PART 1

Basic Notions

1

Introductory Problems

Logistic problems are all around us. We only need to observe a little and to have some imagination to find them. In this chapter we propose three problems that perfectly illustrate the potential touch of madness of an operations researcher. These examples enable us to approach the issues that may crop up in the industrial world gradually, which will be described more formally in the following chapters. They are drawn from exam papers assigned to undergraduate students that have taken classes in a subject called "optimization problems and procedures". They are kept in their original form on purpose. The questions asked will be answered over the course of this book. A detailed answer key for the exercises, including comments, is supplied in the last chapter of this book. Beginners in combinatorial optimization can play the following little game: could you recognize the correlation between each practical problem and its theoretical equivalent?

1.1. The "swing states" problem

In the United States of America, during the presidential elections, there are certain states called "swing states", which are liable to swing from the Democratic Party towards the Republican or vice versa. It is these states that both parties pay most attention to, especially when the results day is drawing near. Table 1.1 shows the list of these states and the figures of their Electoral College.

The advisers of one of two candidates (you are free to choose either side) ask you to help them make the most of their last week of campaigning. You are also provided with, in Table 1.1, an estimate of the sum that needs to be invested in every state in order to have a chance to command a majority. There is a $500,000 global budget left to invest. The question is simple: which states should you choose in order to win the greatest number of electors?

Swing states	Electoral College	Invested sum (in K$)
North Carolina	15	80
Colorado	9	50
Florida	27	200
Indiana	11	70
Missouri	11	80
New Hampshire	4	30
New Mexico	5	50
Nevada	5	40
Ohio	20	150
Pennsylvania	21	110
Virginia	13	80
Wisconsin	10	60

Table 1.1. *List of "swing states" and estimate of the investment necessary to obtain a majority*

1) What kind of combinatorial optimization problem is the "swing state" problem in relation with?

2) Determine a criterion according to which the states can be ranked from most interesting to least interesting. Deduce a construction heuristic from this and give its principle. What solution do you find?

3) Remove the most expensive state from the last solution. Can you then complete your solution by choosing some other states, thus improving it?

4) Deduce a neighborhood system for this problem.

5) Propose the most appropriate upper and lower bounds of the optimal solution.

1.2. Adel and his camels

Your favorite Operations Research professor (we will call him Mr L.), having barely arrived in Douz[1], meets Adel, a professional camel driver. Mr L., after introducing Adel to Operations Research, works out a deal for a free excursion to the Sahara. In exchange, Mr L. has to show Adel that his knowledge in optimization can help him conduct his business more proficiently. Adel reveals to Mr L. that he has two problems. Before we tackle these, let us examine in detail our camel driver's activity.

Adel owns 12 camels. Every morning the camels need gearing up before being able to carry tourists. Two procedures are required and they have to be carried out in this order. First of all, a veterinary check-up is performed in order to make sure that the animals are in good health. Afterwards, the camels are saddled up in the Bedouin way. The time it takes to perform these two tasks varies according to the age of the animal and the kind of saddle that needs to be placed on it. These times are shown in Table 1.2. For the sake of confidentiality, we cannot tell you the names of the camels, which we will then refer to as numbers (from 1 to 12). Adel has two workers. The first one deals with the veterinary check-up. The second is an expert in Bedouin traditions and only saddles up the camels.

Camels	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Veterinary check-up	10	8	12	10	14	8	4	6	16	8	6	20
Saddling-up	6	12	4	12	10	14	12	8	6	12	14	10

Table 1.2. *Time necessary to gear up the camels (in minutes)*

1 Douz is a small town of 17,000 people, located in the Nefzaoua region (Tunisia). If its main resource is the cultivation of the date palm, Douz is also the starting point for most camel and Méhari excursions or 4×4 dune rides.

1) A group of 12 tourists arrives at 10 o'clock for a camelback ride. Adel wants to fully satisfy his prestigious customers. Thus, he asks Mr L. to help him reduce their waiting time as much as possible. All the camels will have to be ready as soon as possible, given that they will start working at 8 o'clock. Mr L. offers to determine the order in which the camels will have to be geared up in order to minimize the tourists' waiting time. The problem seems so easy to him that he exults:

- What kind of optimization problem has Mr L. recognized and why does he seem so happy?

- What solving technique is Mr L. going to employ?

- What will the tourists' waiting time be?

2) On the following day, 12 other tourists arrive, but this time not all at once (see Table 1.3). Adel hopes that for every tourist that arrives, there will always be a camel ready:

- He thanks Mr L. heartily for determining the order in which the camels have to be geared up and starts calculating the sum of all the tourists' waiting times. What number will Adel find?

Tourists	*A, B, C*	*D, E, F*	*G, H*	*I, J, K*	*L*
Arrival time	8 : 30	9 : 00	9 : 30	10 : 00	10 : 10

Table 1.3. *Time of the tourists' arrival*

- The situation is even worse than Adel thought. The customers arriving at the same time are together, and want to set out on the ride all at once. For example, if the third camel is ready at 8:38, the three clients A, B and C (all having arrived at 8:30) will have to wait for 8 minutes, i.e. $3 \times 8 = 24$ minutes of total waiting time. What is the actual overall waiting time?

- Adel, more and more fascinated by Operations Research, then puts forward the idea that the more readily available camels should be favored. Thus, he is convinced he will find a solution that will satisfy his clients completely and sets to work. Describe and then employ a greedy heuristic procedure in relation to this idea. Will this solution satisfy Adel?

3) Adel, discouraged, turns once again to Mr L. In your opinion, how will the story end (Figure 1.1)?

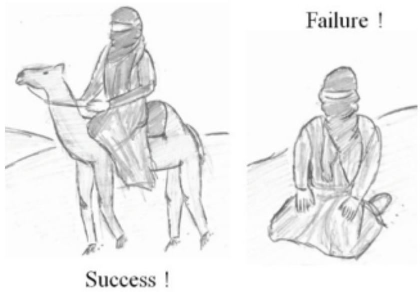

Figure 1.1. *Will Mr L. satisfy Adel's customers?*

1.3. Sauron's forges

Scary Sauron has employed you, one of his most faithful lieutenants, as his chief engineer. Your role consists of supervising the various Mordor forges and in making sure of their constant supply of iron ores so that they can produce weapons at their full capacity. Given the strategic role played by these forges in the upcoming war, there is no need to point out that your life will be inextricably linked to your productivity!

Figure 1.2 represents the map of Mordor and the locations of the different forges (numbered 1-11). The only iron mine of the whole kingdom, which is where you are located with your team and constitutes the source of supply for all the forges, is also represented on this map (as the letter M). You will also find on the map the different paths between these locations as well as the time – in hours – it takes to cover them.

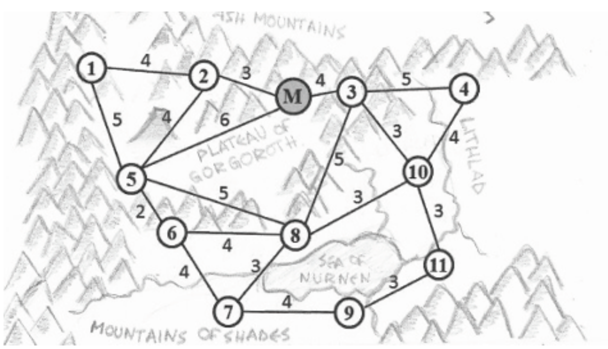

Figure 1.2. *Map of Mordor*

1.3.1. *Problem 1: The inspection of the forges*

Recently appointed, you decide to walk all over Mordor in order to supervise the set of forges:

1) Fill out the distance in Table 1.4. For each non-gray case, you will give the shortest distance from the first location (shown across the rows) to the second (shown in the columns). Why is it not necessary to provide information about every case presented in this table?

2) Your aim is to visit all the forges in Mordor. Even if you completely trust your partner (who will stand in for you at the mine during your absence), it is clear that you would like your absence to be as short as possible. Thus, you try to minimize the travel time (the time spent at the locations remaining constant). What is the name of the problem you have to solve? What solution do you obtain when you apply the "nearest neighbor" heuristic upon leaving the mine? (If several forges are equidistant, you will choose the one corresponding to the smallest number). Can you hope to obtain a better result if you apply the same technique but with a different starting point?

	M	1	2	3	4	5	6	7	8	9	10	11
M												
1												
2												
3												
4												
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7												
8												
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10												
11												

Table 1.4. *Distance between the mine and the forges*

3) Propose the best solution possible.

4) In addition to inspecting the forges, you want to carry out the delivery of iron. The volume of the iron that needs to be transported to the forges is

shown in Table 1.5. The wagon you can use to transport this load has a maximum capacity of 100 m^3. Once the wagon has been loaded, its speed is reduced. So if it carries more than 75 m^3, the travel time is doubled. If it transports between 50 and 75 m^3, the travel time is multiplied by 1.5.

Forges	1	2	3	4	5	6	7	8	9	10	11
Volume (m^3)	10	8	10	15	20	12	25	15	15	10	20

Table 1.5. *Volume (in m^3) to be transported to each forge*

Without taking into account the delay on the transport time, what is this problem called? If you retrieve the best solution you found in the last part just as it is, how long will your travel time be?

Propose a division of the cities (which you will justify) into as many subsections as the number of journeys you predict to make. What solution do you find by optimizing each journey? What will be the cost of your solution? (You will propose an estimate of your solution that takes into account the context of the study. You will justify your choice).

1.3.2. *Problem 2: The production of the deadly weapon*

In the war that pits Mordor's forces against the armies of the Good, the development of new weapons plays a fundamental role. One of Sauron's generals has just built a brand new siege machine capable of smashing through the sturdiest ramparts. This enormous weapon, if produced quickly enough, could in itself turn the tide of the war and clinch the victory for your side. All the forges in the kingdom are then employed to this end. The aim is to finish the production of this weapon as soon as possible.

The machine is made up of several parts: a frame, a lever, a sling-shot, a pulley, a counterweight, three axletrees and five projectiles. Each of these parts can be produced in any forge. The production cost *units* of each of the parts is given in Table 1.6.

Parts	Frame	Lever	Sling-shot	Pulley	Counterweight	Axletree	Projectile
Production Cost	40	60	120	100	100	80	60

Table 1.6. *Production cost units for the parts of the siege machine*

Each forge has its own production capacity (or speed), which is given in Table 1.7.

So if forge 5 (with a capacity of 7 units per hour) has to produce the lever (which requires *a cost* of 60 units), it will take it 60/7 = 8.57 hours, i.e. around 9 hours (we will always round up to the next integer to take into account any possible variables during the production).

Forges	1	2	3	4	5	6	7	8	9	10	11
Capacity per hour	6	8	5	8	7	8	10	5	6	5	10

Table 1.7. *Production capacity of the forges per hour*

The machine is assembled at the mine (under your responsibility). The process can start once all the parts have arrived and it lasts 10 hours.

The time it takes to transport a part from the forge, where it has been produced to the mine, is determined thanks to the data presented in Figure 1.2. For example, the lever we have just produced will arrive at the mine in 15 hours (9 hours for the production and 6 for the transport).

The production of a part in a forge can begin straight away (at time 0). Naturally the forge needs the blueprint of the part. However, given the urgency of the task, the Nazgûls take care in person of the delivery of the blueprints (Figure 1.3), which will be carried out in a negligible amount of time.

Figure 1.3. *Nazgûl riding a winged creature*

It is definitely possible to assign the production of several parts to one forge (the number of parts that need producing is larger than the number of forges!). The production of the second part begins as soon as the first one has been completed. The three axletrees or the five projectiles can be fabricated in different forges.

Propose an empirical rule (as simple as possible) that can determine *a priori* where the parts have to be produced. By rigorously applying the rule you have set, what solution do you find? How long will it take you to produce the weapon? Propose an approach that can improve this solution.