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Kinematics and Dynamics of Crank Shaft–Connecting Rod–Piston Linkage

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In an internal combustion engine, the combination of mechanical parts, which allows the force exerted by the combustion of gas to be transformed into rotational movement resulting in vehicle wheel rotation, is referred to as "the moving part". This includes pistons, piston pins, connecting rod bearings, connecting rods and the crank shaft. The small elements related to sealing or assembling the piston and lock rings for piston pin positioning are not discussed. Because of their low mass, these connecting elements barely influence the forces created by the moving part. The connecting rod consists of a set of assembled solid objects: connecting rod beam, cap, bearings, screws, washers and possibly nuts. It will be assumed that there is no movement between these various elements of the connecting rod.

The aim of this chapter is to determine the kinematic relationships between elements of the moving part and forces involved in the junctions. Although these elements are made up of elastic materials and are thus capable of deformation under the effect of force transmission, in this chapter they will be considered non-deformable.

The junctions between the elements of the moving part are generally made with sliding bearings. These require a lubricant film layer in order to function well. The extra hundredths of a millimeter occupied by the layer contribute to the general junction mobility and additional mobility of very small amplitudes. This notable extra mobility significantly complicates the kinematic model of the moving part.

A study on dynamics using a complete kinematic model requires knowledge of junction static and dynamic properties, e.g. all the coefficients of stiffness and damping matrices of each link, the dimensions of each matrix is equal to the degrees of freedom. The developments and examples presented in other chapters of this

book show that the elastohydrodynamic behaviors of lubricated bearings in non-stationary conditions are such that they make it impossible to construct a dynamic model where the bearings would be represented by stiffness and damping matrices known in advance. Once again, the purpose of this chapter is to give the necessary background needed for junction forces calculations. A kinematic and dynamic model where the junctions are reduced to their core mobility is sufficient for acquiring results with the desired precision. In section 1.3, a model is developed that takes into account the extra mobility added by the significant deformability of the connecting rod bearing.

1.1. Kinematic model of crank shaft–connecting rod–piston linkage

1.1.1. *Model description*

The moving part examined is assumed to be those of a single-cylinder engine made up of five non-deformable bodies numbered from 0 to 4 as follows:

– 0: engine block;

– 1: the crank shaft with the center of rotation O and the center of mass G_1;

– 2: the connecting rod AB with the center of mass G_2;

– 3: the element coupling the connecting rod and the piston;

– 4: the piston.

The mechanism plan consists of a closed kinematic chain located in a Cartesian plane O ($\mathbf{x}_0$, $\mathbf{y}_0$). Only the basic mobility of the junctions between these solid bodies will be considered as follows:

– L_{01}: pivoting link between the engine block and the crank shaft with the center O and axis O $\mathbf{z}_0$;

– L_{12}: pivoting link between the crank shaft and the connecting rod with the center A and axis A $\mathbf{z}_0$;

– L_{23}: sliding pivoting link between the connecting rod and the axis with the center B and axis B $\mathbf{z}_0$;

– L_{34}: pivoting link between the piston pin and the piston with the center B and axis B $\mathbf{z}_0$;

– L_{04}: annular linear link[1] between the engine block and the piston with the center O and axis D $\mathbf{x}_0$.

Figure 1.1 shows the kinematic diagram of this model. In this figure, we can see the basis and the parameters used. To simplify notation, the basis vectors use the same notations as the frame axis. For example, the frame $\Re_0$ of the engine block has O as the point of origin and axis O$\mathbf{x}_0$, O$\mathbf{y}_0$ and O$\mathbf{z}_0$. The base of this frame consists of orthonormal vectors $\mathbf{x}_0$, $\mathbf{y}_0$ and $\mathbf{z}_0$. Because of this similarity, the vector notation is not shown in the figure.

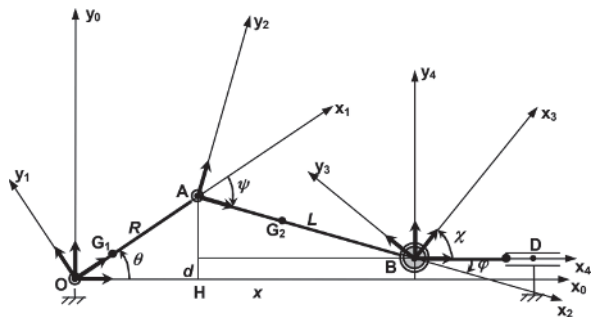

Figure 1.1. *Kinematic model*

The reference frames and basis are defined as follows:

– $\Re_0 \equiv \{O, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0\}$: engine block frame;

– $\Re_1 \equiv \{O, \mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_0\}$: crank shaft frame;

– $\Re_2 \equiv \{A, \mathbf{x}_2, \mathbf{y}_2, \mathbf{z}_0\}$: connecting rod frame;

– $\Re_3 \equiv \{B, \mathbf{x}_3, \mathbf{y}_3, \mathbf{z}_0\}$: small end shaft frame;

– $\Re_4 \equiv \{B, \mathbf{x}_4, \mathbf{y}_4, \mathbf{z}_0\} \equiv \{B, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0\}$: piston frame;

– $B_i \equiv \{\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_0\}$: basis of the frame $\Re_i$.

The geometrical values used are as follows:

– R: the radius of the crank shaft;

– a: distance from center O to the crank shaft's center of mass G_2;

– L: connecting rod length;

– $k = AG_2/L$: position relative to point G_2 of the connecting rod;

1 This choice is necessary for achieving a non-hyperstatic assembly.

– d: default alignment between the cylinder axis and the center of the crank shaft.

The parameters used are as follows:

– θ: crank shaft angle ($\mathbf{x}_0$,$\mathbf{x}_1$) relative to the engine block;

– ψ: connecting rod angle ($\mathbf{x}_1$,$\mathbf{x}_2$) with respect to the crank shaft;

– φ: connecting rod angle ($\mathbf{x}_0$,$\mathbf{x}_2$) with respect to the engine block;

– χ: connecting rod small end axis angle ($\mathbf{x}_0$,$\mathbf{x}_3$) with respect to the engine block;

– x: piston pin position with respect to the center of the crank shaft .

Except for the angle χ of the piston axis that is independent, these parameters are geometrically interrelated to one another as follows:

$$HA = R\sin\theta = d - L\sin\varphi$$

$$x = R\cos\theta + L\cos\varphi$$

$$\varphi = \theta + \psi$$

Knowing that $\overline{R} = \dfrac{R}{L}$ and $\overline{d} = \dfrac{d}{L}$, the following equations can be derived:

$$\sin\varphi = \overline{d} - \overline{R}\sin\theta \qquad [1.1]$$

$$\cos\varphi = \sqrt{1 - (\overline{d} - \overline{R}\sin\theta)^2} \qquad [1.2]$$

$$\varphi = Arc\sin\left(\overline{d} - \overline{R}\sin\theta\right) \qquad [1.3]$$

$$x = L\left(\frac{R}{L}\cos\theta + \cos\varphi\right) = L\left[\overline{R}\cos\theta + \sqrt{1 - \left(\overline{d} - \overline{R}\sin\theta\right)^2}\right] \qquad [1.4]$$

for which only the independent parameters θ and χ are absolutely necessary.

1.1.2. *Expressions of angular velocities*

The crank shaft can be assumed to move at an angular velocity of $\omega = \dot{\theta}$. This velocity will be the kinematic input parameter used for studying the hydrodynamic properties of crank shaft journal bearings. For the bearing at the big end of the connecting rod, this role will be carried out by the angular velocity $\dot{\psi}$. Using the previously derived equation [1.3], the connecting rod angular velocity $\dot{\varphi}$ can be calculated as:

$$\dot{\varphi} = \frac{-\overline{R}\,\dot{\theta}\cos\theta}{\sqrt{1-(\overline{d}-\overline{R}\sin\theta)^2}} = \frac{-\overline{R}\,\dot{\theta}\cos\theta}{\cos\varphi} \qquad [1.5]$$

The angular velocity $\dot{\psi}$ can then be expressed as:

$$\dot{\psi} = \dot{\theta}\left[\frac{\overline{R}\cos\theta}{\sqrt{1-(\overline{d}-\overline{R}\sin\theta)^2}} - 1\right] = \dot{\theta}\left(\frac{\overline{R}\cos\theta}{\cos\varphi} - 1\right) \qquad [1.6]$$

The angular velocity $\dot{\chi}$ cannot be determined solely by kinematic relationships.

1.1.3. *Expressions of velocity for points A, G_2 and B*

The velocity relation between points of non-deformable bodies allows us to calculate the velocities of different points in the mechanism.

$$\mathrm{V}_{01}(\mathrm{A}) = R\,\dot{\theta}\,\mathrm{y}_1 \qquad [1.7]$$

$$\mathrm{V}_{02}(\mathrm{G}_2) = R\,\dot{\theta}\left[\mathrm{y}_1 - \frac{k\cos\theta}{\cos\varphi}\mathrm{y}_2\right] \qquad [1.8]$$

$$\mathrm{V}_{02}(\mathrm{B}) = R\,\dot{\theta}\left(-\sin\theta + \tan\varphi\cos\theta\right)\mathrm{x}_0 \qquad [1.9]$$

The vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ can be easily replaced by their separated components in the basis B_0:

$$y_1 = -\sin\theta x_0 + \cos\theta y_0$$

$$y_2 = -\sin\varphi x_0 + \cos\varphi y_0$$

which gives:

$$V_{02}(G_2) = R\dot{\theta}\left[\left(-\sin\theta + k\tan\varphi\cos\theta\right)x_0 + (1-k)\cos\theta y_0\right] \qquad [1.10]$$

With the help of equations [1.1] and [1.2], the velocities can be expressed as a function of a single parameter θ.

In most engines, the piston is in alignment with the center of the crank shaft, so that d becomes zero. In this case, the angular velocity $\dot{\varphi}$ can be written as:

$$\dot{\varphi} = \frac{-\overline{R}\dot{\theta}\cos\theta}{\sqrt{1-\overline{R}^2\sin^2\theta}} \qquad [1.11]$$

and the velocity of points G_2 and B can be expressed as:

$$V_{02}(G_2) = R\dot{\theta}\left[-(1+k\frac{\overline{R}\cos\theta}{\sqrt{1-\overline{R}^2\sin^2\theta}})\sin\theta x_0 + (1-k)\cos\theta y_0\right] \qquad [1.12]$$

$$V_{02}(B) = -R\dot{\theta}\sin\theta\left[1+\frac{\overline{R}\cos\theta}{\sqrt{1-\overline{R}^2\sin^2\theta}}\right]x_0 \qquad [1.13]$$

1.1.4. *Expressions of connecting rod angular acceleration and points G_2 and B accelerations*

From equation [1.5], the angular acceleration $\ddot{\varphi}$ can be obtained as:

$$\ddot{\varphi} = \frac{\overline{R}}{\cos\varphi}\left[-\ddot{\theta}\cos\theta + \dot{\theta}^2 \frac{\left(\cos^2\varphi\sin\theta + \overline{R}\cos^2\theta\sin\varphi\right)}{\cos^2\varphi}\right] \qquad [1.14]$$

From [3.10] and [3.9], the equations describing acceleration of points G_2 and B can be derived as:

$$\begin{aligned}\Gamma(G_2/\Re_0) = {} & R\ddot{\theta}\left[\left(-\sin\theta + k\tan\varphi\cos\theta\right)x_0 + (1-k)\cos\theta y_0\right] \\ & + R\dot{\theta}^2\left[\left(-\cos\theta - k\tan\varphi\sin\theta - k\overline{R}\frac{\cos^2\theta}{\cos^3\varphi}\right)x_0 - (1-k)\sin\theta y_0\right]\end{aligned} \qquad [1.15]$$

$$\Gamma(B/\Re_0) = R\left[\ddot{\theta}\left(-\sin\theta + \tan\varphi\cos\theta\right) + \dot{\theta}^2\left(-\cos\theta - \tan\varphi\sin\theta - \overline{R}\frac{\cos^2\theta}{\cos^3\varphi}\right)\right]x_0 \qquad [1.16]$$

When the crank shaft turns at a constant velocity ω (no acyclism), these equations can be reduced to:

$$\ddot{\varphi} = \overline{R}\dot{\theta}^2 \frac{\cos^2\varphi\sin\theta + \overline{R}\cos^2\theta\sin\varphi}{\cos^3\varphi} \qquad [1.17]$$

$$\Gamma(G_2/\Re_0) = R\dot{\theta}^2\left[\left(-\cos\theta - k\tan\varphi\sin\theta - k\overline{R}\frac{\cos^2\theta}{\cos^3\varphi}\right)x_0 - (1-k)\sin\theta y_0\right] \qquad [1.18]$$

$$\Gamma(B/\Re_0) = \ddot{x}x_0 = R\dot{\theta}^2\left(-\cos\theta - \tan\varphi\sin\theta - \overline{R}\frac{\cos^2\theta}{\cos^3\varphi}\right)x_0 \qquad [1.19]$$

1.2. Efforts in the links between the crank shaft, the connecting rod and the piston

1.2.1. *Hypothesis and data*

The single-cylinder model defined in section 1.1 is considered. It will be assumed that the input parameter from the point of view of forces consists of pressure exerted by gas on the upper face of the piston. This pressure varies continuously throughout the whole engine cycle. The piston movement, which results from the aforementioned pressure, is transmitted throughout the entire moving part onto the receiving unit. Because of its large inertia, the latter is assumed to turn at a constant speed, making equations [1.17]–[1.19] valid.

The weight of the elements is assumed to be negligible when compared to the other forces acting on various elements of the mechanism.

The calculations of frictional forces between the piston and the cylinder, either at piston ring or piston skirt level, require an understanding of hydrodynamic behavior of lubricated contacts [GAM 06] that goes beyond the scope of this book.

Tian *et al*. [TIA 96] showed that for a device of comparable dimensions, the frictional forces at skirt/liner contact points do not exceed 50 N, except for several degrees around the top dead center (TDC) region, where end compression occurs, and it reaches 100 N. As far as the frictional forces between the liner and piston are concerned, Keribar and Dursunkaya [KER 92] found that during the combustion phase for a piston with a diameter of 125 mm (50% larger than the one considered here), the hydrodynamic forces were about 200 N and the forces resulting from a mixed lubrication model were around 700 N. Similar values were obtained by Mansouri and Wong [MAN 05]. In the following discussion, these frictional forces are not taken into consideration. The validity of the hypothesis will be checked.

The reference frame $\mathfrak{R}_0$ is assumed to be Galilean. The engine block acceleration due to the movement of the vehicle and itself with respect to the latter is not taken into account.

For the studies of bearing hydrodynamic properties, general dynamic equations are used to determine the forces acting in each link. These forces are divided into $\mathbf{x}_0$ and $\mathbf{y}_0$ components. The $\mathbf{z}_0$ component of the resulting forces is not relevant to the study of bearings, which is why in the following description it will not be considered. The $\mathbf{x}_0$ and $\mathbf{y}_0$ component moment projections that originate from the pressure field in the film are zero for aligned bearings and give the righting moments for non-aligned bearings. As the junctions are assumed to have perfect kinematic

properties, this last case is not considered. The projection $\mathbf{z}_0$ component of the moment due to shear stress because of lubricant viscosity is unknown.

Only the equations used for determining the link forces are given.

The given mass parameters are as follows:

– m_1: crank shaft mass;

– m_2: connecting rod mass;

– m_3: piston pin mass;

– m_4: piston mass;

– I_2: moment of inertia of the rod with respect to the axis G $\mathbf{z}_0$;

– I_3: moment of inertia of the axis with respect to the axis B $\mathbf{z}_0$.

1.2.2. *Dynamics equations for the piston*

External forces

– $\mathbf{F}_g = -p_g S\mathbf{x}_0$: force of gas where p_g is the gas pressure and S is the piston surface;

– $\mathbf{F}_{0\to4} = Y_{04}\mathbf{y}_0$: force exerted by the cylinder on the piston normal to the direction of the displacement of the piston;

– $\mathbf{F}_{3\to4} = -X_{43}\mathbf{x}_0 - Y_{43}\mathbf{y}_0$: force exerted by the axis on the piston.

Equations

– $m_4\mathbf{\Gamma}(B/\Re_0) = \mathbf{F}_g + \mathbf{F}_{0\to4} + \mathbf{F}_{3\to4}$

which projected on basis B_0 gives:

$$\begin{cases} m_4\ddot{x} = -p_g S - X_{43} \\ 0 = Y_{04} - Y_{43} \end{cases} \qquad [1.20]$$

1.2.3. *Dynamics equations for the axis*

External forces and moments

– $\mathbf{F}_{4\to3} = X_{43}\mathbf{x}_0 + Y_{43}\mathbf{y}_0$: force exerted by the piston on the axis;

– $\mathbf{F}_{2\to3} = X_{23}\mathbf{x}_0 + Y_{23}\mathbf{y}_0$: force exerted by the rod on the axis;

– $\mathbf{M}_{4\to3}(\mathrm{B}).\mathbf{z}_0 = M_{43}$: frictional moment exerted by the piston on the axis;

– $\mathbf{M}_{2\to3}(\mathrm{B}).\mathbf{z}_0 = M_{23}$: frictional moment exerted by the rod on the axis.

Equations

– $m_3\mathbf{\Gamma}(\mathrm{B}/\Re_0) = \mathbf{F}_{4\to3} + \mathbf{F}_{2\to3}$

– $\mathrm{I}_3\ddot{\chi}\ \mathbf{z}_0 = \mathbf{M}_{4\to3}(\mathrm{B}) + \mathbf{M}_{2\to3}(\mathrm{B})$

which can be combined as follows:

$$\begin{cases} m_3\ddot{x} = X_{43} + X_{23} \\ 0 = Y_{43} + Y_{23} \\ I_3\ddot{\chi} = M_{43} + M_{23} \end{cases} \qquad [1.21]$$

1.2.4. *Dynamics equations for the connecting rod*

External forces and moments

– $\mathbf{F}_{3\to2} = -X_{23}\mathbf{x}_0 - Y_{23}\mathbf{y}_0$: force exerted by the axis on the connecting rod;

– $\mathbf{F}_{1\to2} = X_{12}\mathbf{x}_0 + Y_{12}\mathbf{y}_0$: force exerted by the crank shaft on the connecting rod;

– $\mathbf{M}_{3\to2}(\mathrm{B}).\mathbf{z}_0 = -M_{23}$: frictional moment exerted by the axis on the connecting rod;

– $\mathbf{M}_{1\to2}(\mathrm{A}).\mathbf{z}_0 = M_{12}$: frictional moment exerted by the crank shaft on the connecting rod.

Equations

– $m_2\mathbf{\Gamma}(\mathrm{G}_2/\Re_0) = \mathbf{F}_{3\to2} + \mathbf{F}_{1\to2}$

– $I_2\ddot{\varphi}\ \mathbf{z}_0 = \mathbf{M}_{3\to2}(\mathrm{B}) + \mathbf{G_2B} \wedge \mathbf{F}_{3\to2} + \mathbf{M}_{1\to2}(\mathrm{A}) + \mathbf{G_2A} \wedge \mathbf{F}_{1\to2}$

which can be combined to give:

$$\begin{cases} m_2 R \dot{\theta}^2 \left(-\cos\theta - k\tan\varphi \sin\theta - k\overline{R}\dfrac{\cos^2\theta}{\cos^3\varphi} \right) = -X_{23} + X_{12} \\ -m_2(1-k)R\dot{\theta}^2 \sin\theta = -Y_{23} + Y_{12} \\ I_2 \ddot{\varphi} = -M_{23} + M_{12} - L\left[(1-k)Y_{23} + kY_{12}\right]\cos\varphi + L\left[(1-k)X_{23} + kX_{12}\right]\sin\varphi \end{cases} \quad [1.22]$$

There are seven junction strength components and three moment components in equations [1.20]–[1.22]. The unknowns related to angular acceleration of axis $\ddot{\chi}$ should be added to these unknowns, giving a total of 11 unknowns in eight equations. The moments M_{12}, M_{23} and M_{43} related to viscous friction can only be determined by carrying out hydrodynamic calculations.

To resolve this issue, the first possibility is to assume that the viscous friction has a negligible effect on junction force component calculations. Assuming M_{12}, M_{23} and M_{43} are zero, the angular acceleration of axis $\ddot{\chi}$ should also be zero. This leaves seven unknowns in seven equations.

If it is assumed that the moments of viscous friction are not negligible, a more complicated hydrodynamic model must be created in order to incorporate equations [1.20]–[1.22] (creation of a model like this is outlined in section 3.4).

1.2.5. *Dynamics equations for the crank shaft*

External forces

– $\mathbf{F}_{0\to1} = -X_{10}\mathbf{x}_0 - Y_{10}\mathbf{y}_0$: force exerted by the engine block on the crank shaft;

– $\mathbf{F}_{2\to1} = -X_{12}\mathbf{x}_0 - Y_{12}\mathbf{y}_0$: force exerted by the connecting rod on the crank shaft.

Equations

– $m_1\mathbf{\Gamma}(G_1/\Re_0) = \mathbf{F}_{0\to1} + \mathbf{F}_{2\to1}$

which can be expressed as:

$$\begin{cases} -m_1 a \dot{\theta}^2 \cos\theta = -X_{10} - X_{12} \\ -m_1 a \dot{\theta}^2 \sin\theta = -Y_{10} - Y_{12} \end{cases} \quad [1.23]$$

If it is assumed that the crank shaft is statically balanced, this would mean that the center of mass is located on the O z_0 axis and the equations become:

$$\begin{cases} 0 = -X_{10} - X_{12} \\ 0 = -Y_{10} - Y_{12} \end{cases} \quad [1.24]$$

1.2.6. *Efforts for frictionless links*

If the viscous friction is ignored, the system can be reduced to the following five equations:

$$\begin{cases} X_{43} = -m_4 \ddot{x} - p_g S \\ X_{43} + X_{23} = m_3 \ddot{x} \\ X_{23} - X_{12} = m_2 R \dot{\theta}^2 K \\ Y_{23} - Y_{12} = m_2 (1-k) R \dot{\theta}^2 \sin\theta \\ \left[(1-k)X_{23} + kX_{12}\right]\sin\varphi - \left[(1-k)Y_{23} + kY_{12}\right]\cos\varphi = \dfrac{I_2}{L}\ddot{\varphi} \end{cases}$$

where the factor K is defined as:

$$K = \cos\theta + k\tan\varphi\sin\theta + k\overline{R}\frac{\cos^2\theta}{\cos^3\varphi} \quad [1.25]$$

If m_{34} represents the sum of the piston mass and pin mass:

$$m_{34} = m_3 + m_4 \quad [1.26]$$

the solution to the system is

$$X_{43} = -m_4 \ddot{x} - p_g S$$

$$X_{23} = m_{34} \ddot{x} + p_g S$$

$$X_{12} = -m_2 R \dot{\theta}^2 K + m_{34} \ddot{x} + p_g S$$

$$Y_{23} = k m_2 R \dot{\theta}^2 \left[(1-k)\sin\theta - K\tan\varphi\right] + \left(m_{34}\ddot{x} + p_g S\right)\tan\varphi - \frac{I_2}{L\cos\varphi}\ddot{\varphi}$$

$$Y_{12} = -m_2 R \dot{\theta}^2 \left[(1-k)^2 \sin\theta + kK\tan\varphi\right] + \left(m_{34}\ddot{x} + p_g S\right)\tan\varphi - \frac{I_2}{L\cos\varphi}\ddot{\varphi}$$

The forces of interaction between the elements of the moving part, expressed using B_0, are thus:

$$F_{1\to 0}\begin{cases} m_2 R \dot{\theta}^2 K - m_{34}\ddot{x} - p_g S \\ m_2 R \dot{\theta}^2 \left[(1-k)^2 \sin\theta + kK\tan\varphi\right] - \left(m_{34}\ddot{x} + p_g S\right)\tan\varphi + \dfrac{I_2}{L\cos\varphi}\ddot{\varphi} \end{cases} \quad [1.27]$$

$$F_{1\to 2}\begin{cases} -m_2 R \dot{\theta}^2 K + m_{34}\ddot{x} + p_g S \\ -m_2 R \dot{\theta}^2 \left[(1-k)^2 \sin\theta + kK\tan\varphi\right] + \left(m_{34}\ddot{x} + p_g S\right)\tan\varphi - \dfrac{I_2}{L\cos\varphi}\ddot{\varphi} \end{cases} \quad [1.28]$$

$$F_{2\to 3}\begin{cases} m_{34}\ddot{x} + p_g S \\ k m_2 R \dot{\theta}^2 \left[(1-k)\sin\theta - K\tan\varphi\right] + \left(m_{34}\ddot{x} + p_g S\right)\tan\varphi - \dfrac{I_2}{L\cos\varphi}\ddot{\varphi} \end{cases} \quad [1.29]$$

$$F_{4\to 3}\begin{cases} -m_4\ddot{x} - p_g S \\ -k m_2 R \dot{\theta}^2 \left[(1-k)\sin\theta - K\tan\varphi\right] - \left(m_{34}\ddot{x} + p_g S\right)\tan\varphi + \dfrac{I_2}{L\cos\varphi}\ddot{\varphi} \end{cases} \quad [1.30]$$

1.3. Load diagram correction in the case of large deformations

In the previous sections, the relationship of the force exerted by the crank shaft on the connecting rod is determined based on the pressure of gas in a combustion chamber with a configuration that uses tight junctions with non-deformable solids. For high power engines (for example, Formula 1 engines), the connecting rods are as small as possible to reduce their force dynamic effects. The dynamic load on the bearings still remains high and when passing through the TDC, the traction on

the connecting rod is such that the clearance in the bearing exceeds its nominal value several times. In this case, the load diagram can be adjusted as discussed in the following.

1.3.1. *Kinematics of crank shaft–connecting rod–piston system with mobility*

Figure 1.2 shows the crank shaft–connecting rod system when it has mobility in the "small end". The connecting rod end A at the bearing housing center is located at a different position than the end A_1 at the center of the pin. The position of A_1 in the reference frame A $\mathbf{x_2y_2z_2}$ in the connecting rod is given using ε_x and ε_y.

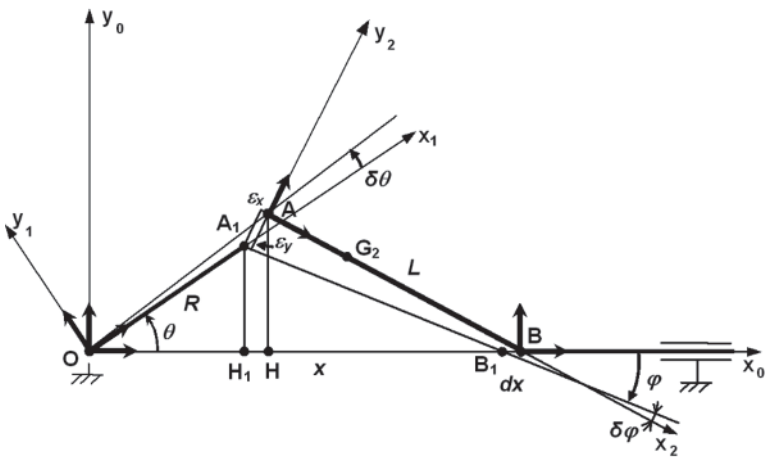

Figure 1.2. *Crank shaft–connecting rod–piston system with mobility in the connecting rod big end link*

By using the previously established notations, the geometric relationships for H_1A_1 and HA can be written as follows:

$$R\sin\theta = -L\sin\varphi$$

$$R\sin\theta - L\sin\varphi - \varepsilon_x \sin\tilde{\varphi} - \varepsilon_y \cos\tilde{\varphi} = -L\sin\tilde{\varphi}$$

with:

$$\tilde{\varphi} = \varphi + \delta\varphi \qquad [1.31]$$

where $\delta\varphi$ is the modification of φ due to mobility. Assuming the displacement of A_1A is small with respect to R, the equations can be written as a function of ε_x and ε_y:

$$\delta\varphi \simeq \frac{\varepsilon_y \cos\varphi + \varepsilon_x \sin\varphi}{\varepsilon_y \sin\varphi - (L - \varepsilon_x)\cos\varphi} \simeq -\frac{1}{L}\left(\varepsilon_y + \varepsilon_x \tan\varphi\right) \qquad [1.32]$$

The position and speed of the center A of the connecting rod "small end" bearing can be written as:

$$\mathrm{OA} = R\mathrm{x}_1 - \varepsilon_x \mathrm{x}_2 - \varepsilon_y \mathrm{y}_2$$

$$\mathrm{V}_{02}(\mathrm{A}) = R\omega \mathrm{y}_1 - \dot{\varepsilon}_x \mathrm{x}_2 - \dot{\varepsilon}_y \mathrm{y}_2 - \varepsilon_x \dot{\mathrm{x}}_2 - \varepsilon_y \dot{\mathrm{y}}_2 \qquad [1.33]$$

The vectors $\mathbf{y}_1$, $\mathbf{x}_2$ and $\mathbf{y}_2$ can easily be expressed in terms of their components in the basis B_0 and are given as:

$$\mathrm{y}_1 = -\sin\theta \mathrm{x}_0 + \cos\theta \mathrm{y}_0$$

$$\mathrm{x}_2 = \cos\tilde{\varphi}\, \mathrm{x}_0 + \sin\tilde{\varphi}\, \mathrm{y}_0 \simeq (\cos\varphi - \delta\varphi \sin\varphi)\mathrm{x}_0 + (\sin\varphi + \delta\varphi\cos\varphi)\mathrm{y}_0$$

$$\mathrm{y}_2 = -\sin\tilde{\varphi}\, \mathrm{x}_0 + \cos\tilde{\varphi}\, \mathrm{y}_0 \simeq -(\sin\varphi + \delta\varphi\cos\varphi)\mathrm{x}_0 + (\cos\varphi - \delta\varphi\sin\varphi)\mathrm{y}_0$$

The derivatives of vectors $\mathbf{x}_2$ and $\mathbf{y}_2$ are given as:

$$\dot{\mathrm{x}}_2 = \dot{\tilde{\varphi}}\left[-\sin\tilde{\varphi}\, \mathrm{x}_0 + \cos\tilde{\varphi}\, \mathrm{y}_0\right] = \left(\dot{\varphi} + \dot{\delta\varphi}\right)\left[-\sin\tilde{\varphi}\, \mathrm{x}_0 + \cos\tilde{\varphi}\, \mathrm{y}_0\right]$$

$$\dot{\mathrm{y}}_2 = -\dot{\tilde{\varphi}}\left[\cos\tilde{\varphi}\, \mathrm{x}_0 + \sin\tilde{\varphi}\, \mathrm{y}_0\right] = -\left(\dot{\varphi} + \dot{\delta\varphi}\right)\left[\cos\tilde{\varphi}\, \mathrm{x}_0 + \sin\tilde{\varphi}\, \mathrm{y}_0\right]$$

with:

$$\dot{\varphi} = -\bar{R}\omega \frac{\cos\theta}{\cos\varphi} \qquad [1.34]$$

$$\dot{\delta\varphi} = -\frac{1}{L}\left(\dot{\varepsilon}_y + \dot{\varepsilon}_x \tan\varphi + \varepsilon_x \frac{\dot{\varphi}}{\cos^2\varphi}\right) \qquad [1.35]$$

These relationships allow $\mathbf{V}_{02}(\mathrm{A})$ to be expressed in the following form:

$$\mathrm{V}_{02}(\mathrm{A}) = u_A \mathrm{x}_0 + v_A \mathrm{y}_0 \qquad [1.36]$$

where u_A and v_A can be obtained by substituting the previous equations in [1.33]:

$$\begin{aligned} u_A &= -R\omega\sin\theta + \left(\dot{\varepsilon}_y + \dot{\varepsilon}_x\,\delta\varphi\right)\sin\varphi - \left(\dot{\varepsilon}_x - \dot{\varepsilon}_y\,\delta\varphi\right)\cos\varphi \\ &\quad + \dot{\tilde{\varphi}}\left[\left(\varepsilon_x - \varepsilon_y\delta\varphi\right)\sin\varphi + \left(\varepsilon_y + \varepsilon_x\delta\varphi\right)\cos\varphi\right] \end{aligned} \qquad [1.37]$$

$$\begin{aligned} v_A &= R\omega\left(\cos\theta\right) - \left(\dot{\varepsilon}_x - \dot{\varepsilon}_y\,\delta\varphi\right)\sin\varphi - \left(\dot{\varepsilon}_y + \dot{\varepsilon}_x\,\delta\varphi\right)\cos\varphi \\ &\quad + \dot{\tilde{\varphi}}\left[\left(\varepsilon_y + \varepsilon_x\delta\varphi\right)\sin\varphi - \left(\varepsilon_x - \varepsilon_y\delta\varphi\right)\cos\varphi\right] \end{aligned} \qquad [1.38]$$

The velocity of the connecting rod mass center G_2 can be derived as:

$$\begin{aligned} \mathrm{V}_{02}(\mathrm{G}_2) &= \mathrm{V}_{02}(\mathrm{A}) + \Omega_{02} \wedge \mathrm{AG}_2 = \mathrm{V}_{02}(\mathrm{A}) + \dot{\tilde{\varphi}}\mathrm{z}_0 \wedge kL\mathrm{x}_2 \\ &= u_A \mathrm{x}_0 + v_A \mathrm{y}_0 + kL\dot{\tilde{\varphi}}\mathrm{y}_2 \end{aligned}$$

or:

$$\mathrm{V}_{02}(\mathrm{G}_2) = \left[u_A - kL\dot{\tilde{\varphi}}\left(\sin\varphi + \delta\varphi\cos\varphi\right)\right]\mathrm{x}_0 + \left[v_A + kL\dot{\tilde{\varphi}}\left(\cos\varphi - \delta\varphi\sin\varphi\right)\right]\mathrm{y}_0 \qquad [1.39]$$

When the connection at A has no mobility (ε_x, ε_y and $\delta\varphi$ are zero as well as their derivatives), the velocity of G_2 can be simply written as:

$$\mathrm{V}_{02}(\mathrm{G}_2) = R\omega\left[\left(-\sin\theta + k\tan\varphi\cos\theta\right)\mathrm{x}_0 + (1-k)\cos\theta\,\mathrm{y}_0\right]$$

The velocity of point B can be calculated by choosing k equal to 1 in the following expression:

$$V_{02}(B) = \left[u_A - L\dot{\tilde{\varphi}}(\sin\varphi + \delta\varphi\cos\varphi) \right] x_0 \qquad [1.40]$$

After deriving [1.39] and [1.40] and eliminating the second-order parameters with respect to $\delta\varphi$, the equations describing acceleration for points G_2 and B can be obtained as:

$$\begin{aligned}\Gamma(G_2/\Re_0) = &\left[\dot{u}_A - kL\left(\left(\ddot{\varphi} + \ddot{\delta\varphi} - \dot{\varphi}^2\,\delta\varphi \right)\sin\varphi + \left(\ddot{\varphi}\,\delta\varphi + \dot{\varphi}^2 + 2\dot{\varphi}\,\dot{\delta\varphi} \right)\cos\varphi \right) \right] x_0 \\ &+ \left[\dot{v}_A + kL\left(\left(\ddot{\varphi} + \ddot{\delta\varphi} - \dot{\varphi}^2\,\delta\varphi \right)\cos\varphi - \left(\ddot{\varphi}\,\delta\varphi + \dot{\varphi}^2 + 2\dot{\varphi}\,\dot{\delta\varphi} \right)\sin\varphi \right) \right] y_0\end{aligned} \qquad [1.41]$$

$$\Gamma(B_2/\Re_0) = \left[\dot{u}_A - L\left(\left(\ddot{\varphi} + \ddot{\delta\varphi} - \dot{\varphi}^2\,\delta\varphi \right)\sin\varphi + \left(\ddot{\varphi}\,\delta\varphi + \dot{\varphi}^2 + 2\dot{\varphi}\,\dot{\delta\varphi} \right)\cos\varphi \right) \right] x_0 \qquad [1.42]$$

$\ddot{\tilde{\varphi}}$ is obtained by deriving $\dot{\varphi} + \ddot{\delta\varphi}$ (equations [1.34] and [1.35]) assuming the crank shaft turns at a constant speed ω (no acyclism):

$$\ddot{\varphi} = \overline{R}\omega \frac{\omega\cos\varphi\sin\theta - \dot{\varphi}\sin\varphi\cos\theta}{\cos^2\varphi}$$

$$\ddot{\delta\varphi} = -\frac{1}{L}\left[\ddot{\varepsilon}_y + \ddot{\varepsilon}_x\tan\varphi + \frac{1}{\cos^2\varphi}\left(\varepsilon_x\ddot{\varphi} + 2\varepsilon_x\dot{\varphi}^2\tan\varphi + 2\dot{\varepsilon}_x\dot{\varphi} \right) \right] \qquad [1.43]$$

The parameters $\dot{u}_A$ and $\dot{v}_A$ can be derived by using equations [1.37] and [1.38] and eliminating the second-order parameters with respect to $\delta\varphi$ as follows:

$$\begin{aligned}\dot{u}_A &= -R\omega^2\cos\theta + \left[\left(\varepsilon_x - \varepsilon_y\delta\varphi\right)\cos\varphi - \left(\varepsilon_y + \varepsilon_x\delta\varphi\right)\sin\varphi\right]\dot{\varphi}^2 \\ &+ \left[\ddot{\varepsilon}_y + \ddot{\varepsilon}_x\,\delta\varphi + 2\left(\dot{\varepsilon}_x - \dot{\varepsilon}_y\,\delta\varphi\right)\dot{\varphi} + 2\left(\dot{\varepsilon}_x - \varepsilon_y\,\dot{\varphi}\right)\dot{\delta\varphi}\right]\sin\varphi \\ &- \left[\ddot{\varepsilon}_x - \ddot{\varepsilon}_y\,\delta\varphi - 2\left(\dot{\varepsilon}_y + \dot{\varepsilon}_x\,\delta\varphi\right)\dot{\varphi} - 2\left(\dot{\varepsilon}_y + \varepsilon_x\,\dot{\varphi}\right)\dot{\delta\varphi}\right]\cos\varphi \\ &+ \ddot{\varphi}\left[\left(\varepsilon_x - \varepsilon_y\delta\varphi\right)\sin\varphi + \left(\varepsilon_y + \varepsilon_x\delta\varphi\right)\cos\varphi\right] + \ddot{\delta\varphi}\left(\varepsilon_x\sin\varphi + \varepsilon_y\cos\varphi\right)\end{aligned} \qquad [1.44]$$

$$\begin{aligned}\dot{v}_A &= -R\omega^2\sin\theta + \left[\left(\varepsilon_y + \varepsilon_x\delta\varphi\right)\cos\varphi + \left(\varepsilon_x - \varepsilon_y\delta\varphi\right)\sin\varphi\right]\dot{\varphi}^2 \\ &- \left[\ddot{\varepsilon}_x - \ddot{\varepsilon}_y\,\delta\varphi - 2\left(\dot{\varepsilon}_y + \dot{\varepsilon}_x\,\delta\varphi\right)\dot{\varphi} - 2\left(\dot{\varepsilon}_y + \varepsilon_x\,\dot{\varphi}\right)\dot{\delta\varphi}\right]\sin\varphi \\ &- \left[\ddot{\varepsilon}_y + \ddot{\varepsilon}_x\,\delta\varphi + 2\left(\dot{\varepsilon}_x - \dot{\varepsilon}_y\,\delta\varphi\right)\dot{\varphi} + 2\left(\dot{\varepsilon}_x - \varepsilon_y\,\dot{\varphi}\right)\dot{\delta\varphi}\right]\cos\varphi \\ &+ \ddot{\varphi}\left[\left(\varepsilon_y + \varepsilon_x\delta\varphi\right)\sin\varphi - \left(\varepsilon_x - \varepsilon_y\delta\varphi\right)\cos\varphi\right] + \ddot{\delta\varphi}\left[\varepsilon_y\sin\varphi - \varepsilon_x\cos\varphi\right]\end{aligned} \qquad [1.45]$$

For simplification reasons, the following notation is defined:

$$\Gamma(\mathrm{G}_2/\Re_0) = \left(\gamma_x^{\mathrm{G}_2} + \delta\gamma_x^{\mathrm{G}_2}\right)\mathrm{x}_0 + \left(\gamma_y^{\mathrm{G}_2} + \delta\gamma_y^{\mathrm{G}_2}\right)\mathrm{y}_0 \qquad [1.46]$$

where $\gamma_x^{\mathrm{G}_2}$ and $\gamma_y^{\mathrm{G}_2}$ represent the components of acceleration of B in the case of a connection at A without mobility (section 1.1.4):

$$\begin{cases}\gamma_x^{\mathrm{G}_2} = -R\omega^2\left(\cos\theta + k\tan\varphi\sin\theta + k\overline{R}\dfrac{\cos^2\theta}{\cos^3\varphi}\right) \\ \gamma_y^{\mathrm{G}_2} = -R\omega^2(1-k)\sin\theta\end{cases} \qquad [1.47]$$

From these relationships and equations [1.41], the following equations can be deduced:

$$\begin{aligned}\delta\gamma_x^{G_2} &= \left[-\left(\varepsilon_y\cos\varphi+\varepsilon_x\sin\varphi\right)\dot{\varphi}^2+\left(\varepsilon_x\cos\varphi-\varepsilon_y\sin\varphi\right)\ddot{\varphi}\right]\delta\varphi \\ &+\left[\left(\ddot{\varepsilon}_x-2\dot{\varepsilon}_y\dot{\varphi}\right)\sin\varphi+\left(\ddot{\varepsilon}_y+2\dot{\varepsilon}_x\dot{\varphi}\right)\cos\varphi+kL\left(\dot{\varphi}^2\sin\varphi-\ddot{\varphi}\cos\varphi\right)\right]\delta\varphi \\ &+2\left[\left(\dot{\varepsilon}_y+\varepsilon_x\dot{\varphi}\right)\cos\varphi+\left(\dot{\varepsilon}_x-\varepsilon_y\dot{\varphi}\right)\sin\varphi-kL\dot{\varphi}\cos\varphi\right]\dot{\delta}\varphi \\ &+\left(\varepsilon_x\sin\varphi+\varepsilon_y\cos\varphi-kL\sin\varphi\right)\ddot{\delta}\varphi\end{aligned} \quad [1.48]$$

$$\begin{aligned}\delta\gamma_y^{G_2} &= \left[\left(\varepsilon_x\cos\varphi-\varepsilon_y\sin\varphi\right)\dot{\varphi}^2+\left(\varepsilon_x\sin\varphi+\varepsilon_y\cos\varphi\right)\ddot{\varphi}\right]\delta\varphi \\ &+\left[\left(\ddot{\varepsilon}_y+2\dot{\varepsilon}_x\dot{\varphi}\right)\sin\varphi-\left(\ddot{\varepsilon}_x-2\dot{\varepsilon}_y\dot{\varphi}\right)\cos\varphi-kL\left(\dot{\varphi}^2\cos\varphi+\ddot{\varphi}\sin\varphi\right)\right]\delta\varphi \\ &+2\left[\left(\dot{\varepsilon}_y+\varepsilon_x\dot{\varphi}\right)\sin\varphi-\left(\dot{\varepsilon}_x-\varepsilon_y\dot{\varphi}\right)\cos\varphi-kL\dot{\varphi}\sin\varphi\right]\dot{\delta}\varphi \\ &+\left(\varepsilon_y\sin\varphi-\varepsilon_x\cos\varphi+kL\cos\varphi\right)\ddot{\delta}\varphi\end{aligned} \quad [1.49]$$

$$\mathrm{V}_{02}(\mathrm{B})=\left[u_A-L\dot{\tilde{\varphi}}\left(\sin\varphi+\delta\varphi\cos\varphi\right)\right]\mathrm{x}_0$$

Similarly, it can be written that:

$$\Gamma(\mathrm{B}/\Re_0)=\left(\ddot{x}+\ddot{\delta}x\right)\mathrm{x}_0 \quad [1.50]$$

where $\ddot{x}$ represents the component of acceleration of B in the case of a connection at A without mobility (section 1.1.4):

$$\ddot{x}=-R\omega^2\left(\cos\theta+\tan\varphi\sin\theta+\overline{R}\frac{\cos^2\theta}{\cos^3\varphi}\right)$$

$\ddot{\delta}x$ can be obtained using $\gamma_x^{G_2}$ (equation [1.19]) assuming that k is equal to 1.

1.3.2. *Dynamics of crank shaft–connecting rod–piston system with mobility*

For junction force component calculations, using the mass data defined in section 1.2.1 and assuming that viscous friction is negligible, the following equations can be derived by the laws of dynamics:

– *Dynamic equations for the piston*

$$\begin{cases} m_4\left(\ddot{x}+\ddot{\delta}x\right)=-p_g S-X_{43} \\ 0=Y_{04}-Y_{43} \end{cases}$$

– *Dynamic equations for the axis*

$$\begin{cases} m_3\ddot{x}=X_{43}+X_{23} \\ 0=Y_{43}+Y_{23} \end{cases}$$

– *Dynamic equations for the connecting rod*

$$\begin{cases} m_2\left(\gamma_x^{G_2}+\delta\gamma_x^{G_2}\right)=-X_{23}+X_{12} \\ m_2\left(\gamma_y^{G_2}+\delta\gamma_y^{G_2}\right)=-Y_{23}+Y_{12} \\ I_2\ddot{\tilde{\varphi}}=-L\left[(1-k)Y_{23}+kY_{12}\right]\cos\tilde{\varphi}+L\left[(1-k)X_{23}+kX_{12}\right]\sin\tilde{\varphi} \end{cases}$$

The system can be reduced to the following five equations:

$$\begin{cases} X_{43}=-m_4\left(\ddot{x}+\ddot{\delta}x\right)-p_g S \\ X_{43}+X_{23}=m_3\left(\ddot{x}+\ddot{\delta}x\right) \\ X_{23}-X_{12}=-m_2\left(\gamma_x^{G_2}+\delta\gamma_x^{G_2}\right) \\ Y_{23}-Y_{12}=-m_2\left(\gamma_y^{G_2}+\delta\gamma_y^{G_2}\right) \\ \left[(1-k)X_{23}+kX_{12}\right]\sin\tilde{\varphi}-\left[(1-k)Y_{23}+kY_{12}\right]\cos\tilde{\varphi}=\dfrac{I_2}{L}\ddot{\tilde{\varphi}} \end{cases}$$

If m_{34} represents the sum of the masses of the piston and the piston pin:

$$m_{34} = m_3 + m_4$$

the system solution is given as:

$$X_{43} = -m_4\left(\ddot{x} + \ddot{\delta}x\right) - p_g S$$

$$X_{23} = m_{34}\left(\ddot{x} + \ddot{\delta}x\right) + p_g S$$

$$Y_{23} = -km_2\left(\gamma_y^{G_2} + \delta\gamma_y^{G_2}\right) + \left[m_{34}\left(\ddot{x} + \ddot{\delta}x\right) + p_g S + km_2\left(\gamma_x^{G_2} + \delta\gamma_x^{G_2}\right)\right]\tan\tilde{\varphi} - \frac{I_2}{L}\frac{\ddot{\tilde{\varphi}}}{\cos\tilde{\varphi}}$$

$$X_{12} = m_2\left(\gamma_x^{G_2} + \delta\gamma_x^{G_2}\right) + m_{34}\left(\ddot{x} + \ddot{\delta}x\right) + p_g S$$

$$Y_{12} = (1-k)m_2\left(\gamma_y^{G_2} + \delta\gamma_y^{G_2}\right) + \left[m_{34}\left(\ddot{x} + \ddot{\delta}x\right) + p_g S + km_2\left(\gamma_x^{G_2} + \delta\gamma_x^{G_2}\right)\right]\tan\tilde{\varphi} - \frac{I_2}{L}\frac{\ddot{\tilde{\varphi}}}{\cos\tilde{\varphi}}$$

For TEHD bearing studies, the force $\mathbf{F}_{1\to 2}$ is expressed in the basis B_2 attached to the connecting rod as:

$$\begin{aligned}\mathrm{F}_{1\to 2} &= X_{12}\mathrm{x}_0 + Y_{12}\mathrm{y}_0 \\ &= \left[X_{12}\cos\tilde{\varphi} + Y_{12}\sin\tilde{\varphi}\right]\mathrm{x}_2 + \left[Y_{12}\cos\tilde{\varphi} - X_{12}\sin\tilde{\varphi}\right]\mathrm{y}_2\end{aligned}$$

which gives:

$$\begin{aligned}\mathrm{F}_{1\to 2} = &\left[(1-k)m_2\left(\left(\gamma_x^{G_2} + \delta\gamma_x^{G_2}\right)\cos\tilde{\varphi} + \left(\gamma_y^{G_2} + \delta\gamma_y^{G_2}\right)\sin\tilde{\varphi}\right)\right. \\ &\left. + \left(km_2\left(\gamma_x^{G_2} + \delta\gamma_x^{G_2}\right) + m_{34}\left(\ddot{x} + \ddot{\delta}x\right) + p_g S - \frac{I_2}{L}\ddot{\tilde{\varphi}}\sin\tilde{\varphi}\right)\frac{1}{\cos\tilde{\varphi}}\right]\mathrm{x}_2 \\ &+ \left[(1-k)m_2\left(\left(\gamma_y^{G_2} + \delta\gamma_y^{G_2}\right)\cos\tilde{\varphi} - \left(\gamma_x^{G_2} + \delta\gamma_x^{G_2}\right)\sin\tilde{\varphi}\right) - \frac{I_2}{L}\ddot{\tilde{\varphi}}\right]\mathrm{y}_2\end{aligned} \qquad [1.51]$$

If W_x and W_y represent the components of a calculated load diagram for a without mobility configuration at A, the equilibrium equations describing the pressure field (see section 2.3.1 [BON 14]) must take into consideration the loading change:

$$\iint_{\Omega}\left[p(x,z)+p_c(x,z)\right]\sin\frac{x}{R}dxdz=\mathrm{F}_{1\to2}\bullet\mathrm{x}_2=W_x+\delta\mathrm{F}_{1\to2}\bullet\mathrm{x}_2$$

$$\iint_{\Omega}\left[p(x,z)+p_c(x,z)\right]\cos\frac{x}{R}dxdz=\delta\mathrm{F}_{1\to2}\bullet\mathrm{y}_2=W_y+\delta\mathrm{F}_{1\to2}\bullet\mathrm{y}_2$$

where $\boldsymbol{\delta}\mathbf{F}_{1\to2}$ represents the correction of the load, which is derived from the connection mobility between the crank shaft and the connecting rod. Considering the approximate first-order equations of $\cos\tilde{\varphi}$ and $\sin\tilde{\varphi}$, $\boldsymbol{\delta}\mathbf{F}_{1\to2}$ can be expressed using the basis B_2 as:

$$\begin{aligned}\delta\mathrm{F}_{1\to2}=&\Big[(1-k)m_2\left(\cos\varphi\,\delta\gamma_x^{\mathrm{G}_2}+\sin\varphi\,\delta\gamma_y^{\mathrm{G}_2}-\left(\gamma_x^{\mathrm{G}_2}\sin\varphi-\gamma_y^{\mathrm{G}_2}\cos\varphi\right)\delta\varphi\right)\\&+\left(km_2\,\delta\gamma_x^{\mathrm{G}_2}+m_{34}\,\ddot{\delta x}-\frac{I_2}{L}\sin\varphi\,\ddot{\delta\varphi}\right)\frac{1}{\cos\varphi}\\&+\left(\left(km_2\gamma_x^{\mathrm{G}_2}+m_{34}\,\ddot{x}+p_gS\right)\sin\varphi-\frac{I_2}{L}\ddot{\varphi}\right)\frac{\delta\varphi}{\cos^2\varphi}\Big]\mathrm{x}_2\\&+\left[(1-k)m_2\left(-\sin\varphi\,\delta\gamma_x^{\mathrm{G}_2}+\cos\varphi\,\delta\gamma_y^{\mathrm{G}_2}-\left(\gamma_x^{\mathrm{G}_2}\cos\varphi+\gamma_y^{\mathrm{G}_2}\sin\varphi\right)\delta\varphi\right)-\frac{I_2}{L}\ddot{\delta\varphi}\right]\mathrm{y}_2\end{aligned}\qquad[1.52]$$

The load components without mobility can be given by:

$$W_x=(1-k)m_2\left(\gamma_x^{\mathrm{G}_2}\cos\varphi+\gamma_y^{\mathrm{G}_2}\sin\varphi\right)+\left(km_2\gamma_x^{\mathrm{G}_2}+m_{34}\,\ddot{x}+p_gS-\frac{I_2}{L}\ddot{\varphi}\sin\varphi\right)\frac{1}{\cos\varphi}$$

$$W_y=(1-k)m_2\left(-\gamma_x^{\mathrm{G}_2}\sin\varphi+\gamma_y^{\mathrm{G}_2}\cos\varphi\right)-\frac{I_2}{L}\ddot{\varphi}$$

Using the equation for W_x, it is possible to express $\boldsymbol{\delta}\mathbf{F}_{1\to 2}$ with respect to basis B_2 regardless of the gas pressure as:

$$\begin{aligned}\delta F_{1\to 2} = &\Big[(1-k)m_2\left(\cos\varphi\,\delta\gamma_x^{G_2} + \sin\varphi\,\delta\gamma_y^{G_2} - \left(\gamma_x^{G_2}\sin\varphi - \gamma_y^{G_2}\cos\varphi\right)\delta\varphi\right)\\ &+\left(km_2\,\delta\gamma_x^{G_2} + m_{34}\,\ddot{\delta x} - \frac{I_2}{L}\sin\varphi\,\ddot{\delta\varphi}\right)\frac{1}{\cos\varphi}\\ &+\left(\left(W_x - (1-k)m_2\left(\gamma_x^{G_2}\cos\varphi + \gamma_y^{G_2}\sin\varphi\right)\right)\tan\varphi - \frac{I_2}{L}\ddot{\varphi}\right)\delta\varphi\Big]\mathrm{x}_2 \qquad [1.53]\\ &+\left[(1-k)m_2\left(-\sin\varphi\,\delta\gamma_x^{G_2} + \cos\varphi\,\delta\gamma_y^{G_2} - \left(\gamma_x^{G_2}\cos\varphi + \gamma_y^{G_2}\sin\varphi\right)\delta\varphi\right) - \frac{I_2}{L}\ddot{\delta\varphi}\right]\mathrm{y}_2\end{aligned}$$

1.4. Examples of link efforts between the elements of crank shaft–connecting rod–piston system

1.4.1. *Data*

A single-cylinder model as defined in section 1.1 is examined with given parameters summed up in Table 1.1.

Figure 1.3 shows a diagram of the pressure applied to the piston surface. This corresponds to a four-stroke engine, with the beginning of the intake phase chosen as the zero engine angle. To demonstrate the effect of inertia forces on bearing load, it is assumed that the diagram of gas pressure is the same in both cases. In reality, differences of more than 20% above the maximum pressure can be measured [SVO 84].

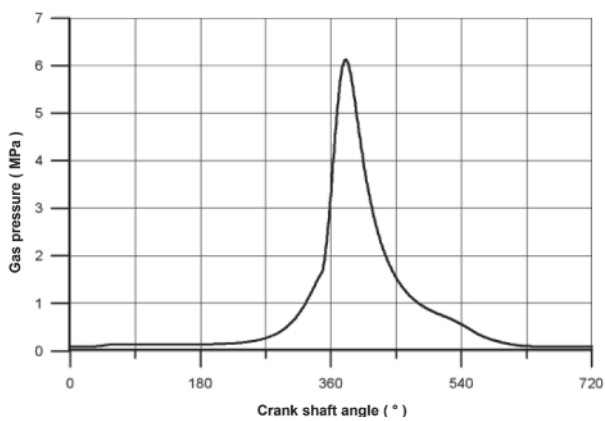

Figure 1.3. *Diagram of gas pressure*

For hydrodynamic calculations, the forces must be projected on the basis relevant to the frame of each bearing housing.

For example, the force $\mathbf{F}_{1\to2}$ split into components using the basis B_2 connected to the connecting rod gives:

$$\mathbf{F}_{1\to2}/B_2 \begin{cases} -m_2 R\dot{\theta}^2\left(K\cos\varphi\left(1+k\tan^2\varphi\right)+(1-k)^2\sin\theta\sin\varphi\right)+\dfrac{m_{34}\ddot{x}+p_g S}{\cos\varphi}-\dfrac{I_2}{L}\ddot{\varphi}\tan\varphi \\ (1-k)m_2 R\dot{\theta}^2\left(K\sin\varphi-(1-k)\sin\theta\cos\varphi\right)-\dfrac{I_2}{L}\ddot{\varphi} \end{cases}$$

To illustrate the different load diagram shapes, the following three motor rotation frequencies are considered: 2,500, 5,000 and 7,500 revolutions per minute (rpm).

Crank shaft radius	R	46.5	mm
Misalignment defect	D	0	mm
Connecting rod length	L	144	mm
Relative position of the con rod mass center	K	0.2628	
Piston surface	S	5,372	mm^2
Connecting rod mass	m_2	0.672	kg
Connecting rod moment of inertia (G z_0)	I_2	2,660	$kg{\cdot}mm^2$
Piston and axis mass	$m_3 + m_4$	0.422	kg

Table 1.1. *Data for a monocylinder*

1.4.2. *Load diagrams for the connecting rod big end bearing*

Figure 1.4 shows $\mathbf{F}_{1\to2}/B_2$ in a polar diagram for a frequency of 2,500 rpm. At this relatively low speed, the inertial forces are low with respect to the load. They correspond to the olive-shaped part on the left-hand side of the diagram. The connecting rod is essentially subjected to compression produced during gas compression and expansion phases corresponding to engine angle values between 300° and 540° (Figure 1.3). As the engine speed increases, the inertial load increases its amplitude as shown in Figures 1.5 and 1.6.

At 7,500 rpm, the inertial load becomes dominant. The traction on the connecting rod at the TDC passage at the end of the exhaust phase is higher (36, 455 N at an engine angle of 0°) than when the compression is at its maximum (29, 573 N at an engine angle of 511°). At an engine angle of 180°, corresponding to the passage at the bottom dead center at the end of the intake phase, the compression is due solely to inertial forces and reaches 26,557 N.

The force $\mathbf{F}_{1\to2}$ be expressed in terms of components with respect to the basis B_1 attached to the crank shaft (Figure 1.7). This helps to highlight the position of the force exerted by the connecting rod on the crank shaft pin: it is shown in Figure 1.7 that the force exerted by the connecting rod on the crank pin is never at a force direction angle between 0° and 90°. This area will therefore be the preferred one for lubricant supply placement.

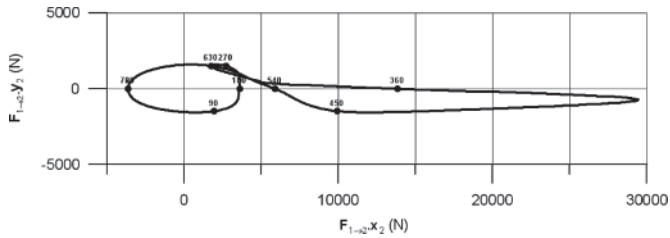

Figure 1.4. *Load polar diagram for a connecting rod big end bearing at 2,500 rpm with respect to the basis attached to the crank shaft*

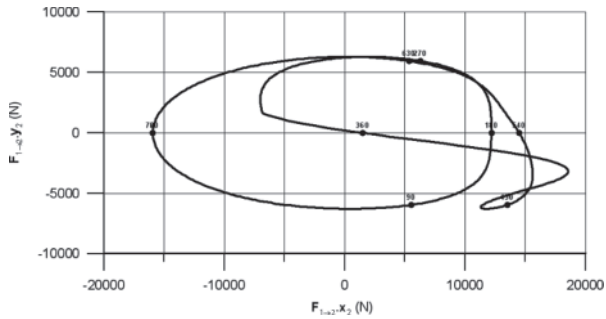

Figure 1.5. *Load polar diagram for a connecting rod big end bearing at 5,000 rpm*

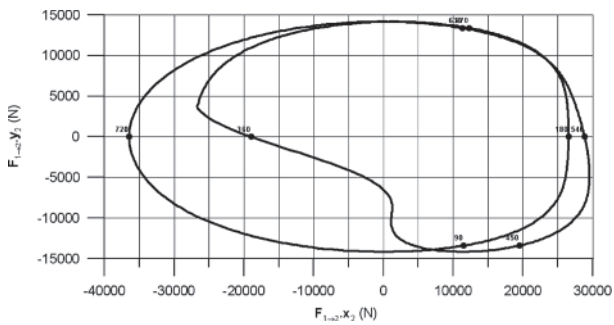

Figure 1.6. *Load polar diagram for a connecting rod big end bearing at 7,500 rpm*

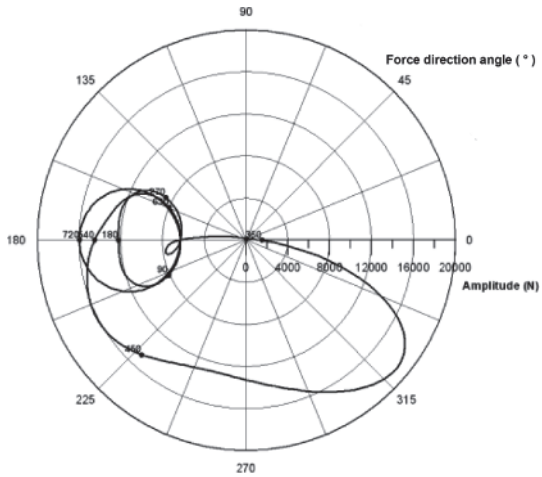

Figure 1.7. *Load polar diagram for a connecting rod big end bearing at 7,500 rpm with respect to the basis attached to the crank shaft*

1.4.3. *Load diagrams for a connecting rod small end bearing*

The polar diagrams showing the force exerted by the connecting rod small end are shown in Figures 1.8–1.10 projected on the basis B_2 attached to the rod for the three regime conditions. At 2,500 rpm, the force is almost exclusively orientated toward the connecting rod.

At 5,000 rpm, the traction on the connecting rod due to the piston mass reaches 6,648 N at the TDC. At 7,500 rpm, the inertial forces are much more pronounced with a traction of 15,544 N at the TDC.

The maximum thrusts on the connecting rod among the three conditions were 31,483, 26,667 and 18,741 N, respectively, at a 383° engine angle, where the pressure is the highest in the combustion chamber (Figure 1.3).

The important difference between these three values well reflects the pressure loss at higher speed due to the inertia of the piston.

The level reached by the forces exerted on the connecting rod small end is about 100 times higher than the magnitude of frictional forces between the cylinder liner on one side and the piston skirt on the other side. Thus, assuming that the mentioned forces are relatively negligible can be justified.

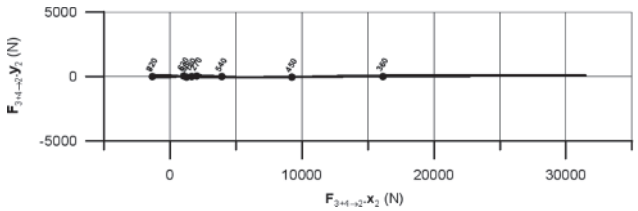

Figure 1.8. *Load polar diagram for a connecting rod small end bearing at 2,500 rpm*

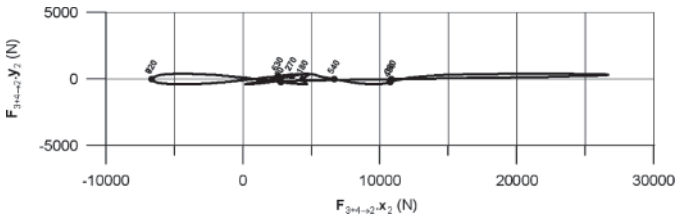

Figure 1.9. *Load polar diagram for a connecting rod small end bearing at 5,000 rpm*

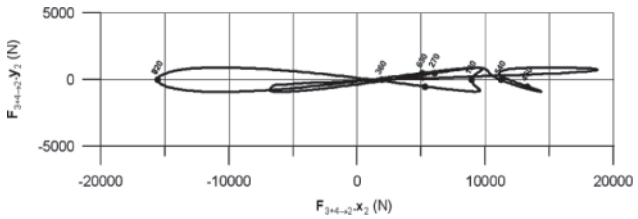

Figure 1.10. *Load polar diagram for a connecting rod small end bearing at 7,500 rpm*

1.4.4. *Load diagrams for a crank shaft main bearing*

The crank shaft of a monocylinder is usually connected to the engine block with two bearings. The diagrams shown in Figures 1.11–1.13 combine the efforts of both bearings in the case of a statistically balanced crank shaft. The pear-shaped part of the curve covering the angular interval 630°–720°/0°–270° corresponds to a completely inertial load at the exhaust and intake phases of the engine cycle. The pressure on the bearing cap marked at 2,500 rpm (29,143 N at an engine angle of 381°) is then gradually absorbed by the inertial forces when increasing the rotational frequency (18,032 N at 5,500 rpm) and becomes negligible compared to the original inertial force at 7,500 rpm.

For this type of high speed, the force acting on the bearing cap is 26,557 N at 180° engine angle. At an engine angle of 0°, the moving part assembly gives rise to a 36,455 N force directed against the engine block.

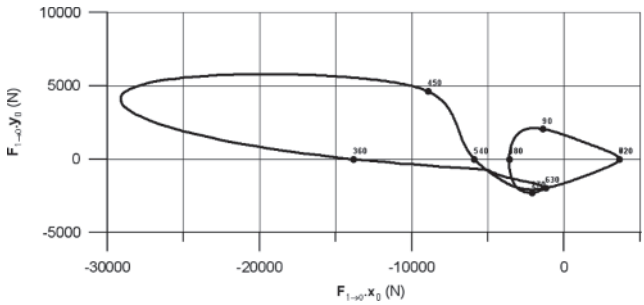

Figure 1.11. *Load polar diagram for a crank shaft main bearing at 2,500 rpm*

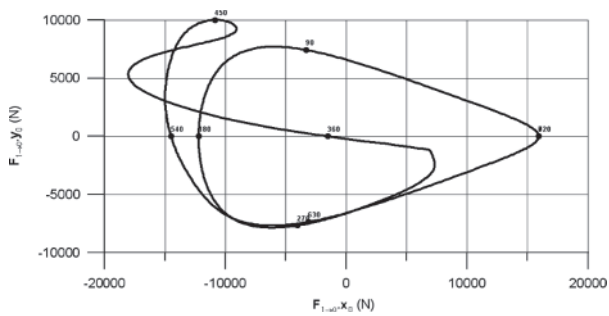

Figure 1.12. *Load polar diagram for a crank shaft main bearing at 5,000 rpm*

1.4.5. *Engine torque*

Figure 1.14 shows the engine torques for the three regimes. They oscillate between – 530 Nm at 35° engine angle for 7,500 rpm and 853 Nm at 395° engine angle for 2,500 rpm. These significant variations can be caused by the crank shaft speed fluctuations (acyclism), especially if the inertia moment of the wheel is not sufficient enough.

The order of magnitude of the engine torque is more than 100 times greater than the frictional torque in the different bearings of the moving part. Therefore, omitting the latter value in load calculations should not make a major difference.

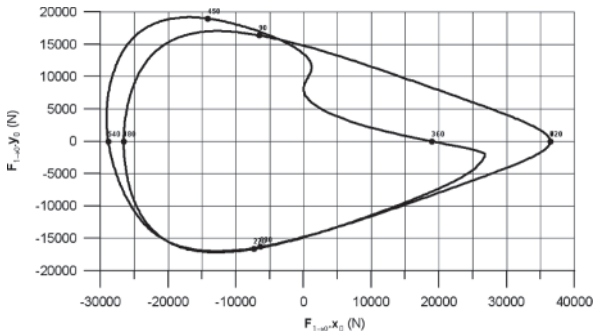

Figure 1.13. *Load polar diagram for a crank shaft main bearing at 7,500 rpm*

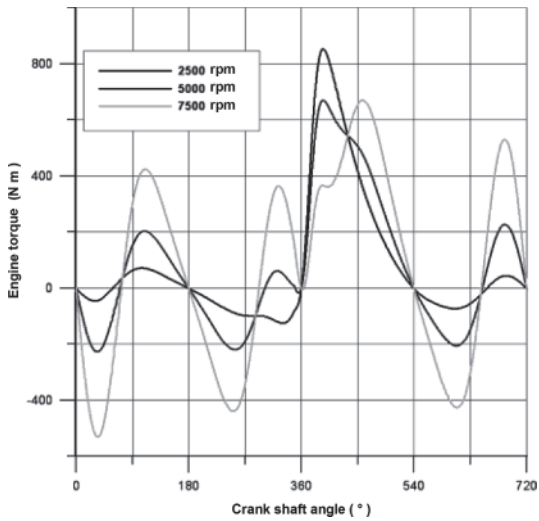

Figure 1.14. *Engine torque for the three rotational speeds*

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