{Ax} = \partial\zeta_A/\partial x_A$ and $\gamma_{Ay} = \partial\zeta_A/\partial y_A$. In the 2D case, $\zeta_A \equiv \zeta(x_A)$, and in the 3D case, $\zeta \equiv \zeta(x_A, y_A)$. The normal derivative $\partial/\partial N_A$ is defined by:

$$\frac{\partial F}{\partial N_A} = \boldsymbol{N_A} \cdot \boldsymbol{\nabla} F. \qquad [1.118]$$

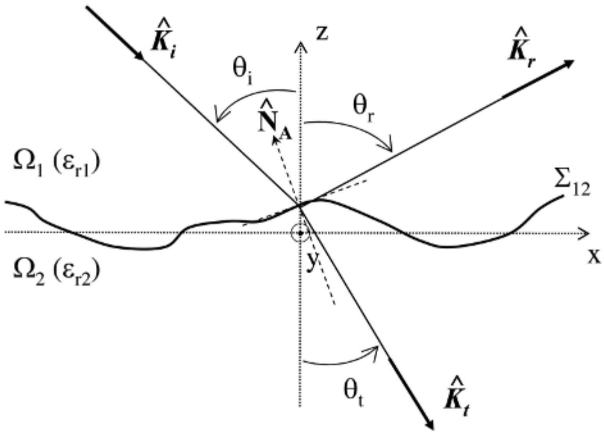

Figure 1.11. *Presentation of the problem (view in the plane* $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{z}})$*)*

The equation for $\boldsymbol{R} \in \Omega_1$ corresponds to the *Huygens principle*, as the field at any point inside Ω_1 can be calculated from the knowledge of the field on the surface $E_1(\boldsymbol{R_A})$ inside Ω_1 and of its normal derivative. To do so, it is necessary to use the appropriate Green function $G_1(\boldsymbol{R}, \boldsymbol{R_A})$ (as well as its normal derivative). The equation for $\boldsymbol{R} \in \Omega_2$ corresponds to the *extinction theorem*, as the integral over the rough surface cancels out the incident field.

For the dielectric case, for which a transmitted wave inside the lower medium Ω_2 exists, the Huygens principle and the extinction theorem are obtained using the same method:

$$\left.\begin{array}{l} \boldsymbol{R} \in \Omega_1, \quad 0 \\ \boldsymbol{R} \in \Omega_2, \; E_2(\boldsymbol{R}) \end{array}\right\} = - \int_{\Sigma_A} d\Sigma_A \left(E_2(\boldsymbol{R_A}) \frac{\partial G_2(\boldsymbol{R}, \boldsymbol{R_A})}{\partial N_A} - G_2(\boldsymbol{R}, \boldsymbol{R_A}) \frac{\partial E_2(\boldsymbol{R_A})}{\partial N_A} \right), \qquad [1.119]$$

where $E_2(\boldsymbol{R_A}) = E_t(\boldsymbol{R_A})$ is the total field on the surface Σ_A inside the transmission medium, which is equal to the field scattered in transmission $E_t(\boldsymbol{R_A})$.

The determination of these equations for a given Green function is related to the knowledge of the total field and its normal derivative on the surface.

1.4.2.2. *Expressions in the vectorial case*

The expressions in the scalar case have been generalized to 3D in the vectorial case by Stratton and Chu [STR 41, KON 90]. These equations, which are usually called Stratton–Chu equations, describe the Huygens principle and the extinction theorem, and can be expressed in the form:

$$\left.\begin{array}{l}\boldsymbol{R} \in \Omega_1,\ \boldsymbol{E_1}(\boldsymbol{R}) \\ \boldsymbol{R} \in \Omega_2,\quad \boldsymbol{0}\end{array}\right\} = \boldsymbol{E_i}(\boldsymbol{R}) + \int_{\Sigma_A} d\Sigma_A \Big\{ i\omega\mu_0\, \bar{G}_1(\boldsymbol{R}, \boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{H_1}(\boldsymbol{R_A})] + \boldsymbol{\nabla} \wedge \bar{G}_1(\boldsymbol{R}, \boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{E_1}(\boldsymbol{R_A})] \Big\}, \quad [1.120]$$

for the field $\boldsymbol{E_1}(\boldsymbol{R})$ inside Ω_1, and

$$\left.\begin{array}{l}\boldsymbol{R} \in \Omega_1,\quad \boldsymbol{0} \\ \boldsymbol{R} \in \Omega_2,\ \boldsymbol{E_2}(\boldsymbol{R})\end{array}\right\} = -\int_{\Sigma_A} d\Sigma_A \Big\{ i\omega\mu_0\, \bar{G}_2(\boldsymbol{R}, \boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{H_2}(\boldsymbol{R_A})] + \boldsymbol{\nabla} \wedge \bar{G}_2(\boldsymbol{R}, \boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{E_2}(\boldsymbol{R_A})] \Big\}, \quad [1.121]$$

for the field $\boldsymbol{E_2}(\boldsymbol{R})$ inside Ω_2, where $\bar{G}_\alpha$ represents the vectorial Green function, usually called dyadic Green function, which is defined as:

$$\bar{G}_\alpha(\boldsymbol{R}, \boldsymbol{R_A}) = \left(\boldsymbol{I} + \frac{\boldsymbol{\nabla\nabla}}{k_\alpha^2} \right) G_\alpha(\boldsymbol{R}, \boldsymbol{R_A}), \quad [1.122]$$

where $\alpha = \{1, 2\}$, and $\boldsymbol{H_\alpha}(\boldsymbol{R_A})$ is expressed in terms of $\boldsymbol{E_\alpha}(\boldsymbol{R_A})$ from the Maxwell equation [1.2], which can be rewritten in a Cartesian frame and by assuming non-magnetic media ($\mu_r = 1$) in the form:

$$\boldsymbol{H_\alpha}(\boldsymbol{R_A}) = \frac{1}{i\omega\mu_0}\, \boldsymbol{\nabla} \wedge \boldsymbol{E_\alpha}(\boldsymbol{R_A}). \quad [1.123]$$

In the vectorial case, for a given Green function, the determination of these equations is related to the knowledge of the tangential components of the electric and magnetic fields, $\boldsymbol{N_A} \wedge \boldsymbol{E_\alpha}(\boldsymbol{R_A})$ and $\boldsymbol{N_A} \wedge \boldsymbol{H_\alpha}(\boldsymbol{R_A})$. These quantities play a role in the passage relations on the surface, which are written in the general form by [1.12] and [1.13], respectively.

1.4.2.3. *Kirchhoff–Helmholtz equations*

In what follows, the equations that describe the Huygens principle will be used in a slightly different form for which only the scattered field (and not the total field) is expressed at any point $\boldsymbol{R}$ in terms of the total field on the surface: these equations are then called *Kirchhoff–Helmholtz equations*. For a scalar problem, they are expressed in reflection and transmission by:

$$\forall \boldsymbol{R} \in \Omega_1,\ E_r(\boldsymbol{R}) = +\int_{\Sigma_A} d\Sigma_A \left(E_1(\boldsymbol{R_A})\frac{\partial G_1(\boldsymbol{R},\boldsymbol{R_A})}{\partial N_A} - G_1(\boldsymbol{R},\boldsymbol{R_A})\frac{\partial E_1(\boldsymbol{R_A})}{\partial N_A}\right), \quad [1.124]$$

$$\forall \boldsymbol{R} \in \Omega_2,\ E_t(\boldsymbol{R}) = -\int_{\Sigma_A} d\Sigma_A \left(E_2(\boldsymbol{R_A})\frac{\partial G_2(\boldsymbol{R},\boldsymbol{R_A})}{\partial N_A} - G_2(\boldsymbol{R},\boldsymbol{R_A})\frac{\partial E_2(\boldsymbol{R_A})}{\partial N_A}\right), \quad [1.125]$$

respectively. For the vectorial case, they are expressed in reflection and transmission by [TSA 01b]:

$$\forall \boldsymbol{R} \in \Omega_1, \boldsymbol{E_r}(\boldsymbol{R}) = +\int_{\Sigma_A} d\Sigma_A \Big\{ i\omega\mu_0\, \bar{G}_1(\boldsymbol{R},\boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{H_1}(\boldsymbol{R_A})] + \boldsymbol{\nabla} \wedge \bar{G}_1(\boldsymbol{R},\boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{E_1}(\boldsymbol{R_A})] \Big\}, \quad [1.126]$$

$$\forall \boldsymbol{R} \in \Omega_2, \boldsymbol{E_t}(\boldsymbol{R}) = -\int_{\Sigma_A} d\Sigma_A \Big\{ i\omega\mu_0\, \bar{G}_2(\boldsymbol{R},\boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{H_2}(\boldsymbol{R_A})] + \boldsymbol{\nabla} \wedge \bar{G}_2(\boldsymbol{R},\boldsymbol{R_A}) \cdot [\boldsymbol{N_A} \wedge \boldsymbol{E_2}(\boldsymbol{R_A})] \Big\}, \quad [1.127]$$

respectively.

1.4.3. *Green functions (2D/3D)*

The Green function is a mathematical tool that makes it possible to propagate a wave from a point of a given medium to another point; for our purpose, it is used to relate the scattered wave on the surface to the scattered wave at any point of the space, and that checks the radiation condition at infinity. Its derivation is not detailed here; for more details, see, for instance, [DUR 03] and [SOU 01a]. Its general expression is given for a wave propagating from any point $\boldsymbol{R_1}$ to any point $\boldsymbol{R_2}$ inside the medium Ω_α (with $\alpha = \{1, 2\}$), in 2D and 3D (for the scalar case) by:

$$G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{i}{4} H_0^{(1)}(k_\alpha ||\boldsymbol{R_2} - \boldsymbol{R_1}||), \text{ where } \boldsymbol{R} = x\hat{\boldsymbol{x}} + z\hat{\boldsymbol{z}}, \quad [1.128]$$

$$G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{\exp(ik_\alpha ||\boldsymbol{R_2} - \boldsymbol{R_1}||)}{4\pi ||\boldsymbol{R_2} - \boldsymbol{R_1}||}, \text{ where } \boldsymbol{R} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}, \quad [1.129]$$

respectively, where $H_0^{(1)}$ is the Hankel function of first kind and order zero. The expression in 3D and in the vectorial case is given by the dyadic Green function, which is expressed from [1.129] by:

$$\bar{G}_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \left(\boldsymbol{I} + \frac{\boldsymbol{\nabla}\boldsymbol{\nabla}}{k_\alpha^2}\right) G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}), \text{ where } \boldsymbol{R} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}. \quad [1.130]$$

1.4.3.1. *Weyl representation of the Green function*

Another possible equivalent representation of the Green functions is the representation in the Fourier domain under the form of a sum (a spectrum) of plane waves. Also called Weyl representation of the Green function, it is expressed in 2D by:

$$G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{i}{2} \int_{-\infty}^{+\infty} \frac{dk_x}{2\pi} \, \frac{e^{ik_x(x_2 - x_1) + if(k_x)|z_2 - z_1|}}{f(k_x)}, \quad [1.131]$$

where $\boldsymbol{R} = x\hat{\boldsymbol{x}} + z\hat{\boldsymbol{z}}$, with:

$$f(k_x) = \begin{cases} \sqrt{k_\alpha^2 - k_x^2} & \text{if } k_\alpha^2 \geq k_x^2 \\ i\sqrt{k_x^2 - k_\alpha^2} & \text{if } k_\alpha^2 < k_x^2 \end{cases}, \quad [1.132]$$

where k_α is the wavenumber inside the medium Ω_α, and in 3D (in the scalar case) by [BOU 04a, FUN 94, BAS 78]:

$$G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{i}{2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{d\boldsymbol{k}}{(2\pi)^2} \, \frac{e^{i\boldsymbol{k}\cdot(\boldsymbol{r_2} - \boldsymbol{r_1}) + if(\boldsymbol{k})|z_2 - z_1|}}{f(\boldsymbol{k})}, \quad [1.133]$$

where $\boldsymbol{R} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$, $\boldsymbol{k} = k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}}$ and $\boldsymbol{r} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$, with:

$$f(\boldsymbol{k}) = \begin{cases} \sqrt{k_\alpha^2 - ||\boldsymbol{k}||^2} & \text{if } k_\alpha^2 \geq ||\boldsymbol{k}||^2 \\ i\sqrt{||\boldsymbol{k}||^2 - k_\alpha^2} & \text{if } k_\alpha^2 < ||\boldsymbol{k}||^2 \end{cases}. \quad [1.134]$$

Note that the case $k_\alpha^2 < ||\boldsymbol{k}||^2$ ($k_\alpha^2 < k_x^2$ in 2D) corresponds to the contribution of the evanescent waves. However, generally we study the scattered field in the far-field zone of the surface. In this case, the evanescent waves can be neglected, and the integration of $k_{x,y}$ in the interval $]-\infty;+\infty[$ is reduced to the interval $[-k_\alpha;+k_\alpha]$. Then, the variable $f(\boldsymbol{k})$ is always positive and is equal to $f(\boldsymbol{k}) = \sqrt{k_\alpha^2 - ||\boldsymbol{k}||^2}$. Using the same method in the 2D case, $f(k_x) = \sqrt{k_\alpha^2 - k_x^2} > 0$.

The above expressions of the Green function can then be simplified: the term inside the exponential can be rewritten in the form of a scalar product between the vector $\boldsymbol{R_2} - \boldsymbol{R_1}$ and the propagation wave vector $\boldsymbol{K}$ from the point $\boldsymbol{R_1}$ to the point $\boldsymbol{R_2}$. $\boldsymbol{K}$ can then be written in the form:

$$\boldsymbol{K} = k_\alpha \frac{\boldsymbol{R_2} - \boldsymbol{R_1}}{||\boldsymbol{R_2} - \boldsymbol{R_1}||}. \qquad [1.135]$$

In the 3D case, $\boldsymbol{K}$ is also expressed by:

$$\boldsymbol{K} = \boldsymbol{k} + k_z\hat{\boldsymbol{z}}, \quad \text{with } k_z = \boldsymbol{K} \cdot \hat{\boldsymbol{z}} = \text{sign}(\boldsymbol{K} \cdot \hat{\boldsymbol{z}})\, f(\boldsymbol{k}). \qquad [1.136]$$

From equation [1.135], the vector $\boldsymbol{K}$ points in the same direction as $\boldsymbol{R_2} - \boldsymbol{R_1}$, but in the opposite way. Thus, $k_z = \boldsymbol{K} \cdot \hat{\boldsymbol{z}}$ and $z_2 - z_1 = \boldsymbol{R_2} - \boldsymbol{R_1} \cdot \hat{\boldsymbol{z}}$ have the same sign. As a result, the term inside the exponential may be rewritten in the form [ISH 96, BAH 01]:

$$\exp[i\,\boldsymbol{K} \cdot (\boldsymbol{R_2} - \boldsymbol{R_1})], \qquad [1.137]$$

and the Weyl representation of the Green function can be rewritten in 2D in the form:

$$G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{i}{4\pi} \int_{-\pi/2}^{+\pi/2} d\theta \, \exp[i\,\boldsymbol{K} \cdot (\boldsymbol{R_2} - \boldsymbol{R_1})], \qquad [1.138]$$

and in 3D in the form [ISH 96]:

$$G_\alpha(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{ik_\alpha}{8\pi^2} \int \sin\theta d\theta \int d\phi \, \exp[i\,\boldsymbol{K} \cdot (\boldsymbol{R_2} - \boldsymbol{R_1})], \qquad [1.139]$$

with (θ,ϕ) as the angular directions corresponding to the wave vector $\boldsymbol{K}$.

It is also possible to express the Green function that represents the propagation of a wave from a point $\boldsymbol{R_1}$ to a point $\boldsymbol{R_2}$ after the reflection onto a perfectly flat surface of elevation z_S by [BAS 78, TSA 75, SOU 01a]:

$$G_{\alpha,r}(\boldsymbol{R_2}, \boldsymbol{R_1}) = \frac{i}{2} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{d\boldsymbol{k}}{(2\pi)^2}\, r(\boldsymbol{k})\, \frac{e^{i\boldsymbol{k} \cdot (\boldsymbol{r_2} - \boldsymbol{r_1}) + if(\boldsymbol{k})(z_2 + z_1 - 2z_S)}}{f(\boldsymbol{k})}, \qquad [1.140]$$

where $r(\boldsymbol{k})$ is the Fresnel reflection coefficient.

1.4.3.2. *Far-field approximation*

In the case when the point $\boldsymbol{R_2} \equiv \boldsymbol{P}$ is in the far-field zone relative to $\boldsymbol{R_1}$, the Green function may be expressed approximately in 2D and 3D in the form:

$$G_\alpha(\boldsymbol{P}, \boldsymbol{R_1}) \simeq \frac{i}{4}\sqrt{\frac{2}{\pi k_\alpha R}}\ \exp\left[i(k_\alpha R - \boldsymbol{K} \cdot \boldsymbol{R_1} - \pi/4)\right], \qquad [1.141]$$

$$G_\alpha(\boldsymbol{P}, \boldsymbol{R_1}) \simeq \frac{\exp\left[i(k_\alpha R - \boldsymbol{K} \cdot \boldsymbol{R_1})\right]}{4\pi R}, \qquad [1.142]$$

respectively, where $R = ||\boldsymbol{P}||$ and $k_\alpha = ||\boldsymbol{K}||$.

1.4.4. *Scattered powers and scattering coefficients*

To determine the EM power (or intensity) scattered by random rough surfaces, usually, a coefficient that relates the power density scattered in a given direction to the incident power is used. This coefficient differs in its name and definition according to various scientific communities (optics, radar, etc.). In what follows, the classical definition in the rough surface scattering community is derived, and the relationship with the other definitions is given.

1.4.4.1. *Incident power, scattered power(s) and energy conservation*

It is necessary to know the incident power onto the surface as well as the power scattered by this surface, not only in reflection in the incident medium, but also in transmission. First, the incident power onto the surface is taken as the average power on the rough surface Σ_{12}, corresponding to the incident power on the average plane S_{12} of the rough surface. Knowing that the elementary flux dF of a vector $\boldsymbol{V}$ through an element of (flat) surface $\boldsymbol{dS}$ is equal to:

$$dF = \boldsymbol{V} \cdot \boldsymbol{dS}, \qquad [1.143]$$

the flux of the average incident Poynting vector $\langle \boldsymbol{\Pi_i} \rangle$ received by an element of flat surface $dS_{12} = dxdy$ with normal $\hat{\boldsymbol{N}} = \hat{\boldsymbol{z}}$ is then equal to $dP_i = -\langle \boldsymbol{\Pi_i} \cdot \hat{\boldsymbol{N}} \rangle\, dx\, dy = -(\langle \boldsymbol{\Pi_i} \rangle \cdot \hat{\boldsymbol{z}})\, dx\, dy$ (the minus sign is due to the fact that dP_i must be positive). Then, the total incident power P_i on the average plane is [TSA 01a]:

$$P_i = \int_{-L_x/2}^{+L_x/2} \int_{-L_y/2}^{+L_y/2} -(\langle \boldsymbol{\Pi_i} \rangle \cdot \hat{\boldsymbol{z}})\, dx\, dy = L_x L_y \cos\theta_i\, ||\langle \boldsymbol{\Pi_i} \rangle||, \qquad [1.144]$$

where $||\langle \boldsymbol{\Pi_i} \rangle|| = \frac{\langle |E_i|^2 \rangle}{2\, Z_1}$, with $Z_1 = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_{r1}}}$ for a plane wave inside a non-magnetic perfect dielectric medium.

The flux of the average scattered Poynting vector $\langle \boldsymbol{\Pi_s} \rangle$ (in reflection $\langle \boldsymbol{\Pi_r} \rangle$ or in transmission $\langle \boldsymbol{\Pi_t} \rangle$) by the rough surface through an element of surface dS with normal $\hat{\boldsymbol{n}}$ is equal to $dP_s = +(\langle \boldsymbol{\Pi_s} \rangle \cdot \hat{\boldsymbol{n}})\, dS$. The scattered wave being spherical in general, in the spherical frame (R,θ_s, ϕ_s), the element of surface dS is expressed in the hypothesis when the scattered wave is in the far-field zone of the surface by:

$$\boldsymbol{dS} = dS\, \hat{\boldsymbol{n}} = R^2 \sin\theta_s\, d\theta_s d\phi_s\, \hat{\boldsymbol{n}}, \tag{1.145}$$

where R is the distance of the scattered field $\boldsymbol{E_s}$ to the origin, θ_s is the elevation angle of the scattered wave and ϕ_s is its azimuthal angle. Then, the total scattered wave P_s (in reflection P_r or transmission P_t) by the rough surface is [TSA 01a]:

$$P_s = \int\!\!\int (\langle \boldsymbol{\Pi_s} \rangle \cdot \hat{\boldsymbol{n}})\, R^2 \sin\theta_s\, d\theta_s\, d\phi_s = \int\!\!\int ||\langle \boldsymbol{\Pi_s} \rangle||\, R^2 \sin\theta_s\, d\theta_s\, d\phi_s, \tag{1.146}$$

where $||\langle \boldsymbol{\Pi_s} \rangle|| = \frac{\langle |E_s|^2 \rangle}{2\, Z_\alpha}$, with $Z_\alpha = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_{r\alpha}}}$ for a plane wave inside a (non-magnetic) perfect dielectric medium. For a wave scattered in reflection, $s \equiv r \Rightarrow \alpha = 1$, and for a wave scattered in transmission, $s \equiv t \Rightarrow \alpha = 2$.

Then, it is possible to study the energy conservation, which should be checked in theory, that is $(P_r + P_t)/P_i = 1$. By analogy with a flat surface, the reflectivity is usually defined by P_r/P_i and the transmissivity by P_t/P_i. They are then defined for perfect dielectric media by:

$$\frac{P_r}{P_i} = \int\!\!\int \frac{R^2\, ||\langle \boldsymbol{\Pi_r} \rangle||}{P_i} \sin\theta_r\, d\theta_r\, d\phi_r \tag{1.147}$$

$$= \int\!\!\int \frac{R^2\, \langle |E_r|^2 \rangle}{L_x L_y \cos\theta_i\, |E_i|^2} \sin\theta_r\, d\theta_r\, d\phi_r,$$

$$\frac{P_t}{P_i} = \int\!\!\int \frac{R^2\, ||\langle \boldsymbol{\Pi_t} \rangle||}{P_i} \sin\theta_t\, d\theta_t\, d\phi_t \tag{1.148}$$

$$= \int\!\!\int \frac{R^2\, \langle |E_t|^2 \rangle}{L_x L_y \cos\theta_i\, |E_i|^2} \frac{Z_1}{Z_2} \sin\theta_t\, d\theta_t\, d\phi_t.$$

Thus, by analogy with the case of a flat surface, the principle of energy conversation must lead to the result $(P_r + P_t)/P_i = 1$. As a result, the study of $(P_r + P_t)/P_i$ is a good means (among others) to study the validity of an EM model, depending on various parameters such as the incidence angle and the characteristic statistical parameters of the studied rough surface.

1.4.4.2. *Scattering coefficient, radar cross-section (RCS) and bidirectional reflectance distribution function (BRDF)*

By definition, the scattering coefficient is equal to the proportion of the wave scattered by the surface (relatively to the incident power) in a solid angle defined by $\sin\theta_s \, d\theta_s d\phi_s$, around the observation direction $\hat{\boldsymbol{K}}_{\boldsymbol{s}}$ given by (θ_s, ϕ_s) [TSA 01a][14]:

$$\frac{P_s(\hat{\boldsymbol{K}}_{\boldsymbol{i}})}{P_i(\hat{\boldsymbol{K}}_{\boldsymbol{i}})} = \int_{\phi_s} \int_{\theta_s} \sigma_s(\hat{\boldsymbol{K}}_{\boldsymbol{s}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}}) \, \sin\theta_s \, d\theta_s d\phi_s. \qquad [1.149]$$

It is defined in the far-field zone $R \to +\infty$. Thus, by identification, the scattering coefficients in reflection and transmission are defined by [TSA 01b]:

$$\sigma_r(\hat{\boldsymbol{K}}_{\boldsymbol{r}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}}) = \lim_{R \to +\infty} \frac{R^2 \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{r}} \rangle||}{P_i} = \lim_{R \to +\infty} \frac{R^2 \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{r}} \rangle||}{L_x L_y \cos\theta_i \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{i}} \rangle||}, \qquad [1.150]$$

$$\sigma_t(\hat{\boldsymbol{K}}_{\boldsymbol{t}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}}) = \lim_{R \to +\infty} \frac{R^2 \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{t}} \rangle||}{P_i} = \lim_{R \to +\infty} \frac{R^2 \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{t}} \rangle||}{L_x L_y \cos\theta_i \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{i}} \rangle||}. \qquad [1.151]$$

For 2D problems, by using the same way [DÉC 04], the scattering coefficients are written in reflection and transmission by:

$$\sigma_r(\theta_r, \theta_i) = \lim_{R \to +\infty} \frac{R \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{r}} \rangle||}{P_i} = \lim_{R \to +\infty} \frac{R \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{r}} \rangle||}{L_x \cos\theta_i \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{i}} \rangle||}, \qquad [1.152]$$

$$\sigma_t(\theta_t, \theta_i) = \lim_{R \to +\infty} \frac{R \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{t}} \rangle||}{P_i} = \lim_{R \to +\infty} \frac{R \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{t}} \rangle||}{L_x \cos\theta_i \, ||\langle \boldsymbol{\Pi}_{\boldsymbol{i}} \rangle||}. \qquad [1.153]$$

The radar cross-section (RCS), widely used in the radar domain, has a similar definition. However, contrary to the scattering coefficient that is dimensionless, as its name suggests, the RCS is proportional to a surface, so it is homogeneous to m^2. It is defined by:

$$RCS = 4\pi \lim_{R \to +\infty} R^2 \, \frac{||\langle \boldsymbol{\Pi}_{\boldsymbol{r}} \rangle||}{||\langle \boldsymbol{\Pi}_{\boldsymbol{i}} \rangle||}. \qquad [1.154]$$

Thus, the relation between the RCS and the scattering coefficient in reflection is as follows:

$$RCS = 4\pi \, L_x L_y \cos\theta_i \; \sigma_r(\hat{\boldsymbol{K}}_{\boldsymbol{r}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}}). \qquad [1.155]$$

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14 A slightly different definition is often met (particularly in the Anglo-Saxon literature), where a multiplicative factor $1/4\pi$ appears on the right-hand side of equation [1.149]. Consequently, the scattering coefficient is multiplied by 4π.

The difference between these two definitions may be explained by the fact that in radar, the target (illuminated object) is often unknown; consequently, its size is not known, so the incident total power on the target is unknown. That is why the RCS is defined from the *density* of the incident power. Also note that the scattering coefficient is often called normalized radar cross-section (NRCS), as it corresponds to a normalization of the RCS by the illuminated object size.

In the optical domain, the bidirectional reflectance distribution function (BRDF) has a slightly different definition from the scattering coefficient (or NRCS) in reflection σ_r. It can be shown [CAR 03a] that the BRDF is expressed by:

$$BRDF(\hat{\boldsymbol{K}}_{\boldsymbol{r}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}}) = \frac{\sigma_r(\hat{\boldsymbol{K}}_{\boldsymbol{r}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}})}{\cos\theta_r}. \qquad [1.156]$$

Using the same method, the bidirectional transmittance distribution function (BTDF) is defined by [CAR 03a]:

$$BTDF(\hat{\boldsymbol{K}}_{\boldsymbol{t}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}}) = \frac{\sigma_t(\hat{\boldsymbol{K}}_{\boldsymbol{t}}, \hat{\boldsymbol{K}}_{\boldsymbol{i}})}{\cos\theta_t}. \qquad [1.157]$$

In the remainder of the book, these general concepts are used for calculating the scattered field and intensity of random rough surfaces by using asymptotic models.