
General Presentation of Vehicle Routing Problems

Vehicle routing problems (VRPs) represent an important family of problems encountered in the fields of logistics, as well as in many other applications. In general, a number of customers have to be served with a fleet of vehicles. They can be modeled as an integer programming problem, solved by combinatorial optimization tools. However, exact methods cannot solve instances that consider a large set of customers, as encountered in most real cases. It is, therefore, often necessary to resort to approximate paradigms generally carried out through metaheuristics.

This first chapter introduces what the logistics management and the combinatorial optimization are, before giving a formal definition of the CVRP, with notations useful throughout the remainder of the book.

1.1. Logistics management and combinatorial optimization

In the last few decades, a great interest has grown up in the area of logistics among both industry and academia, for different reasons [BRA 98]. First, companies are facing fierce competition in today's global markets. They need to innovate to keep their position, and they realize the savings that can be achieved by a better planning and management of their logistic systems.

Furthermore, the evolution of lifestyles is significant. Modes of consumption are changing and expectations of consumers switch to products with short lifecycles, and the advancement in communications and transportation technologies, such as mobile communication and overnight delivery, motivates continuous development of the management of logistic systems.

These changes attract attention of the academic community, whose approach consists of determining characteristics of the problems and developing solution methodologies, as well as providing specific guarantees of effectiveness.

1.1.1. History of logistics

Logistics is not a recent trend in managing the flow of goods from an origin to a destination, with the aim to meet some requirements. Logistics made an important stride during the construction of the pyramids in ancient Egypt, for example. It played a key role in global sea trade with the invention of rowing vessels around 300 B.C. Logistics was also one of the main factors for the victory of most wars throughout history.

In military context, logistics is responsible for supplying the troops. It deals with the inventory management and transportation. However, this type of requirement also predominates in carriers and wholesalers activities. Thus, it is natural that modern logistics appears in industry.

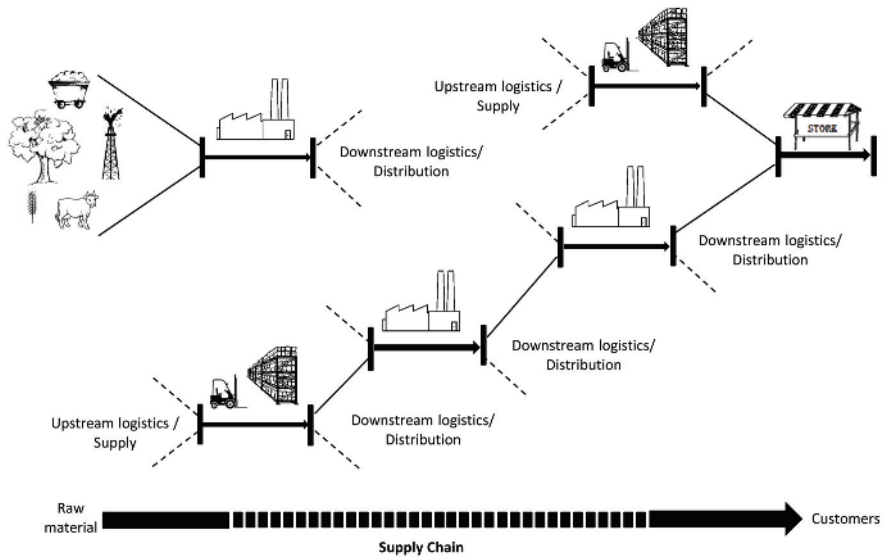


Figure 1.1. Example of a supply chain

Nowadays, the function extends from production to distribution, leading to the supply chain (Figure 1.1). In this chain, upstream activities take place prior to a particular link, when the latter orders for material to suppliers in the aim to bring its

added value. On the contrary, downstream activities involve the sale of a material to other businesses, governments or private individuals. The extreme link in the upstream part usually concerns raw materials, while the extreme downstream link is related to the final customer. However, each other link in the midstream is both customer of predecessor actors and supplier of successors. Midstream can be a manufacturer, a cooperative warehouse, a regional consolidation center, a city hub, local depot, etc.

Most of the freight transport in the chain is carried in containers, although bulk transport is used more for large volumes of durable goods. The reason is that this option is often the most efficient and cost-effective way to supply the products. However, for the smaller quantities generally required at the final destination, the supply chain is often less efficient. This characteristic is known as the “last mile problem”, which can represent up to 28% of the total cost to move goods. In addition, if transport plays an important role in economic growth and globalization, it causes air pollution and a large amount of traffic. Hence, a good transport planning is essential to control the costs, as well as the flow and limit nuisances.

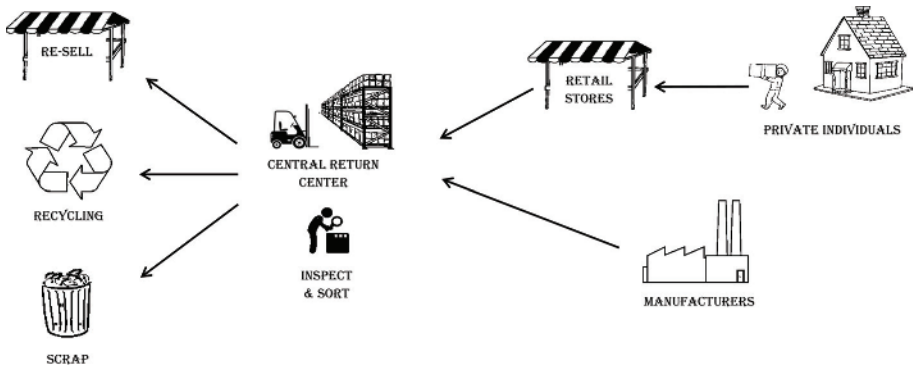


Figure 1.2. Possible outline of a reverse logistics chain

In an even more global view, the network also integrates reverse flows. These cover all operations related to the recycling of products and materials thrown away by the public or by industries (obsolete products, mixed waste and even hazardous). The so-called *reverse logistics* brings together the movements of products from consumers to producers through a distribution chain (Figure 1.2). The growing concern for integrating environmental requirements into green supply chain management concepts and practices makes it even more relevant. The reverse logistics process refers to activities undertaken to reduce, manage and dispose of waste from industrial activities. It meets the need to decommission the products after use and treat the destruction, by transforming or recycling in order to reduce costs,

and valuing the recovered products. Several related activities, therefore, involve: collecting waste, the location of recycling points/storage, inventory management and integration of products from the collection at the related industries. It also includes the optimization of the Ecodesign to facilitate future recycling.

Other issues have arisen recently about *city logistics* which are obviously related to the last mile problem described before. In fact, the freight distribution in urban area has to deal with several aspects. First, traffic may be difficult because of congestion at some rush hours, which makes the travel time dependent on the time of the day. Another particularity is the accessibility constraint. It might be quite complicated to deliver the goods in some areas because of the lack of parking for example, or because of city restrictions on the use of trucks in favor to smaller vehicles. In the same vein, economic and environmental problem concerns might lead to choose alternative types of transport for urban freight distribution (such as electric vehicles), as well as to adopt new commercialization behaviors. For example, the growth of e-commerce brings new questions and some retail companies have studied the use of drones to deliver online purchased goods to consumers.

Hence, many activities are involved in the supply chain, from the network design, to logistics of transportation, passing through warehouse management, international commerce or information systems. Transportation is one of the main parts of logistics. It can be made through several modes such as air, rail, road, water, cable, pipeline and space and may require particular infrastructures (Figure 1.3). These include links in the network (roads, railways, canals or pipelines, for instance) and terminals such as airports, railway stations, warehouses and depots. A wide range of issues emerges in this context, sweeping topics as diverse as the routing, inventory, cross-docking or network structure.

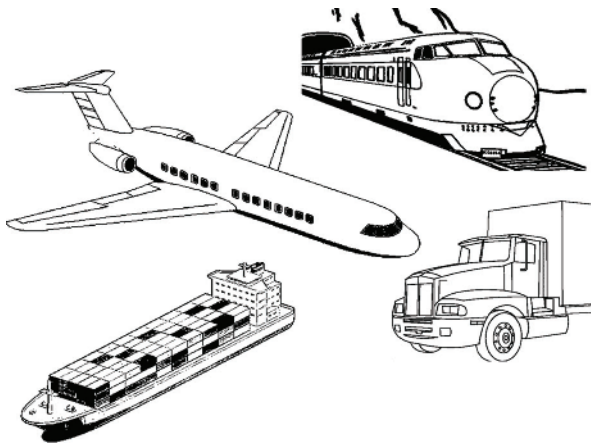


Figure 1.3. Example of transportation modes

1.1.2. *Logistics as a science*

The logistics function has risen to such an important place that it is now a profession in itself, and even a science. The goal in logistics management is to be efficient and cost-effective across the entire system [BRA 98]. Therefore, the objective is not simply to minimize locally transportation cost or reducing inventories. Every facility that has an impact on system effectiveness must be taken into consideration, from suppliers to retailers through manufacturing facilities, warehouses and distribution centers.

In fact, logistics management encompasses many of the firm's activities, from the strategic level through the tactical to the operational level:

- the strategic level deals with decisions that have usually a long-term effect. Concerning logistics, this includes, for instance, decisions regarding the number, location and capacities of warehouses and manufacturing plants;

- the tactical level typically includes decisions that are updated anywhere between once every quarter and once every year. This includes purchasing and production decisions, inventory policies and transportation strategies including the frequency with which customers are visited;

- the operational level refers to day-to-day decisions such as scheduling, routing and loading trucks.

Therefore, logistics activities obviously deserve to be recognized as a science, and this has begun to be true from the middle of the 20th Century [TAY 07].

1.1.3. *Combinatorial optimization*

The science of logistics can be seen as the study of the physical flow of products and services through the supply chain. Therefore, the chain can be seen as a network, or a graph, in which a flow has to go through, with some constraints that need to be encountered and an objective, often relative to a cost function, to optimize. Thus, most of the decision-making to manage the logistics can be taken by modeling the problem in terms of a mathematical program to optimize.

Optimization is a branch of mathematics particularly applied in operations research and management science. It consists of finding one or more best (optimal) solutions from all feasible solutions. Optimization problems can be divided into two categories depending on whether the variables are continuous or discrete. The latter case is known as a combinatorial optimization problem. Solving such problems can be a difficult task. The difficulty arises from the fact that feasible solutions belong to a finite but high cardinality set. In fact, finding a global optimum to the problem requires proving that a particular solution dominates all feasible points by arguments

other than the calculus based on derivative approaches of convex programming. Therefore, different approaches exist. The simplest one relies on the enumerative techniques, but an exhaustive search is often not possible due to the time required. Other options are, for example, relaxation and decomposition techniques, and cutting planes approaches based on polyhedral combinatorics. An algorithm is usually required to search the solution space, and most often, it cannot find and prove the optimality in polynomial time. In such a case, the problems are said to be “NP-hard” (non-deterministic polynomial-time hard).

In fact, in many cases, combinatorial optimization problems are NP-hard. Consequently, metaheuristics are mainly developed for real-world problems, which often attain notably high levels of complexity, although they are not able to certify the optimality of the solutions they find.

1.2. Vehicle routing problems

Vehicle routing problems are well-spread combinatorial optimization problems. They can be encountered in various areas, even if their main application stands on logistics of transportation.

1.2.1. Problems in transportation optimization

Truckload transportation from sources to destinations represents a first family of transportation problems, where the amount of goods fully fills a vehicle (or a vehicle carries goods directly from the source to the destination, without any other service on the way). The shortest paths between these sources and destinations are computed, mainly through a graph where vertices correspond to intersections and the edges correspond to road segments, each weighted by the length of its road segment, for instance. Then, a typical transportation problem deals with sources, where a supply of some commodity is available, and destinations where the commodity is demanded. The first studies on this subject appeared in the 1930s. An example is Tolstoy who published an article called *Methods of finding the minimal total kilometrage in cargo-transportation planning in space*, for the freight between sources and destinations along the railway network of the Soviet Union [TOL 30].

In this book, transportation is considered as *less-than-truckload*: vehicle capacity is large enough to allow servicing several customers without returning to the depot. This leads to interesting combinatorial optimization problems which need to handle routing aspects. In fact, the classical vehicle routing problem (capacitated VRP – CVRP) is an important problem in the fields of logistics and transportation. It consists of the determination of the optimal set of routes to be performed by a vehicle fleet to serve the demand of a given set of customers. With the traveling salesman

problem (TSP), it is one of the most important and studied combinatorial optimization problems. Theoretical research on vehicle routing started in 1959 by Dantzig and Ramser with the truck dispatching problem, and it was the beginning of a proliferation of work in this field.

1.2.2. *Vehicle routing problems in other contexts*

In fact, VRPs belong to a family of problems outreaching the field of transportation optimization, with applications in additional areas, particularly services. In such contexts, a vehicle is more a generic term to represent a mobile that visits a number of sites in order to complete certain tasks. The latter can be pick up or delivery tasks, as well repairs, meter reading or any other activity.

Therefore, a problem can be seen as a VRP when allowed movements describe a graph, and the result must be to visit some arcs or nodes by one or several circuits in this graph, especially with the same start and end point, while respecting a set of constraints. Nowadays, many more examples arise, from helicopters sent to evacuate the casualties after a disaster, to a laser beam that engraves transistors making up the integrated circuits, including inspection of three-dimensional (3D) structures (such as bridge girders) by a robot.

With these numerous applications, utilization of optimization software, based on operations research and mathematical programming techniques, extends to efficiently manage the supply of goods and services. Technological innovations such as geographic information systems, radio frequency identification and parallel computing entail new challenges.

1.2.3. *Characteristics of vehicle routing problems*

Vehicle routing problems cover a wide variety of problems. Let us describe the typical characteristics of these problems by considering their main components, constraints and possible objectives to be optimized.

1.2.3.1. *Components*

Four components constitute a vehicle routing problem. Without limiting the generality, the terminologies mainly used for these components are:

- the network, which is generally described through a *graph*;
- the sites to be visited (customers to serve, tasks to process, etc.), denoted as *customers* which have a specific request often called a *demand*;
- the *fleet of vehicles* that represents the mobiles performing the task;

– the *depot(s)*, usually from where the vehicles start and come back.

The network is made up of vertices and arcs/edges representing the links between vertices. In a logistics context, it is characterized by the transport infrastructure. It consists of the fixed installations including the road junctions, and nodes which are terminals (such as seaports, stations, warehouses and depots). Nonetheless, in other applications, the network is not always materialized by a physical structure, and arcs are used to describe the allowed movements. The original graph (which is often very sparse) is generally transformed into a complete graph by removing the links between nodes that do not need to be visited (such as keeping only vertices corresponding to the customers and the depots). The links then represent the shortest path between vertices. They can be directed (particularly when they can be traversed in only one direction because of the presence of one-way streets, for instance) or undirected. Each link is weighted by a cost, which generally represents its length and/or travel time, possibly dependent on the vehicle type or on the period during which the arc is traversed.

Demand can be of many types from a product to be supplied to a service to be given. The latter case includes passenger transport that may be public (where operators provide scheduled services) or private. Much of the recent research in logistics is related to this type of transport, where the demand stands for getting people from an origin to their destination by an alternative mode to their private vehicles, either because of traffic congestion, or their concern about environmental issues for instance. Solutions include the use of feeder buses, car sharing programs and even pod cars. Other types of services also arise in vehicle routing problems, and can be related to health care (such as home health care and aid supplies during humanitarian relief), maintenance (repairs or inspection) or production (setting fastener materials on airplane cabin), among others.

A depot represents the location where mainly the vehicles are parked, reset, unloaded or recharged. It is generally the starting and/or ending node for the vehicles.

Vehicles traveling on the network embody service providers. They can be of various types, and may include trucks, aircraft, boats and trains, as well as bicycles, buses, helicopters and even pedestrian, drones, laser beams or robot arms. Depending on the demand, the task can be performed by the vehicle itself or, most often, by the operator in the vehicle (a caregiver in health service, a technician for a maintenance task, etc.). However, in the latter case, the vehicle and the operator are considered as a whole. Routes performed by vehicles are often classified as less-than-truckload. This means that the capacity enables the vehicle to perform several tasks without returning to the depot. Procedures may impose some constraints including financing, legalities and policies. In the transport industry, operations and ownership of infrastructure can be either public or private, depending on the country and transportation mode.

Then, from a general point of view, each customer has a demand, often represented like an amount of goods, which must be delivered or collected and there may be a service time, possibly dependent on the service provider. In some variants of the problem, it is not possible to fully satisfy the demand. Thus, the amounts to be delivered or collected can be reduced, or a subset of customers can be left unserved (*VRP with profit* or *orienteeing problems – OP*), which often affects the objective function (penalization of non-visited customers or maximization of profits associated with visits). If the visit can be made only on specific periods of the day, these periods are called *time-windows (VRPTW)*. Finally, some customers may have accessibility constraints (e.g. access limitations or loading and unloading requirements) and in these cases, only a subset of the available vehicles that can be used to serve these customers, as in the *truck and trailer routing problem (TTRP)*. In this problem, some customers cannot be visited by the complete vehicle and the trailer must be detached and parked to reach them.

The routes performed to serve customers start and end at one or several depots (*multi-depot VRP - MDVRP*). If the vehicles do not return to their home depot, routes can end at the last visited customer (*open VRP*). Sometimes, vehicles may stop at intermediate depots. Other variants consider several levels of the supply chain (*two-echelon VRP - VRP-2E*).

Each depot may be characterized by a limited capacity. Their locations are usually fixed but it may be a decision variable as in the *location-routing problem – (LRP)*. In this case, each depot can have a set-up cost if at least one route is assigned to it.

A homogenous or heterogeneous fleet of vehicles (*HFVRP*) can be associated with each depot. A type of vehicle is identified by (1) its capacity, (2) fixed and variable costs associated with its utilization, (3) possible subdivision into compartments, each one having a specific capacity and particular types of goods they can contain *multi-compartment VRP (MC-VRP)*, (4) and the subset of arcs they can traverse (accessibility constraints).

1.2.3.2. Constraints

The characteristics of the VRPs components, the nature of the demand and additional regulations (such as working periods during the day, number and duration of breaks, maximum duration of driving periods, etc.) impose to comply with a number of operational and regulatory constraints.

Examples of constraints are given here:

- depots and vehicles may have limited capacities, which require the current load to not exceed the related limit;
- when several depots are available, possibly with limited capacity, and/or the fleet of vehicles is limited, routes are assigned accordingly and in particular, no more

vehicle can be used. However, in some applications, each vehicle can operate more than one route during the considered time period as in the *multi-trip VRP* (VRPMT);

- the customer demand must be satisfied, and in case of transportation, this can require either only the delivery or the collection task, or both possibilities. The demand is sometimes allowed to be split to be served by several vehicles during the time period as it occurs in the *Split-Delivery VRP* (SDVRP). In other cases, if the time horizon is composed of several time periods, the customers may have to be visited several times over this horizon (for example in the *periodic VRP* (PVRP));

- usually, a route beginning at a given depot must finish at this depot, but sometimes it can last for more than one period;

- customers have to be served within both their time windows and the working periods of the drivers associated with the vehicle routes in which they are scheduled;

- precedence and synchronization constraints can affect the visit order of the customers. For instance, in the VRP variant called *dial-a-ride problems* (DARP), a demand task is made up of a number of people to be transported from an origin to a destination by one vehicle, implying that the origin point must be visited before the destination node. Collection and delivery of goods are also performed in routes of the *VRP with Backhauls*, but constraints associated with the loading and unloading operations mean that all deliveries must be performed before the collections. *Synchronization* occurs when a customer needs at least two simultaneous visits (e.g. by technician with different skills) to be served.

Generally, data are supposed to be perfectly known in advance. However, this is not always the case and only partial knowledge of the customer demands or the costs (and travel times) associated with arcs of the graph may be available. In these situations, it is necessary to consider stochastic or time-dependent dynamic versions of the problem.

Different optimization problems can be combined with the CVRP. The LRP quoted before encompasses a facility location problem with routing decisions. Inventory decisions can be added to the problem so that vehicles can supply customers according to their stock level *inventory routing problem* (IRP). Loading constraints may also be part of the problem as in the *VRP with two/three-dimensional loading constraints* (2L-VRP and 3L-VRP).

Several other constraints can be considered, so the list cannot be exhaustive. The resulting problems can be called *rich vehicle routing problems* [HAR 06, HAS 07].

1.2.3.3. Objectives

The classical objective in the CVRP is the minimization of the total cost, which is dependent on the global traveled distance (or on the global travel time) and on the fixed costs, associated with the use of vehicles. Further variants also include the fixed costs of using depots (LRP) or inventory costs (IRP). If the constraints are partially

satisfied, penalties are generally applied but must be minimized (as when a partial service of the customers is delivered). In some cases, the minimization of the number of vehicles required to serve all the customers can be added. Balancing of the routes, in terms of travel time and/or vehicle load, can be interesting to obtain fair timetabling between drivers. Some contexts require unusual criteria to optimize. This is the case in humanitarian logistics, when the objective relies on the time required to bring aid to victims. Instead of minimizing the total time, the *cumulative CVRP* (CCVRP) aims at minimizing the sum of the arrival time, that is equivalent to the mean arrival time at each customer.

In this manner, any weighted combination of these objectives can be considered for the vehicle routing problems. However, particularly when they are conflicting, multi-objective optimization may be more appropriate.

In summary, a large class of problems is hiding behind the VRPs. Next section provides a formal definition of the basic version.

1.2.4. The capacitated vehicle routing problem

The basic version of the VRP, the *Capacitated VRP* (CVRP), is a routing problem in which each customer has a known and deterministic demand that must be satisfied by a single visit. A fleet of identical capacitated vehicles starts and ends at a single central depot and the load in each vehicle does not exceed the related limit. The objective is to minimize the total cost to serve all the customers, which includes the global traveled distance and, when the fleet size is a decision variable, the fixed costs of vehicles. Figure 1.4 illustrates an instance of possible solution.

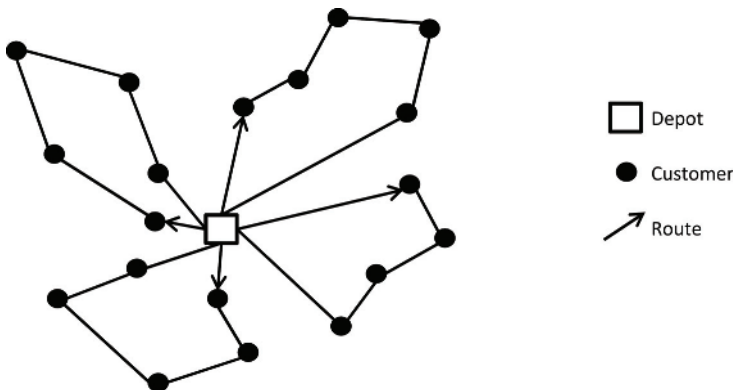


Figure 1.4. Example of a CVRP solution

1.2.4.1. Mathematical model

The CVRP belongs to the class of *node routing problems*, in which tasks are associated with nodes of the network, by contrast to *arc routing problems* where the demands are associated with edges or arcs. It can be formally defined on a graph. Let $G = (V, E)$ be a complete undirected graph. The node set V is made of a depot (node 0) from where a set K of a homogeneous fleet of vehicles of capacity Q can start, and n customers with a given demand $q_i, i = 1, 2, \dots, n$ for a good. Each edge $[i, j]$ from E represents on shortest path between nodes i and j in a real road network. Its associated cost c_{ij} is known. Depending on the authors, the number of available vehicles can be fixed or not, a service time s_i can be added for each customer i , and each route cost can de-limited by L (corresponding to a limited working time, for instance).

Various integer linear programs exist to model the CVRP [TOT 02, GOL 08]. The main difficulty stands on the way to eliminate subtours, i.e. cycles that do not go through the depot. Hereafter, a simple mathematical formulation is given in which two nodes represent the depot (nodes 0 and $n+1$, respectively, the starting and ending point of the routes) and each edge $[i, j]$ is replaced by two arcs (i, j) and (j, i) belonging to set A . The binary variables x_{ij}^k are equal to 1 if vehicle k goes through arc (i, j) .

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} \cdot x_{ij}^k \quad [1.1]$$

$$\sum_{j \in V \setminus \{i\}} \sum_{k \in K} x_{ij}^k = 1 \quad \forall i \in V \setminus \{0, n+1\} \quad [1.2]$$

$$\sum_{j \in V \setminus \{i\}} x_{ji}^k = \sum_{j \in V \setminus \{i\}} x_{ij}^k \quad \forall i \in V \setminus \{0, n+1\} \quad \forall k \in K \quad [1.3]$$

$$\sum_{i \in V \setminus \{0, n+1\}} \sum_{j \in V \setminus \{i\}} q_i \cdot x_{ij}^k \leq Q \quad \forall k \in K \quad [1.4]$$

$$t_i^k + s_i + c_{ij} \leq t_j^k + M(1 - x_{ij}^k) \quad \forall i \in V \quad [1.5]$$

$$x_{ij}^k \in \{0, 1\} \quad \forall (i, j) \in A \quad \forall k \in K \quad [1.6]$$

$$t_i^k \geq 0 \quad \forall i \in V \quad \forall k \in K \quad [1.7]$$

The objective-function [1.1] sums the total cost of the routes. Constraints [1.2] and [1.3] are the continuity constraints: a single vehicle visits customer i and this vehicle leaves it. Vehicle capacities hold due to constraints [1.4]. Variables t_i^k give the arrival time of vehicle k at customer i . Equations [1.5] remove subtours: if vehicle k goes from i to j ($x_{ij}^k = 1$), the right-hand side with the big positive constant M turns to t_j^k , so vehicle k can go to j only after having served i and traveled from i to j ; otherwise (if $x_{ij}^k = 1$), the constraint is trivially true.

1.2.4.2. *Solution methods*

This basic version, already NP-hard [LEN 81], was introduced in 1959 by Dantzig and Ramser [DAN 59] under the name of the *truck dispatching problem*. Since then, the number of publications on the subject has exploded as exposed in [LAP 09].

Several surveys are dedicated to vehicle routing problems. Laporte [LAP 09] provided in 2009 a paper drawing a picture on the state of the researches made on the vehicle routing problem, 50 years after its introduction by Dantzig and Ramser [DAN 59]. Other overview papers focusing either on exact resolution methods and/or heuristic ones are published [BAL 07, LAP 02]. The text book, edited by Toth [TOT 02], covers entirely the subject through a selection of six contributed chapters. A second edition updates the first book, and including emerging applications, such as disaster relief and green vehicle routing [TOT 14].

Some recent surveys dealing with VRP variants have also appeared in the literature. For instance, [BAL 12] and [BAL 08b] provide overviews dealing with the VRPTW in the first paper and the heterogeneous fleet VRP in the second paper. Vidal *et al.* [VID 12b, VID 13a] present surveys on multi-attribute vehicle routing problems. Gendreau *et al.* [GEN 15] offer a review for time-dependent routing problems. Montoya-Torres *et al.* [MON 15] contributed with a state-of-the-art on multi-depot vehicle routing problems. The two-echelon VRP is the subject considered in the recent survey from Cuda *et al.* [CUD 15], and the location routing literature is summarized in the article from Prodhon and Prins [PRO 14]. Labadie and Prins [LAB 12a] also present a paper including the most important results on a large variety of vehicle routing variants, with an emphasis on problems occurring in developing countries. Labadie and Prodhon [LAB 14] give an overview of vehicle routing problems dealing with more than one objective function.

The book coordinated by Golden *et al.* [GOL 08] offers an overview of different vehicle routing extensions, summarizing the new trends in terms of methods and models since the early 2000.

Despite this rich and diverse amount of research, exact methods are still limited to problems involving up to 100 clients [BAL 08a], when real cases can achieve more than 1,000 customers. Hence, metaheuristics are the most appropriate way to deal with these problems.

1.3. Conclusion

This chapter gives a general presentation of vehicle routing problems (VRPs) and the main related fields, which are logistics and combinatorial optimization. Good transport planning is essential to controlling the costs, as well as the flow and other side constraints (regulations, nuisances, etc.). For instance, the last mile problem

often behind VRPs includes the challenge of delivering goods in urban areas where congestion and safety problems are often encountered.

In addition to its great interest in today's global markets, and the savings that can be achieved by a better planning and management of logistic systems, and beyond its attractiveness as a benchmark problem in combinatorial optimization, the family of VRPs also draws attention of both the academic and economic communities for the variety of its applications.

In addition to classical supply of goods, VRPs can model a large range of problems on various scales. At a microscopic level, how to engrave transistors on an integrated circuit with a laser beam is a question that can be handled as a VRP. In production sites, several issues arise such as the optimization of robot trajectories to perform welds. Warehouse management brings its topics like the pick up of goods in shelving. From a macroscopic point of view, subjects can be related to the collection of satellite images.

In real contexts, VRP models include a large number of customers to be served and the related integer program cannot be solved in acceptable time by combinatorial optimization tools. Therefore, in this case, heuristic approaches are more suitable. Chapter 2 introduces significant constructive heuristic methods and local improvement moves classically applied in VRPs to provide feasible solutions quickly to metaheuristics.