
Preliminaries in Decision-Making

1.1. Introduction

Supply chain (SC) is the framework that gathers different commercial entities perceived to be effective in the planning operations and cost-saving activities. We try in the present chapter to define the main concepts used within the SC and that are related to the decision-making between some entities of the SC, which is generally viewed as a course of actions to be handled and scheduled by the manager. The decision-making process starts by defining and stating the problem. Then, the problem designer should be able to depict the problem features that characterize the above-mentioned statements in order to select the appropriate solution approach. As illustrated in Figure 1.1, two main categories of solution approach exist: optimization approach and game theory approach. Once the solution approach is selected, it has to be evaluated by the use of specifically designed metrics. Simulations are then conducted in order to produce comparative study of the solution approach.

Hence, the decision-making process is decomposed into elementary steps to be handled by appropriate experts and validated to measure the real gap between the theoretical plan and its implementation. This system realization, implementation and validation makes its design (from a theoretical point of view) and building (from a practical standpoint) more coherent and much more efficient once compared to the initial problem specification and system concept that can be pointed out from the following list:

1) *System*: a set of components intercorrelated by precedence and resource requirements in order to accomplish one or several objectives.

2) *Closed system*: a system that does not need any external interaction to accomplish its objective(s).

3) *Open system*: a system that continuously needs external interactions to accomplish its objective(s).

4) *Suboptimality*: the quality of the solution related to the accomplishment of the system objective(s) and to be the best, in which case it is called “optimal”, or close to the best, in which case it is called “suboptimal”. The quality of such a solution is closely dependent on the complexity of the process.

Once the decision-making process is defined and clearly specified, the problem should be analyzed and quantitatively expressed in terms of its inputs and outputs.

The remainder of this chapter is organized as follows. In section 1.2, we define the decision-making problems. Section 1.3 deals with the optimization modeling of the decision problem. Section 1.4 presents the game theory modeling for the decision-making problems.

1.2. Decision-making problems

A decision-making problem is the quantitative modeling of a problem situation. Generally speaking, a decision-making problem is split into the following three main components:

- the decision maker(s);
- the objective(s) to be reached;
- the set of structural constraints (system constraints and decision variables) that bound the feasible set.

Depending on these components, we can point out the solution approaches that solve a decision-making problem. To do so, it is required for the decision maker (DM) to study the problem complexity in order to identify the class to which the decision problem lies. We can point out two main solution approaches for a decision problem: the optimization or the game theory approach.

For the optimization modeling, two main classes of decision-making problems are:

1) *Constrained decision problems* modeled as the optimization of an objective function $z(x)$ expressed while fulfilling a set of structural constraints that bound the decision space. So, three components can characterize a constrained decision problem, namely, the objective function, the set of constraints and the decision variable requirements.

2) *Unconstrained decision problems* that consist of minimizing or maximizing a function $z(x)$ that is generally nonlinear. The main concern is the finding of the

solution value that corresponds to the local optima of $z(x)$. In this case, there is neither consideration of system constraints nor of the range of the solution x .

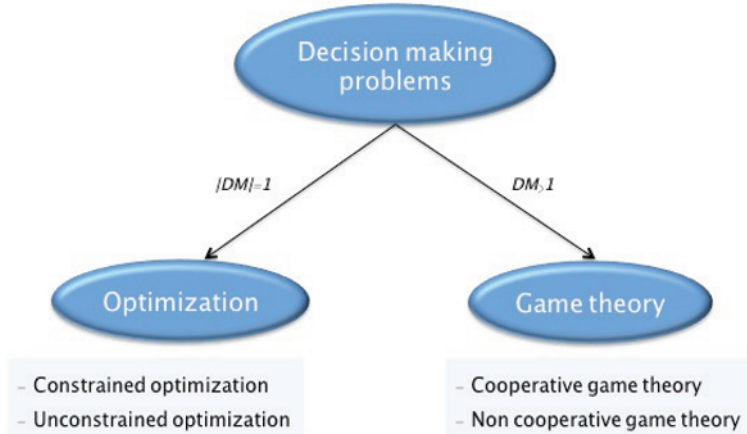


Figure 1.1. *Taxonomy of decision-making problems*

From a game theoretical standpoint, we point out two main classes:

1) *cooperative games* that model a collaborative decision-making process where a group of players (decision makers) can coordinate their actions and share their winnings. In fact, the cooperative game theory deals with how players can synchronize their decisions and divide the spoils after they have made binding agreements;

2) *non-cooperative games* that address the problem with multiple decision makers where each one has to choose among various options from several possible choices. However, the preferences that each decision maker has on his actions depend on the actions of the others. Thus, his action depends on his beliefs about what the others are willing to do. The main idea of non-cooperative game theory is thus to analyze and understand such a multi-person decision-making process.

1.3. Optimization modeling of a decision problem

An optimization problem is a formal specification of a set of proposals related to a specific framework that includes one decision maker, one or several objectives to be reached and a set of structural constraints. A possible structure of an optimization modeling is shown in Figure 1.2. Optimization has been practiced in numerous fields of study as it provides a primary tool for modeling and solving complex and hard constrained problems. Throughout the 1960s, integer programming formulations and

approximate approaches received considerable attention as useful tools in solving optimization problems. Depending on the problem structure and its complexity, appropriate solution approaches were proposed to generate appropriate solutions in a reasonable computation time. Several optimization studies are formulated as a problem whose goal is to find the best solution, which corresponds to the minimum or maximum value of a single objective function. The challenge of solving combinatorial problems lies in their computational complexity since most of them are NP-hard [GAR 79]. This complexity can mainly be expressed in terms of the relationship between the search space and the difficulty to find a solution. The search space in combinatorial optimization problems is discrete and multidimensional. The dimensionality of the search space greatly influences the complexity of the decision problem.

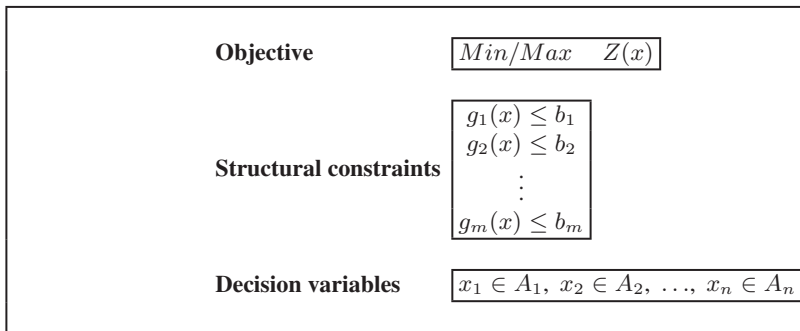


Figure 1.2. *Structure of an optimization problem*

1.3.1. Notation

We list below the major symbols used for defining an optimization problem:

Symbols	Description
n	the number of decision variables
k	the number of objectives
$x = (x_1, \dots, x_n)^T$	the vector of decision variables
$c_{(p,n)}$	the cost matrix
A	the matrix of constraints
B	resource limitations
E_O	the set of efficient solutions in the objective space
E_D	the set of efficient solutions in the decision space

Assuming the linearity of an optimization problem, its mathematical modeling is outlined as follows:

$$\text{Max } p.x \quad [1.1]$$

$$S.t. \ A.x \leq B \quad [1.2]$$

$$x \in \mathcal{X} \quad [1.3]$$

where $x = (x_1, \dots, x_n)^T$ denotes the vector of decision variables, p , b and A are the constant vectors and matrix of coefficients, respectively.

Many variants of this formulation can be pointed out:

- *Continuous linear programming (CLP)*: the optimization model [1.1]–[1.3] is a CLP if the decision variables are continuous. For continuous linear optimization problems, efficient exact algorithms such as the simplex-type method [BÜR 12] or interior point methods exist [ANS 12].

- *Integer linear programming (ILP)*: the optimization model [1.1]–[1.3] is an ILP if $\mathcal{X}$ is the set of feasible integer solutions (i.e. decision variables are discrete). This class of models is very important as many real-life applications are modeled with discrete variables since their handled resources are indivisible (such as cars, machines and containers). A large number of combinatorial optimization problems can be formulated as ILPs (e.g. packing problems, scheduling problems and traveling salesman) in which the decision variables are discrete and the search space is finite. However, the objective function and constraints may take any form [PAP 82].

- *Mixed integer programming (MIP)*: the optimization model [1.1]–[1.4] is called MIP, when the decision variables are both discrete and continuous. Consequently, MIP models generalize CLP and ILP models. MIP problems have improved dramatically of late with the use of advanced optimization techniques such as relaxations and decomposition approaches, branch and bound and cutting plane algorithms when the problem sizes are small [GAR 11, WAN 14, COO 11]. Metaheuristics are also a good candidate for larger instances. They can also be used to generate good lower or upper bounds for exact algorithms and improve their efficiency.

1.3.2. Features of an optimization problem

Optimization problems can be classified in terms of the nature of the objective function and the nature of the constraints. Special forms of the objective function and the constraints give rise to specialized models that can efficiently model the problem under study. From this point of view, various types of optimization models can be highlighted: linear and nonlinear, single and multiobjective optimization problems and continuous and combinatorial programming models. Based on such features, we have to define the following points:

- *The number of decision makers*: if one DM is involved, the problem dealt with is an *optimization problem*. Otherwise, we are concerned with a *game* that can be cooperative or non-cooperative, depending on the DMs' standpoints.

– *The number of objectives*: this determines the nature of the solution to be generated. If only one objective is addressed in the decision problem, the best solution corresponds to the optimal solution. However, if more than one objective is considered, we are concerned with generating a set of efficient solutions that correspond to some tradeoffs between the objectives under study.

– *The linearity*: when both the objective(s) and the constraints are linear, the optimization problem is said to be linear. In this case, specific solution approaches can be adapted as the simplex method. Otherwise, the problem is nonlinear, in which case the resolution becomes more complex and the decision space is not convex.

– *The nature of the decision variables*: if the decision variables are integer, we deal with a combinatorial optimization problem.

1.3.3. A didactic example

Let us consider the following optimization problem involving two decision variables x_1 and x_2 . We show in this illustrative example, inspired from [KRI 14a], how the solution changes in terms of the nature of the decision variables that can be either continuous or binary and the number of objectives $k = 1, 2$. Hence, four optimization problems follow:

SINGLE OBJECTIVE		MULTI-OBJECTIVE	
CONTINUOUS DECISION VARIABLES	$\begin{array}{ll} \text{Max} & 2x_1 + x_2 \\ \text{S.t.} & 5x_1 + 7x_2 \leq 100 \\ & x_1 - 3x_2 \leq 80 \\ & x \geq 0 \end{array}$	$\Rightarrow \boxed{\begin{array}{l} (x_1, x_2) = (20, 0) \\ z(x) = 40 \end{array}}$	$\begin{array}{ll} \text{Max} & 2x_1 + x_2 \\ & x_1 + 5x_2 \\ \text{S.t.} & 5x_1 + 7x_2 \leq 100 \\ & x_1 - 3x_2 \leq 80 \\ & x \geq 0 \end{array}$
			$\Downarrow$ $\boxed{\begin{array}{l} E_D = \{(20, 0), (0, 14.285)\} \\ E_O = \left\{ \begin{pmatrix} 40 \\ 20 \end{pmatrix}, \begin{pmatrix} 14.285 \\ 71.428 \end{pmatrix} \right\} \end{array}}$
BINARY DECISION VARIABLES	$\begin{array}{ll} \text{Max} & 2x_1 + x_2 \\ \text{S.t.} & 5x_1 + 7x_2 \leq 100 \\ & x_1 - 3x_2 \leq 80 \\ & x \in \{0, 1\} \end{array}$	$\Rightarrow \boxed{\begin{array}{l} (x_1, x_2) = (1, 1) \\ z(x) = 3 \end{array}}$	$\begin{array}{ll} \text{Max} & 2x_1 + x_2 \\ & x_1 + 5x_2 \\ \text{S.t.} & 5x_1 + 7x_2 \leq 100 \\ & x_1 - 3x_2 \leq 80 \\ & x \in \{0, 1\} \end{array}$
			$\Downarrow$ $\boxed{\begin{array}{l} E_D = \{(1, 1)\} \\ E_O = \left\{ \begin{pmatrix} 3 \\ 6 \end{pmatrix} \right\} \end{array}}$

As mentioned previously, the resolution of the single objective optimization problem yields to the finding of the optimal solution that varies depending on the nature of the decision variables. However, if a second objective is added, the resolution generates a set of Pareto-optimal solutions, as it is the case for $k = 2$.

1.4. Game theory modeling of a decision problem

Game theory has become an important tool in the decision-making areas. Indeed, it is considered as an alternative way that achieves cost-saving objectives. These are realized using various collaboration schemas. Such schemas try to form coalitions of players in order to minimize or maximize a given objective. The coalition formation problem (CFP) covers two fundamental behavioral social notions: conflict (competition) and cooperation. As such, the theory of games was divided into two distinct types: non-cooperative game theory and cooperative game theory. In the non-cooperative theory, a game is a detailed model of all the moves available to the players. However, the preferences that each decision maker has on his actions depend on the actions of the others. By contrast, cooperative theory abstracts away from this level of detail, and only describes the outcomes that result when the players come together in different combinations [BRA 07]. Once the type of game theory is defined, the decision of each player to join the coalition or not is based on its shared payoff generated due to that coalition formation regarding its individual payoff. This shared payoff is calculated using an allocation method. The main objective of the game theory is to find a stable coalition structure where no player has an interest in deviating from the coalition.

1.4.1. Notation

We list below the major symbols used for defining a game theoretical problem:

Symbols	Description
$v(S)$	the payoff function
n	the number of players
S	the subset of coalitions
$N = \{1, 2, \dots, n\}$	the set of players
i	the player's index

Theoretical games can be defined as a set of n players/participants that can coordinate their objectives and synchronize their strategies in order to achieve cost saving while sharing information and allocating payoffs.

Let $N = \{1, 2, \dots, n\}$ be a finite set of players with different cooperation possibilities. Each subset $S \in N$ is referred to each formed coalition or alliance. We recall that a coalition is the group of players that synchronize their strategies and goals. The number of coalitions is $2^n - 1$. For each S , $\|S\|$ refers to the number of agents within the coalition. $v(S)$ be the characteristic function that associates a real number with each coalition S which can be interpreted as the maximum value of cost savings that the members of S would divide among themselves. Given a coalition S , we define an allocation: $(x_1, x_2, \dots, x_n)$ as a division of the overall created value. It specifies for each player $i \in S$ the payoff portion x_i that this player will receive when he cooperates with other players. We give in what follows some features of an allocation:

- an allocation $(x_1, x_2, \dots, x_n)$ is *individually rational* if $x_i \geq v(i)$ for all i ;
- an allocation $(x_1, x_2, \dots, x_n)$ is *efficient* if $\sum_{i=1}^{\|N\|} x(i) = v(N)$;
- an *imputation* is an efficient and individually rational allocation;
- a *stability* of a coalition structure is given if $\sum_{i \in S} x_i \leq v(S)$.

The individual rationality means that a division of the overall value (i.e. an allocation) must give each player as much value as that player receives without interacting with other players (single coalition). Efficiency means that all the value that can be created can, in fact, be divided among collaborating players.

1.4.2. The coalition formation problem

The CFP models many situations in multiple domains including multi-agent systems, economics, industry or politics. One approach is concerned with cases where a group of players is interested in accomplishing, individually or cooperatively, one common task. The other is interested in sets of autonomous, self-motivated players who act in order to achieve their own task or increase their own profit. A solution to the CFP consists of:

- dispatching the players into coalitions forming a “coalition structure”. This step is called the coalition structure generation (CSG). This step tries to search for the coalition structure corresponding to the maximum total sum of coalition values;
- dividing the gains among the players in such a way that no player is tempted to deviate. This is called the payoff division (PD).

Game theorists modeled the CFP as a cooperative game in its two forms, either as a characteristic function game or as a partition function game, in order to obtain stable payoff vectors [STE 68] and [KAH 84]. However, game theory did not provide any mechanism to form coalitions of the players.

Games with externalities assume that the payoff of a coalition depends on what other coalitions form [BLO 96]. A function assigning to each coalition a payoff depending on the whole coalition structure is called a “partition function”. Such a game is called a “partition function game” [MAR 03].

All created works addressed the CFP in terms of two main features: the superadditivity and the non-superadditivity. A game is superadditive if each two subgroups of players receive at least as much when cooperating as by acting individually. Therefore, the CSG step will obviously generate the grand coalition.

Table 1.1 reports the main research that has addressed the CFP. These works are concerned in solving either the CSG step or the PD step. We continue this research by addressing both features of the problem (CSG and PD) mainly for non-superadditive games.

	<i>Superadditive games</i>	<i>Non-superadditive games</i>
<i>CSG</i>	<i>The grand coalition forms in all cases</i>	[DAN 04], [SAN 99] [SEN 00] [YAN 07] [LAR 00]
<i>PD</i>	[STE 68] [KAH 84] [BEL 06] [GIL 59] [AUM 74] [SHA 53] Hart (1997)	[YAN 07]

Table 1.1. *Classification of the CF papers in terms of the superadditivity*

When the game is not superadditive, the proposed approaches try to generate the coalition structures that maximize the total welfare as in [YAN 07], without dividing the obtained payoff among the players.

The PD was addressed mainly for superadditive games. Various stability concepts were proposed in the literature to define a dividing strategy with two alternative designs:

- 1) non-cooperative game: prisoner’s dilemma “Nash equilibrium” *Pollute, Risk*;
- 2) cooperative game: coalition formation... “Nash equilibrium, Shapley, Core” *Profit, Cost*.

The payoff vector x should fulfill the following rationality concepts. To do so, let N be a set of players, $S \subseteq N$ a coalition of players, CS a coalition structure and C the set of all coalition structures of N .

1) *Individual rationality*: a payoff vector $x = (x_1, \dots, x_n)$ is said to be individually rational if it satisfies:

$$x_i \geq v(\{i\}) \quad \forall i \in N \quad [1.4]$$

2) *Coalitional rationality*: a payoff vector $x = (x_1, \dots, x_n)$ is said to be coalitionally rational if it satisfies:

$$\sum_{i \in S} x_i \geq v(S) \quad \forall S \subseteq N \quad [1.5]$$

3) *Efficiency*: an efficient payoff vector $x = (x_1, \dots, x_n)$ for non-superadditive games can be written as:

$$\sum_{i \in N} x_i = \max_{CS \in C} \sum_{S \in CS} v(S) \quad [1.6]$$

1.4.3. The stability concepts

Nash equilibrium: this is a fundamental concept in the theory of games and the most widely used method of predicting the outcome of strategic interaction in the social sciences. A game consists of the following three elements: a set of players, a set of actions available to each player and a payoff (or utility) function for each player. The payoff functions represent each player's preferences over action profiles, where an action profile is simply a list of actions, one for each player. A pure-strategy Nash equilibrium is an action profile with the property that no single player can obtain a higher payoff by deviating unilaterally from this profile.

Core stability: to define the core, some additional notations will be useful. For any subset S of the set of players N , let $x(S) = \sum_{i \in S} x(i)$. In other words, the term $x(S)$ denotes the sum of the values received by each of the players i in the subset S .

DEFINITION 1.1 AN ALLOCATION.— $(x_1, x_2, \dots, x_n)$ is *collectively rational* if $x(S) \geq v(S)$ for all i .

DEFINITION 1.2 THE CORE.— [SHA 53] This is the set of efficient allocations satisfying the collective rationality.

$$Cr(N, v) = \{x(N) = v(N) \text{ and } x(S) \geq v(S)\} \quad [1.7]$$

In a game (N, v) , if any group of players, say S , anticipated capturing a lower value in total than the group could create on its own, i.e. if $x(S) < v(S)$, then this

group of players would do better to create a coalition apart, S , and divide the value $v(S)$ by themselves. This would not happen under core allocations. In summary, the core has the interesting interpretation that the total created value is allocated in such a way that no group of players would have the incentive to leave the system (the grand coalition N) and form a coalition apart because they collectively receive at least as much value as they could obtain for themselves as a coalition. The grand coalition is then immune to coalitional deviations, this concept has been called the core stability.

DEFINITION 1.3 THE GRAND COALITION N .– This is said to be stable or core stable if it has a non-empty core.

1.5. Allocation methods

In this section, we turn our attention to describing the three allocation rules that we use in the rest of the dissertation. This includes equal allocations, proportional allocations and Shapley value allocations.

1.5.1. Shapley value allocation

This section is devoted to introducing the concepts and axioms of Shapley value [SHA 53], one of the most central solution concepts in game theory. Shapley value is a solution that prescribes a single payoff for each player, which is the average of all marginal contributions of that player to each coalition of which he is a member. It satisfies the following axioms:

- efficiency: the payoffs must add up to $v(N)$, which means that all the grand coalition surplus is allocated;
- symmetry: if two players are substitutable because they contribute the same to each coalition, the solution should treat them equally;
- additivity: the solution to the sum of two games must be the sum of what it awards to each of the two games;
- dummy player: if a player contributes nothing to every coalition, the solution should pay him nothing.

There is a unique single-valued solution to games satisfying efficiency, symmetry, additivity and dummy. The function that assigns to each player i the payoff [SHA 53]:

$$Sh(N, v)(i) = \sum_{s \in S} \frac{(\|S\| - 1)! - (\|N\| - \|S\|)!}{\|N\|!} (v(s) - v(s \setminus i)) \quad [1.8]$$

The Shapley value awards to each player the average of his marginal contributions to each coalition. The marginal contribution of a player i with respect to

a given ordering is defined as his marginal worth to the players before him in the order, $v(1, 2, \dots, i-1, i) - v(1, 2, \dots, i-1)$ where $1, \dots, i-1$ are the players preceding i in the given ordering. The Shapley value is obtained by averaging the marginal contributions for all possible orderings. In taking this average, all orders of the players are considered to be equally likely.

The Shapley value is usually viewed as a good normative answer to the question posed in cooperative game theory, that is, those who contribute more to the groups that they belong to should be paid more. However, the Shapley value may be not stable in the sense of the core. For instance, it may allocate a negative value to some players. Besides, the Shapley value may lie outside the core unless for some special games like convex games [SHA 71]. For a recent study on Shapley value's stability, see [BEA 08].

Equal allocations: the simplest allocation of savings would be to give an equal portion to each player.

$$v(i) = \frac{v(N)}{n} \quad [1.9]$$

Proportional allocation: another simple way of allocating savings would be to distribute them proportionally to the initial inputs (contributions) of different players. For example, consider a savings game (N, v) such that $v(S) = (\sum_{i \in S} C(i)/C(S))$ for any coalition S , the function C is the cost characteristic function. The savings may be allocated proportionally to the stand-alone cost (individual cost) of each player. This division protocol is termed the cost-based proportional rule. Each player i gets,

$$v(i) = \frac{v(i)}{\sum_{j \in S} v(j)} v(N) \quad [1.10]$$

1.6. Conclusion

We outlined in this chapter the main concepts related to the decision-making process. As such, we pointed out two main classes of decision-making problems, in terms of the number of involved DMs. For the single DM we surveyed the most important tools that characterize the resolution of such a class seen as an optimization problem that can address one or multiple objectives subject to structural constraints. The resolution of an optimization problem depends on numerous features mainly the complexity of the problem and its size. Alternatively, if multiple DMs are involved in the decision problem, the problem modeling corresponds to a game that can be cooperative or non-cooperative, depending on players' standpoints. We discussed some stability concepts related to game theory. We also detailed, for cooperative games, the need for a coalition formation and the incentive of forming coalitions followed by numerous payoff division protocols. All such concepts will be used in the subsequent chapters that focus on supply chain activities involving single or multiple DMS.