Chapter 1

Towards a Unified Description of Multiphase Flows

1.1. Continuous approach and kinetic approach

In classic fluid mechanics theories, fluid is usually considered as a "continuous medium", described locally and at each instant using a certain number of characteristic variables, and its evolution is represented by "balance equations". These balance equations are partial differential equations in three-dimensional space and time, whose original writing uses the Eulerian point of view: a geometric point in space is designated, and at this point, the characteristics of the fluid at each instant are observed, in particular its velocity. This leads to the so-called continuity equation and then to Euler and Navier–Stokes equations and necessitates the definition of the Cauchy stress tensor, containing the pressure and tensor of viscous stresses. The description is, therefore, not complete until these new variables can be given by specific "constitutive laws", which represent the nature and small-scale properties of the fluid in question. These constitutive laws are not necessarily based solely on a theory; they can, with more generality, be of empirical origin, but in any case they must follow the principles of thermodynamics.

There is another way to find these same equations and laws by considering the fluid medium from the beginning as a set of atoms or molecules in steady motion and colliding frequently in the void space, under the action of the laws of classic mechanics for material points. This approach, often called the kinetic approach, was introduced by Boltzmann and Maxwell and has been used to recover classical Eulerian continuity equations and Navier–Stokes equations since the work of Chapman and Enskog in the early 20th Century. In this context, the characteristic

variables that we might call "macroscopic" variables are defined in a small volume around each point in space, and in a small interval of time around a given instant. When the volume and the interval of time considered are very small in relation to the spatial and temporal scales of variation expected for macroscopic variables, in the macroscopic experiments and situations with which we are concerned, these macroscopic variables no longer depend on the real size of the volume nor the time interval, and have continuity properties with respect to space and time. It is in these conditions that we can consider the medium as continuous, on the macroscopic scale and with our limited view.

The base equation of the kinetic approach is the Boltzmann equation, which concerns the distribution function of the velocities of molecules at a given point M (x) and a given instant t. To be exact, and for a gas medium containing only one kind of molecule, this function, $f(u,x,t)$, is defined as the number of molecules that can be found, in the medium, in a small volume around point x approximately dx, at an instant t approximately dt, and which have velocity u approximately du (here, of course, the variables x and u are three-component vectors, as are dx and du). The number of molecules per unit of volume of the medium is, thus, defined as the integral in space of the values of velocities (from minus infinity to plus infinity) of this distribution function, and what we might call the macroscopic velocity of the medium is the integral, in this same space of velocities, of the product uf, divided by the integral of f itself. All the molecules of the gas being identical and of known mass, the volumetric mass of the medium is the product of the number of molecules per unit of volume and the mass of a molecule, and the velocity of the medium thus appears as the ratio between the momentum of the medium and its mass, both per unit of volume. The historical development of this approach is discussed in [CHA 60] and a very complete recitation of the developments in [HIR 54]. In the framework of the approximation of continuous media, this approach allows us to find the same forms of the balance equations of mass and momentum by using the first continuous approach discussed above. However, in addition, the fact that we can describe in more detail the microscopic structure of the material enables us to obtain the constitutive laws in a theoretical manner, that is expressions of all the variables that appear in the balance equations, namely the Cauchy stress tensor, pressure, viscous stress tensor, etc. These expressions are calculable only because of certain hypotheses on the microscopic characteristics of the gas being considered (for example the approximation of molecules as hard spheres of a certain mass). The energy balance equation of the continuous medium can also be found by defining the heat conduction flow in particular. When the approximation of the continuous medium is no longer valid, as is the case for rarefied gases, the Boltzmann equation remains valid and the balance equations of mass, momentum and energy still exist with the same overall form, but they contain additional terms, and the usual terms can no longer be calculated using the same approximations. In certain conditions, it is no longer useful to define a macroscopic velocity for the medium, and the

calculation of the velocity distribution function itself, by using adequate approximations to represent collisions between molecules in the Boltzmann equation, is recommended.

When undertaking a theoretical study of multiphase fluid media, we might ask ourselves which of the two approaches is of more interest, especially when we wish to focus on a case in which one of the phases is dispersed into numerous parcels of different sizes.

In fact, it seems that the first studies involving clouds of drops in a gaseous medium used the kinetic approach to study the liquid phase composed of the set of drops by examining a distribution function where the variables were both the velocities and sizes of the drops, or $f(u,v,x,t)$, where v is the volume of the drops. At a given point and at a given instant, the number per unit of volume (of the medium) of drops of volume v, approximately dv, is still the integral of f on the space of velocities, and the total number of drops of all sizes is the integral of f both on the space of velocities and that of volumes. The function f follows a type of Boltzmann equation here again, in which the most difficult term to approximate is the term due to the collisions of particles. We may refer, for example to, [WIL 58]. However, it has never been suggested to represent the gaseous phase in which these drops are dispersed using a Boltzmann equation that, when the quantity of drops is relatively large, might include the influence of these drops on the motion of the fluid. The description of this gaseous phase has been kept in the form of a continuous medium, where the influence of the drops is represented by additional interaction terms.

For less well-defined multiphase media such as those containing bubbles and gas pockets that can be of various shapes, a "macroscopic continuous medium" approach has been applied from the beginning, by focusing on the characteristic variables for a small volume around each point in space. The basic hypothesis has been made that this volume is very large in relation to the size of the inclusions of the dispersed phases, and these variables therefore represent an "equivalent continuous medium" corresponding to our multiphase medium. The fictive volume within which the macroscopic variables are defined is often called the "representative elementary volume". We may refer, for example to, [SOO 89] and [DEL 81]. In these conditions, the balance equations for these variables have the classic form but the necessary constitutive laws are more complicated; they cannot be calculated and must be obtained through experimentation. This approach is valid, but the definition of the characteristic variables of the medium, which must have a certain continuity, as well as those of the various constitutive laws, must be precisely made. Obtaining these laws is (too) highly dependent on experimentation, and the domain of validity of these laws is difficult to know with precision. Moreover, the condition that each inclusion must be much smaller than the representative elementary volume has very often been

invalidated, or noted visually as soon as modern viewing methods were able to be used. Does this flaw seriously affect the approach, or is it responsible for only "small" errors? This remains an ongoing debate and, most likely, must depend on each of the various cases considered.

More recently, in the 1980s, it appeared possible to develop an approach that might be called intermediary, which defined the equivalent continuous medium more accurately and thus made it possible to obtain more information about the form of the constitutive laws, which in turn made it possible, importantly, to better examine situations with large inclusions, and could therefore be adapted to widely diverse practical situations. In fact, if the size of each inclusion in a phase is large enough to include a large number of molecules that interact on the interface with extremely numerous molecules of the other phase in contact, then it is possible to treat all of the phases, fluid or solid, like classic continuous media. The multiphase medium is, thus, a "piecewise continuous medium" in which each piece of continuous medium is separated from the other by an interface where the properties can show a discontinuity. Using this perspective, we enable the shape and size of the inclusions in each phase to vary in the medium in a totally arbitrary manner. However, the balance equations for phases in continuous media are unusable as such for representing the full multiphase medium, for the same reason that equations of material points mechanics cannot be used to represent a gas, which is a group of molecules. We do not precisely know the initial conditions of each molecule in the gas, nor the initial position or shape of each inclusion in each phase, even if these inclusions are not too small. To establish some macroscopic characteristics at the scale of a multiphase medium, and to deduce the macroscopic equations that they must satisfy, it is necessary to define, through an averaging operation that will be specified later, a "mean continuous" medium that will be the equivalent continuous medium. This approach has grown slowly since its introduction. It was presented for the first time in [DRE 83]. However, it is very similar to the approach used for flows in porous media by Marle in the beginning of 1967 [MAR 67]. At roughly the same time, Nigmatulin began using the same approach in the foundation chapters of his book [NIG 91], which also presents highly useful and detailed developments for several different applications. General equations for piecewise continuous media have been studied in detail by Kataoka [KAT 86], taking into account mass exchanges as well as exchanges of momentum or energy between these phases. This generality is one of the strong points of the approach.

This averaged piecewise continuous medium approach is the approach that we will follow in this book. In this way, we will avoid defining a distribution function for the sizes and velocities of parcels, but it will be important to know other macroscopic variables of the medium besides its volumetric mass and velocity. We will be interested at least by the "volume fraction" occupied in the medium by each of the phases, and also by the mean temperature or internal energy of each phase,

which are to be defined precisely. It will also be necessary to have a thorough understanding of at least one macroscopic variable related to the size of the inclusions of a phase that is dispersed in another carrier phase. First, and in the most general of situations, it is through the "mean area of interface per unit of volume" of the medium that this knowledge will be introduced and studied. For the case of dispersed phases with "particles" of various sizes, if necessary, we can also understand the distribution of sizes and velocities by defining and studying the number of particles of volume v (approximately dv). This will come down to considering a multiphase medium with an infinite number of phases, with each category of particle in the given volume able to be considered as a different phase.

In Chapter 2, we will describe the basic equations that represent a piecewise continuous medium instantaneously and locally. The definition of averages and the obtaining of equations related to equivalent continuous media will be discussed in the following chapters.

1.2. Eulerian–Lagrangian and Eulerian formulations

If we wish to study only situations in which, in a fluid, small quasi-spherical particles of another fluid or a solid are dispersed, one practical method that comes immediately to mind is simply to study the motion of the center of gravity of these particles, taking into account the friction between them and the fluid that surrounds them, which depends on their volume and shape, using specific formulas that experiments can validate. If it is not only their motion that interests us, but also their temperature, for example, we can also study this quantity by supposing that it is homogeneous in the particle in question. This brings us back to consider the total internal energy of the particle, and its dependence on exchanges of heat and possibly of mass with the outside environment, without necessarily having to specify whether these changes occur more on one side or the other side of the particle, and using more or less empirical formulas.

This point of view, which separates the carrier fluid, still considered as a continuous medium, from a certain number of isolated objects, has been highly developed. It is not too complicated to implement when the particles have a well-defined geometric shape and retain this shape permanently. It has been used to construct an Eulerian representation of a mean particulate medium (for example [JAC 00]) and also, especially, for the so-called Eulerian-Lagrangian approach, in which the continuous fluid medium is still described by Eulerian equations but the particles of the other phase are observed in a Lagrangian manner (see [OES 06]). Suspensions containing liquid drops or solid particles can be studied using this perspective, as long as the droplets or particles do not break apart or cluster together too much. Exchanges between particles and the continuous medium have been

studied in great detail, and these studies constitute a body of knowledge that is quite generally useful; we recommend, for example, [SIR 92] and [SIR 99]. The book by Crowe also provides a broad recapitulation of the knowledge and practices using this approach [CRO 01].

This particular point of view is not easy to generalize for types of multiphase media other than highly dispersed environments, and it is not inclined to allow a general theoretical approach, not least because it immediately introduces asymmetry into the description. Moreover, Lagrangian–Eulerian representation introduces significantly greater difficulties of application in situations where, as the number of particles is higher, these particles interact more with each other than with the fluid, and in situations where these particles do not have a constant and simple shape. There are, of course, methods for calculating the Lagrangian motion of solid parcels for granular media, taking into account the multiple contacts between these "grains", but these contacts are firmly localized; the fluid between the grains is not taken into account, and increasing the number and complicating the shape of the grains greatly increases the calculation time. The Lagrangian point of view has certain advantages, not the same as the Eulerian point of view does; it is beneficial to make use of these advantages, but the theoretical separation of a Lagrangian part and an Eulerian part has no general justification.

The unified approach we are looking for will be completely Eulerian: the macroscopic continuous medium equivalent to the multiphase medium, which is non-continuous by nature, will be described by Eulerian balance equations. The necessary models will, therefore, be studied first of all in a Eulerian context. Nevertheless, it is of interest, as we will see in Chapter 18, to use a Lagrangian translation of certain equations and models, which is closer to the Lagrangian–Eulerian representation, often used in practice. We will then show how to link the balance equations of the two approaches. In addition, the possibility of coupled calculations, in the various geometric zones of the space, between a model expressed in Eulerian form and a model expressed in Lagrangian–Eulerian form is also a promising practical solution.