Chapter 1

Understanding Uncertainty

1.1. Uncertainty and reality

Uncertainty is inherent to real life, whether it be natural or the result of human activities. Mankind has long been aware of the need to master this uncertainty; however, this awareness does not always lead to the development of a tried and tested methodology, particularly in the domain of mechanics. All too often, the existence of models that are entirely satisfactory in a well-established theoretical framework hides an inability to link these data or behavior models to existing information. Take, for example, the traditional notion of "safety" factors that, in simple terms, are the result of the expert analysis of a situation including unknown factors, without explicit description of the contents of this expert analysis. In this section, we will consider the necessity of developing awareness of uncertainty, highlighting the limitations of current knowledge and proposing a classification of uncertainty, which will be developed in the following sections.

1.1.1. *Awareness of uncertainty*

From the moment mankind became aware of the capacity for learning, we have been interested in the nature of observed events and the prediction of future events. Moving beyond animal reflexes, conditioned by the correspondence between the internal clock and the astronomical clock, which creates awareness of daily and seasonal cycles, human beings were able to contemplate the various events that interfered with these cycles: events that were considered to be uncertain – i.e. the results of chance or an accident – vague, little known or unknown. Our capacities for observation and reflection then came into play with attempts to understand and predict the rules subjacent to all non-ordinary events. Any deviation from predictable behaviors was attributed to chance. This led to a new question: does chance have a cause? The first answers came from religious sources, with the idea that the gods might find it amusing to interfere with human life, or to send down trials as punishment. The way to manage uncertainty, therefore, was through prayer and sacrifices, intended to appease the wrath of the gods.[1] The earliest philosophers, beginning with Socrates and Plato, considered the nature of knowledge, distinguishing between "visible" and intelligible knowledge. "Visible" knowledge, i.e. the knowledge accessible to the senses, is based on conjecture and conviction, whereas intelligible knowledge is based on science and believed to be genuine. Over the course of time, rational explanations emerged as a result of the human capacity to understand and explain phenomena. In the 18th Century, however, Emmanuel Kant expressed a concern that, however it might be expressed, science could never fully reflect real life. As our knowledge of the mysteries of nature progresses, we become increasingly aware of the limitations of our knowledge as it stands □. Kant's warning went unheeded by 19th Century scientists. Pierre-Simon de Laplace believed in causal determinism, as expressed in the introduction to his Philosophical Essay on Probabilities:

> *An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.*

Probability is only used to counteract gaps in our knowledge, and epistemic progress should lead to precision in predictions. At the end of the century, scientists were able to announce the completion of the "scientific conquest", with the exception of certain small details; early in the next century, however, these "small" details proved to be rather more significant than was first thought. At the end of the 19th Century, physics was based on

1 From a contribution by J.-Y. DANTAN [DAN 09].

two pillars: Newtonian mechanics, and Maxwell's ideas on electromagnetism [KLE 08]. Each theory appeared to be correct, but their principles were incompatible. The downfall of the 19th Century scientists was spectacular, as Henri Poincaré remarked in the early 20th Century with the introduction of the notion of chaotic behaviors: *A very small cause which escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance*. This reflection was the starting point for the idea of deterministic chaos, but Poincaré did not overstep the boundaries of the framework established by Laplace. The *deterministic fallacy* was highlighted by a sensitivity to initial conditions that can never be known with a sufficient level of precision. This error was made increasingly apparent in the work of John Von Neumann and Norbert Wiener on trajectories subject to noise, which led to the development of stochastic chaos as a tool for predicting reality. Deterministic calculation was finally condemned and limitations to reasoning were established by Kurt Gödel's incompleteness theorem. Nowadays, we know that our information is, and will always be, incomplete, proving a point made by Emmanuel Kant: *Someone's intelligence can be measured by the quantity of uncertainties that he can bear*. The acceptance of uncertainty in technologies was difficult, as expressed by Henry Le Chatelier, founder of the modern French chemical industry, in the early 20th Century [BAY 95]: *the hypothesis of chance offers an escape route to the incompetent, who shy away from taking a scientific approach*. However, uncertainty is real; it can be mapped, and this map can be explored using a scientific approach.

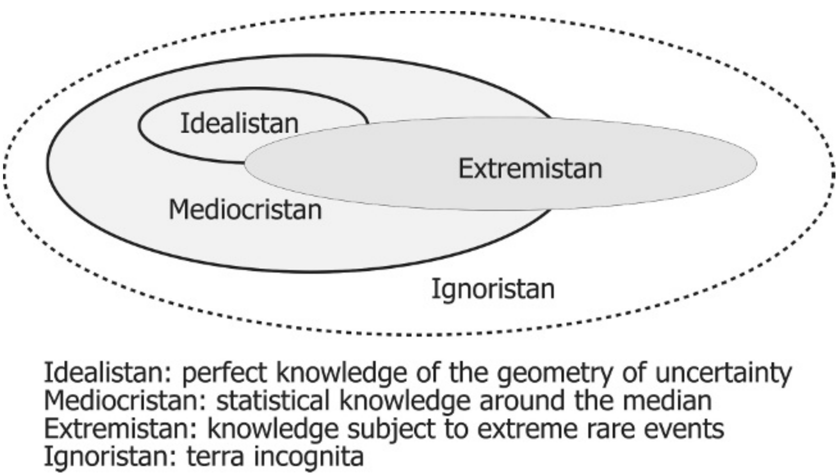

Figure 1.1. *Idealistan, Extremistan, Mediocristan and Ignoristan territories*

1.1.2. *Territories of uncertainty*

As a concept, the notion of the *geometry of chance* owes its existence to Blaise Pascal. "Chance" may be said to exist when an event has a random outcome. The geometry of chance has, regrettably, been absorbed into probability theory, i.e. proof theory; a bijection should not be made between chance and probability. Chance has a structure, and under certain conditions, this structure may be represented using probability theory. In such cases, it is important to identify the structure. Even before Pascal, all who thought about chance in any depth linked the notion to its implication in games. Games are a human invention, with rules and a set of events for which a probability space may be easily identified. These considerations mean that a probabilistic modeling is relevant in such cases. The generalization of these methods to abstract sets led to the perfect mathematical establishment of probability theory by Kolmogorov. However, while all is perfect in theory, it is impossible, with the exception of academic examples or game theory, to construct a geometry of chance. In mechanics, we are faced with a need to master uncertainty using information that must always be insufficient. Uncertainty is an intrinsic part of real life, and the structure of uncertainty is, in itself, uncertain.

Taleb [TAL 07] referred to the probabilistic model of economic chance as the *ludic fallacy*; as in the case of mechanics, this model of chance must always be based on incomplete information. Taleb established a distinction between two territories of uncertainty. The first, "Mediocristan"[2], refers not to a situation of mediocrity but to the median. In this territory, all events are located around the median, often close to the mean. In this case, new observations do not lead to significant modifications to acquired knowledge. Examples of this type include the average weight of the inhabitants of a country or manufacturing dimensions; at most, there will be slow evolutions over time. In this territory, the dominance of statistics and probabilistic prediction is unchallenged.

The second territory is "Extremistan", and in this area, extremely rare events can create significant modifications to the parameters of uncertainty. This is the case, for example, for seismic levels in France, established based on the background noise resulting from observations, which may be subject to sudden changes in the future. We wish to know whether the noise perceived

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2 Mediocristan refers to that which is located around the mean or the median. The term "Medianistan" may be considered preferable, as it eliminates the negative connotations of "mediocre"; however, we will retain the original vocabulary selected by Taleb.

over a period of several years can be extrapolated to a sufficient level. These considerations are dealt with in the domain of extreme statistics and in connection with risk: the consequences of passing a certain level are, however, seen to be acceptable by society or by individuals. In the last case, the mastery of uncertainty involves the decision to accept or reject the rare event, and not through a probabilistic hypothesis, which would need to be compared to that used for other events of the same type, which are not taken into account – such as the probability of a meteorite or satellite falling to Earth at a given location. The probability that a satellite, at the end of its lifecycle, with a trajectory that is completely unpredictable a few hours before falling, will fall on a given surface of 1 km^2 is 1.96×10^{-9} (the inverse of the area of the globe).

Taleb's mapping may be supplemented by the addition of "Idealistan", a territory where our knowledge of the structure of chance is perfect, inhabited by game theory and by designers who believe that reality is presented on a computer screen, and "Ignoristan", inhabited by all unimagined events. In this case, we need to use ignorance exploration techniques, such as failure mode, effects and criticality analysis (FMECA), functional analysis, quality assurance and any number of imaginable tools, as discussed by Ligeron [LIG 06]. Figure 1.1 illustrates the territories of uncertainty where a form of classification may serve as a guide.

Thunnissen [THU 03] explored the perception of the notion of uncertainty in a large number of disciplines as seen in the published literature. He highlighted major differences in understanding according to domains, producing a classification that will be discussed below.

The first cause of uncertainty is *ambiguity*, which is the result of a lack of precision in language, in a project, in design and in feedback, where descriptions are often too vague. Words are random variables for which the common meaning differs between different communities and is subject to deviation. Uncertain ambiguity should not be confused with the reasoned choice not to provide precise values during the design phase, so as to allow a certain degree of freedom. Within the aeronautics industry, a simplified language (*simplified English*) (http://en.wikipedia.org/wiki/simplified_english) was created in order to guarantee perfect understanding in all areas, particularly for maintenance purposes. From a formal perspective, the theory of fuzzy logic allows us to attribute a degree of trust to an affirmation when dealing with ambiguities.

The second cause of uncertainty results from the *random* or *aleatoric character* of certain information. The word "random" arises from the French term "root", which means "to run wildly", generally used for horses; it is the equivalent of the French term "aléatoire", from a Latin word referring to the game of dice. The word "stochastic" arises from a Greek term, which means "soothsayer".

Aleatoric uncertainty (or stochastic uncertainty, used as an equivalent term) concerns variations inherent to a physical or mechanical system and its environment. Is this uncertainty real, or simply the result of our ignorance? It is intrinsic to the quantum model, as stated in [AGU 13], but in mechanical engineering, this uncertainty is purely the result of our inability to access and use all data.

This uncertainty is therefore impossible to control, and constitutes a background noise, which may vary within a domain of more or less fixed boundaries. Expert opinions or physical considerations help us to set these limits, and probability theory provides a structure for the distribution of variations, a structure that we then need to identify. Random uncertainty is irreducible and objective. The dimensions of pieces during fabrication and the properties of materials are examples of this kind of uncertainty. Probability theory produces a perfect model of random knowledge, but the structure of this random aspect remains approximative, except in the case of game theory. "Aleatoric" is not, therefore, necessarily equivalent to "probabilistic".

Epistemic uncertainty concerns any lack of information, which may be remedied by measurements, appropriate actions or decisions. It is therefore reducible and subjective, and may be classified in relation to two main sources:

– The *stochastic modeling of design variables*: each variable possesses a description that depends on the level of knowledge. This depends on the advancement of the project: "the material is steel", and on the depth of physical knowledge: "its characteristic elastic limit is 240 MPa". According to the advancement of the project and the scale of observation, it may be represented using different entities (section 2.1).

– The *physical modeling of behaviors*: the description of a phenomenon using a mathematical model benefits from a rigorous and coherent approach, but involves the use of hypotheses that may be more or less well validated. The solution to the model, which is generally numerical, only gives an approximation of the solution and may even include errors in the programming of the algorithms. While errors may be avoided by careful quality assurance, the representation of gaps (and not errors!) between the model and the physical reality remains a subject for research. Recalibration methods are most effective on a case-by-case basis, but they do not allow us to map gaps across the domain of validity. The lack of phenomenological knowledge is also a source of epistemic uncertainty, and plays a particularly important role in cases of innovation in a mechanical system or in a new environment. In these cases, the designer only has access to subjective estimations (section 2.2), and we should remember that the behavior may fall within the territory of Extremistan.

The solution to epistemic uncertainty, i.e. the production of a certain representation, is clearly impossible in cases where an uncertainty is represented by a continuous variable in a domain, even if the domain is bounded. In this case, the inclusion of successive information allows us to construct a model with parameters that are themselves tainted with random uncertainty, and the introduction of new information causes the process to repeat itself. The representation of epistemic uncertainty thus appears to be fractal and always results in random uncertainty. Epistemic uncertainty would, however, be solvable if (following Laplace) a higher form of intelligence proved able to acquire the information needed for exhaustive knowledge of the space of possibilities; that being said, Poincaré reminds us of the issue of sensitivity to initial conditions. One example is provided by clearance mechanisms, where a random drawing selects one of a finite number of identifiable combinations [DAN 12].

Other sources of uncertainty exist, which will not be examined in depth in this book. *Volitional uncertainty*[3] results from human behaviors over which the designer has little or no control when specifying possible choices. It concerns design choices where the exhaustiveness of solutions has not been reached, the requirements that individuals may introduce independently, the way in which the will of the user may be expressed in given conditions and crude errors. Finally, *interactive uncertainty* concerns the simultaneous occurrence of certain events or disciplines, or potential "forking" from one behavior to another. If we accept that a single, clearly identified cause almost never leads to a highly undesirable event, as its simple identification leads to protection measures being taken, extreme events are generally the result of multiple causes, none of which would have extreme effects if taken separately.

The distinction between aleatoric and epistemic uncertainties tends to disappear when uncertainty is represented by a model. If an uncertainty is

3 Volition is an act in which something is determined by the will. In other terms, it designates the finality of a process where a human being makes use of will. This is the event by which an individual is "able to act" in relation to an internal or external result. In common parlance, volition may be assimilated to the formulation of a choice (http://en.wiktionary.org/wiki/volition).

aleatoric and represented by a chosen probability density function (PDF), then no additional information should be able to change this choice, even though it was chosen based on incomplete information. The choice of a PDF is therefore a matter of epistemic uncertainty, in which we should include physical knowledge, for example whether or not the density is bounded. We also need to include the results of the composition of uncertainties: Weibull's law for materials, for example. Epistemic uncertainties may be represented by probability densities when sufficient information is available to define the first statistical moments. In this case, the parameters of the densities are known and a probabilistic model may be chosen. As highlighted in [KIU 09], the distinction between epistemic and aleatoric uncertainties is a result of choices made during the modeling process.

Finally, for any type of uncertainty, modeling converges toward a PDF; this attitude may be compared to placing the mastery of uncertainty under the "light"[4] of probability theory. Failing that, we need to be able to master heterogeneous information combining cognitive, heuristic and algorithmic knowledge.

1.1.3. *Conclusion*

The awareness of uncertainty leads to recognition of the *deterministic fallacy*. Uncertainty is inherent in life and in mechanics, and it must be mastered. To do this, we need to clearly distinguish between situations: those where we have a large amount of information – which belong to Mediocristan and for which we have a probability map – and those that result from rare events, or even extrapolations – which belong to Extremistan – with a corresponding risk map. A situation is uncertain if the outcome may take different forms, which may or may not be identified within the probability structure that defines the associated random variables. While this theory defines a perfect mathematical framework based on a stochastic data model, exhaustive knowledge is never possible as mechanics is not subject to the strict rules of game theory, nor do we possess samples of the size used in opinion polls, for example. Avoiding the *ludic fallacy* is significant. The outcome of an uncertain test is therefore represented based on the available

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4 The Lost Key: Night time, in a street, near a street lamp. A man is standing, looking at the ground, and appears to be looking for something. Another man walks by and asks: "What are you looking for?" – "I'm looking for my key". – "You've lost your key?" – "Yes". – "You lost it here?" – "No". – "But if you lost it somewhere else, why are you looking here?" – "Because there's light here" [CAR 98, p.354].

knowledge using different models, which we will examine in 2.1, whose limit may be a random variable or stochastic process.

The following sections will be devoted to the notions of robustness and reliability, and to the consideration of uncertainty within the design process. We will then go on to explore the territories of uncertainty in more detail (sections 2.1 and 2.2).

1.2. Robustness and reliability

A clear understanding of uncertainty is essential for mechanical engineers who wish to develop robust and reliable products.[5] In this section, we will analyze the relationship between uncertainty, robustness and reliability, attempting to clarify issues of vocabulary, which constitute a primary cause of uncertainty. We will then consider the question of optimization.

1.2.1. *Robustness*

1.2.1.1. *Definition and measure of robustness*

In the popular meaning, robustness is the quality of something that is robust and the word "robust" is associated with strength: that which has the necessary strength to resist events is robust. Antonyms include words such as weak and feeble: to be in a robust state of health is to have strong resistance to uncertain potential attacks by disease. A design is robust if it has the capacity (the strength) to resist uncertain and/or unexpected events that the object may encounter in the course of its lifecycle. The word "robustness" takes on different meanings for different disciplines (http://en.wikipedia.org/wiki/robustness): robust algorithms in numerical analysis converge toward the right solution in all circumstances; robust estimators in statistics converge to the desired parameter as the size of the sample tends toward infinity; and in mechanics, robustness refers to the solidity of a material or product (although this notion is more closely connected with the idea of reliability). Robustness also refers to *performance stability in engineering*. We will retain this latter perspective, as it characterizes the quality of adaptation of a product or performance to chance factors, highlighting its stability, but not necessarily its optimality. We will therefore adopt the following definition:

5 Subject discussed with N. GAYTON.

In an engineering context, a robust design is one that determines the nominal parameters of a product or a system so that its performance is sufficiently insensitive to any uncertain event, which it may encounter in the course of its lifecycle.

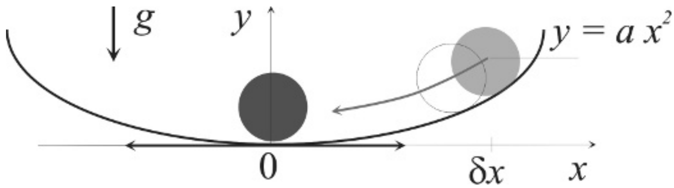

Figure 1.2. *Illustration of the principles of robustness: ball bearing in a cup*

This definition is simple to understand in the case of dynamic systems, where any perturbation will create an opposing force drawing the state back to the point of equilibrium (e.g. a ball bearing in a cup, an automobile suspension system, the elastic stability of shells or control systems). It is more difficult to understand in a static context. Two different modes of robustness coexist. The first mode is linked to the behavior model (*robustness of behavior*) and the second mode is linked to the nominal parameters of the system (*robustness of parameters*). Let us take the example of a ball bearing in a parabolic cup. The position parameter is x, which implies a performance $y = a\,x^2$. Clearly, any solution $x \neq 0$ will be unstable in the gravitational field g as the mechanical behavior of the system leads to a return to the initial position following any variation δx, unless δx exceeds a threshold where the ball leaves the cup. This shows that robustness may be limited by the variation of parameters.

The formal expression of this property is based on the notion of virtual variation. Let us consider a set of design parameters $X_1, X_2, ..., X_n$, a *response* defined by a *variable of interest* $Y = \mathbf{M}(X_1, X_2, ..., X_n)$ and $(\delta X_1, \delta X_2, ..., \delta X_n)$, an admissible virtual variation of the design parameters. Thus:

$$Y(X + \delta X) = Y(X) + \sum_i \left.\frac{\partial \mathbf{M}}{\partial X_i}\right|_X \delta X_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial^2 \mathbf{M}}{\partial X_i\,\partial X_j}\right|_X \delta X_i\,\delta X_j + ...$$

The first term of the expansion expresses the level of the response. To know whether or not this is sufficient, we need to consider the second mode of robustness, and this question is examined in the following section in connection with reliability. The second term (the gradient) represents the *sensitivity* s_i of the variable of interest Y as a function of the parameters X_i:

$$s_i = \left.\frac{\partial \mathbf{M}}{\partial X_i}\right|_X$$

The components s_i of the sensitivity vector, which are dependent on the dimension of the variables, are therefore not directly comparable and it is preferable to introduce the notion of *elasticity* defined by:

$$e_i = \left.\frac{\partial \mathbf{M}}{\partial X_i}\frac{X_i}{\mathbf{M}(X_1, X_2, ..., X_n)}\right|_{(X_1, X_2, ..., X_n)^{(r)}}$$

where $(X_1, X_2, ..., X_n)^{(r)}$ is a point of reference: the mean, the median or the design point associated with the maximum likelihood of the failure [LEM 09]. The elasticity measure is therefore a one-dimensional vector, which depends on the calculation point.

Finally, the Hessian matrix $H_{ij} = \frac{\partial^2 \mathbf{M}}{\partial X_i\, \partial X_j}$ shows stability, whether positively or negatively defined, according to the context. As an illustration, let us consider the case of a given need (or internal strength or stress) S that must be satisfied by dimensioning a resource (or resistance) $Y(X) > S$ (Figure 1.3); the opposite situation is also possible. A performance with zero sensitivity is connected to each of the design points P_0, P_1 and P_2, but only P_0 is robust as the resource increases as a result of a variation δP_0. Matrix H is thus defined as positive; it would be negative in the case of a need with a given resource. Behavioral robustness cannot exist without extrema.

In cases where dimensioning implies zero sensitivity, robust behavior is only present if, for any admissible virtual variation in parameters:

$$\forall \delta X_i \text{ admissible}$$

$$\delta dY = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 \mathbf{M}}{\partial X_i\, \partial X_j} \delta X_i\, dX_j \left|\begin{array}{lll} < 0 & \text{if} & \text{need} \\ > 0 & \text{if} & \text{resource} \end{array}\right.$$

where dX_j is a differential increment. Note that this criterion is local for small perturbations around the calculation point. If the expression is null, we must look for higher order derivatives. Points P_1 and P_2 give stronger resources and their use helps to satisfy reliability criteria.

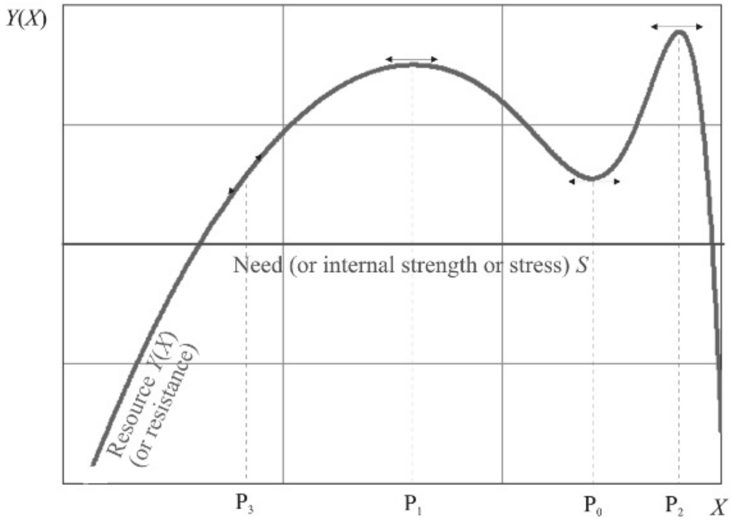

Figure 1.3. *Illustration of robustness – case of a resource and a fixed need*

If performance around a criterion is monotone, then matrix H is defined as neither positive nor negative and is considered to be null in cases of linear behavior. The notion of robust behavior therefore disappears. However, the sensitivity indicates the direction of variation to take toward a better response, as illustrated by point P_3. Thus, the choice of height for a dam may not be robust in behavioral terms, but will simply ensure a certain reliability in relation to the uncertain factor of water levels.

This analysis demonstrates the notion of "*sufficient insensitivity*", introduced in the definition. A design presents *robust behavior* if the sensitivity is such that any perturbation will increase the deviation between resource and need. Robustness therefore depends on the possibility of including non-monotone evolution of parameters in a design. It does not depend on uncertainy, but on *sensitivity analysis*. Where this is not possible, a design will be conditionally robust in connection with identified parameters within a given interval, showing a link with the idea of reliability.

1.2.2. *Reliability*

1.2.2.1. *Definition and measure of reliability*

The term "reliability" expresses ideas of trust and confidence. One definition, used in slightly different forms in a variety of domains, is as follows:

> *Reliability is the capacity of a part of equipment or a product to carry out its function in the required conditions of use over a fixed period of time.*

A quantitative measure is sometimes added to this qualitative definition, referring to probabilities; this will be discussed below.

Reliability is an essential component of reliability, availability, maintainability and safety (RAMS), which includes reliability objectives alongside availability and maintainability criteria in relation to the expected performance, while respecting safety constraints linked to the risks presented by the product to its users or to its environment.

Practical reliability is subject to correct usage procedures, and is associated with quality assurance; theoretical reliability is a means of prediction using mathematical models to estimate the chance of successful performance (demonstration of a resistance function, conformity of production to specifications, allocation of reliability to components in a system, etc.).

The idea of success and failure encourages the use of probability to measure reliability, to the point where certain definitions link the two notions; however, we should remember the importance of *notional probability*, i.e. probability conditioned by the available stochastic model. If these precautions are respected, then probability becomes an essential measure of reliability. In Mediocristan, experimental feedback from relatively large samples may be used to validate the experimental frequency – probability relationship. This is not the case in Extremistan, where feedback is limited to censored occurrences that did not lead to failure. Reliability can thus only be seen as a measure of the state of a system, for which a sensitivity analysis can provide us with information concerning the most influential parameters.

In *reliability analysis*, a variable of interest Y is compared to a threshold $\mathcal{T}$ that may be set at zero. In this case, we are no longer looking at stability, but at a distance from the threshold. The probabilistic measure of reliability is thus the probability of the event:

$$E = \{\text{response } Y = \mathbf{M}(X) > \mathcal{T}\}$$

where Y represents a positive performance. The probability of success becomes:

$$P_{\text{success}} = \text{Prob}\{Y = \mathbf{M}(X) > \mathcal{T}\}$$

and the probability of failure P_f becomes:

$$P_f = 1 - P_{\text{success}} = \text{Prob}\{Y = \mathbf{M}(X) \leq \mathcal{T}\}$$

P_f is often referred to as a reliability measure; however, it is actually a measure of *unreliability*.

In mechanics, a stress S is covered by a resistance $Y(X) = \mathbf{M}(X) > S$, and we use $G(X, S) = G(\mathbf{M}(X),\ S)$ to denote the function such that $G(X, S) > 0$ represents success and $G(X, S) \leq 0$ represents failure. The conditional measure of reliability is thus P_f =Prob$\{G(X, S) \leq 0\}$, given the stochastic model of X and S.

The example in Figure 1.4 illustrates this idea. The stress is presumed to be fixed at a value of S_i for each point, such that we obtain a positive margin $M_i = Y(P_i) - S_i$. Let $f_Y(y)$ be the probability density of the resource at P_i. At P_0, there are no realizations of P_0 that give $G(Y, S_0) < 0$, without looking for an extremely broad distribution. At P_1 and P_3, we have $G(Y, S_{1 \text{ or } 3}) < 0$, and the gray zone illustrates failures, one associated with zero sensitivity the other associated with monotone behavior.

Figure 1.5 shows the respective densities of an uncertain requirement $f_S(x)$ and uncertain resource $f_Y(x)$, and the hatched zone shows the probability of a resistance being inferior to the stress. The notional probability calculation is thus given by:

$$P_f = \cup dP_f = \int_{\mathbb{R}} f_S(x)\, F_Y(x)\, dx$$

where $F_Y(x)$ is the distribution function of Y.

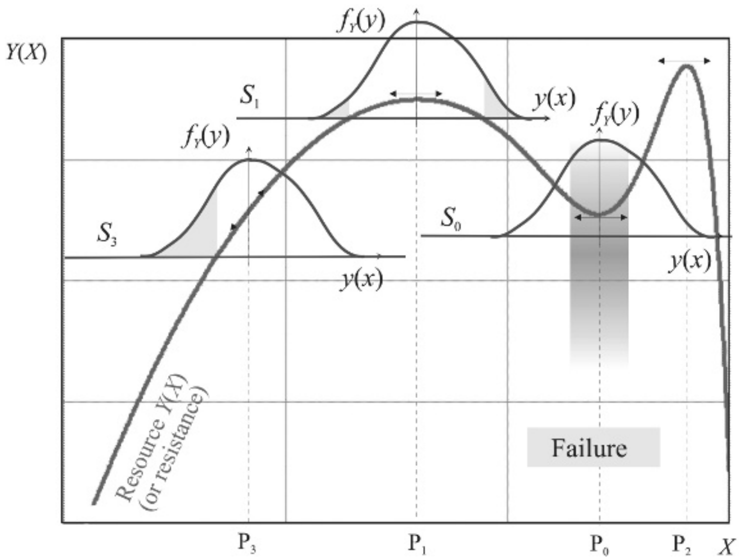

Figure 1.4. *Illustration of reliability in the case of a resource responding to a given requirement*

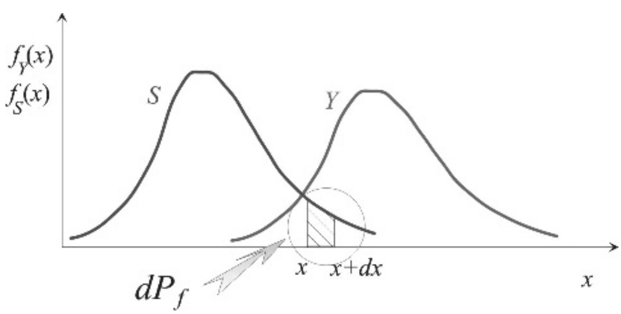

Figure 1.5. *Probability densities of the stress and the resistance*

According to the complexity of the response and the threshold, the calculation of P_f, which is a "simple" integral, will require tailored methods; the direct application of the Monte Carlo simulation [LEM 09] is too expensive in terms of calculation resources.

Although *robustness* is characterized by sensitivity, *reliability* is characterized by the *distance* between the performance function and a fixed threshold. In probability theory, this distance is measured by the probability of failure or by an equivalent reliability index β such that $P_f = \Phi(-\beta)$ where Φ designates the normal distribution with zero mean and variance 1. This index presents the advantage of a logarithmic scale, which is better suited to design sensitivity in relation to risk taking. Other distance measures may be used, the simplest of which uses a safety factor that imposes a deviation between characteristic values of the resistance and the stress □.

1.2.3. *Relationship between robustness and reliability*

1.2.3.1. *Three situations of robustness and reliability*

Let us consider the illustration in Figure 1.6 and examine the solutions as a function of the design points for stresses S_i. We will define two types of parameter variation intervals as follows:

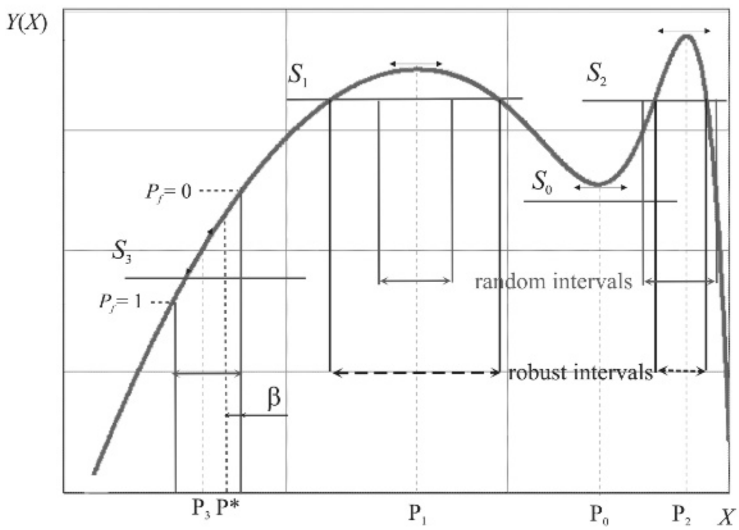

Figure 1.6. *Comparison of robust and reliable situations*

– An interval is said to be robust if the bounds of parameter variations are considered to be certain, without prejudice for the existence of a distribution within this interval.

– An interval is said to be random if it corresponds to the support of the distribution of the parameter.

Let us consider the following three situations represented by a design with points P_i:

– Point P_0 shows the robustness of the ball bearing in the cup. Its reliability is absolute in a large vicinity and the margin is always positive. The design is robust in relation to behavior. Point P_0 is chosen as a dimensioning point and the margin at this point is zero. It represents an ideal robust and reliable situation.

– Point P_1 shows a situation for which large intervals of robustness may be defined. The reliability is total if the random interval is contained within the robust interval. The design is robust in relation to its parameters. The dimensioning point is P_1 and the margin is such that the random and robust intervals are mixed together. The generalization of this approach will be considered in section 2.1.2.3, associating robustness with convexity.

– For point P_2, the robust interval is too limited to contain the random interval; the design is not robust and presents a certain level of reliability. The dimensioning point is P_2 or in the vicinity of P_2 and the margin is defined by the required reliability level.

– Point P_3 is neither robust nor reliable. The dimensioning point P^* is chosen at a distance $\beta = P_3P^*$ in order to give a certain level of reliability. In the figure, if the random interval is bounded, then the dimensioning may be totally reliable.

Robust and reliable design involves two subjacent hypotheses. The first hypothesis requires a behavior model with a positive (respectively, negative) curve, and the second hypothesis requires the existence of a guaranteed interval of robustness. The only remaining uncertainty is thus that of the model. Situations of this kind are extremely rare in practice, and designers must accept a certain level of risk, a level which depends both on the occurrence of the undesirable event and on its consequences.

1.2.3.2. *Reliability and sensitivity*

In an ideal world, we would be able to design in such a way that the resistance, or the stress, would always be a function presenting extrema. In practice, most of the design points we encounter are in the situation of P_3 with an increasing or decreasing slope, measured by its sensitivity s_i, as in the case of robustness.

Distance β is a measure associated with probability. An approximation of this measure is given by the following calculation, which demonstrates the role of sensitivity. Let us carry out the first-order Taylor expansion around the mean m_X of the variables:

$$G(X,S) = G(X,S)|_{m_X} + \underbrace{\left.\frac{\partial G(X,S)}{\partial X}\right|_{m_X}}_{\text{Sensitivity } s_i} (X - m_X) + ...$$

Thus:

$$\text{mean of } G : m_G = G(X,S)|_{m_X}$$

$$\text{variance of } G : \ \text{var}_G = \left[\left.\frac{\partial G(X,S)}{\partial X}\right|_{m_X}\right]^t [\text{cov}_X] \left[\left.\frac{\partial G(X,S)}{\partial X}\right|_{m_X}\right]$$

The first idea of a reliability measure by an index was proposed by Rzhanitzyn [RZH 49], and then further studied by Cornell and Benjamin [BEN 70]. This index is the relationship between the mean and the standard deviation (the inverse of the coefficient of variation):

$$\beta = \frac{m_G}{\sqrt{\text{var}_G}}$$

β is a one-dimensional measure showing the number of standard deviations between the mean (or median) point and the failure point at a given level of reliability, generally a few units, corresponding to the common engineering practice of shifting experimental results by a few standard deviations. It is linked to the probability of failure by the following relationship:

$$P_f \approx \Phi(-\beta) \qquad [1.1]$$

where $\Phi()$ is the normal distribution function. If G is a linear function of X Gaussian variables, then approximation [1.1] is precise. Calculated at a robust optimum point, the variance of G is null as $s_i = 0$ and $P_f = 0$.

This first approach illustrates the connection between robustness and reliability through the use of sensitivity. It was extended to include broader hypotheses [DIT 96, LEM 09], which led to the specification of the sensitivity at the design point (at the point of maximum likelihood of failure, rather than at the mean) and to the specification of the estimator trust interval, i.e. a reliability measure that is robust when faced with uncertainty.

1.2.3.3. *Robustness in systems*

Given the impossibility of designing a robust product based on the behavior of a single component, we must consider the possibility of redundancy, enabling systems to continue to operate, at least at reduced capacity. Redundancy may be active, for example if two elements are responsible for the same function, where one would be sufficient. In the course of normal operations, the two elements receive a relatively small load; in the (highly unlikely) case of failure of one of these elements, the second component would take over the full load. Clearly, these elements need to be independent. Passive redundancy is where a secondary element, in a state of standby, is only activated in case of failure of the primary element.

This idea of redundancy is illustrated by practices in the automotive industry in relation to tire failures:

– Case 1: vehicles are equipped with a spare wheel, which is identical to the wheels used on the vehicle. The wheel is changed and normal operations resume (unless a second tire blows).

– Case 2: vehicles are equipped with a flat spare wheel, which offers reduced levels of performance when compared to the normal situation. The wheel is changed and operations resume at a limited level, i.e. with lower acceptable maximum speeds.

– Case 3: vehicles do not carry spare wheels, but the driver has access to a call center. Operations may therefore be resumed, but only after a certain period of time has elapsed.

In aeronautics, the *fail safe* concept means that a failing element must not have consequences for safety, but not that normal operations must continue. Dimensioning of this type requires the use of redundant effort transmission pathways.

These examples show that the fulfillment of a required function must be subject to a systemic analysis of the consequences of potential failure.

1.2.4. ***Optimizing robustness and reliability***

We have now considered robustness and reliability based on performance satisfaction, without looking at notions of performance optimization. Moreover, some only consider a design to be robust if it is optimal, requiring us to measure optimality. Taguchi [TAG 89] worked on this question in the late

1980s, creating a methodology that reduces development time and costs before a product is launched [CLE 00]. This methodology is based on optimizing robustness during design, using data and a measurement obtained through experimentation. Two types of parameters are included in the performance function Y:

– $\{X\}$, vector of the control parameters available to the designer and used for optimization;

– $\{\xi\}$, vector of the noise parameters over which the designer has little influence.

Let $\hat{Y}$ be the target and κ a set of experiments with fixed $\{X\}$. The root mean square deviation measures the dispersion due to the parameters $\{\xi\}$:

$$\text{RMS} = \text{E}\left[\left(Y(\{X\},\{\xi\}_\kappa) - \hat{Y}\right)^2\right] = \sigma_Y^2 + (m_Y - \hat{Y})^2 \approx \frac{1}{\kappa}\sum_{i=1}^{\kappa}\left(Y(\{X\},\{\xi\}_\kappa) - \hat{Y}\right)^2$$

where m_Y and σ_Y^2 are the mean and the variance of Y, respectively. Taguchi proposed the use of the signal/noise ratio as a measure:

$$\text{SN} = -10\log_{10}(\text{RMS})$$

where the optimal robust design is the solution $\{X\}^{opt}$ that maximizes SN with noise $\{\xi\}$.

A key contribution made by this method is the solution of the optimization problem using a carefully chosen range of experiments. While Taguchi's focus on the most robust design, rather than simply robust design, is commendable, the value of the SN measurement is debatable. Moving beyond this experimental method, Beyer and Sendhoff [BEY 07] highlight the need to develop a measure of robustness including representations (deterministic, probabilistic and possibilistic) and tools (worst case, stochastic calculation and genetic algorithms), which we will discuss in greater detail in the following chapter. The authors also highlight the fact that the maximization of performance at the point of design and the minimization of variance across a wide domain of operating conditions often constitute two contradictory aims.

The two keys to robust optimization are the choice of a function measuring robustness, for example the signal/noise relationship or the reliability index, and the means of solving the optimization problem, by experimentation or by algorithms such as those identified by researchers in reliability-based design optimization (RBDO) [ENE 94b, KHA 02, CHO 07]. Algorithms without tests, however, are as limited in their value as tests without models.

1.2.5. *Conclusion*

A product is robust if it is sufficiently insensitive to all uncertain events. It is only totally robust if the dimensioning point is situated in a particularly favorable version of a behavior model, which it is not always possible to design. Otherwise, it can only be totally robust if the parameter definition intervals are absolutely certain. *The essential measure associated with robustness is sensitivity*.

A product is totally reliable if the undesirable event is impossible, either because the product is totally robust or because the random intervals are contained within the intervals of robustness. Otherwise, the product will be reliable to a certain level, generally measured using a notional probability. *The measure of reliability is associated with a distance*.

Table 1.1 gives a summary of possible situations. It highlights the importance of the behavior model for robustness and the uncertainty model for reliability. The notions of robustness and reliability, illustrated here by the case of a resource needed to balance a requirement, must be extended to all functions of design variables, including estimators such as probability itself and the statistical moments of variables.

		Robustness	**Reliabilty**	
Behavior with minimum (zero sensitivity) $s = 0$		Total	Perfect	Insensitivity to uncertainty, except that of the behavior model, margin null
Behavior with maximum (zero sensitivity) $s = 0$	Robust interval containing the random interval	Around the design point	Perfect	Requires certainty of the robust interval, margin defined by the robust interval
	Random interval containing robust interval		Reliability measure	Requires knowledge of the random interval, margin defined by the reliability measure
Monotonic behavior $s \neq 0$	Random interval		Reliability measure	

Table 1.1. *Situations of robustness and reliability for the dimensioning of a resistance*

The schema presented in this section remains static, and needs to be extended to cover the full product design and lifecycles, introducing robustness into design, production, service in normal mode and operations in degraded states in relation to maintenance activities.

1.3. Designing for robust production

In the real world, engineering takes place in an uncertain environment (section 1.1) and the aim of designers is to define products that will be robust and reliable (section 1.2) over the course of their lifecycle. Our focus in this section includes the methodological elements of an approach that aims to create a robust product design process, in order for the production to be robust and for the product to be robust within its environment and conditions of use. After a brief consideration of various development cycle models, we will look at the classic V cycle in greater detail, highlighting sources of uncertainty in the model itself and in the course of a typical step in the cycle. The implementation of this approach will be illustrated using diagrams for predicting robustness and mastering uncertainty. The scientific and technical content of this approach will be discussed in later sections in relation to the modeling of uncertain data (section 2.1) and of behaviors (section 2.2).

1.3.1. *Robustness and lifecycles*

The consideration of uncertainty through realistic design [DAN 09] forms part of a global product lifecycle management (PLM) approach, from design to production, which allows permanent validation and virtual certification of the model produced as it will be created and as it will be used[6]. Mastering uncertainty is strategically important when making decisions and implementing actions in the product design and the manufacturing process, in order to reduce the length of the design cycle, optimize industrial procedures and estimate risks, i.e. the consequences of uncertainty, before any decision is made.

To clarify the role of uncertainty in design, we must begin by positioning key concepts in the context of a product development approach. Several models of development cycles are used, including the V cycle, the cascade and the spiral (Boehm) cycles [FOR 91, TOL 98, THÉ 03]. These cycles

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6 Text based on a contribution by J.-Y. DANTAN, discussed with R. BIGOT and A. ETIENNE.

include all of the phases of verification and validation. The integration of these phases reminds us of the presence of uncertainty: we must verify that specified requirements have been correctly understood, and validate the proposed responses. The spiral cycle places a greater emphasis on risk management than the other two models. The *V* cycle remains the best-known of these models, and will be used as a basis for the analysis presented in this section.

"Designing for robust production" is based on several approaches with slightly different objectives: robust design, reliability-based design, risk-based design and tolerancing:

– The objective of robust design is to identify the solution that minimizes the effect of uncertainty on performance, while guaranteeing required performance levels and respecting requirements.

– The objective of reliability-based design is to demonstrate an acceptable probability level for system failure (i.e. the probability that certain system requirements will not be fulfilled during a given period of time).

– The objective of risk-based design is to identify a solution that minimizes the maximal criticality induced by failures during a given period of time.

– The objective of tolerancing is, for a fixed solution, to determine geometric specifications and allocate tolerances that minimize cost (or another performance criterion), while guaranteeing the required levels of performance and the respect of requirements.

These approaches are not mutually exclusive, and provide complementary views of the effects of different uncertainties. They are included in different phases and stages of the *V* cycle, with a main focus on robust design.

1.3.2. *Description of the V cycle*

The *V* cycle shows the relative positions of the different phases of product development, from specification to validation. In Figure 1.7, the horizontal axis represents time and the vertical axis represents the level of integration: system – parts – components. The descending portion of the *V* cycle includes a functional analysis/specifications and a design phase; the point of the *V* corresponds to the manufacturing and production phase, and the rising part of the cycle shows the different phases of verification and validation. Figure 1.7 (adapted from [AFI 13]) gives a classic view of the *V* cycle, including steps for "client requirements", "responses: physical solutions", etc., and showing a mechanical engineering approach to product development activities: “technical definition of requirements”, “concept research”, etc. [SCA 04].

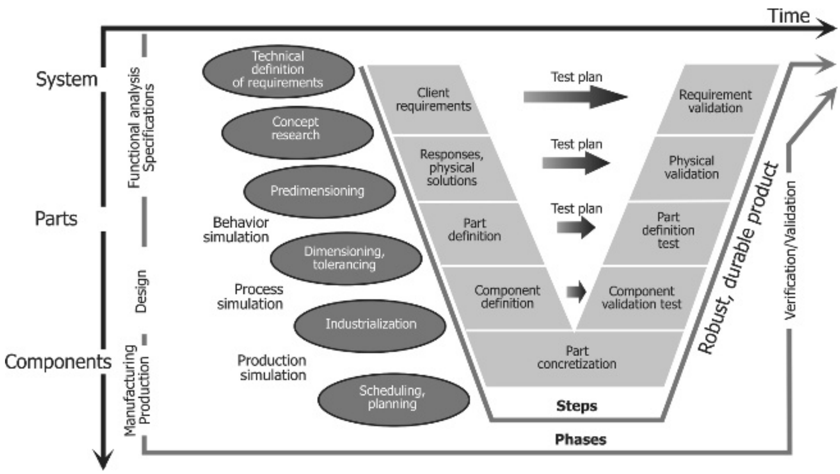

Figure 1.7. *V cycle, completed from a mechanical engineering perspective*

The functional analysis/specifications phase focuses on the definition of requirements, functions, product specifications, etc.; the preparation of a quality plan, validation plan, etc.; and the definition of the desired product reliability level. This phase only covers one step of the V cycle: “client requirements”, also referred to in mechanical engineering as “technical requirement definition” or “functional analysis” (Figure 1.7).

The design phase then begins by a search for physical solutions (concepts), then the definition of parts (predimensioning), the definition of components (dimensioning, tolerancing, etc.), not forgetting industrialization, sequencing and production planning. This phase runs alongside the definition of test plans and risk analysis activities. The concretization phase (or realization, in the context of systems) is often known as the manufacturing or production phase in the case of manufactured products. The final phase, verification and validation, involves the verification of all components, organs and/or systems in relation to the design, and functional validation – an important phase – in which we consider the functions and quality level of the product in relation to the specification and the requirement analysis:

– Verification: “confirmation through tangible proof that the specified requirements have been satisfied”; verification is a response to the question “are we making the product right?”.

– Validation: “confirmation through tangible proof that the requirements for a specific use or planned application have been satisfied”; validation is a response to the question “are we making the right product?”.

This brief overview of the *V* cycle introduces the need to consider uncertainty in all of the phases, from specification to validation, particularly for the “definition of the desired product reliability level”, “risk analysis” and “verification and validation”.

1.3.3. *Uncertainty in the V cycle*

Studies of uncertainty are generally classified according to the origins of the uncertainty:

– Uncertainty relating to the product and its conditions of use (with a direct impact on the performance of the product).

– Uncertainty relating to the overall development process (with an impact on the advancement of the project – uncertainty linked to the project, concerning human operators, activities, planning, etc.).

The analysis given here will concentrate on the first type of uncertainty, although, in reality, the borderline between the two types is extremely flexible. We will now look more closely at the verification and validation phases, highlighting the impact of uncertainty.

1.3.3.1. *The verification phase*

The purpose of the verification phase, for each component of a product or mechanical system, is to detect potential non-conformity to specifications arising from imprecisions inherent in the production process (commonly referred to as manufacturing imperfections) and from uncertainties present in the industrialization phases. The interpretation of component specifications during the industrialization phase can be a cause of non-conformity, but ignorance of certain aspects of the impact of fabrication phenomena on the component is also important. This leads to the classification of uncertainties mentioned in section 1.1.

Uncertainty linked to ambiguity and to the interpretation of data originating in the early phases of the design process (component specifications). The activity of specification interpretation by actors in the production process is not simply a matter of encoding or decoding information, and involves the use of normalized language and "trade" practices. The designer and creator are involved in dual processes of production and interpretation of these specifications, which may be affected by three factors:

– Under-determination or the incomplete nature of component specifications: in this case, it is impossible to reconstruct a meaning from specifications, or this meaning is too general.

– Over-determination, or an excess of specifications: in this case, the interpreter is unable to understand the whole of the specifications, which include several implicit or explicit complementary or contradictory meanings; two types of implicit meanings exist: presuppositions and inferences.

– The use of unclear language, linked to the "vague" nature of certain terms.

Uncertainty linked to earlier phases: manufacturing defaults or imperfections. This is the best-known type of uncertainty. For the purposes of clarification in relation to these cases, we may use an axiom developed by Srinivasan [SRI 99] and Mathieu [MAT 07]:

– *Production variability axiom*: "no-one is capable of producing a component in exact conformity with its functional definition (absolute accuracy); moreover, no one is able to exactly reproduce a pre-existing component (imperfection)".

In other words, it is not possible to produce a component with characteristics identical to those of the target defined by the designer, nor for two components in the same batch to present strictly identical characteristics. This variability, or the manufacturing imperfections, is taken into account in the tolerancing phase with the aim of creating a robust design. At the component level, indicators are used to characterize this variability:

– The dispersion of a characteristic: a bounded interval representing the variability of a characteristic. This value is representative of a percentage of the population (99.73% according to the French standard NFX 06-033).

– Capability[7]: an indicator of the quality of production in relation to a requirement. The required performance, in relation to a parameter – typically a dimension – is characterized by a nominal value $\bar{X}$ and a tolerance interval IT_X. The quality obtained is measured using a sample X_i, $i = 1, ..., n$, with an observed mean of m_X, a standard deviation σ_X and a mean shift $\delta_X = m_X - \bar{X}$. Two levels are defined. The first, denoted as C_p, shows the

7 Reread by N. GAYTON.

dispersion of production and is considered to be equal to $C_p = IT/6\sigma$. The second, denoted as C_{pk}, gives the centering of the production in the tolerance $C_{pk} = \frac{IT_X - 2\,|\delta_X|}{6\sigma}$. Values of $C_p \geq 1.67$ and $C_{pk} \geq 1.3$ are often used in the automotive industry. Capability levels show both weak dispersion and good centering. Capability levels may be used to deduce the rate of non-conformity, i.e. the number of non-conforming pieces per 10^6 units of production, based on a hypothesis of a distribution law for the sample [GAY 09], generally a Gaussian distribution, which is selected for the simplicity of calculation rather than for objective reasons □.

Uncertainty linked to models, knowledge and processes involved in the activity

– One of the problems encountered during industrialization (involving the modeling of knowledge in order to establish a fabrication process) resides in the fact that certain aspects of this knowledge are not specifically expressed by experts. This lack of precise information results in uncertainty associated with different decisions made during the expert reasoning process [DER 98]. This uncertainty essentially arises from the contextual aspects of the expression of rules. In the case of sequencing machining orders for crossed bores, for example, the general rule is to carry out the smallest boring operation first, but this rule may be supplemented or replaced by different complementary rules: for instance, "if the angle of incidence between the two bores is high and the smallest diameter is large, it is possible to create the larger bore first", or "if the tolerance of the smaller bore is tight, then the smallest bore must be created before the larger bore", etc. The purpose of these rules is to establish connections between different characteristics of the component and the sequencing of operations via a number of contextual criteria. In the first rule, the limit between small and large, in terms of the diameter of the bore, depends on the angle of incidence and remains unclear.

– A second problem is linked to the validation of certain industrialization solutions using numerical simulations of the behavior of the component during shaping; these simulations use models that only constitute partial representations of the real behavior. The granularity of definition of the models used affects the precision of the results, and consequently the understanding and prediction of the phenomena under consideration.

The verification phase, applied at the part, product or system level, has the same essential aims as when it is applied at the component level: its purpose is to detect non-conformity to specifications. This verification is necessary not only due to the imprecisions inherent in manufacturing, but also due to the uncertainties involved in the design phases: concept identification, predimensioning, dimensioning, tolerancing, etc.

The classification of the origins of uncertainty used in these cases is similar to that used in component verification:

– Origins linked to the ambiguity and interpretation of data from previous stages of the design process (product specifications).

– Origins linked to later stages: manufacturing imperfections or imprecisions.

– Origins linked to the models, knowledge and processes used during the various stages of the V cycle, a question covered in some details in sections 2.1 and 2.2.

1.3.3.2. *The validation phase*

The validation phase is different from the verification phase in terms of objectives; the first is functional, whereas the second is of a technical nature. The validation phase serves to ensure that all client requirements have been respected, i.e. to validate the functions expressed in the client specification and described in detail in the functional analysis. This validation is required due to the difficulty of extracting client requirements and transforming them into expected functions, then into technical specifications. The requirement elicitation process involves a variety of communication, negotiation and collaboration activities between future users and the designers of a product, who are most affected by uncertainties linked to ambiguity and the interpretation of data. As little precision is required in general communications, we have a tendency to use imprecise terms and expressions. However, in a context where precision is essential, this lack of clarity can create uncertainty.

This first analysis allows us to position uncertainty as a whole within the product development cycle, and to understand the complexity of mastering uncertainty, due to the wide variety of possible origins. It is important to note that none of the current approaches based on robust design, risk-based design, etc., allow us to tackle uncertainty and its effects in their entirety. The ambiguity inherent in verbal descriptions of processes progressively imposes a rigorous formalization of vocabulary, as in the case of medicine, in order to provide a correct diagnosis, i.e. the existence or non-existence of a pathology.

1.3.4. *Uncertainty linked to a step in the V cycle*

As we have seen, mastering uncertainty is a fundamental and particularly important aspect for all of the activities involved in the V cycle. Proactive mastery of uncertainty in the course of these activities requires certain changes in relation to deterministic practices.

It is clearly impossible to identify and model all of the uncertainties involved at the start of the product development process. Certain uncertainties generally decrease over the course of the process, new uncertainties may appear, and a certain number are irreducible. To better understand these uncertainties, we will analyze a generic step in the product development process, illustrated in Figure 1.8.

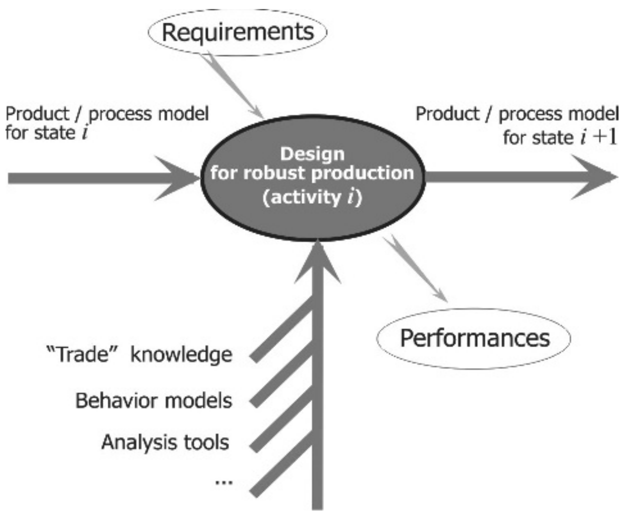

Figure 1.8. *Diagram of a generic step in the product development process*

The aim of a step in this process is to pass from a state i in the definition of the product or the manufacturing process to a state $i+1$, i.e. to obtain additional elements in terms of the definition and modeling of the product or process. This addition is driven by requirements and carried out through the use of domain-specific models, tools or knowledge; it is evaluated using performance measures.

This formalization of a design activity allows us to propose a first taxonomy of uncertainties, using the element connected to the uncertainty as a classification criterion. We can identify a number of different categories.

– Uncertainty *linked to data values*: during a step in designing a product and/or its production process, the generation or evaluation of solutions requires the representation, and therefore the parametering, of the product and/or the process (product/process model of state i or state $i + 1$ – Figure 1.8). These parameters may relate to the geometric description of the product, to materials, to operating conditions of the production process, etc. They are subject to variations in production (imperfections). The variability of these parameters characterizes uncertainty linked to the values of the data used to represent the product or production process during design phases. Figure 1.9 shows the impact of uncertainty linked to a parameter X on performance Y for a minimization of the objective function. See section 2.1 for a discussion of possible representations.

– Uncertainty *linked to the formalization of knowledge*: the difficulty of formalizing the expertise of actors in the product design and manufacturing processes is due to insufficient consideration of context. Expert reasoning and knowledge are based on a set of subjective data (e.g. the choice of the process used to create a hole is a function of not only a set of explicit data, such as the diameter of the hole, but also a set of subjective data, such as the similarity of the hole to other holes, accessibility, etc.). Experts tend to base their reasoning on previous cases, and formalize their knowledge in accordance with these cases. The notions of case similarity and the domain of validity of knowledge are difficult to specify, and pose problems similar to those of granularity in modeling. The limitations of the description of the context of expression result in a non-unified formalization of knowledge. The evaluation of the relevance of a piece of knowledge in relation to a given situation may be characterized by the maturity model. Figure 1.9 illustrates the impact of data on the formalization of knowledge relating to a given objective function, but uncertainty exists in terms of the choice of this function. We cannot be sure if an expert has formalized knowledge in the correct way to allow it to be used in models without generating model errors, i.e. physical mistakes.

– Uncertainty *linked to models* – models or descriptions relating to reality: in the real world, no one is able to create a model or description in exact conformity to reality. A model is, by definition, the partial representation of reality required for a process – the granularity of the definition of the model is chosen in relation to requirements, and cannot tend toward infinity. It is generally the result of a compromise between the limitations imposed on processing resources by high-performance treatments and attaining results with the right level of precision. For this reason, there is a gap between models and reality, which itself is filtered through a model, representing the uncertainty linked to modeling. Work on model shifts, often (incorrectly) referred to as model errors, aims to establish limits or, better, a measure reflecting the uncertainty of a model across a whole domain of definition. These points are discussed in section 2.2.

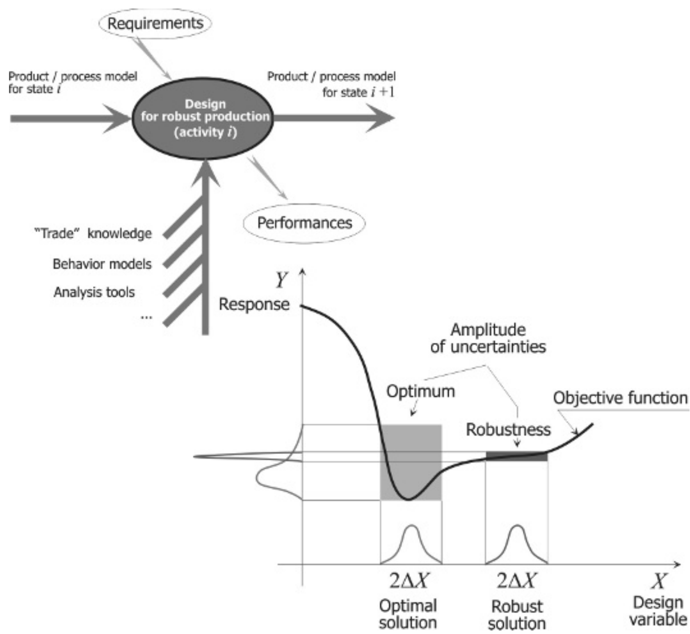

Figure 1.9. *Illustration of uncertainties linked to data values*

– Uncertainty *linked to the modeling of the product and the process* and uncertainty linked to behavior models: for understanding and prediction purposes, "realistic" models and simulations play an essential role in analyzing complex systems such as manufactured products and production process. The granularity of the definition of the models in question (geometric description, description of mechanical behavior, etc.) affects the precision of the results, and thus impacts our understanding and prediction of the studied phenomena. This uncertainty occurs on two levels in relation to design activities: first, activity input/output models, and second, behavior models for specific activities (mostly used in performance evaluation). As we see in Figure 1.10, uncertainty in behavior models, represented by an enveloping zone showing the relationship between the design parameters X and the performance Y on conclusions and decisions that may be made, has a non-negligible impact.

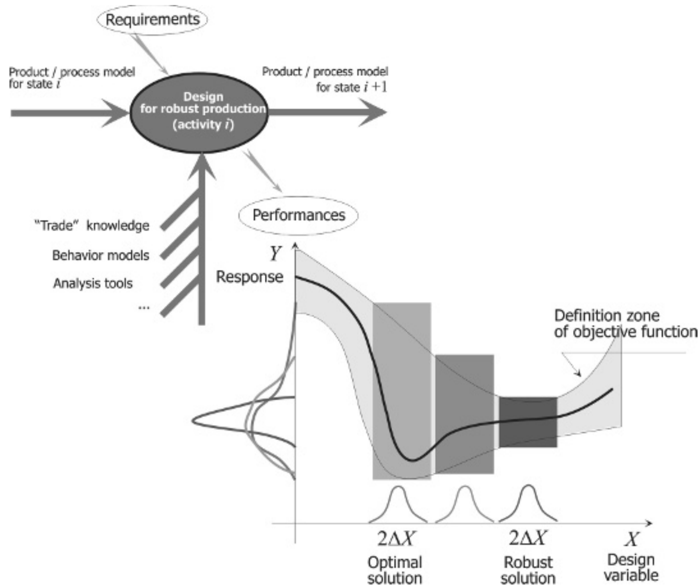

Figure 1.10. *Illustration of uncertainties linked to models and to data values*

– Uncertainty *linked to model couplings*: during multidisciplinary analysis of a product and a production process, the representation of these aspects and the level of granularity used in modeling will not be the same for all experts. The coupling of descriptions involved in multiexpert treatments creates problems in terms of model compatibility; this problem is compounded by limitations linked to the performance of treatment processes (time and precision). This issue is similar to modeling-related uncertainty and to the gap between models and reality; however, in this case, the distance exists between two different models.

– Uncertainty *linked to numerical processing*: in addition to the errors, deviations or simplifications involved in modeling choices, numerical solution is also liable to generate uncertainty, either in "direct" numerical processing of the model (numerical precision, convergence of algorithms) or due to the use of meta-models or response surfaces to avoid difficulties with long calculation times. An approach has been developed in recent years to deal with this type of uncertainty, known as "V&V" for "Model Verification & Validation" [THA 04] □.

1.3.4.1. *Conclusion*

The generation and evaluation of a state $i + 1$ from a state i in order to satisfy design requirements requires an appropriation, formalization and modeling of knowledge. These aspects are driven by the need for predictive performance demonstration. The designer must make use of available information on an as-needed basis. Data interpretation creates knowledge of each aspect, and formalization involves the selection of an appropriate scenario, which is then represented by models. This question is covered in detail in section 2.2. The aggregation aspect of the phase under consideration raises questions concerning performance-related uncertainties; this will be discussed in section 2.3.

In the following sections, we will provide a description of the approach to robustness and to mastering uncertainty used during a V design cycle.

1.3.5. *Robustness and uncertainty*

The aim of the product development process is to create a robust and reliable product[8]. The generic approach proposed in section 1.3.4 may be divided into five actions, categorized by purpose.

1) *Analyze the robustness of a product and the production process in relation to uncertainty (state $i + 1$, Figure 1.10):* analyze the sensitivity of the performance of the "manufactured" product to uncertainty (as a whole) or to certain specific uncertainties. Measure the difference between expected performances and those exhibited by the "manufactured" product.

Example: analyze the sensitivity of the operational strength of a part to variations in material characteristics inherent to the manufacturing process.

2) *Analyze the robustness of a solution to calculations in relation to calculation models*: analyze the sensitivity of calculated results to uncertainty linked to data values, uncertainty linked to descriptions of the product, process and behavior, and/or uncertainty linked to the treatment.

Example: analyze the sensitivity of the results of a simulation to differences between reality and the behavior model.

3) *Reduce the sensitivity of the performances of the "manufactured" product*: in relation to uncertainty (as a whole) or to certain specific types of uncertainty. Reduce the gap between the expected and real performances of

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8 Text based on a contribution by T. YALAMAS.

a manufactured product by modifying the solution (product and/or production process and/or processing tool and/or granularity of modeling, etc.).

Example: improve the behavior model in order to reduce the sensitivity of simulation results to differences in relation to reality.

4) *Optimize the solution(s) (product and/or production process and/or processing tool and/or granularity of modeling, etc.)*: to minimize the sensitivity to uncertainty of the performances of the manufactured product, or to guarantee a certain level of sensitivity to uncertainty in the functions or characteristics of the manufactured product.

Example: optimize the targets of "product" parameters in order to minimize the sensitivity of product performances to parameter variations.

5) *Tolerance*: limit overall or specific uncertainties in order to guarantee a probable performance level for the solution (product and/or production process and/or processing tool and/or granularity of modeling, etc.).

Example: limit variations in material characteristics in order to guarantee a certain level of operational strength or allocate reliability across a system.

Each of these actions may be the target of specific developments, and we will discuss certain aspects in greater detail in relation to sensitivity and reliability analysis (section 2.2) and multicriterion optimization (section 3.1). However, robustness and reliability cannot be isolated within specific parts of the development cycle, but must be integrated into a global approach based on two main components: robustness and uncertainty.

1.3.5.1. *The robustness approach*

Figure 1.11 shows mastering robustness in a design process as a process in its own right, split into different steps corresponding to phases of the design process.

This illustration demonstrates the complementarity of approaches, often carried out by different entities within a group, which all aim to ensure that the system responds to a set of requirements. Note the importance of strong communication links between services to ensure that all of the available information is used in the best possible way.

In the downward section of the *V* cycle, classic tools such as functional analysis, preliminary risk analysis and failure mode, effects and criticality analysis (FMECA) are used; these are relatively common [COL 11] and do not require detailed treatment here. The use of simulation for mastering uncertainty is somewhat less common, and will be discussed in greater detail below.

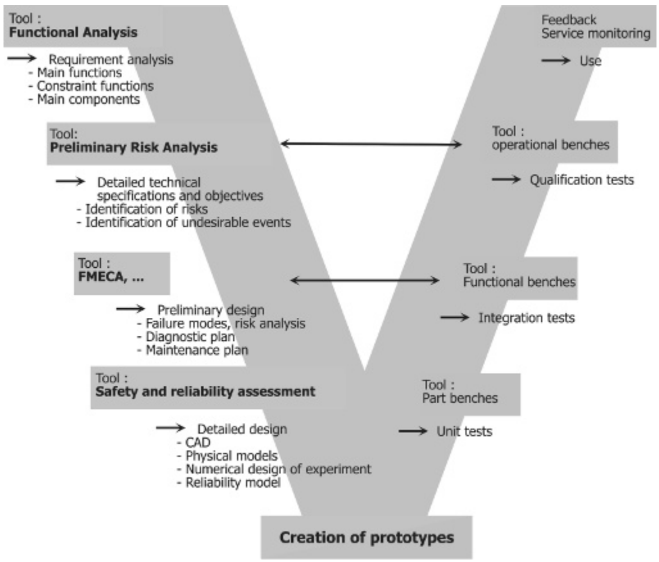

Figure 1.11. *Robust and reliable design approach using the* ***V*** *cycle*

1.3.5.2. *The uncertainty approach*

Much work has recently been carried out on modeling uncertainty and its propagation through a physical (mechanical, hydraulic, chemical, etc.) model, involving discussions between actors from a number of different establishments (notably *Electricité de France* (EDF), European Aeronautic Defence and Space Company (EADS), Phimeca Engineering S.A (PHIMECA) and the Commissariat à l'Energie Atomique (CEA) in France). These discussions resulted in the publication of a book on the subject, [DE 08]. This methodology, taken in a probabilistic context, consists of four clearly identified steps (Figure 1.12).

– *Step A – Physical model*

From an engineering perspective, this is the most widely known step. It consists of the following:

- Identifying the system in question and modeling it from a deterministic standpoint (analytical model, finite element model, etc.). In cases where detailed modeling would generate excessive calculation costs, it is also possible to replace the deterministic model at this step by a predefined meta-model (or response surface) obtained using polynomial chaos [SUD 07b], support vector regression (SVR) [DEH 08b] or by Kriging [DUB 11b, ECH 12]), or to use model reduction techniques (see section 2.3).

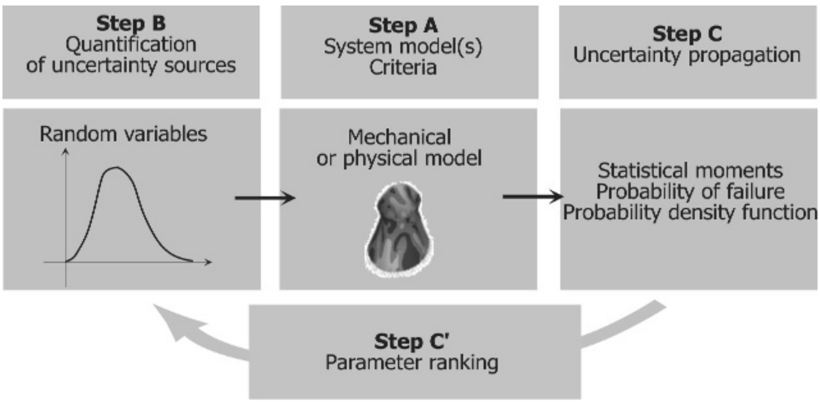

Figure 1.12. *Methodological principles for the treatment of uncertainty, based on [DE 08]*

- Identifying the model output values that are of interest (maximal constraints in a domain, constraints at a point, displacement components, etc.).

- Formulating a criterion using one or more quantities of interest, where necessary (e.g. for reliability analysis).

– *Step B – Quantification of uncertainty*

At this step, we distinguish between cases based on the availability or unavailability of data required to construct a probabilistic model (see section 2.1).

- In cases where experimental data or a database is available, we may use the usual statistical methods to identify the most suitable random variable distribution.

- In the absence of data, *a priori* physical knowledge of the parameter being modeled should be combined with the experience of the probabilistic engineer in order to construct a physically acceptable probabilistic model.

Note that model updating methods [BER 11] allow the use of measures obtained *a posteriori* to refine an existing probabilistic model in order to improve the predictive capacities of the model.

– *Step C – Propagation of uncertainty*

Three types of analysis are generally used:

- Scatter analysis (Figure 1.13(a)): its aim is to characterize (in the statistical sense of the term) one or more quantities of interest. It involves the determination of the statistical moments (e.g. mean and standard deviation) of this quantity of interest, up to an order that depends on the desired precision and on the chosen method.

- Distribution analysis: (1.13(b)): in this case, the aim is to estimate the complete probability distribution of the quantities of interest.

- Reliability analysis (Figure 1.13(c)): the purpose of reliability analysis is to obtain as precisely as possible the probability that the parameter in question will exceed a threshold value: the probability of failure in relation to the dimensioning criterion [LEM 09].

Note that implementation difficulties increase as the distance from the central trend (Mediocristan) increases, and that we wish to estimate the tails of distributions (Extremistan) (from left to right in Figure 1.13).

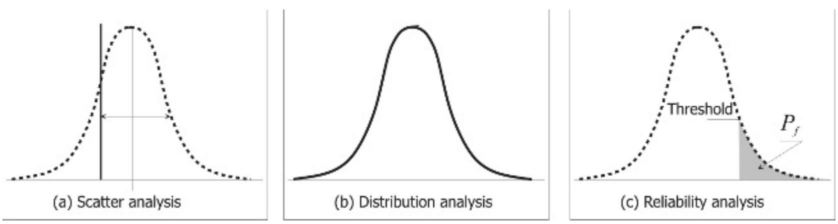

Figure 1.13. *Step C: Propagation of uncertainty*

– *Step C' – Ranking uncertainties*

- The purpose of this step is to identify parameters, or combinations of parameters, where variability has the greatest impact on the quantity of interest. This step is generally associated with the previous step as the methods used in step C also provide information for ranking. A distinction is made between local sensitivity indexes, generally associated with reliability analysis, and global sensitivity indexes, which are often associated with a variance breakdown analysis known as analysis of variance (ANOVA) [SAL 04, SAL 10]. Figure 1.14 shows a classic representation of the weightings of four variables □.

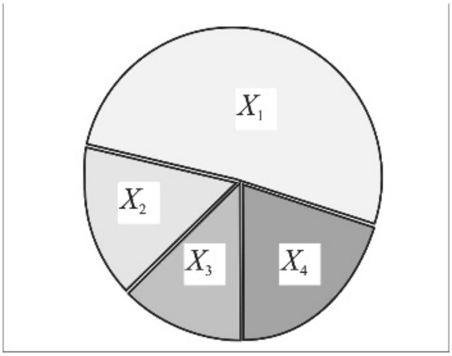

Figure 1.14. *Step C': Illustration of uncertainty weightings for variables: X_1 to X_4*

1.3.6. *Conclusion*

In designing products, parts or systems for robust production, we work in an environment of dynamic and varied uncertainty, which comes from a wide range of sources. These sources include ambiguity in projects, which are often described in insufficient detail; knowledge, which must always be limited; and uncertainty related to the use that the user will make of the product. This uncertainty is dynamic, as it evolves as a function of decisions and the advancement of knowledge. Designing for robust production requires us to integrate all of the necessary phases and steps, from those based on cognitive sciences to those involving different branches of physics, mechanics and mathematical modeling. Individual aspects cannot usefully be considered in isolation. For this reason, designing for robust production requires the involvement of multidisciplinary teams, with each member contributing fully to each step and phase in the process.

The V cycle and the associated approaches for demonstrating robustness and reliability provide bases for reflection, which is now at a stage requiring better formalization. The Wikipedia article on the subject cites [KEY 78], which uses a hexagram diagram, Figure 1.15 (left), to represent the interdisciplinary links between the cognitive sciences. Similarly, we can create a hexagram of the links between modeling sciences, shown in

Figure 1.15 (right). The algebra of the operators connecting these entities needs to be constructed in order for design for robust production to become a science in its own right, providing the elements required for decision acceptability from all relevant perspectives: societal, economic, judicial, technical, etc., as was seen at the IMdR/AFM conference in May 2012 [LEM 12].

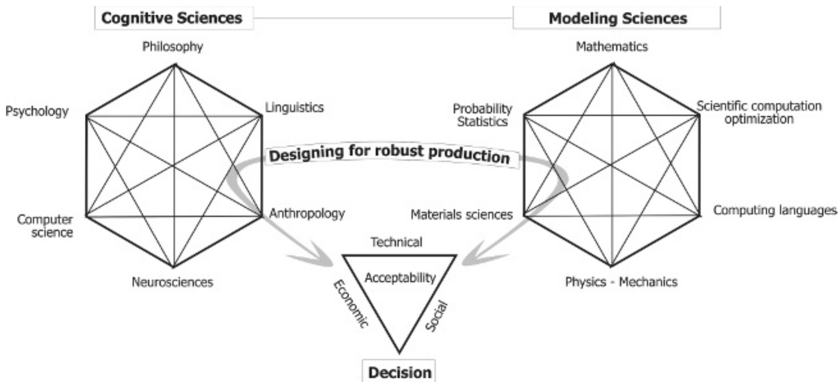

Figure 1.15. *Symbolic representation of the cognitive sciences (left) and the modeling sciences (right). Designing for robust production requires sustained and effective dialog between the two domains in order to produce acceptable decisions*

In the following chapter, we will consider a number of points relating to the field of modeling.