PART 1

Transmission Lines

1

Electromagnetic of TEM Transmission Lines

1.1. General waves

Generally, electromagnetic waves are given by the solution of well-known Maxwell–Faraday formulas. These formulas give electric and magnetic fields $(\vec{\mathcal{E}},\vec{\mathcal{H}})$.In the case without charges we have the following expressions:

$$\vec{rot}\,\vec{\mathcal{E}} = -\mu \frac{\partial \vec{\mathcal{H}}}{\partial t}$$

$$\vec{rot}\,\vec{\mathcal{H}} = \varepsilon \frac{\partial \vec{\mathcal{E}}}{\partial t}$$

$$div\vec{\mathcal{E}} = 0$$

$$div\vec{\mathcal{H}} = 0$$

1.2. Transverse electromagnetic (TEM) waves

The waves are propagating in a medium which is

– linear;

– homogeneous;

– isotropic;

– without loss.

The medium being linear means that the permittivity constant ε and the permeability constant μ are independent of the frequency.

In the case of a harmonious state, the electric and magnetic fields are of the form:

$$\begin{cases} \vec{\mathcal{E}}(M,t) = \vec{E}(x,y)\, e^{j\omega t \pm \gamma z} \\ \vec{\mathcal{H}}(M,t) = \vec{H}(x,y)\, e^{j\omega t \pm \gamma z} \end{cases}$$

where the propagating constant is:

$$\gamma = \alpha + j\beta$$

α is the attenuation and β is the propagation.

When we consider only positive propagating waves, we have a solution in $-\gamma$, and the resulting electric and the magnetic waves are:

$$\begin{cases} \vec{\mathcal{E}}(M,t) = \vec{E}(x,y)\, e^{-\alpha z} e^{j(\omega t - \beta z)} \\ \vec{\mathcal{H}}(M,t) = \vec{H}(x,y)\, e^{-\alpha z} e^{j(\omega t - \beta z)} \end{cases}$$

The amplitude decreases by the factor $e^{-\alpha z}$. Also, phase is constant if:

$$\omega t - \beta z = ct$$

This means that:

$$\omega dt - \beta dz = 0$$

And we have the phase speed:

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

Now we decompose the electromagnetic wave (Figure 1.1) with:

– a part parallel to the propagating way z : $E_z(x,y)\vec{u}$;

– a part perpendicular to the propagating way z : $\vec{E}_T(x,y)$.

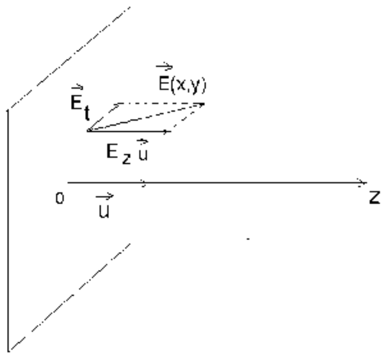

Figure 1.1. *Decomposition of the EM field*

In the case of transverse electromagnetic (TEM) fields, the waves are transversal; then:

$$\begin{cases} E_z = 0 \\ H_z = 0 \end{cases}$$

which means that:

$$\begin{cases} \vec{u}.\vec{E} = 0 \\ \vec{u}.\vec{H} = 0 \end{cases}$$

And the transversal fields satisfy the following group of equations:

$$div_T \vec{E}_T = 0 \qquad div_T \vec{H}_T = 0$$
$$rot_T \vec{E}_T = 0 \qquad rot_T \vec{H}_T = 0$$
$$\gamma\left(\vec{u} \wedge \vec{E}_T\right) = j\omega\mu \vec{H}_T \qquad \gamma\left(\vec{u} \wedge \vec{H}_T\right) = -j\omega\varepsilon \vec{E}_T$$

To ensure the compatibility of these equations, we must have

$$\gamma = \pm j\omega\sqrt{\mu\varepsilon} = \pm j\beta$$

And the TEM fields are lossless

$$\alpha = 0$$

From the quantity

$\alpha = 0$ and $\beta = \omega\sqrt{\mu\varepsilon}$

This gives

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

The TEM fields are propagating at the same speed as the uniform plane waves c.

From these equations, one can deduce the values of transversal electric and magnetic fields.

$$\begin{cases} \vec{H}_T = \dfrac{1}{Z}\left(\vec{u} \wedge \vec{E}_T\right) \\ \vec{E}_T = Z\left(\vec{H}_T \wedge \vec{u}\right) \end{cases}$$

where $Z = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the wave impedance.

TEM fields have the same properties as the uniform plane waves. The only difference is that the TEM fields $\vec{E}_T$ *and* $\vec{E}_T$ *depend x and y.* $\vec{E}_T$, $\vec{E}_T$ $\vec{u}$ *and a trirectangle and direct trihedral.*

1.3. Solutions of the transverse electromagnetic waves

We have to satisfy:

$$\begin{cases} \vec{rot}\vec{E}_T = \vec{0} \\ div_T \vec{E}_T = 0 \end{cases}$$

From the first equation, there is a scalar potential V that $\vec{E}_T = -\vec{grad} V$.

In the second formula, we obtain:

$$div_T \left(\vec{grad} V \right) = 0$$

This means that the Laplacian of the potential V is zero:

$$\Delta_T V = 0$$

– If TEM waves exist, then there are at least two conductors.

– The conductors are at the same potential.

– The force lines (E field) are orthogonal to the equipotential.

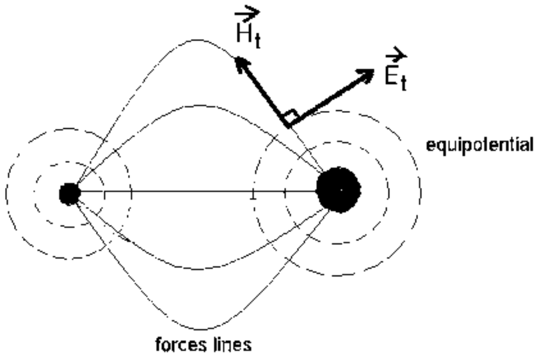

Figure 1.2. *Two conductors*

– If magnetic and electric fields are orthogonal, then the determination of the EM fields of TEM conductors is the same as a problem of electrostatic. We have to resolve:

$$\begin{cases} \vec{E}_T = -\vec{grad}V \\ \Delta V = 0 \\ \vec{n} \wedge \vec{E}_T = 0 \\ \vec{H}_T = \dfrac{1}{Z}\left(\vec{u} \wedge \vec{E}_T\right) = -\dfrac{1}{Z}\vec{u} \wedge \vec{grad}V \end{cases}$$

1.4. Characteristic parameters of the TEM lines

1.4.1. *Capacitance per unit of length*

The charge density of the conductors in Figure 1.3 is:

$$\rho_S = D_n = \vec{n}_v\,\vec{D} = \varepsilon\,\vec{n}_v\vec{E}$$

$$\rho_S = -\varepsilon\,\vec{grad}V.\vec{n}_v = \varepsilon\,\vec{grad}V.\vec{n}$$

$$\rho_S = \varepsilon\left|\vec{grad}V\right|$$

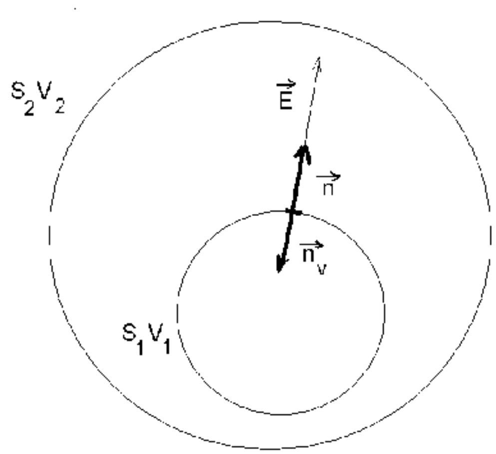

Figure 1.3. *Capacitance by length*

On the surface S_1, the charge is $Q = \iint_{S_1} \rho_S \, dS = \varepsilon \iint_{S_1} \left| grad\vec{d}V \right| dS$

On the surface S_2, the charge is $-Q$.

The potential between the two conductors S_1 and S_2 is:

$$V = \int_{S_1}^{S_2} \vec{E} d\vec{l} = -\int_{S_1}^{S_2} grad\vec{d}V \, d\vec{l} = -\int_{S_1}^{S_2} dV = V_{S_1} - V_{S_2} = V_1 - V_2$$

Then, in the plane the capacity is

$$C = \frac{\varepsilon \iint_{S_1} \left| grad\vec{d}V \right| dS}{\int_{S_1}^{S_2} grad\vec{d}V \, d\vec{l}}$$

1.4.2. *Characteristic impedance*

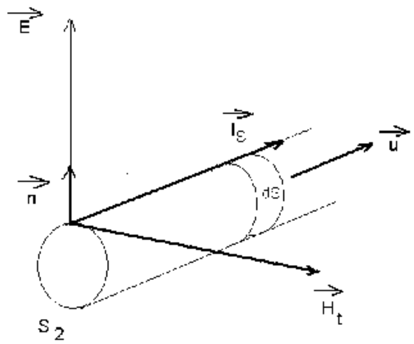

Figure 1.4. *Characteristic impedance*

The superficial density of current is longitudinal because:

$$\vec{I}_S = \vec{n} \wedge \vec{H}_T = |H_T|\vec{u}$$

And the total current is:

$$I = \iint_{S_2} \vec{I}_S \, d\vec{S} = \iint_{S_2} \left(\vec{n} \wedge \vec{H}_T\right) \vec{u}.dS = \iint_{S_2} |H_T| \, \vec{u}.\vec{u}.dS = \iint_{S_2} |H_T| dS$$

Then:

$$I = \iint_{S_2} |H_T| dS = \frac{1}{Z} \iint_{S_2} \left| gra\vec{d}V \right| dS = \frac{1}{Z}\frac{Q}{\varepsilon} = \frac{CV}{\varepsilon Z}$$

And by definition of the characteristic impedance

$$Z_c = \frac{V}{I} = \frac{\varepsilon Z}{C}$$

where $Z = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the wave impedance of the plane waves.

We also have

$$C = \varepsilon \frac{Z}{Z_c}$$

1.4.3. *Conductance per unit of length*

The lines of H_t are the equipotential. Let us calculate the flux ϕ of the vector $\vec{B}$.

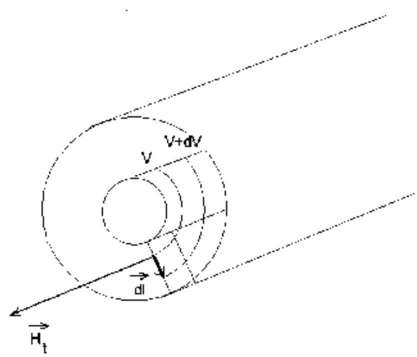

Figure 1.5. *Conductance*

We have:

$$|H_T|dl = \frac{|E_T|}{Z}dl = \frac{|grad\vec{d}\,V|}{Z}dl = \frac{dV}{Z}$$

And the flux:

$$|B|dl = \mu|H_T|dl = \mu\frac{dV}{Z} = \sqrt{\mu\varepsilon}dV$$

$$\phi = \int_{S_2}^{S_1} \sqrt{\mu\varepsilon}\, dV = \sqrt{\mu\varepsilon} V = \frac{\mu V}{Z}$$

But:

$$\phi = L I = \sqrt{\mu\varepsilon}\, V$$

$$L = \sqrt{\mu\varepsilon}\frac{V}{I} = \sqrt{\eta\varepsilon}\, Z_c$$

This can be written as:

$$L = \frac{\mu\varepsilon}{C}$$

Notice that the speed of the propagation of the TEM waves is the same as that of the classical theory of lines:

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{LC}}$$

In the case of the TEM lines, we have, in fact, to remember only the expressions of the capacity, of the inductance and then of the characteristic impedance:

$$C = \frac{\sqrt{\mu\varepsilon}}{Z_c}$$

$$L = \sqrt{\mu\varepsilon} Z_c$$

$$Z_c = \sqrt{\frac{L}{C}}$$

And we have the justification of the Kirchoff theory in the case of the TEM waves.

1.5. The power

1.5.1. *Density*

We define the density of magnetic power and of electric power by

$$\begin{cases} \overline{W}_M = \frac{1}{4} L\, I\, I^* = \frac{1}{4} \iint\limits_S \mu\, \vec{H}_T\, \vec{H}_T^* dS \\ \overline{W}_E = \frac{1}{4} C\, V\, V^* = \frac{1}{4} \iint\limits_S \varepsilon\, \vec{E}_T\, \vec{E}_T^* dS \end{cases}$$

1.5.2. *Flux*

If a phenomenon is periodic, its average value over a long time tends to be its average value in one period.

$$\overline{P} = \Re_e \left\{ \frac{1}{2} \iint \left(\vec{\tilde{E}} \wedge \vec{\tilde{H}}^* \right) \vec{u}\, dS \right\} = \Re_e \left\{ \frac{1}{2} \iint \left(\vec{E} \wedge \vec{H}^* \right) \vec{u}\, dS \right\}$$

where

$$\vec{\tilde{E}} = \vec{E}\, e^{-j\beta z}$$

$$\vec{H}^* = \frac{1}{Z} \vec{u} \wedge \vec{E}^* = \frac{1}{Z} \left\{ \left(\vec{E}.\vec{E}^* \right).\vec{u} - \left(\vec{E}.\vec{u} \right).\vec{E}^* \right\}$$

Since $\vec{E}$ is perpendicular to $\vec{u}$, only the first part of the double vectorial product is different from zero, and using $\vec{E} = -grad\vec{\ }V$, we get

$$\overline{P} = \Re_e \frac{1}{2Z} \iint \vec{E}.\vec{E}^* dS = \Re_e \iint \left| grad\vec{\ }V \right|^2 dS$$

Now Green's relation gives the integral with the $\vec{grad}$ of two functions Φ, Ψ as:

$$\begin{cases} \iint_S (\vec{grad}\,\Phi . \vec{grad}\,\Psi + \Psi\,\Delta\Phi)\,dS = \oint_C \Psi\,\vec{grad}\,\Phi . \vec{n}\,dl \\ where\ C\ is\ a\,close\ outline\ around\ surface\ S. \end{cases}$$

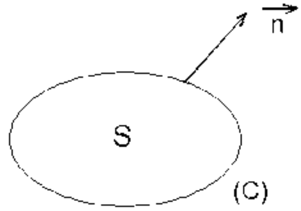

Figure 1.6. *Green schemas*

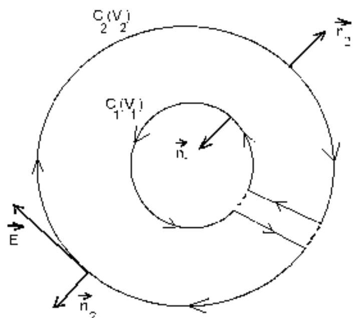

Figure 1.7. *Green in the case of two conductors*

To use Green's formula, we take

$\Phi = \Psi = V$

And using

$\Delta V = 0$

starting with an integral of order 2, we arrive at an integral of order 1 as:

$$\iint_S \left| grad\vec{d}V \right|^2 dS = \oint_C grad\vec{d}V.\vec{n}\, dl$$

and

$$\oint_C = \oint_{C_1} + \oint_{C_2}$$

The integrals around C_1 and around C_2 are easy to calculate as:

$$\oint_{C_1} = V_1 \oint_{C_1} grad\vec{d}V.\vec{n}_1 dl = V_1 \oint_{C_1} \left| grad\vec{d}V \right| dl = \frac{QV_1}{\varepsilon}$$

In the same manner:

$$\oint_{C_2} = -\frac{QV_2}{\varepsilon}$$

And the total integral is then:

$$\oint_C = \frac{Q}{\varepsilon}(V_1 - V_2)$$

And we can obtain the average power flux:

$$\bar{P} = \Re_e \left\{ \frac{1}{2Z} \frac{QV}{\varepsilon} \right\} = \Re_e \left\{ \frac{V^2}{2Z_C} \right\}$$

In general, this is a real quantity

$$\bar{P} = \frac{1}{2} \frac{V^2}{Z_C}$$

1.6. Problems

1.6.1. *The band-line*

1) A band-line is made up of two parallel conductors (Figure 1.8). When we consider the simplified model (without the border effects), we can calculate the potential.

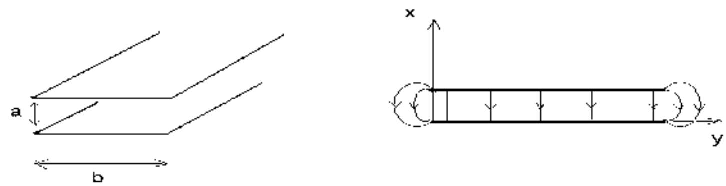

Figure 1.8. *The band-line and the waves*

2) Let us compute the total charge, the capacity by length, the inductance by length and the characteristic impedance.

SOLUTIONS.–

1) To simplify the problem, we neglect the border effects. It is the same if we have a magnetic wall at the two sides (Figure 1.9).

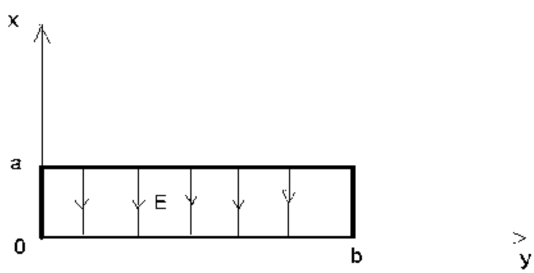

Figure 1.9. *The electric waves of the band-line when neglecting the border effects*

We have to verify

$$\Delta V = 0 \quad \text{or} \quad \frac{\partial^2 V}{\partial x^2} = 0$$

The problem is only a function of x:

$$\frac{dV}{dx} = -E$$

$$V(x) = -E\,x + C^{te}$$

The conditions are:

$$\left\{\begin{matrix} x = 0,\ V = 0 \\ x = a,\ V = a \end{matrix}\right\} \Rightarrow E = -\frac{V}{a}$$

and:

$$V(x) = V\frac{x}{a}$$

2) The total charge is:

$$Q = \int_0^b \rho_S\,dl = -\varepsilon\,E\int_0^b dl = -\varepsilon\,E\,b = \varepsilon\,V\frac{b}{a}$$

The capacity by length is:

$$Q = C\,V\ ; \quad C = \varepsilon\frac{b}{a}$$

The inductance by length is:

$$L = \frac{\mu\,\varepsilon}{C}; \quad L = \mu\frac{a}{b}$$

The characteristic impedance is:

$$Z_C = \sqrt{\frac{L}{C}}\ ;\quad Z_C = \sqrt{\frac{\mu}{\varepsilon}}\frac{a}{b} = Z\frac{a}{b}$$

We recover the same values by using the electrostatic theory.

1.6.2. *The coaxial cable*

1) Give the expressions of the potential of a coaxial cable.

2) Deduce the electric and magnetic waves.

3) What is the characteristic impedance, the capacity and the inductance by unit length and density power?

SOLUTIONS.–

1) Electric waves are radial and magnetic waves are perpendicular to electric waves (Figure 1.10).

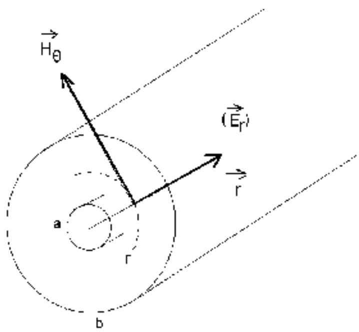

Figure 1.10. *The waves of the coaxial cable*

We have to verify in circular cylindrical coordinates:

$$\Delta V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$$

Symmetry revolution gives that V is independent of θ:

$$\frac{\partial^2 V}{\partial \theta^2} = 0$$

Then:

$$\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0 \quad \Rightarrow \quad r\frac{dV}{dr} = C^{te} \quad \Rightarrow \quad dV = C^{te}\frac{dr}{r}$$

A general solution is:

$$V(r) = C^{te} Logr + C_1^{te}$$

And with the limiting conditions:

$$V(a) = V, \;\; V(b) = 0$$

we have

$$V(r) = \frac{Log\left(\frac{b}{r}\right)}{Log\left(\frac{b}{a}\right)}$$

2) Computing the waves, the electric wave is purely longitudinal:

$$\vec{E} = \vec{E}_r, \quad E = -\frac{dV}{dr}$$

$$E = \frac{V}{r\,Log\left(\frac{b}{a}\right)}$$

The magnetic wave depends only on θ:

$$\vec{H} = \vec{H}_\theta, \quad H = \frac{1}{Z} E$$

$$H = \frac{V}{r\, Z\, Log\left(\frac{b}{a}\right)}$$

3) The characteristic impedance is:

$$Z_C = \frac{Z}{2\pi} Log\left(\frac{b}{a}\right)$$

where $Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$ in the air

The capacity and the inductance by unit length are, respectively:

$$C = \varepsilon \frac{Z}{Z_C} = \frac{2\pi\varepsilon}{Log\left(\frac{b}{a}\right)}$$

$$L = \frac{\mu\varepsilon}{C} = \frac{\mu}{2\pi} Log\left(\frac{b}{a}\right)$$

The density power is

$$\overline{P} = \frac{1}{2}\frac{V^2}{Z_C} = \frac{\pi}{Z}\frac{V^2}{Log\left(\frac{b}{a}\right)}$$

1.7. Bibliography

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