Chapter 1

The Need

1.1. The need to carry out studies into vibrations and mechanical shocks

During their service life, many materials are subjected to vibratory environments, during their transport [OST 65], [OST 67], because they are intended to equip themselves with means of transport (airplanes, road vehicles, etc.) or because they are placed beside vibratory sources (engines, wind mills, roads, etc.). These vibratory environments (vibrations and shocks) create dynamic strains and stresses in the structures which can, for example, produce intermittent or permanent breakdowns in electrical equipment, plastic deformations or fractures by up-crossing an ultimate stress of the material (yield limit, rupture limit), optical misalignments of systems or may contribute to the fatigue and the wear of the machine elements.

It is therefore necessary to take all of these points into consideration during the design phase of structures and of mechanical equipment. The approach is normally made up of several steps:

– measuring the vibration phenomena;

– analyzing the results of the measurements, bearing in mind that this analysis will be used for different objectives, including:

- the characterization of the frequency contents of the vibration (the search for predominant frequencies, amplitudes, etc.), for example, to compare the natural frequencies of the structures,

- comparing the relative severity of several different vibratory environments (transport on various vehicles) or comparing the severity of such vibration environments with a standard,

- confirming *a posteriori* the validity of a dimensioning or test specification which is established starting from *fallback level* values, from data collected at the time of a preceding project or starting from values resulting from normative documents;

– the transformation of measurements into dimensioning specifications for research departments; these are presented in the simplest possible form requiring a synthesis of all the measured data;

– during and at the end of the design phase, at the time of the qualification, realization of tests intended to validate the behavior of the materials developed from these environments.

The vibrations most frequently encountered in the real environment are of a random nature. Along with shocks, they constitute the main part of mechanical excitations. These two environments can be severe, shocks by their amplitude and random vibrations by their duration.

In certain situations, however (near turning machines), it is possible to observe sinusoidal vibrations which are often polluted by noise. This is especially the case for vibrations which are produced by propeller airplanes and helicopters. In these cases, the random noise which is produced is significantly important compared to the sinusoidal lines (fundamental and harmonics).

Whenever such rotating machines are switched on and off, their frequency varies, in a continuous way, generating a vibration similar to a swept sine. This type of environment is primarily used in laboratory tests in order to carry out research into the resonance frequency of structures.

The mechanical excitations which are then analyzed, resulting from measurements of the environment or test laboratory, belong to one of the following groups:

– sinusoidal vibrations;

– swept sine vibrations;

– random vibrations;

– mechanical shocks;

or a combination of these vibrations:

– sine on random (one or several lines);

– a swept sine on random (with a sweeping on one or several frequency bands);

– a narrowband random vibration swept on a wideband noise, etc.

The vibrations which are produced in the real world have quite different frequency domains:

– between approximately 1 and 500 Hz for road vehicles;

– between approximately 10 and 2,000 Hz for airplanes and spacecraft;

– between approximately 1 and 35 Hz for earthquakes;

– more than 10,000 Hz for shocks which are created by metal–metal impacts, several tens of thousands of Hz for shocks which are created by pyrotechnic devices.

Vibrations are often classed into three different categories, depending on their frequency. The different categories are as follows:

– very low frequency for frequency values between 0 and 2 Hz;

– medium frequency for frequency values between 2 and 20 Hz;

– high frequency for frequency values between 20 and 2,000 Hz.

These values in conventional matter are given only as an indication and do not have any theoretical legitimacy. The low frequency concept can in fact be definite only according to the natural frequency of the system which undergoes the vibration. The frequency of a vibration will be low for a mechanical system if it induces any dynamic response (no attenuation and no amplification).

1.2. Some real environments

1.2.1. *Sea transport*

The sources of vibrations on board ships have various origins and natures. They are primarily due to:

– the propeller (periodic vibrations);

– the propelling unit and the auxiliary groups (periodic vibrations);

– the equipment used on board (for example, winches);

– the effects of the sea (random vibrations).

The measured levels are in general the lowest amongst all the means of surface transport.

1.2.1.1. *Vibrations produced by the ship's propeller*

The rotation of the propeller can excite the modes of the ship's frame in different ways:

– the accelerations transmitted to the hull via the line shafts;

– forces exerted on the ship's rudder;

– hydroelastic coupling between the propeller and the shafts' line;

– fluctuations in pressure distributed on all parts of the back hull, having as an origin the wake in which the propeller works. These fluctuations in pressure are dependent on:

- the variations of propeller's push. When the propeller provides a push, the back of each blade is subjected to a "negative pressure" (suction) compared to the environmental pressure, and the front face is subjected to an overpressure,

- the number, area and thickness of the blades. The fluctuations in pressure are a linear function of the average thickness of the blades and decrease very quickly when the number of blades increases,

- the presence of a variable vapor pocket on the surface of the blade and in its slipstream, as a consequence of cavitation.

Around the propeller is formed a cavity filled with vapor within the liquid, due to a local pressure lower than the saturating steam pressure. When the vapor bubbles reach higher pressure zones, they condense brutally. This phenomenon, known as cavitation, involves very strong mechanical actions (vibrations, noises, etc.).

Cavitation is the source of the majority of vibration problems encountered on ships. It is equivalent to an increase of the thickness of blades and, as a result, increases the pressure fluctuations. The variation of the volume of the cavitation pocket over time is a second source of pressure fluctuation. The fundamental frequency is around 20 Hz for fixed blade propellers from 5 to 6 m in diameter and 10 Hz for propellers from 8 to 10 m in diameter. The natural frequencies of the blades decrease when the diameter increases.

1.2.1.2. *Vibrations produced by the ship's engine*

The vibrations which are produced by a ship's engine are caused by the alternate movements of the piston, connecting rod and crankshaft systems.

They can excite the modes of the ship's frame, especially for medium-sized ships. Their vibratory frequency generally lies between 3 and 30 Hz.

1.2.1.3. *Vibrations produced by the state of the sea*

Vibrations due to the swell

The swell heave leads to the creation of vibrations of a long duration and of very low frequency (less than 2 Hz) in both the longitudinal (pitching) and transverse direction (rolling). These random oscillations are always of a seismic nature.

Their frequency varies between 0.01 Hz (when the sea is very calm) and 1.5 Hz (during bad weather). Their associated accelerations range from approximately 0.1 m/s^2 to 9 m/s^2.

Vibrations of the whole of the ship due to the state of the sea

In general, two types of vibrations are considered:

– hydrodynamic shocks applied to the front of the ship lead to the vibration of the whole of the ship, which works like a beam. This phenomenon occurs whenever the ship navigates the sea with its front first, with relative movements of the stem sufficiently significant to create impacts. These impacts can be distinguished as follows:

- shocks which are produced on the flat part at the bottom of the ship, when the ship makes contact with the sea, after it emerges from the water,

- shocks on planking of the stem, without emergence, without the ship resurfacing from the sea,

- areas of seawater;

– excitations which are caused by the swell's variable hydrodynamic forces, which lead to a steady state free vibration of the entire ship.

These vibrations generally have low or very low frequencies and, to a lesser extent, some can have high frequencies [VIB 06]. The frequencies range from 0.01 Hz to 80 Hz, with a maximum value of between 3 Hz and 30 Hz. The vibrations are periodic or random.

1.2.2. *Earthquakes*

The rapid release of the deformation energy which is accumulated in the Earth's crust or mantle (the underlying layer) is felt as a vibration on the Earth's surface: an earthquake. The vibration (the tremor) lasts in general for a few tens of a second. Their amplitude on the ground level can reach several m/s^2.

The shock response spectrum was created in the 1930s in order to group together the different effects that earthquakes of different amplitudes have on buildings. The amplitudes are taken from actual acceleration signals which were measured from real earthquakes (see Volume 2).

1.2.3. *Road vibratory environment*

The road transport vibratory environment is complex. It can be described as a mixture of permanent vibrations and discrete superimposed vibrations. The permanent part is comprised of variable proportions of the following types of vibrations:

– wideband noise, with a distribution of the instantaneous values which is generally Gaussian;

– very narrowband excitation with amplitude distribution very close to a Gauss law (for example, in response to a suspension);

– excitation of only one frequency and of constant amplitude (a poorly balanced rotor).

The discrete components can be recurring (i.e. with a periodicity), for example at the time of the passage of joints of a road made up of concreted plates, or intermittent (only one or some occurrences), for example during the crossing of a railway crossing.

Four main sources of vibrations can be distinguished: the suspension system, tires, the driving system and parts of the vehicle's framework [FOL 72]. The spectrum's characteristics depend on the state of the road or the type of terrain on which the vehicle is being used, the speed at which the vehicle is traveling and the vehicle's suspension.

The vehicle suspension generates vibrations at quite high amplitudes with frequencies between 3 and 6 Hz. The tires produce recurring components between 15 and 25 Hz. The engine and the driving train produce continuous excitation with frequencies between 60 and 80 Hz. The structural responses can range from 100 Hz to 120 Hz [FOL 72]. Other frequency domains can reach frequencies of up to 1,000 Hz according to the type of vehicle that is being used, due for example to the operation of electrical brakes.

The road vibratory environment is mainly made up of the following components:

– longitudinal movements which are linked to the acceleration and slowing down of a vehicle;

– lateral movements which correspond to driving around bends;

– vibrations which occur along the vertical axis, related to rolling on the road;

– longitudinal and lateral movements which are both associated with vertical non-symmetric excitation.

The first two environments are relatively weak and quasi-static. The last two are dependent on the state of the road. The frequencies of the spectrum can reach up to approximately 30 Hz, with low frequencies being able to produce large displacements. Frequencies larger than 30 Hz can also exist, being able to excite local resonances of structures [HAG 63]. The vibrations according to the vertical axis are generally dominant.

The rms acceleration of these vibrations ranges between 2 and 7 m/s^2 approximately [RIS 08].

The spectrum measured on the tracked vehicles is comprised of a random broadband noise and other higher energy bands of random vibrations which are created by the interaction of the caterpillar with the track and the toothed wheels. It is preferable to simulate these types of vibrations by using a swept sine on a wideband noise.

1.2.4. *Rail vibratory environment*

The permanent excitation measured during the rail transport is of a slightly smaller amplitude than that measured on the road [VIB 06].

The origin of the vibrations is primarily related to defects which exist on railway lines, for example, gaps between the rails, distance between the rails, switch point areas, etc. These examples are only a few of those that exist.

The vertical axis is in general the most excited, but the vibrations according to the transverse axes can also be severe, at least for particular frequency bands. The highest levels correspond to the frequency of the suspension (between 1 and 10 Hz), to the frequency of the train's framework (between 10 and 100 Hz) and to the areas where there are joints which hold the rails together (between 10 and 30 Hz). The switch point areas produce the strongest excitations [FOL 72] like the shocks between coaches during the process of putting the train together – attaching the wagons of the trains (the most severe levels of all types of surface transport).

1.2.5. *Propeller airplanes*

The vibrations measured on the propeller planes have a spectrum that is made up of a wideband noise and of several sinusoidal or narrowband lines. Wideband noise comes from the flow of air that occurs around the airplane and also from the multiple periodic components which are produced by all the elements in rotation in the propeller.

The peaks come from the flow of air that exists between the blades of the propellers, creating periodic aerodynamic pressure fields on the structure of the plane. The narrowbands are centered on a frequency which corresponds to the number of propeller blades multiplied by the engine's rotation speed and on its harmonics.

The most visible lines are generally the fundamental frequency as well as the first two or three harmonics. The amplitude of these rays depends on the stage of the flight, i.e. take-off, ascent, cruise, landing, etc., and also depends on the point at which the measurement is taken.

The same spectrum can also be observed around the airplane's engine. The majority of engines have an almost constant rotation speed. This rotation speed can be modified by supplying fuel to the engine, or by changing the angle of the propeller's blades. The frequency of the peaks is also quite stable. Their width is linked to the small change in rotation speed and to the fact that the vibrations which are generated are not purely sinusoidal vibrations.

Other engines function with a more variable rotation speed. In this case, simulation in a laboratory is instead carried out by specifying a test defined by a swept sine on a wideband random vibration.

1.2.6. *Vibrations caused by jet propulsion airplanes*

1.2.6.1. *During take-off and ascent*

The strongest vibrations occur along the vertical axis of an airplane during its take-off and ascent. The weakest vibrations occur along the airplane's horizontal axis.

Depending on the type of airplane, the typical frequency has a value of between 60 and 90 Hz, with a root mean square of about 5 m/s^2.

1.2.6.2. *The cruising phase*

The amplitudes of the vibrations are much lower during the cruising phase of the plane than is the case during the take-off and ascent phases. Nevertheless, the amplitudes remain stronger along the vertical axis. These values are much lower along the other two axes. There is also a constant frequency of between 60 and 90 Hz.

1.2.7. *Vibrations caused by turbofan aircraft*

We observe here a tendency towards a continuous rise of the levels of amplitude between 20 and 1,000 Hz, then a decrease of the amplitudes.

Once again, the strongest vibrations occur along the vertical axis and the weakest vibrations along the longitudinal axis. The vibration signal tends to be made up of a sine wave which is superimposed onto a wideband Gaussian noise.

This type of vibration occurs on the fighter airplane and is produced by many sources, including:

– the engine's noise which is then transmitted by the airplane's bodywork;

– aerodynamic flow;

– dynamic responses due to operations (airbrakes, missile launches, etc.).

In addition to these vibrations, shocks (which are sometimes severe) also occur, related to landing, taking-off, catapult-launchings, etc.

1.2.8. *Helicopters*

The vibrations which are produced by helicopters are made up of a random wideband noise and sinusoidal lines which are produced by the helicopter's main rotor, tail rotor and engine. The frequency of the sinusoidal lines does not vary much, the rotation speed of all of these components remaining relatively constant (variation of approximately 5%). The fundamental frequency which can be found in the sinusoidal lines corresponds to the rotation speed of the rotor and to its harmonic frequencies.

The amplitude of the lines is a function of the type of the helicopter and the point of measurement (proximity of the source).

The helicopter produces the most severe environment among all the means of air transport, producing high amplitudes at low frequency. The permanent random wideband component is very complex and has an extremely large amplitude.

The dynamic environment of the helicopters is different from that created by fixed wing airplanes. There is little difference here between take-off and cruising, and the amplitudes are generally larger.

The rotation speed does not vary much during flight for helicopters, except during hovering flight. Random vibrations (approximately Gaussian) are superimposed on sine lines, with a significant component at very low frequency. These lines are difficult to identify (frequency and amplitude) and extract. The amplitude of the rays varies depending on whether the vibration was recorded close to the rotors and engine, or not.

The vertical axis is in general the most severe. The fundamental frequency of the vibration depends on the rotation speed of the blades and also on the number of blades present.

The first component, between 15 and 25 Hz for the main rotor, is easily identifiable on the three axes and is more important according to the longitudinal and transverse axes [FOL 72]. The back rotor produces higher frequencies in general, between 20 and 100 Hz approximately, according to the type of apparatus and the number of blades.

The tail rotor tends to produce frequencies of a higher value, i.e. between 20 Hz and 100 Hz. These values depend on the type of helicopter and on the number of blades on the helicopter's propeller.

Example 1.1.

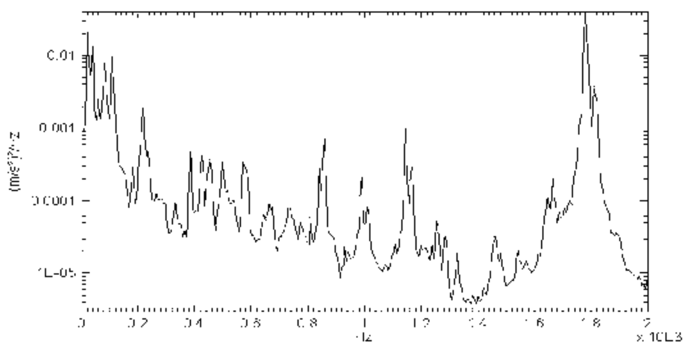

Figure 1.1. *Frequency contents of a vibration measured on a helicopter (power spectral density)*

1.3. Measuring vibrations and shocks

Different physical parameters can be *a priori* used for characterizing a vibration: an acceleration, a velocity, a displacement, a force or even a stress directly. All these parameters are measurable, but the most frequently used is undoubtedly acceleration. The main reason for this is due to the diversity of the different sensors which are available, their different acceleration and frequency ranges and their different sizes.

A sensor is an energy converter. Accelerometers are composed of a seismic mass suspended by an elastic element. Measuring the force F at which the mass m is subjected allows the acceleration G to be derived. The dynamic mass may carry compressive, bending or shearing forces. The different types of accelerometers differ in the force measurement principle.

Accelerometers are mechanically one-degree-of-freedom systems (see Figure 1.2). The system's mass response, which is subject to a certain level of acceleration applied at its base, will be studied in later chapters.

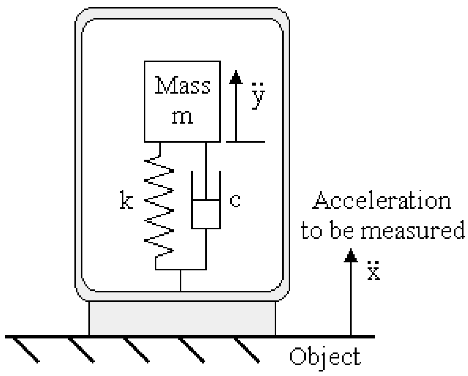

Figure 1.2. *Mechanical principle of an accelerometer*

Several physical principles are used to convert movement into an electric signal. These principles are as follows [ERE 99], [WAL 07]:

– the piezoelectric effect: a crystal which has a dynamic stress applied to it produced, in response to the acceleration which is to be measured, electrical charges which are converted into tension;

– a variation of capacity between two very near microstructures. This variation in capacity is also transformed into a variation of tension;

– the piezoresistive effect (change in resistance with acceleration);

– etc.

Figure 1.3. *Example of piezoelectric accelerometer (PCB 357B81, 2000 g, 20 pC/g, 9kHz shear ceramic) (courtesy of PCB Piezotronics)*

Figure 1.4. *Example of piezoresistive accelerometer (MEMS, 20000 g – 0 to 10 kHz – 2.83 g, -54 to 121°C, shock measurements) (courtesy of PCB Piezotronics)*

The resulting signal can be analog (continuous tension proportional to acceleration) or digital.

These sensors are characterized by their bandwidth (frequency domain, which is a function of the sensor's resonance frequency), by their effective range, by their sensitivity (V/g) and their size (or masses). Some make it possible to measure acceleration according to three axes.

Accelerometer	Advantages	Disadvantages	Field of Use
Piezoelectric	- Usable at high temperature (up to 700°C) - Generally low costs - Large measurement scale (from 10–5 to 105 g) - Sensitive to weak amplitude vibrations - Low volume - Response in a wide frequency, from 0.5 Hz to 40 kHz	- Does not filter DC components - Badly adapted to pyroshocks beyond 100,000 g	- Impulse or non-impulse vibratory phenomena - Characterization of structure and equipment behavior - Measurements at high temperatures - Seismic measurements - Shock measurements - Low frequency vibratory phenomena (vibratory comfort analysis)
Piezoresistive	- Does not filter DC components - Low volume - Adapted to the measurement of amplitude shocks (greater than 100,000 g)	- Temperatures lower than 130°C - More expensive than piezoelectric - Less sensitive to weak levels than piezoelectric	- Low amplitude and low frequency vibration acceleration measurements (up to several thousand Hz) - Shock measurements - Characterization of structures and equipment (quasi-static measurement): vehicle behavior, suspension during road tests, crash-tests, etc.
Capacitive	- Does not filter DC components - Very high resolution (up to 10^{-6} g) - High output signal	- Cost - Fragility - Volume - Temperatures lower than 150°C	- Low amplitude and low frequency inertial phenomena measurements - Examples: trajectory correction, stabilization of platforms, etc.

Table 1.1. *Advantages and disadvantages of the different types of accelerometers*

Advantages	Disadvantages	Applications
Large dynamic field Large frequency range Resistance to high level shocks Low supply costs Less sensitive to the electromagnetic environment Easy to use High impedance output Large length of cable possible without noise Output parameters fixed by construction	Field limited by temperatures of use Maximum: 170°C Integrated electronics subjected to same environment as sensor Low frequency response determined by construction	Modal analysis Motors In-flight tests Drop tests Earthquake behavior tests HALT/HASS Cold environment tests

Table 1.2. *Advantages and disadvantages of piezoelectric accelerometers with integrated electronics*

MEMS are Micro Electro Mechanical Systems which use small silicon surfaces (the material used for CMOS technology). They are measured in micrometers.

Theoretically, MEMS accelerometers do not have a zero derivative. One drawback with MEMS accelerometers in shock measurement is their considerable amplification at resonance (for example, 1000:1). This can lead to a rupture in response to high frequency inputs (for example, metal–metal impacts, pyroshocks, etc.). This defect can be improved by incorporating a small damping film.

Signal conditioners

Conditioners are used to carry out a load/voltage or voltage/voltage conversion, with an amplification and attenuation gain. Some conditioners also make it possible to integrate the signal in order to obtain at the output velocity or displacement signals. Signal pre-filtering functions often enable us to optimize the signal before saving and/or analysis.

Measurements must be carried out under real conditions of use if possible, for example, the same vehicle (if the material is embarked), the same interfaces, etc.

Some simple rules must be respected:

– the vibration should be measured at the input of material, the sensor being placed onto its support near an area which is very close to the material's fixation point, preferably on the most rigid surface available [STA 62]. It would be best to avoid placing the sensor on a sheet of metal or on the hood, etc.;

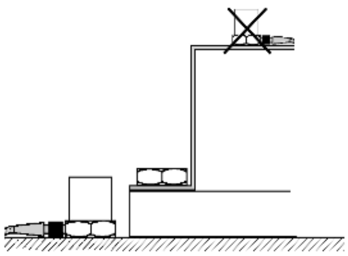

Figure 1.5. *Position of the sensor for measuring vibrations experienced by equipment*

– a sufficient number of sensors should be used so that a better understanding of how the material works is obtained. However, caution is required, as we do not want to have too many sensors present in order to avoid modifying its mechanical behavior.

It is important to evaluate the representativeness of measurement compared to the physical phenomenon. Is one measurement enough? Does the variability of the results require the realization of several recordings, statistical processing, etc.?

1.4. Filtering

1.4.1. *Definitions*

Filters are used to remove components of undesired frequencies in a measured signal, shock or random vibration. They can also be used to extract the useful components of a signal in a given frequency domain. The filter transfer function (the ratio of the response divided by the input to each frequency) should have a value of 1, or as close to 1 as possible for the frequencies which are to be kept. For all of the other frequencies this value should be zero. The transition zone needs to be as small as possible.

There are two types of filters that exist:

– analog filters. These filters use electronic circuits. The original signal is analog (current, tension), such as the filter's response signal, or filtered signal as it is otherwise known. Examples of such filters include the Butterworth filter, the Tchebycheff filter and the Bessel filter;

– digital filters. Using these filters makes it possible to process signals which have already been digitized and which rely on the use of data processing calculations.

1.4.1.1. *Low-pass filter*

A low-pass filter is a filter which lets low frequencies pass through the filter without making any modification to them. This type of filter then rejects frequencies which have a value of more than f_c. This frequency is known as the cutoff frequency.

An ideal low-pass filter has a constant gain of 1 in its frequency range, and a zero gain in its stop band. For the frequency values of between zero and f_c, the shape of this filter is rectangular. In practice, the transition from a value of 1 to a value of zero is done with a more or less important slope according to the quality of the filter.

The most simple analog low-pass filter (order 1 filter) has the characteristic

$$H(j\,f) = \frac{1}{1 + \frac{j\,f}{f_c}} \qquad [1.1]$$

The gain equals

$$\left|H(j\,f)\right| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \qquad [1.2]$$

where f_c is the *cut-off frequency*, the frequency for which the gain has decreased by 3 dB.

We use the larger n-order filters instead, in which the gain is given by (Butterworth filter):

$$\left|H(j\,f)\right| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}} \qquad [1.3]$$

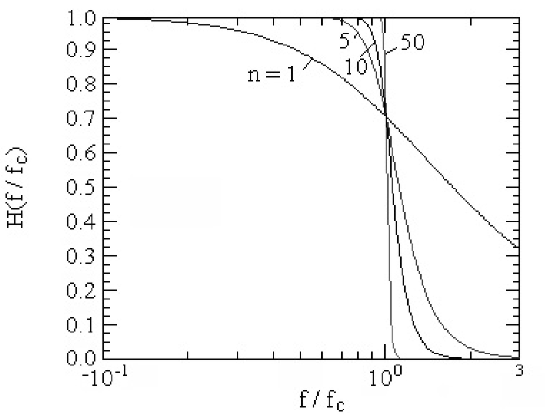

Figure 1.6. *Low-pass filter – gain versus* $f\ /\ f_c$ *, for different values of the filter order n*

The larger the order of the filter, the quicker the return to zero (Figure 1.6). It is easy to show than the decrease slope is about equal to –6 n dB / octave. A 20 order filter is therefore necessary to obtain a decrease of –120 dB / octave.

1.4.1.2. *High-pass filter*

A high-pass filter is a filter which lets high frequencies pass through the filter, and rejects the low-value frequencies which have a value that is less than the cutoff frequency. An ideal high-pass filter has a constant gain of 1 for frequencies which are greater than f_c and a zero gain for frequencies which are lower than f_c .

The n-order filter gain is as follows

$$\left|H(j\,f)\right| = \frac{\left(\frac{f}{f_c}\right)^n}{\sqrt{1+\left(\frac{f}{f_c}\right)^{2n}}} \qquad [1.4]$$

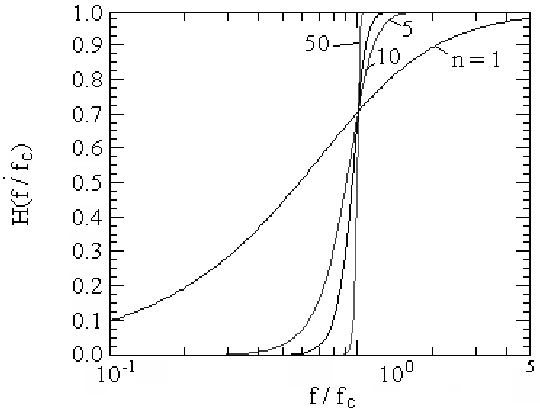

Figure 1.7. *High-pass filter - Gain versus* $f\ /\ f_c$ *, for different values of the filter order n*

1.4.1.3. *Band-pass filter*

A band-pass filter is a filter which only lets frequencies within a certain range pass through the filter. This range includes frequencies which are greater than the low cutoff frequency and which are lower than the high cutoff frequency. The ideal filter gain is zero for all frequencies except for the frequencies which can be found in this particular range. Here the value of the filter gain is 1.

1.4.1.4. *Band-stop filter*

A band-stop filter is a filter which prevents some frequencies, which can be found in a certain interval, from passing through the filter.

The band-stop filter is made up of a band-pass filter and a high-pass filter, and whose cutoff frequency is greater than the cutoff frequency of the low-pass frequency. The band-stop filter can be used to remove any parasite frequencies.

1.4.2. *Digital filters*

The digital filters can be grouped into two different categories:

– Finite impulse response (FIR) filters. These filters are said to be finite because their impulse response is stabilized ultimately to zero. The response which is provided by these filters depends entirely on the entry signal. There is no counter-reaction. FIR filters are said to be non-recursive. Each point of the filtered signal is calculated from the entry signal at the same time and also from preceding points of the signal. These filters are always stable.

The method used consists of numerically carrying out filtering by means of a convolution product, which makes it possible to produce any filter, but requires longer calculations.

Its specifications must specify:

- the ripple ratio in the passing band,
- the all-off rate in the rejected band,
- the width of the transition band.

– Infinite impulse response (IIR) filters. These filters use analog filtering techniques. Their impulse response does not settle. This type of filter is said to be recursive: the response which is provided by this type of filter depends on both the input signal and the output signal because of the existence of a feedback loop. Each point from the filtered signal is calculated from the original signal at the same time, from the amplitudes of the preceding points of the original signal and from the preceding values of the filtered signal. These filters require fewer calculations to be carried out in comparison to their FIR equivalents.

The response of a digital filter can be written as follows:

$$y(n) = \sum_{j=0}^{N} a_j \, x(n-j) - \sum_{k=0}^{M} b_k \, y(n-k) \qquad [1.5]$$

where a_j and b_k are coefficients, x is the current point of the original signal (the input signal) and y is the current point of the filtered signal (the output signal).

The b_k coefficients have a value of zero for the FIR filters.

The order of a non-recursive filter is the largest number of values of the original signal that are necessary to calculate one point of the filter's response.

The order of a recursive filter is equal to the largest number of values from the original signal from the response which is taken into account in this calculation. In general, the number of values considered in the original signal and the response is the same. Thus, each point of index n of the response of the second order filter is calculated starting from the last two points of the original signal (i.e. indices n–1 and n) and of the two preceding points of the response (indices n–2 and n–1).

The slope of the filter at its cut-off frequency is dependent on the order of the filter:

$$\text{Slope in dB/oct} = 6 \times \text{Order} \qquad [1.6]$$

If no particular precaution is taken, it is possible that the filters might introduce a type of phase difference (or delay) when compared to the original signal. It is possible to remove this dephasing during the calculation of the response.

Advantages and disadvantages of digital filters

Advantages	Disadvantages
Not sensitive to environmental conditions (temperature, humidity, etc.)	Filtering limited to 100 MHz
Can process low frequency signals with precision	Analog to digital conversion necessary
Designed and tested directly on a computer	Requires an analog anti-aliasing filter for sampling and restitution
As they are programmable, their characteristics can be changed easily without changing the hardware	Performance of the filter directly proportional to the power of the calculation unit (processor or DSP)
No problem with deriving their components	
Some filters can only be realized digitally (FIR)	
Known and controlled precision	
Reproducible without fine-tuning	

*Advantages and disadvantages of FIR (**F**inite **I**mpulse **R**esponse) filters*

These non-recursive filters have no feedback.

Advantages	Disadvantages
Always stable	Larger calculations with respect to an equivalent IIR filter
Linear phase coefficient symmetry	Delay of the filter can be significant
No phase distortion	
Possible to create all sorts of filters (through calculation of the inverse Fourier transform from a gauge in the frequency range)	

Advantages and disadvantages of IIR (Infinite Impulse Response) filters

These recursive filters have feedback.

Advantages	Disadvantages
Much less calculation with respect to an FIR	Need to check stability
	Nonlinear phase (phase distortion)

1.5. Digitizing the signal

In order to be processed by a computer, the measured signals must be digitized and represented as a time–amplitude couple. How is it possible to choose the number of points per second that need to be digitized, i.e. to choose the sampling frequency?

Digitization consists of:

– sampling, which consists of representing an analog signal using a series of n values quantified at integer multiple instants of a time interval δt, the sampling period;

– and quantization, which consists of approaching each value of the signal using an integer multiple of a basic quantity Δ, called the quantization step.

1.5.1. *Signal sampling frequency*

In 1920, H. Nyquist, from Bell Laboratories, was the first person to demonstrate, without any practical application, that "if a function does not contain any frequency which is larger than f_{max} Hz, then it is completely determined by sampling it with a frequency equal to 2 f_{max}" [SHA 49].

This theory is often associated with Claude Shannon, who worked in the same laboratory. It was Shannon who in 1948 used this theory once again, but this time on applications which were part of the world's first computers.

If we want to analyze any signal with a frequency value of up to f_{max}, it is therefore necessary to make sure that there are no frequencies which have a value that is greater than the value of f_{max} before it is finally digitized at a value of 2 f_{max}. These frequencies can sometimes resemble a real physical object or can simply be a noise. In Volume 3 we will see that these frequencies lead to a phenomenon known as spectrum folding (or aliasing). As far as this phenomenon is concerned, the signal is filtered with the help of a low-pass analog filter, whose cutoff frequency value is f_{max}.

NOTE.– *The Nyquist frequency can be shown as* $f_{Nyquist} = f_{samp.} / 2$.

Thus, it should be considered that the true contents of the filtered signal extend to the frequency corresponding to this attenuation (f_{-40}), which is calculated as follows.

In practice, however, the low-pass filters are not perfect as they do not always reject the frequencies which are above the requested cutoff level. Let us take the example of a low-pass filter which decreases by 120 dB per octave once the cutoff frequency has been passed It is estimated that the signal is sufficiently attenuated with –40 dB. Thus, it should be considered that the true contents of the filtered signal extend to the frequency corresponding to this attenuation (f_{-40}), which is calculated as follows [1.7].

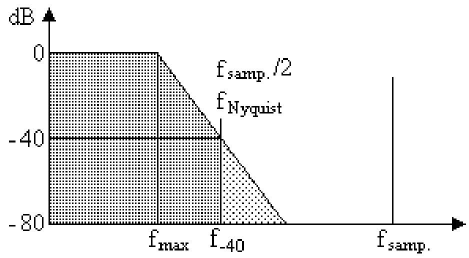

Figure 1.8. *Taking into account of the real characteristics of the low-pass filter for the determination of the sampling rate*

A reduction of 120 dB per octave means that:

$$-120 = \frac{10 \log \dfrac{A_1}{A_0}}{\log \dfrac{f_1}{f_0}} \log 2 \qquad [1.7]$$

where A_0 and A_1 are the amplitudes of the non-reduced signal (with a frequency of f_{max}) and the reduced signal to −40 dB (with a frequency of f_{-40}) respectively.

This yields:

$$-120 = \frac{10\,\frac{-40}{10}}{\log \frac{f_{-40}}{f_{max}}} \log 2 \qquad [1.8]$$

and:

$$\frac{f_{-40}}{f_{max}} = 10^{\frac{\log 2}{3}} \approx 1.26 \qquad [1.9]$$

If f_{-40} is the largest frequency signal, then according to Shannon's theorem we obtain the following equation: $f_{samp.} = 2 f_{-40}$, i.e.:

$$\frac{f_{samp.}}{f_{max}} \approx 2.52 \qquad [1.10]$$

f_{-40} is the Nyquist frequency and is written as $f_{Nyquist}$.

A number like 2.5 times would be adequate, but in order to comply with the computer world, 2.56 is usually the number employed (sometimes 2.6) [BRA 11], [SHR 95]. This result has sometimes led us to state that Shannon's theorem imposes a sampling rate equal to 2.6 times the largest frequency of the signal to be analyzed.

Using this theorem makes it possible to determine the minimum sampling frequency that is required, so that a signal keeps its full frequency contents.

According to this theorem, the sampled signal possesses all of the characteristics of the original signal without any loss of information. This means that it is possible to reconstruct the original signal from the sampled signal (see section 1.5). However, the sampled signal tends not to have the same effects on a mechanical system when it is compared to the original signal.

Example 1.2.

Let us consider the sinusoid from Figure 1.9. The sinusoid has a frequency of 100 Hz and is sampled with a sufficiently large frequency to represent the signal correctly. Figure 1.10 shows the same sampled sinusoid which is sampled at a frequency of 200 Hz (two times the frequency of the sinusoid). The signal's frequency can be read without ambiguity, but the signal is very deformed. It is easily understood that it will not have the same effects on any mechanical system on which it will be applied.

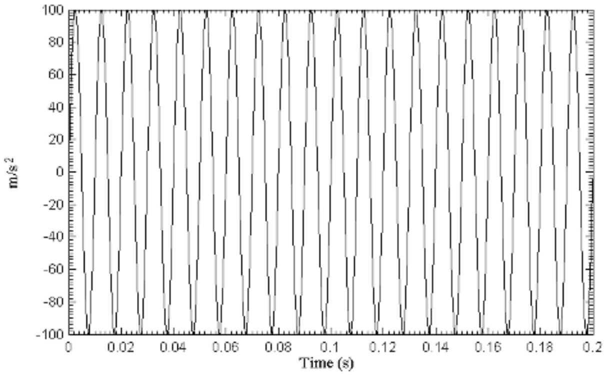

Figure 1.9. *Sampled sinusoid (100 Hz) with a frequency of 8,000 Hz (1,600 points over 200 ms)*

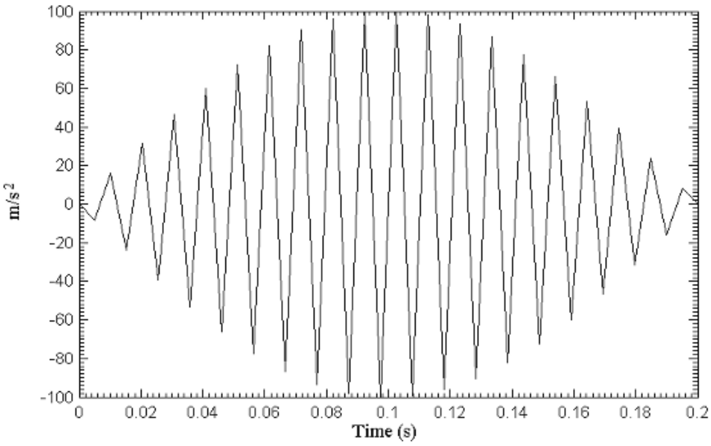

Figure 1.10. *Sampled sinusoid (100 Hz) with a frequency of 200 Hz (40 points over 200 ms)*

A.G. Marshall and F. R. Verdun [MAR 90] have shown that it is necessary to sample a signal with a frequency equal to 20 times its maximum frequency to be able to correctly reproduce its initial form. T. E. Rosenberger and J. DeSpirito [ROS 93] proposed to use a factor of 5 as a set standard.

The best practice today for each signal which is used to digitally calculate the responses produced by a mechanical system is to sample it with a frequency that is:

– ten times larger than the mechanical system's natural frequency for shocks (Volume 2);

– seven times higher than the signal maximum frequency if it is a vibration (Volume 5).

In Volume 3 we will see that Shannon's sampling frequency is sufficient for the calculation of power spectral densities.

1.5.2. *Quantization error*

The variation field of the signal $\left[-X_m, X_m\right]$ is divided into intervals of width Δ.

A signal x(t) is quantified correctly (without clipping) with a converter on n bits if its amplitude x_m is in the interval $\left[-X_m, X_m\right]$ where $X_m = 2^{n-1}\,\Delta$. In the opposite case, the signal will be clipped [HAY 99].

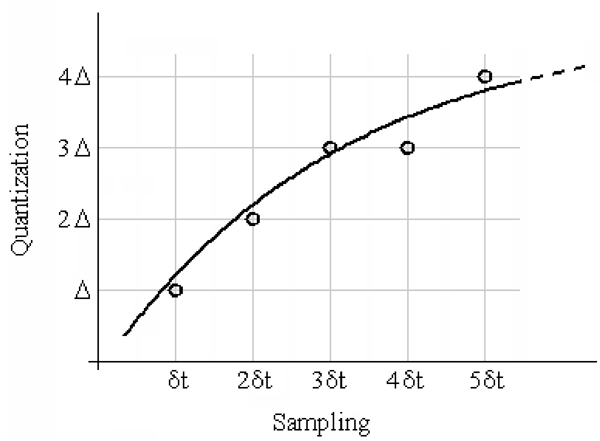

Figure 1.11. *Sampling and quantization*

Δ is called the *quantization step size* or the *resolution of the quantizer*, and the quantizer is said to be a *uniform* or a *linear* quantizer.

We have

$$\Delta = 2\, X_m\, 10^{\frac{\text{resolution (dB)}}{20}} \qquad [1.11]$$

Each signal value can thus be written as

$$x = \sum_{i=0}^{n-1} a_i\, 2^i \qquad [1.12]$$

where a_i is equal to 0 or 1.

This operation cannot be carried out without error. The difference between the actual analog value and quantized digital value is called the quantization error. This error is either due to rounding or truncation.

Assume that each error is independent of the rest, and that the error amplitude is evenly distributed in the range $-\Delta/2$ to $\Delta/2$, where Δ is the step in the analog-to-digital converter (ADC) process, its probability density p(x) being equal to $1/\Delta$.

We may then calculate the mean square value of the error:

$$\sigma^2 = \int_{-\infty}^{\infty} x^2\, p(x)\, dx = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2\, \frac{1}{\Delta}\, dx = \frac{\Delta^2}{12} \qquad [1.13]$$

The noise standard deviation (rms quantization error) is equal to $\sigma = \dfrac{\Delta}{2\sqrt{3}} \approx 0.29\, \Delta$, i.e.

$$\sigma = \frac{2\, X_m}{2^n\, \sqrt{12}} \qquad [1.14]$$

Figure 1.12 shows the variations of this error as a function of the number of bits n for different values of X_m.

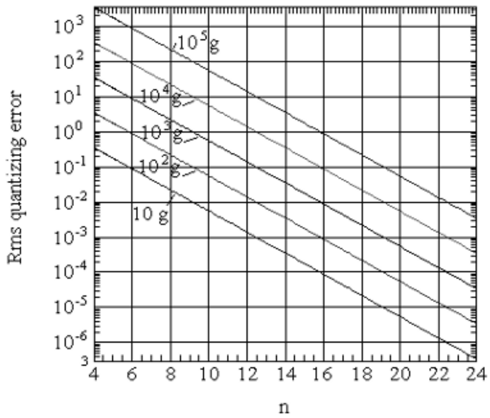

Figure 1.12. *Rms quantizing error versus bits number n*

Number of bits	8	10	12	16	20	24
Number of levels	256	1024	4096	65536	1.05 10^6	1.68 10^7
Absolute error (mV)	40	10	2.5	0.15	0.01	0.0006
Relative error (%)	0.4	0.1	0.025	1.5 10^{-3}	1 10^{-4}	6 10^{-6}

Table 1.3. *Quantization error for an input range of the analog-digital converter of 0 V to 10 V*

The quantization effects can be reduced by a low-pass filtering of the digital signal [BAC 87], with a cut-off frequency a little larger than that of the filter used before the digitization (anti-aliasing filter).

If n is the number of binary bits in the converter, the dynamic range is given by

$$D_R = \frac{\text{rms of largest sine}}{\text{rms of quantisation noise}} = \frac{\dfrac{\Delta}{\sqrt{2}}}{\dfrac{\Delta}{\sqrt{12}}} = \sqrt{6}\,\frac{\Delta}{\dfrac{2\,\Delta}{2^n}} = \sqrt{1.5}\;2^n \qquad [1.15]$$

i.e. in decibels:

$$D_R = 20 \log_{10}(\sqrt{1.5}\ 2^n) \approx 1.76 + 6.02\ n \qquad [1.16]$$

Today, current ADC have 24 bits.

n	11	12	14	16	18	20	22	24
D_R	68	74	86	98	110	122	134	146

Table 1.4. *Dynamic range versus bit number*

Example 1.3.

Let us consider a pyroshock measured with a sensor ± 100 000 g. With an ADC 11 bits (+ sign bit), the quantization step is equal to 200 000 / 2^{11} = 97.6g.

Influence on the calculation of a PSD

The error related to the quantization appears as a white noise having a PSD of amplitude [BAC 87]:

$$e_{PSD} = \frac{X_m^2}{3\ f_{samp}\ 2^{2n}} \qquad [1.17]$$

where f_{samp} is the sample rate of the signal.

1.6. Reconstructing the sampled signal

Sampling a signal transforms a continuous analog curve into a series of points. Shannon's theorem states that the sampling frequency must be equal to twice the largest signal's frequency. This sampling leads to the creation of high frequencies.

It is possible to reconstruct the signal by removing these high frequencies by applying a rectangular window into the frequency domain (a low-pass filter), and at the same time by increasing the number of points of the signal [LAL 04], [SMA 00], [WES 10]. This can be carried out using the following remarks.

The inverse Fourier transform of a rectangular window becomes a function in the form sin x/x in the time domain.

Let us suppose that the functions mentioned below are continuous. Consider a function defined in the frequency interval $-f_{max}, f_{max}$ (after a low-pass filtering if the studied signal refers to a measurement) by n points with a sampling rate of $f_{samp.} \geq 2 f_{max}$.

If we only consider the physical case in which frequencies only have positive values then this function can be expressed in the form of a Fourier integral:

$$\ddot{x}(t) = \frac{1}{2\pi} \int_0^{\Omega_{max}} \ddot{X}(\Omega)\, e^{i\Omega t}\, d\Omega \qquad [1.18]$$

where $\Omega = 2\pi f$ and $\Omega_{max} = 2\pi f_{max}$.

In this frequency band, the function $\ddot{X}(\Omega)$ can be developed into a Fourier series:

$$\ddot{X}(\Omega) = \sum_{n=0}^{\infty} a_n\, e^{-\frac{i n \Omega}{\Omega_{max}}} \qquad [1.19]$$

yielding:

$$\ddot{x}(t) = \sum_{n=0}^{\infty} \frac{a_n}{2\pi} \int_0^{\Omega_{max}} e^{i\Omega(t-t_n)}\, d\Omega \qquad [1.20]$$

where $t_n = \dfrac{n\pi}{\Omega_{max}}$.

After integration:

$$\ddot{x}(t) = \sum_{n=0}^{\infty} \frac{a_n}{\pi} \frac{\sin \Omega_{max}(t-t_n)}{t-t_n} \qquad [1.21]$$

Since:

$$\lim_{t \to t_j} \ddot{x}(t) = \frac{a_j \, \Omega_{max}}{\pi} \tag{1.22}$$

it becomes:

$$\ddot{x}(t) = \sum_{n=0}^{\infty} \ddot{x}(t_n) \frac{\sin \Omega_{max} (t - t_n)}{\Omega_{max} (t - t_n)} \tag{1.23}$$

Knowing that $f_{max} = \dfrac{f_{samp.}}{2}$ and that the signal's temporal step is equal to $\delta t = \dfrac{1}{f_{samp.}}$, this expression can be written as:

$$\ddot{x}(t) = \sum_{n=0}^{\infty} \ddot{x}(n \, \delta t) \frac{\sin\left[\dfrac{\pi}{\delta t}(t - n \, \delta t)\right]}{\dfrac{\pi}{\delta t}(t - n \, \delta t)} \tag{1.24}$$

In order to reconstruct the signal at a given time t, the procedure thus consists of centering a function of the form sinc = sin x/x on each point of the signal and adding all the sinc functions thus defined [BRA 11].

Theoretically, in order to perfectly reconstruct a signal, it is necessary for the signal to have an infinite number of points. In practice, the number of sampling points is necessarily limited and its sum of all of these functions is truncated. Due to this fact, the reconstructed signal can differ slightly from the original signal. This is, however, only a small error which can be ignored whenever the initial sampling frequency is multiplied by 10.

Example 1.4.

Consider a sinusoid which has an amplitude of 100 m/s^2 and a frequency of 100 Hz. The sinusoid is sampled with a frequency of 250 points/s (50 points over 0.2 s).

The signal is reconstructed using equation [2.7]. The number of points of the new curve is multiplied by 20 (i.e. 1,000 points over 0.2 s). The reconstructed signal is compared with the signal sampled with 50 points in Figure 1.13 and, just like a reference, the reconstructed signal is also compared with the original sinusoid which has a very large sampling frequency (5,000 points/s).

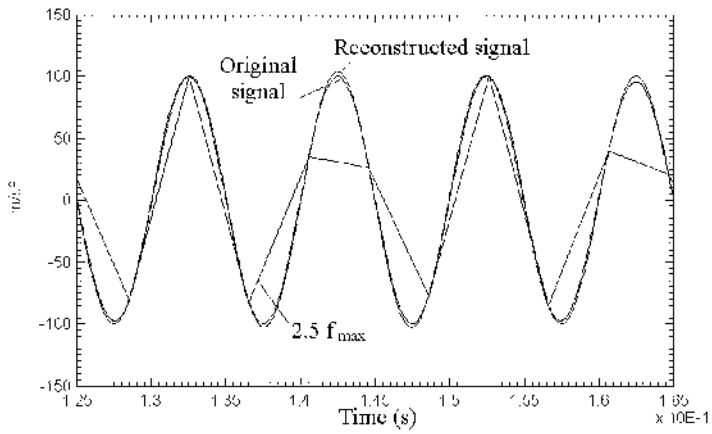

Figure 1.13. *Sampled sinusoid with a frequency of 250 Hz, which is superimposed on a reconstructed signal and on the original signal*

The reconstructed signal is very similar to the ideal signal.

1.7. Characterization in the frequency domain

The recorded signal is generally made up of several types of successive signals, such as random stationary vibrations, shocks, non-stationary vibrations, etc. It is necessary to split the signal so that, with the appropriate mathematical tools, it becomes possible to study the individual components of the signal.

The mechanical shocks are generally characterized by the effects they have on a one-degree-of-freedom linear system according to its natural frequency, i.e. the *shock response spectrum* (see Volume 2).

The frequency content of the random vibrations is studied, when they are stationary, by using a spectrum called *power spectral density* obtained by taking the average of all the Fourier transforms of several samples of the signal (see Volume 3).

Vibrations, just like shocks, can be analyzed by using another spectrum, the *extreme response spectrum*, giving the largest response of a linear one-degree-of-freedom system over the studied duration (see Volume 5).

If we take the duration of the vibrations (which can be quite long) into consideration, they are capable of damaging the mechanical parts of a system by the fatigue which is created by the repetition of stress cycles (see Volume 4). To take this mode of failure into account, a second spectrum is defined, the *fatigue damage spectrum*, which gives the fatigue damage experienced by this same one-degree-of-freedom system according to its natural frequency when it is subjected to the vibration for a given duration of time. These two spectra can be calculated for any type of vibration, for random stationary and non-stationary vibrations in particular or for a large number of repeated shocks (Volume 5).

1.8. Elaboration of the specifications

The dimensioning of a material and the realization of a qualification test with this material require environmental specifications which can result from normative documents or are developed from measurements of the real environment. The MIL STD 810 standards in the USA, GAM EG 13 in France and the international NATO standard recommend this last method, called "*test tailoring*". This approach involves:

– analyzing the conditions in which a material is used (life profile);

– linking environment measurements with each of the different conditions in which the material is used;

– synthesizing all the data thus joined together; and

– for tests, establishing the test program in the most representative and least expensive way.

Each of these operations which make up the four step approach is extremely important, but the most technical is the synopsis which will lead us, for the vibrations, to define a test of the same severity as all vibrations and shocks of the life profile. This test must be able to produce the same failures in the material that would also be created if the material were to be used in a real environment.

Two different synopsis methods exist nowadays. One of these methods involves using envelopes from power spectral densities, whilst the other method aims to reproduce the largest instantaneous stress which is produced by the vibrations, as well as the fatigue damage which is caused by the accumulation of all the different stress cycles. Volume 5 will deal with the second of these methods, which is based on the behavioral laws of fatigue of materials described in Volume 4. As the structure is generally not known at the time of the writing of specifications, the search for a specification respecting these two criteria is carried out by studying the response of a simple mechanical system, the one-degree-of-freedom linear system already used to characterize shocks. This choice highlights the advantage of standardizing the methods that are used to analyze vibrations and shocks.

1.9. Vibration test facilities

1.9.1. *Electro-dynamic exciters*

1.9.1.1. *Principle*

An electro-dynamic exciter converts electrical energy into mechanical energy. The force which is generated on the table supporting the specimen to be tested is produced by the presence of a constant magnetic field which acts upon a conductor coil. The conductor coil is linked to the table and has a variable current that runs through it. The conductor being placed perpendicularly to the magnetic field is subjected to a force perpendicular to the flow and the current.

The constant magnetic field in the air-gap where the coil moves is created by a DC current circulating in two fixed coils.

1.9.1.2. *Main components*

An electro-dynamic exciter is made up of:

– a table supporting the specimen to test, made from an aluminum alloy. This table is connected to an armature by suspension, which makes it possible for the table to move in the axial direction, minimizing the movements in the other directions;

– a mobile coil which is firmly attached to the table and which is placed inside the magnetic circuit's air-gap. This coil is guided using hydrostatic bearings;

– an armature, which forms the polar parts of the magnetic circuit;

– field coils;

– a fixed frame to which the exciter is connected by using two pivots allowing its rotation (for the large exciters).

However, a certain number of extra components are required within the electro-exciter, such as water circulating pumps for the cooling process, different security devices, a control system, etc.

The exciter is installed in a seismic mass with the aim of protecting the room from the vibrations.

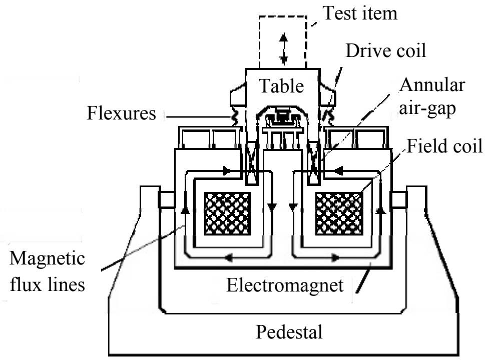

Figure 1.14. *Composition of an electro-dynamic exciter*

1.9.1.3. *Moving assembly*

The moving assembly includes:

– the specimen-holder table which is made out of a cast aluminum alloy. The mobile coil is firmly attached to the table. The table is connected by eight tighteners to a central tube guided by using two hydrostatic bearings;

– the mobile coil which is made up of two superimposed coils:

- the interior coil is made out of hollow aluminum and has a variable current running through it. This coil is cooled by water. It is this coil which transforms electrical energy into mechanical energy,

- the exterior coil is stuck onto the main coil. The exterior coil has a DC current running through it, intended to compensate for the axial loadings;

– the mounting fixture and the test specimen.

1.9.1.4. *Control system*

To obtain a given acceleration on the table at the specimen input, it is necessary to generate an electrical signal which takes account of the transfer function of the facility, the non-linearities of the shaker, the dynamic interactions, the fixture, etc.

The compensation for the transfer function is obtained from feedback making it possible to create the required level of acceleration on the table according to the frequency.

The acceleration signal which is to be generated is sinusoidal, random or a shock.

The control system, which was originally analog, is nowadays digital.

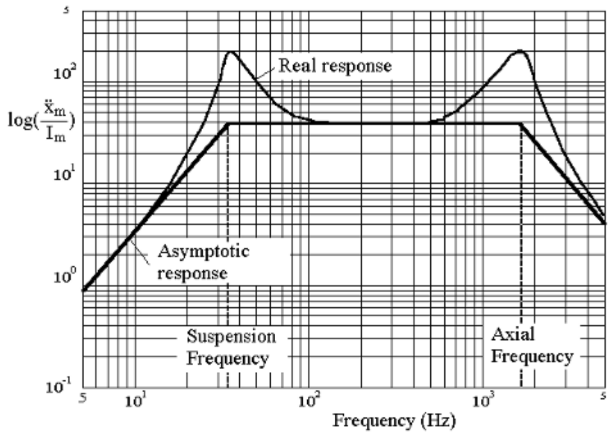

Figure 1.15. *The acceleration/current transfer function of an exciter*

Figure 1.16 shows a diagram which highlights the way in which an exciter provides feedback.

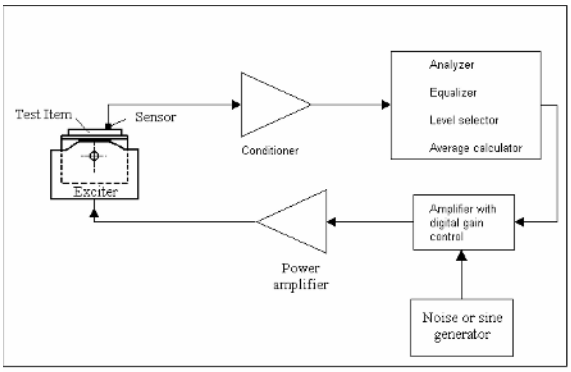

Figure 1.16. *Diagram showing the principle of the feedback process*

1.9.1.5. *Main characteristics*

The *maximum force* which is generated is generally defined by a peak value for sinusoids and by a root mean square for random materials. As far as the random materials are concerned, in order to ensure that the test is reliable, it is necessary to have a root mean square which is approximately 5.5 times smaller than the maximum force.

The *mass of the moving assembly* includes the masses of the table and of the coils, the masses of the mounting fixture and of the specimen. The moving assembly mass limits the maximum value of the specimen's acceleration. Other characteristics include:

– the *maximum mass* which can be dealt with without any external compensation;

– the *maximum couple* which can be applied to the moving assembly by a horizontal charge;

– the *maximum displacement* that can take place between mechanical stops;

– the *maximum velocity*;

– the maximum frequency range.

1.9.1.6. *The horizontal table*

The exciter's axis is generally the vertical axis. When a specimen needs to experience a vibration in any other direction, the exciter's axis is changed by turning the specimen over in order to vertically place the new test axis, using a square fixture to keep the table horizontal.

If the specimen is too heavy, it is best to keep its axis in a vertical position. The solution therefore involves turning the exciter around (using its pivots) so that it is possible to slide a horizontal table onto a thin layer of oil (see Figure 1.17).

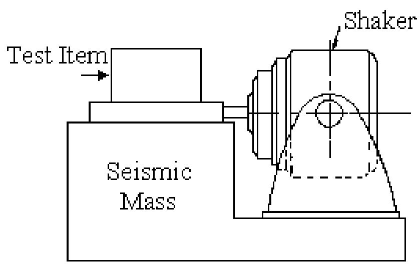

Figure 1.17. *Assembly with a horizontal table*

1.9.2. ***Hydraulic actuators***

1.9.2.1. *Principle*

Electro-hydraulic vibration systems are remote power transmitter systems which use a low pressure fluid that is not very compressible.

These vibration systems are made up of three main parts:

– the generator of pressure (pump), which receives the energy of the external medium (electrical motor) and which communicates it to the fluid;

– the receiver (the actuator), which receives the energy of the fluid and restores it in the external medium;

– focal points, which exist between the pump and the receiver (tubes, valves, etc.).

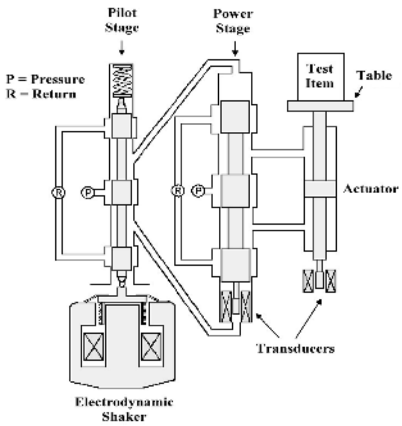

Figure 1.18. *How a hydraulic actuator works*

1.9.2.2. *Description*

The hydraulic actuator is made up of:

– a hydraulic power unit which supplies oil throughout the jack with the help of several pumps that are equipped with a cooling system and an oil reservoir (example: a flow of 600 l/min for 210 bars);

– an electro-hydraulic exciter which converts electrical energy into mechanical energy with the help of a hydraulic amplifier. The electro-hydraulic exciter receives its commands from a servo-valve;

– the servo-valve is responsible for supplying oil within the actuator. The servo-valve is made up of an electro-dynamic exciter attached to the servo-valve distributor;

– a double acting jack made up of a sliding piston in a cylinder, receiving oil on its two sides from the servo-valve distributor.

The piston is guided by hydrostatic bearings at the ends of the cylinder.

1.9.2.3. *How the hydraulic actuator functions*

The servo-valve distributor, which is connected to the mobile coil of the electro-dynamic exciter, moves in relation to the current which runs through the mobile coil.

The distributor's main high pressure supply is connected to one of the pipes that supplies one of the jack's chambers with oil. The other chamber is connected to the low pressure hydraulic return.

Due to the difference in pressure that exits on its two sides, the piston moves at a speed which is proportional to the opening of the pipes in the servo-valve's casing.

1.9.2.4. *Principal characteristics*

– The *maximum force* generated (for example, 120 kN). At higher frequencies, the performances in acceleration are limited. This limitation is due to the maximum dynamic effort which is allowed, and also due to the effects of the hydraulic natural frequency.

– The *mass of the moving assembly,* including the masses of the table and of the piston, the mass of the mounting fixture and of the test specimen. The moving assembly's total mass limits the maximum value of the specimen's acceleration.

– The *maximum displacement*, e.g. of 10 cm (limitation at low frequencies).

– The *maximum velocity*, e.g. of 1.56 m/s. In the medium frequency range, the velocity is limited by the maximum flow of oil throughout the system.

– The *frequency range* (for example, between 0.1 and 300 Hz).

1.9.3. ***Test Fixtures***

It is generally impossible to fix a test object directly to the shaker table itself. The fixture acts as a transition piece between the two. They are generally used to enable us to carry out tests in the three directions.

The real vibratory environment is generally tri-axial. Tests are usually carried out axis by axis, basically due to the cost of tri-axial testing installations. To reduce parastic vibrations as far as possible on the two axes perpendicular to the axis under test, the rule is to place the center of gravity of the specimen and the assembly over that of the testing table.

In real service conditions, equipment is often fixed onto structures which may to a greater or lesser extent deform under the vibrations according to the mass of the specimen. Ideally, the assemblies should reproduce the real fixture conditions, such as stiffnesses, support masses (mechanical impedances). However, these characteristics are generally not specified are not even known.

The assemblies are thus designed instead to be as rigid as possible in order to transmit uniformly to the specimen the forces produced by the exciter at all its fixation points. They are designed so as to not deform the spectrum that will be applied to the specimen. It is thus necessary *a priori* that their resonance frequencies be larger than the maximum specification frequency. It is however difficult to completely suppress the resonance frequencies between 1000 and 2000 Hz. In order to reduce their effects, we can add materials that create a damping reducing the amplitude of the resonance peaks.

Amongst the rules for a good design, the following must be retained [LEV 07]:

– the contact surfaces between the specimen and the test table must be machined to be as flat as possible;

– the joints between the elements making up the assembly must be welded, in a continuous manner (no simple welded joints) and bolts should be avoided;

– the bolts used to fix the specimen to the table must be screwed over a length at least equal to twice their diameter.

The most commonly used materials are steel, aluminum and magnesium, sometimes titanium. The disadvantage of steel is its weight, and magnesium its cost [FIX 87].

The natural frequency is linked with respect to the Young’s modulus E and the density ρ, which varies slightly according to the material and is therefore not a chosen criterion (Table 1.5).

	Steel	Magnesium	Aluminum	Titanium
Young's modulus E (N/m^2)	$21\ 10^{10}$	$4.14\ 10^{10}$	$6.9\ 10^{10}$	$10.7\ 10^{10}$
Density ρ (kg/m^3)	7840	1800	2770	4510
E / ρ ratio (N m / kg)	$2.64\ 10^7$	$2.3\ 10^7$	$2.49\ 10^7$	$2.38\ 10^7$

Table 1.5. *Comparison of the mechanical characteristics of the most commonly used materials for the design of assemblies*

Table 1.6 compares the main ways to build an assembly; the assembly usually being machined or welded.

Mode of manufacture	Advantages	Disadvantages
Machining	Easy to manufacture Good fixture (no joints) Used for small specimens	Costly for large specimens
Casting	Monolithic and homogeneous construction Less handling to be carried out	Only of interest for a small number of assemblies (cost of mold manufacture)
Bolting		Not recommended (behavior of bolts under vibration, loss of rigidity)
Laminating strips of material	Simple to manufacture Possibility of including layers of a damping material (rubber, plastic)	Cost linked to time spent on construction
Welding	Best solution	

Table 1.6. *Advantages and disadvantages of main methods for fabrication of assemblies*