1

Introduction to Structural Dynamics

The aim of this chapter is to convey a non-exhaustive image of all areas considered, from near or far, in this work.

Section 1.1 is dedicated to the general study of structural dynamics. This study intends to attach the essential evaluations to the calculations of dynamic responses, frequencies, appropriate methods and their response functions. All of these aspects are consequently tackled using practical applications.

The dynamic balance equation system of a structure can be solved by using one of the traditional strategies [MOH 05]. The most frequent resolution strategy in dynamics is modal superposition, which is suited to linear structures whose first methods are only the ones that are agitated. In contrast, direct resolution methods incorporate movement equations in order to handle nonlinear structures. These structures can also be applied when the frequency contents of the disturbance cover a large number of methods of the mechanical structure studied.

In section 1.2, a non-exhaustive bibliographic study is put forward regarding the optimization of structures. The objective is to obtain suitable forms from an article by minimizing a given criterion. In every area of structural mechanics, knowing the impact of effective object design is very important in determining its resistance, lifetime and operation. This is one of the challenges faced by industries daily. The development of engineering requires considerable effort to constantly improve the techniques for designing structures. Optimization plays an important role in increasing performance and significantly reducing aerospace and motoring engineering equipment, while simultaneously substantially saving energy.

The last section of this chapter is devoted to describing the different tools that analyze structures with uncertain parameters. The uncertainty of parameters is particularly dangerous in vibratory mechanics. However, consideration of this effect

has the ability to respond to different sorts of needs, among which one can identify two categories: analysis and design. In general, modeled objects and structures respond to a design brief, such as safeguarding security, reliability or comfort guidelines.

When creating a deterministic design, one tends to search for the best possible design from among all potential solutions. This choice is based on cost as well as improvement in product quality. In this case, the objectives of the designer to produce the optimal design are hampered despite the accuracy of the mechanical characteristics of the materials, the geometry and the loading (effects of uncertainties). The resulting optimal design can thus have an unsatisfactory level of reliability. The process that incorporates reliability analysis with the named problem of optimization (Reliability based design optimization or RBDO) aims to envisage structures while establishing the best compromise between cost and effective functioning.

1.1. Composition of problems relating to dynamic structures

The composition of a dynamic problem of small disturbances using Ω of the boundary $\Gamma = \Gamma_u \cup \Gamma_f$ (Figure 1.1) and in a [0, T] time interval is:

$$Div_x\ \sigma(x,t) + g(x) = \rho\,\ddot{u}(x,t) \qquad [1.1]$$

$$\varepsilon = \frac{1}{2}\left(\nabla u + \nabla^t u\right) \qquad [1.2]$$

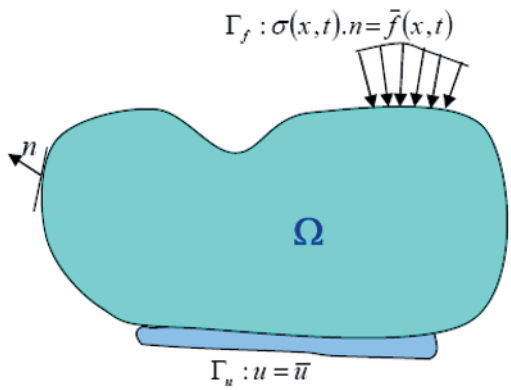

Figure 1.1. *Structure* Ω

Initial conditions:

$$u(x,0)=u_0(x) \quad x\in\Omega \tag{1.3}$$

$$\dot{u}(x,0)=\dot{u}_0(x) \quad x\in\Omega \tag{1.4}$$

Limited conditions:

$$u(x,t)=\overline{u}(x,t) \qquad (x,t)\in\Gamma_u \times [0,T] \tag{1.5}$$

$$\sigma(x,t).n=\overline{f}(x,t) \qquad (x,t)\in\Gamma_f \times [0,T] \tag{1.6}$$

Here, u is the displacement vector, σ and ε are the constrained and deformation tensors, respectively, and ρ is the volumetric density. The vectors $g, \overline{f}$ and $\overline{u}$ represent volumetric strength, exterior strength and imposed movement, respectively, and $\vec{n}$ is the normal vector at the surface.

In terms of isotropic elasticity, the behavior law is written as follows:

$$\sigma_{ij} = \lambda \varepsilon_{kk}\delta_{ij} + 2\,\mu\,\varepsilon_{ij}\,, \tag{1.7}$$

where λ and μ are the functions of Young's modulus and Poisson's coefficient ν, respectively:

$$\lambda=\frac{E.\nu}{(1+\nu)(1-2\nu)} \tag{1.8}$$

$$\mu=\frac{E}{2(1+\nu)} \tag{1.9}$$

The dynamic problem presented above in the case of elasticity can be represented can by the Navier equation as follows:

$$\mu\nabla^2 u_i(x,t)+(\lambda+\mu)\frac{\partial}{\partial x_i}\left(\nabla.u(x,t)\right)=\rho\ddot{u}(x,t)\,, \tag{1.10}$$

where ∇^2 denotes the Laplacian operator: $\nabla^2 = \frac{\partial^2}{\partial^2 x_1} + \frac{\partial^2}{\partial^2 x_2} + \frac{\partial^2}{\partial^2 x_3}$ and $\nabla\cdot$ is the notation for the divergence operator: $\nabla \cdot u = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}$.

1.1.1. *Finite element method*

In the case of complex geometric structures, numerical methods like the finite element method are used. In problems concerning elastodynamics, generally movements are expressed by a combination of vectors [GMÜ 97]:

$$u(x,t) = \left[B(x)\right]\left\{q(t)\right\}, \qquad [1.11]$$

where $\left[B(x)\right]$ is the matrix form of functions and $\left\{q(t)\right\}$ is the vector of discrete real movements, whose components are discrete unknowns of approximation.

After discretization of the problem, a second-order equation system was obtained:

$$\begin{cases} [M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = F(t) \\ \{q(0)\} = \{q_0\}, \quad \{\dot{q}(0)\} = \{\dot{q}_0\} \end{cases}, \qquad [1.12]$$

where N is the number of degrees of freedom of the system; $M(N\times N)$ is the mass symmetrical matrix, which is defined as positive; $C(N\times N)$ and $K(N\times N)$ are the matrices of viscous shock absorption and rigidity, which are symmetrically defined as being non-negative; and F represents the vector of all forces applied.

Equation [1.12] represents a system of differential second-order equations that can be solved by either a direct incorporation method or superposition method.

1.1.2. *Modal superposition method*

If one applies the following transformation to the system presented in equation [1.12]:

$$\{q\} = [\Phi]\{p\}, \qquad [1.13]$$

where $\{p\}$ is the vector of generalized coordinates, $[\Phi]$ is the modal matrix that verifies the attributes of orthogonality: $[\Phi]^T [M][\Phi] = I$ and $[\Phi]^T [K][\Phi] = \left[w^2\right]$ with $\left[w^2\right] = diag\left[w_1^2 \; w_2^2 \cdots\right]$, where w_i is the specific vibration, equation [1.12] becomes:

$$\{\ddot{p}\} + [\Phi]^T [C][\Phi]\{\dot{p}\} + \left[w^2\right]\{p\} = \{P\}, \qquad [1.14]$$

where $\{P\} = [\Phi]^T \{F\}$ is the vector of modal force.

The shock absorption matrix can be proposed as being proportional to the mass and stiffness matrix. This hypothesis was made by Rayleigh and is relatively frequently employed in structural calculations. One can write:

$$[C] = \alpha[M] + \beta[K] \qquad [1.15]$$

$$[\Phi]^T [C][\Phi] = \alpha[I] + \beta\left[w^2\right], \qquad [1.16]$$

which can be transformed into:

$$\{\ddot{p}\} + \left(\alpha[I] + \beta\left[w^2\right]\right)\{\dot{p}\} + \left[w^2\right]\{p\} = \{P\} \qquad [1.17]$$

The unpaired system becomes:

$$\ddot{p}_i + 2\zeta_i w_i \dot{p}_i + w_i^2 p_i = P_i \qquad [1.18]$$

$$2\zeta_i w_i = \alpha + \beta w_i^2, \quad i = 1, 2, \cdots\cdots, N, \qquad [1.19]$$

where ζ_i is the coefficient of reduced shock absorption and the values of α and β are initially unknown, which are calculated using ζ_i.

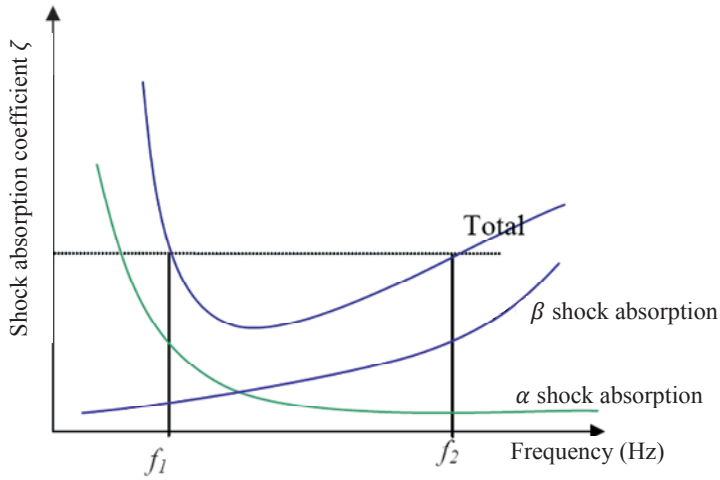

Figure 1.2. *Graph of the shock absorption coefficient*

Figure 1.2 shows the shock absorption coefficient ζ in graphical form. It can be noted that the sum of the two functions is almost a constant to the shock absorption on the frequency band chosen. Therefore, given the modal shock absorption (ζ) and a frequency interval (f_1 and f_2), the two equations can be simultaneously solved to determine α and β:

$$\frac{\alpha}{4\pi f_1} + \beta \pi f_1 = \zeta \qquad [1.20]$$

and

$$\frac{\alpha}{4\pi f_2} + \beta \pi f_2 = \zeta \qquad [1.21]$$

1.1.3. *Direct integration*

There are many methods of integration for differential equations. The general process is to discretize time and formulate what is occurring at the given instance "$t + \Delta t$" in terms of what happens at instance "t" using Taylor developments. The Newmark method will be presented in this section, as well as that of Wilson [KLE 92, EL 13].

1.1.3.1. *Newmark method*

Newmark proposed a method in which speed and movement of $t + \Delta t$ are estimated in terms of $\{\ddot{q}_t\},\{\dot{q}_t\},\{q\}$ and acceleration $\{\ddot{q}_{t+\Delta t}\}$. In addition, movement and speed are developed in a Taylor series with the help of two independent parameters, β and γ, together with time [KLE 92]:

$$\{q_{n+1}\} = \{q_n\} + \{q_n\}\Delta t + \left\{\left(\frac{1}{2} + \beta\right)\ddot{q}_n + \beta\,\ddot{q}_{n+1}\right\}\Delta t^2 \qquad [1.22]$$

$$\{\dot{q}_{n+1}\} = \{\dot{q}_n\} + \{(1-\gamma)\ddot{q}_n + \gamma\,\ddot{q}_n\}\Delta t\,, \qquad [1.23]$$

where $\{q_n\}$, $\{\dot{q}_n\}$ and $\{\ddot{q}_n\}$ are the approximations of $\{q(t_n)\}$, $\{\dot{q}(t_n)\}$ and $\{\ddot{q}(t_n)\}$, respectively, and $t_{n+1} = t_n + \Delta t$, with Δt being time. The two independent parameters, β and γ, assure the accuracy and stability of the solution. When $\gamma \geq 1/2$, $\beta \geq (\gamma + 0.5)/4$.

By transferring these equations onto the movement equation, the following matrix relation can be obtained:

$$\left(M + \Delta t\,\gamma C + \Delta t^2\,\beta K\right)\ddot{q}_{n+1} = C\,\dot{q}_{n+1} + K\,q_{n+1} - F_{n+1} \qquad [1.24]$$

with

$$\dot{q}_{n+1} = \dot{q}_n + (1-\gamma)\Delta t\,\ddot{q}_n \text{ et } q_{n+1} = q_n + \Delta t\,\dot{q}_n + \Delta t^2\left(\frac{1}{2} - \beta\right)\ddot{q}_n \qquad [1.25]$$

Acceleration at the moment t = 0 is created by the balancing conditions and the initial conditions on $\{q\}$ and $\{\dot{q}\}$. The solution of equation [1.24] requires the solution of a linear system at each time interval.

1.1.3.2. *The Wilson method, θ*

The Wilson method is the one in which acceleration varies linearly in the interval $[n\Delta t, (n+1)\Delta t]$. Wilson supposed that this linear variation occurs in the interval $[n\Delta t, (n+1)\Delta t]$. The value of θ recommended by Wilson is 1.4.

If τ denotes time in interval $[0, \theta \Delta t]$, then acceleration in the interval $[t, t + \theta \Delta t]$ is written in [KLE 92] as:

$$\ddot{q}_{t+\tau} = \ddot{q}_t + \frac{\tau}{\theta \Delta t}\left(\ddot{q}_{t+\theta \Delta t} - \ddot{q}_t\right). \quad [1.26]$$

Speed and movement are obtained through successive integrations as follows:

$$\dot{q}_{t+\tau} = \dot{q}_t + \tau \ddot{q}_t + \frac{t^2}{2\theta \Delta t}\left(\ddot{q}_{t+\theta \Delta t} - \ddot{q}_t\right) \quad [1.27]$$

$$q_{t+\tau} = q_t + \dot{q}\,\tau + \frac{1}{2}\ddot{u}_t \tau^2 \frac{\tau^3}{6\theta \Delta t}\ddot{q}_{t+\theta \Delta t} - \ddot{q}_t \quad [1.28]$$

These base equations have been generalized by Hughes. They are given for time $t=\theta\Delta t$ and with the notation $q(n\Delta t) = q_n$:

$$M\,\ddot{q}_{n+\theta} + C\,\dot{q}_{n+\theta} + K\,q_{n+\theta} = F_{n+\theta} \quad [1.29]$$

$$\ddot{q}_{n+\theta} = (1-\theta)\ddot{q}_n + \theta \ddot{q}_{n+1} \quad [1.30]$$

$$\dot{q}_{n+\theta} = \dot{q}_n + \theta \Delta t\left[(1-\gamma)\ddot{q}_n + 2\beta \ddot{q}_{n+\theta}\right] \quad [1.31]$$

$$q_{n+\theta} = q_n + \theta \Delta t\,\dot{q}_n + \frac{(\theta \Delta t)^2}{2}\left[(1-2\beta)\ddot{q}_n + 2\beta \ddot{q}_{n+\theta}\right] \quad [1.32]$$

$$F_{n+\theta} = (1-\theta)F_n + \theta F_{n+1} \quad [1.33]$$

These equations are equal to the ones proceeding for the values β = 1/6 and γ = 1/2.

1.2. Structural optimization

Structural optimization is not a new concept. By searching the archives, one can find a calculation made by Galileo which expresses the law of density of a free cantilever beam by applying a constant distribution of pressure [TRO 87]. It should be noted that the result of this calculation is approved by modern theories. Since Galileo's time, there have been thousands of publications; by not limiting oneself to the modern era of optimization, one can see that since, Schmit introduced the idea of coupling structural analysis by finite elements with nonlinear mathematical programming in order to find different optimal automated designs in the 1960s.

Effectively, he proved that structural design could be formulated using a mathematical programming problem [SCH 01].

Optimization plays a major role in increasing performance and reducing the mass of aerospace and automobile machines, saving a substantial amount of energy. Constant development of design techniques with the help of a computer and optimization strategies can be found here. In order to illustrate the evolution of structural optimization techniques, one can arbitrarily classify structural optimization into three large families: design optimization, shape optimization and topological optimization [EL 13, DUY 96].

1.2.1. *Design optimization*

This allows for the improvement of a structural model by using available resources (called constraints or limitations). Automatic sizing of structures is the particular case which allows us to modify the right section or the horizontal density of the components of a structure whose shape and topology are fixed [AZI 02, ALL 01].

1.2.2. *Shape optimization*

Shape optimization allows changes of process consistent with a previously fixed topology. Traditional process optimization modifies parametric representation of area boundaries [ZHA 92, AFO 02]. By the moving boundaries, one can expect a better solution from among the structures obtained by homeomorphic transformation of the original structure.

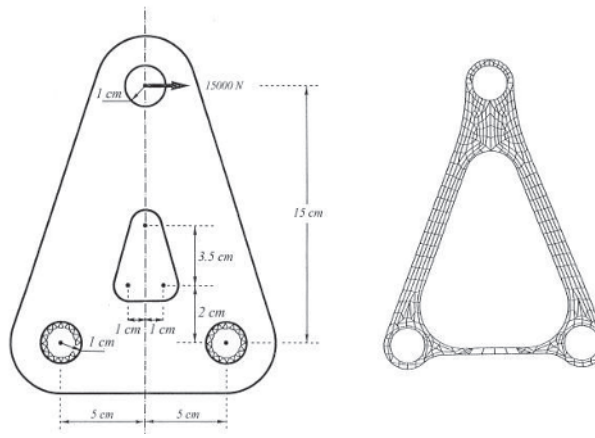

Figure 1.3. *Shape optimization: the initial model (left) and the solution to the problem after 5 versions (right) [ZHA 92]*

1.2.3. *Topological optimization*

Topological optimization allows one to modify the nature of the structure more completely. Here, the geometry of the object is explored without any preconception about the connectivity of the areas or the structural components present in the solution. Optimizing topology naturally leads the shape or the optimal horizontal dimensions of the structure to be determined, such that it is called generalized shape optimization [ROZ 93]. The final structure must satisfy the constraints defined previously by the user (which are generally linked to the restriction of the von Mises maximum distortion) [EL 13].

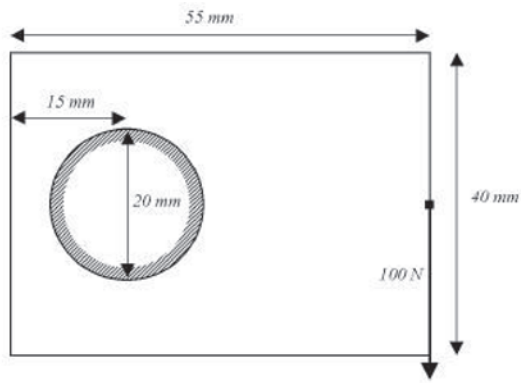

Figure 1.4. *Definition of the Michel trellis problem [REY 99]*

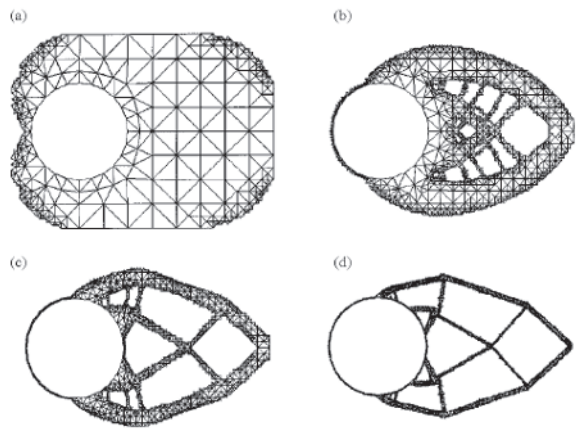

Figure 1.5. *Topological optimization solution applied to the Michell trellis problem [REY 99]*

A conventional reference regarding topological optimization, the Michell trellis problem (the rectangular area is changed in its inferior right section while all those on the left-hand side are fixed), is described in Figure 1.4. Reynold [REY 99] solved the problem by using an adaptive change technique, which is described as follows: once the problematic initial finite element is established, the method starts with meshing refinement by subdividing elements of certain areas where the (von Mises) pressure is minimal. The subdivided elements with minimal pressure will be eliminated, and the process continues until merging. The solution for the structures obtained after 6, 42, 75 and 120 respective repetitions of this process are shown in Figure 1.4. On the 120th repetition, only 8.8% of the entire (initial) section remains.

Topological optimization can also be carried out for trellis. For example, Deb and Gulati [DEB 99] developed a method to discover sections and the optimal topology of a 2D and 3D trellis by using a genetic algorithm. The objective was to reduce the mass to a minimum, under pressure and movement with a predefined value. Figure 1.6 illustrates an example of a 3D trellis in which the genetic algorithm merges with a trellis composed of nine elements (from a configuration of 39 elements).

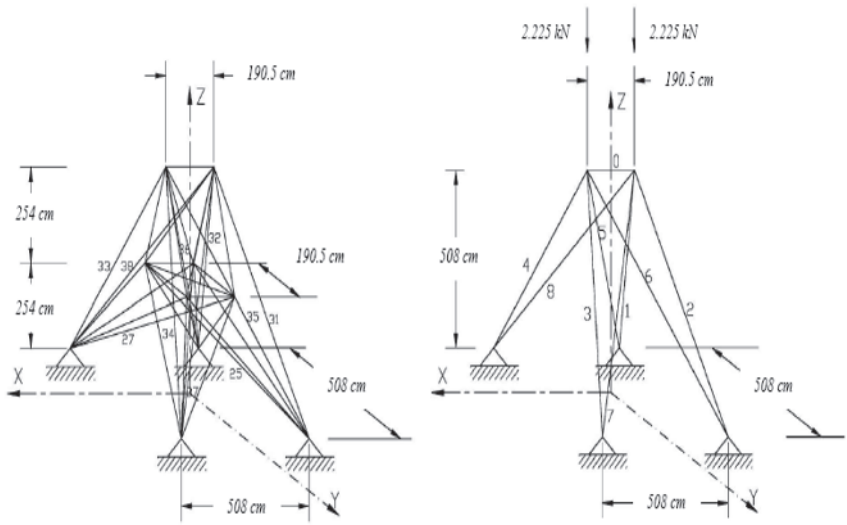

Figure 1.6. *Example of topological optimization [DEB 99]*

1.2.4. *Definitions and formulation of an optimization problem*

An optimizing problem is usually formulated as a minimization problem and is written in the following form:

$$\begin{cases} \min\limits_{x} f(x) \\ \text{such that} \\ g_i(x) \le 0,\ i = 1 \cdots\cdots, m, \\ h_j(x) = 0,\ j = 1 \cdots\cdots, p, \\ x \in S \subset \Re^n, \end{cases} \qquad [1.34]$$

where f is the scalar function to minimize, known as ***cost function*** or ***objective function***, x represents ***the vector of optimization variables***, g_i the ***unequal constraints***, h_j the *equal constraints* and S is ***the variable space*** (also called research space). S indicates the type of variable considered: real, entire, mixed (real and entire in the same problem), discreet, or restricted supply.

The point x_A is called an **inadmissible point** if $x_A \in S$ and if the optimization conditions are satisfied: $g_i(x_A) \le 0,\ i = 1 \cdots\cdots, m$ and $h_j(x_A) = 0,\ j = 1 \cdots p.$ The solution to equation [1.34] is all of the optimas $\{x^*\}$:

– x^* is a global minimum of f, only if $f(x^*) \le f(x)\ \ \forall x \in S$;

– x^* is a local minimum of f, only if $f(x^*) \le f(x)\ \ \forall x \in S / \left\| x - x^* \right\| \le \varepsilon, \varepsilon > 0$.

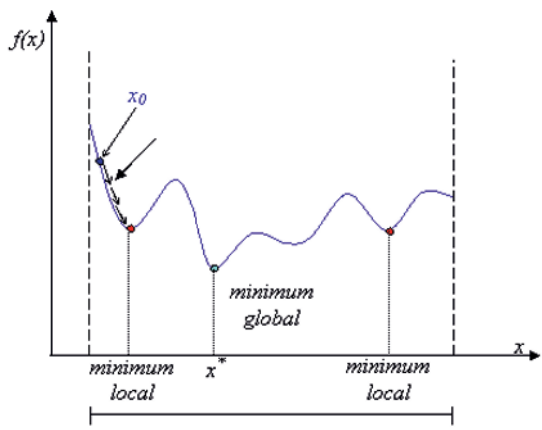

Figure 1.7. *Local minima and global minimum from a function to a variable*

Figure 1.7 shows an example of a function to a variable with local minima and global minimum. From among the local minima, the one with the smallest value of f is the global minimum.

A multimodal function presents several (local) minima, whereas a unimodal function has only one minimum, that is, the global minimum. Figure 1.8 shows a multimodal function with two variables.

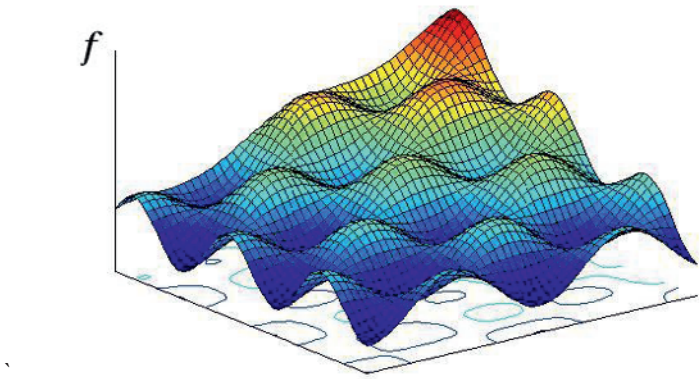

Figure 1.8. *A multimodal function with two variables. For a color version of the figure, see www.iste.co.uk/elhami/stochasticdynamics.zip*

The local method is the one that considers a local minimum. The research considers a local minimum that usually departs from an initial point x_0 with the initial pace Δ_0. Numerous local methods exist. The oldest and most widely used methods are the ones whose direction of descent is inferred from the derivatives of the function (method with the greatest gradient, Newton's method, conjugate gradient method, and near-Newtonian method [EL 13]).

Global methods aim to reach one or several global optima. Typically, the running cost of optimization is conditioned by the number of estimations of the objective function. For example, if one wants to optimize a mechanical system modeled using finite elements, the calculation time will primarily be the amount of time spent using simulations to optimize it.

1.3. Structures with uncertain parameters

On consideration of uncertainties when studying a structure, it is essential to take into account the two sources of uncertainty: those concerning agitation and those

concerning structure. Dessombz [DES 00] classified the uncertainties into four large categories: random parameters:

– ***Random parameters***. Examples include the dimensions of an object whose limits are known or even Young's model or the volumetric mass of a material.

– ***Unknown parameters***. The case of conditions with limits is a typical problem. Embedding signifies a great rigidity, but only a certain type of magnitude is known. Similarly, the different types of assembly, such as welding and gluing, are difficult to model, and it seems that the deterministic values used to represent these methods are largely insufficient.

– ***Variable parameters***. Here, one may distinguish the parameters that could vary with time, are uncontrollable or can be more or less controlled (typically, if materials are damaged or worn). In addition, the parameter whose value is known at any given moment can also be distinguished. In this case, one can put it down to an agitation force (a train passing over a bridge), the quantity of fuel in a tank or even special adjustments.

– ***Uncertainties of the*** model. These include the chosen laws of behavior which do not represent, or only partially represent, physical phenomena, errors due to choices regarding interconnecting finite elements, fine details and chosen elements. These uncertainties are generally difficult to evaluate.

In order to display parametric uncertainty, diverse approaches, probabilistic theory [SAR 04], unclear systems [TIS 15] and convex models [BEN 90] have been illustrated in the literature.

1.3.1. *Monte Carlo simulation*

When approaching problems of uncertainty, whether they are experimental or numerical, the approach naturally used in the first place is based on sampling hypotheses. The Monte Carlo simulation, known and used intensively in several scientific domains, was introduced into structural mechanics by Schinozuka in 1972. The method is kept as a reference, and is simultaneously the simplest and the most expensive.

The method consists of three stages. The first stage is the most important, aiming to generate a great number of systems associated with the stochastic parameters of the physical problem using numerical simulation. The following stage is to solve the deterministic problem for each of the elements of the group, with the aim of obtaining a group that corresponds with the random quantities of response of the

system studied. In the last stage, a great number of output processes are obtained, from which one can calculate the different static values of the response variables.

The computing power needed to carry out all of these simulations severely limits the applications of this method; this is why we use approximation techniques.

1.3.2. *Analytic method*

The different methods used to analyze problems of uncertainty are principally the disturbance method and the Neumann decomposition in series method. The principle of these methods is recalled in [BEN 08]. The use of Neumann series is based on research on the opposite of the operator of the problem, breaking it down into a Neumann series. This limits its application to certain types of differential equations, while the disturbance method does not have this limitation, since it uses decomposition of the output field (in the same way as Taylor). These methods are generally simple to put into practice, but they only provide satisfactory results if the disturbances remain small and if the problems are linear.

1.3.3. *Stochastic finite element method*

A large proportion of the current research deals with the problem of structures with uncertain parameters through stochastic finite element methods. These combine two techniques, namely "traditional" analysis using finite elements and statistical analysis. In general we try to determine the stochastic characteristics of random responses, with the help of knowledge related to hazards including structural parameters, geometrics, limiting conditions and the loads of a system.

Schuëller [SCH 01] exposed different developments in this domain and proposed several references for each aspect of the problem. The explanation of the principle of stochastic finite elements given by Ghanem *et al.* [GHA 96] consists of using decomposition in a series of stochastic processes, shortened in a certain way according to desired accuracy. There are two levels that can be used to describe the stochastic aspect of the problem. The first consists of considering structural characteristics such as known stochastic fields. In this case, one generally uses Karhunen–Loeve decomposition, which is similar to modal superposition used for structural computing, since this decomposition exhibits remarkable properties in terms of orthogonality and merging. The second concerns the stochastic field formed by the solution for structural bonds. In this case, decomposition named polynomial chaos is used [GUE 16]. In this way, stochastic solutions are launched based on orthogonal polynomials, whose variables are Gaussian orthonormals. The properties of this polynomial basis can be used to find, either analytically or using a numerical

average, the standard deviation or distribution of the random solution. The main problem with this method relies in the choice of resolution algorithms for the storage of values. Furthermore, the possibility of implementing parallelism based on the characteristics of the resulting matrices, probably the most important aspect to consider in the development of this method as it offers results of quality immediately, requires substantial computing time.

A different approach in stochastic formulation using the finite element method can be carried out by modifying the existing elements to allow the inclusion of defects, as Combescure [COM 01] presents an axisymmetric element based on a shell element. This approach allows one to analyze structures that possess non-axisymmetric defaults, such as the radius of a cylinder or cone, or a variable density over circumference.

1.3.4. *Fluid logic method*

Fluid logic was discovered by Zadeh [ZAD 65a, ZAD 65b]. Since its creation, it has constantly been the subject of several research projects. Fluid logic is the theory of uncertain systems. Fuzzy set theory or FST, is an extension of the theory of systems. The affiliation function of a traditional system A is defined as:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \qquad [1.35]$$

This indicates that an element x is either in A $\left(\mu_A(x)=1\right)$ or not $\left(\mu_A(x)=0\right)$.

This definition allows one to make a link with traditional probabilistic analysis, whose definition is close enough, except that A is a point in R in this case and not an interval of R. This difference, which is not very easy to interpret from a practical point of view, carries one that demonstrates the limits of probabilistic analysis: if an event A has a probability of $p(A)$, then the probability of an event occurring that goes against event A is known and is worth $p(\overline{A}) = 1 - p(A)$.

This is not true for the theory of possibilities: if $P(A)$ denotes the possibility of an event A, the relationship that links this possibility to the possibility of another event $\overline{A}$ could be written as $P(A) + P(\overline{A}) \geq 1$, because the relationship $\max\left\{P(A), P(\overline{A})\right\} = 1$ is always verified. This translates the fact that if one

considers two opposite events, one of the two is always completely possible, and if an event is considered possible, its limit can also be if one notes the mathematical bases of the theory, as well as numerous computing elements allowing for an initial application of uncertain arithmetic.

1.3.5. *Reliability method*

Most of the approaches studied with respect to structural reliability can be classified into two categories: simulation techniques and analytic methods. These techniques help the research engineer to consider all the possible uncertainties during the design and construction phase in order to calculate the index of reliability or the probability of breakdown in accordance with one or several circumstances of failure [EL 13].

1.3.5.1. *Simulation technique*

Simulation techniques enable a precise evaluation of the probability of failure. These techniques, also named the Monte Carlo method, encompass various simulation methods, such as direct Monte Carlo method, the important drawings method [ENE 93], conditional simulations, directional simulations, adaptive drawings and the Pavé method. The Monte Carlo simulation technique was proposed in the early 1940s to test technological systems by using a cheap simulation technique.

In Monte Carlo simulation, the probability of failure is given by:

$$P_f = \frac{N}{N_f}, \qquad [1.36]$$

where N_f is the number of failures and N is the total number of events simulated. The statistical accuracy of the probability of failure is measured by the covariance coefficient:

$$\mathrm{cov}\left(P_f\right) = \sqrt{\frac{\left(1-P_f\right)P_f}{N}} \Big/ P_f \quad . \qquad [1.37]$$

Equation [1.37] shows that a small probability of failure implies a large number of simulation cycles to maintain an acceptable level of accuracy. Consequently, this causes a rise in operating cost. For a complex problem with several random variables, the Monte Carlo simulation becomes impractical. In order to overcome this difficulty, several more effective alternative methods, such as the important

drawings method [EL 13], have been developed. The fundamental principle of this method is to manage the random parameters with different distributions so their averages are closer to the design than that of the original distribution. Consequently, the efficiency of the simulation is increased, since failures occur more frequently.

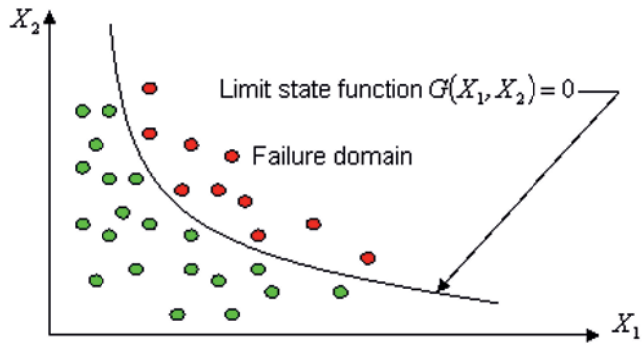

Figure 1.9. *Principle of the Monte Carlo method. For a color version of the figure, see www.iste.co.uk/elhami/stochasticdynamics.zip*

Harbitz and Veritas [HAR 83] have presented a general procedure based on the printing technique to calculate the probability of failure; this process is applied to a fatigue problem. In other words, they demonstrated the efficiency of the drawing method in relation to the Monte Carlo simulation.

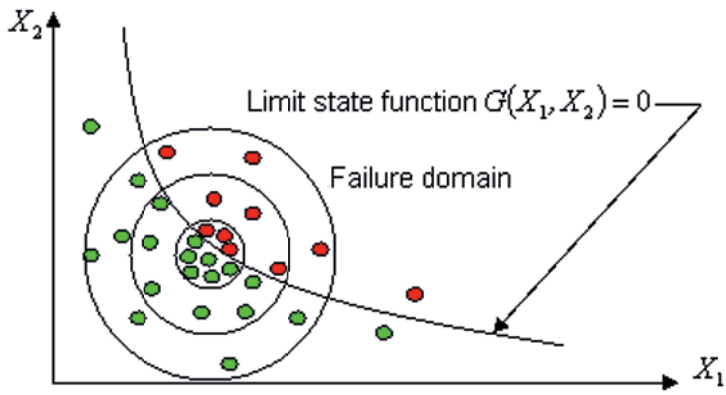

Figure 1.10. *Drawing principle. For a color version of the figure, see www.iste.co.uk/elhami/stochasticdynamics.zip*

1.3.5.2. *Analytical method*

Analytical techniques used to analyze the reliability of structures generally include the first-and second-order reliability methods (FORM: first-order reliability method and SORM: second-order reliability method). The fundamental idea behind these techniques is to transform the random variables into Gaussian variables that are rendered statistically independent in a standardized space. In this space, the FORM or SORM techniques are applied to get closer to the limit state, which circles the failure criterion. The probability of failure is evaluated using a reliability index β and the probability of failure is given by $p_f \approx \Phi^{-1}(\beta)$, where Φ is the distribution function of the standard normal distribution.

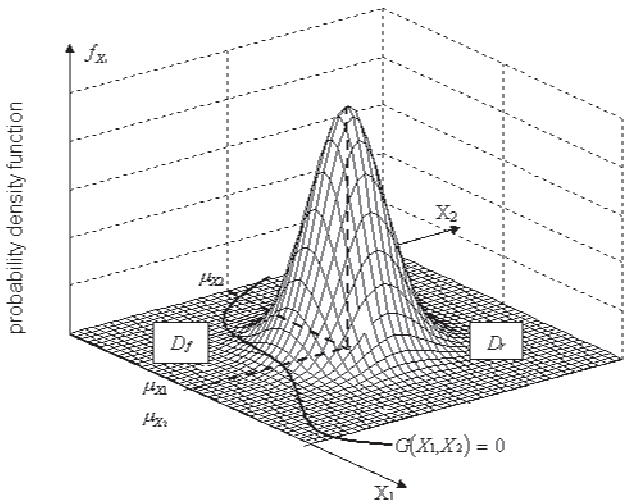

Figure 1.11. *Graphical representation of the physical space of the probability density function for two variables*

In order to evaluate the reliability index for the FORM method, Hasofer and Lind [HAS 74] proposed an iterative algorithm in which the limit state surface is reached through a hyperplane tangent at the design point, which corresponds with the point most likely to fail (MPFP: most probable failure point). The reliability index is defined as the shortest distance separating the origin of the axis system of the state surface limit. Rackwitz and Fiessler [RAC 78] include distribution information. Hasfer and Lind's algorithms combined with that of Racwitz and Fissler are commonly called the HL–RF algorithm.

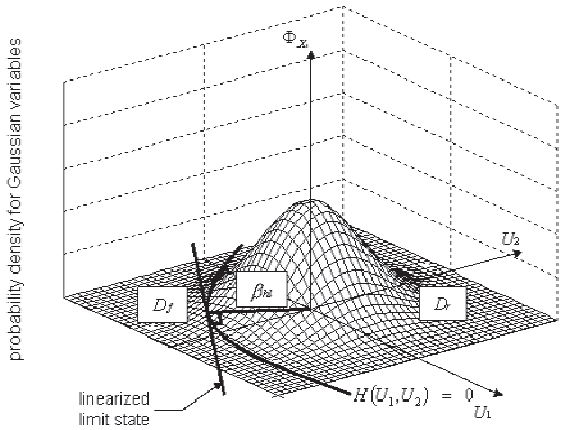

Figure 1.12. *Graphical representation of the reduced space of the probability density function for two variables. For a color version of the figure, see www.iste.co.uk/elhami/stochasticdynamics.zip*

The approach based on FORM to calculate the reliability of system was developed by Hohenbichler and Rackwitz [HOH 83]. First, they broke down the system into a series of parallel systems. Then, they provided a first-order solution to the multi-normal integral for the simple series or parallel systems. Finally, they determined the probability of failure.

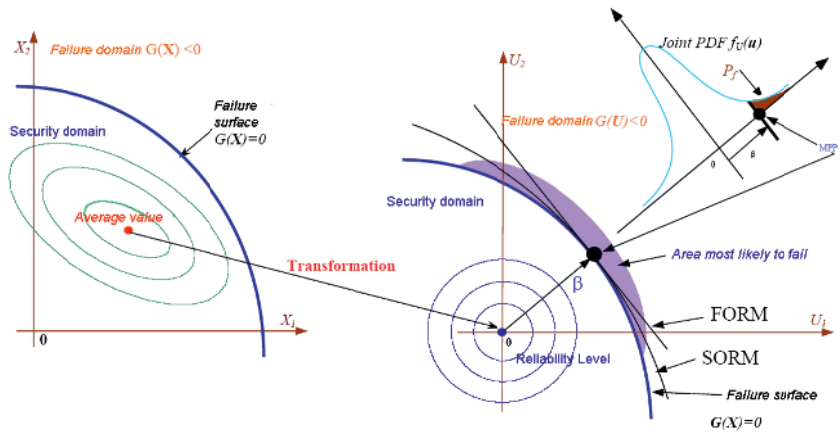

Figure 1.13. *Transformation of probabilistic space into normal space*

FORM is usually accurate for linear limit state functions and not for highly nonlinear limit states. In order to solve this problem of nonlinearity of limit states, SORM was proposed to improve reliability evaluation by using a quadratic estimation of the state surface limit. In [EL 13], one can see an asymptomatic process to predict probabilities of failure for β by applying the quadratic estimation to MPFP. Probability of failure can be expressed as:

$$P_f \approx \Phi(-\beta)\prod_{i=1}^{n-1}\left(1-\beta\frac{\varphi(-\beta)}{\Phi(-\beta)}k_i\right)^{1/2},\ \beta\to\infty \qquad [1.38]$$

where k_i are the principal curves of failure passing through MPFP.

The exact probability of failure for a second-order estimation at MPFP [EL 13] is as follows:

$$P_f = \frac{1}{2}-\frac{1}{\pi}\int_0^{\infty}\sin\left(\beta t+\frac{1}{2}\sum_{i=1}^{n-1}\tan^{-1}\left(-k_i t\right)\right)\frac{\exp\left(-\frac{1}{2}t^2\right)}{t\left[\prod_{i=1}^{n-1}\left(1+k_i^2 t^2\right)\right]^{1/4}}dt \qquad [1.39]$$

1.3.6. *Reliability optimization*

Frangopol [FRA 85] proposed a sensitive analytical technique which was previously applied to design optimization, in which weight was taken as an objective function while target reliability index was taken as a constraint. He developed a vectorial optimization approach for structural design problems demanding multiple limit states and considered simultaneously. He also suggested a vectorial optimization strategy based on reliability in three stages in the solution to the problem of optimization.

Yang and Nikolaidis [YAN 91] established an optimization system reliable for the wing of an airplane subjected to certain loads. FORM was used to predict the reliability index of diverse components, while Ditlevsen adopted a technique to obtain the reliability index of the system. The problem to solve combines two sub-optimization problems with two optimization levels, in which weight was taken as an objective function and the reliability index as a constraint for structures made up of mixed materials.

Enevoldsen and Sorensen [ENE 93] suggested four different processes for solving the problem of reliability optimization of systems in series and parallel. The first two approaches are based on analyzing sensitivity and the last one is based on

sequential methods. Several aspects linked to reliable optimization in structural engineering have been discussed. Several reliable optimization problems have been formulated: the FORM method has been used to evaluate the reliability level of the system and a two-level strategy has been put forward to solve the problem of reliable optimization. The choice of first-order optimization algorithms as well as analyses of sensibility increases the efficiency of the resolution of the optimization problem based on reliability. They also pursued their work by examining several practical outcomes of reliable optimization comprising the use of finite element analysis. They also concluded their findings by describing a strategy to correct and improve the model while evaluating the optimal result.

Royset and Der Kiureghian [ROY 01] offered a decoupling approach through which the optimization problem can be reformulated as a deterministic and semi-infinite problem (characterized by a finite number of design variables and an infinite number of constraints). This approach was then applied to the reliable optimization of structural systems in series with two optimizing compositions. In the first composition, the cost is minimized beneath the reliability and mechanical constraints, while in the other reliability is taken as an objective junction under a mechanical constraint. The advantage of this approach is its flexibility and the fact that any algorithm optimization and any reliability method can be independently adopted for the optimization solution based on reliability, since the optimization and reliability calculations are totally decoupled.

The analytical techniques, primarily FORM/SORM, are always a good choice for evaluating reliability within the problem concerning reliable optimization when the analytical model of the limit state function or the equivalent estimation is available. A one-level optimization strategy proposed by [MOH 05] based on FORM and using optimal criteria was used to solve the two compositions of the problem of reliable optimization. The first problem involves minimizing cost under reliability obligations. The second problem optimizes reliability under cost constraints. The algorithm of the approach has been displayed through three examples, and the results of optimization have been compared to those obtained using other available methods in publications in terms of cost and stability. The inconvenience of this algorithm is that it is limited to a single limit state, and so it cannot be applied to reliable optimization problems.

An effective method was proposed by Kharmanda [KHA 04, KHA 08] through merging the two spaces: physical and normal. This approach is called the hybrid method, as it integrates the two problems into a single one, which is solved simultaneously in spaces with deterministic and random variables. The reliability issue is combined with the optimization problem in order to reach a single objective function and, in this way, reduce the global computing cost. This last method was studied in detail throughout this thesis, and in order to create methods adapted to the

problem of dynamics. We have proposed a method to detect crucial areas in terms of resonance frequencies as well as a new method entitled improved hybrid, which allows us to improve the results of the traditional hybrid method [MOH 05].

1.4. Conclusion

In this chapter, we introduced the issue of structural dynamics while considering their uncertainties.

In section 1.1 the different analytical methods for structural dynamics were explored. In section 1.2 the notion of uncertainty was examined, presenting different methods for stochastic finite elements before analyzing reliability and reliable optimization.