1

Mechanics and Fluid

1.1. Introduction

The mechanics of fluids is a type of mechanics: it looks at the *movement* of matter when under the influence of *forces*. Matter here is in the “fluid state”.

This chapter is approached from the perspective of the foundations of the mechanics of point power. It will also later define what fluid is and which of this matter’s main characteristics are useful to know. These characteristics shall then be brought to “life” in later chapters.

1.1.1. *Mechanics: what to remember*

1.1.1.1. *Who is afraid of mechanics?*

For some curious reason, this branch of physics appears frightening to many students, a curse that thermodynamics also shares. Somewhat recoiled from, the mechanical engineer occupies a special place in the academic world. *Some people even wonder whether mechanical engineers are actually physicists who have a strong handle on mathematics, or are in fact mathematicians lost among physicists. These classifications have not been made any simpler by the addition of digital calculations.*

It cannot be stressed enough that the appearance of mechanics gave birth to mathematical physics.

By pairing movement with mathematics, the Neoplanitician, Galileo, created kinematics. And then, with a stroke of genius, although perhaps slightly mythically, Isaac Newton created dynamics by incorporating the fall of an apple and the Moon’s trajectory into one vision.

Descartes must not be left out of this Pantheon of emerging physics, for he created momentum, was engaged in heated debates with Newton and Leibnitz on this subject as well as others, and discovered kinetic energy through “life force”. Leibnitz and Newton were also the precursors to the differential approach in mechanics.

1.1.1.2. *Principles to remember*

Like a game of chess, the starting rules of mechanics are the simplest. And, like a game of chess, not all paths lead to an easy victory.

a) Remember that a position vector $\vec{r}$ is defined as a vector that links the starting point to another point in space. The coordinates of $\vec{r}$ are evidently the point’s three coordinates:

$$\vec{r} = \vec{r}\left(x, y z\right) \quad [1.1]$$

By definition, the point’s speed is the derivative of the position vector in relation to the time:

$$\vec{V} = \frac{d\,\vec{r}}{dt} \quad [1.2]$$

which, when passing, accelerates the position vector’s second derivative:

$$\vec{\Gamma} = \frac{d\vec{V}}{dt} = \frac{d^2\,\vec{r}}{dt^2} \quad [1.3]$$

Remember that a vector is derived with regard to a scalar by deriving its components:

$$\vec{r} = \left[x(t), y(t), z(t)\right]; \quad \frac{d\,\vec{r}}{dt} = \left[\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right] \quad [1.4]$$

b) In 1687, Isaac Newton’s *Philosophiae Naturalis Principia Mathematica* outlined three laws, which indeed can be reduced into two:

1) *The principle of inertia*;

2) *Fundamental dynamics law*;

3) *The principle of action and reaction*.

Let us take these three principles further:

Law no. 2. Let us begin with the fundamental dynamics principle, when applied to a constant mass (m) material point:

The acceleration that a body undergoes in an inertial frame of reference is proportional to the resulting forces that it undergoes, and is inversely proportional to its mass.

In modern notation (the notion of the vector was acquired in the 20th Century), this is written as:

$$m\frac{d\vec{V}}{dt} = \vec{F} \qquad [1.5]$$

NOTE: Vectorial notation reminds us that a given speed contains three pieces of information: a direction (instantaneous movement support), a route and an hourly speed. A speed cannot be reduced to the datum of $\mathrm{m.s}^{-1}$. A speed vector not only tells me that my car is traveling at $V = 130\,km.hr^{-1}$ (hourly speed), but it also tells me that I am on a highway between Paris and Rome (direction) and that I am going from Paris to Rome (route). However, I would still need the position vector $\vec{r}$ to tell me where the next exit is.

Therefore, an acceleration is also a vector, and there is no reason why it is not collinear to the speed. Central acceleration in a circular movement is (or should be) known to all secondary school students.

Law no. 1. The principle of inertia was actually discovered by Galileo: *In the absence of an external force, all material points continue in a uniform, straight-lined movement.*

NOTE: This is what Captain Haddock realizes in the “Explorers on the Moon”, the illustrated Tintin adventure story by the famous Belgian author, Hergé.

This principle of inertia is in fact a consequence of the fundamental dynamics principle. If the result of forces applied to a material point is zero, then:

$$\vec{F} \equiv \vec{0} \qquad [1.6]$$

and:

$$m\frac{d\vec{V}}{dt}=\vec{0} \qquad [1.7]$$

which implies: $\vec{V}\equiv\vec{C}te$ [1.8]

It means a uniform straight-lined movement.

Law no. 3. If the first principle can be reduced to the second, the third principle of action and reaction is independent: *Every body A exerting a force* $\vec{F}_{AB}$ *on a body B undergoes a force* $\vec{F}_{BA}$ *of equal intensity, but in the opposite direction, exerted by body B*:

$$\vec{F}_{AB}=-\vec{F}_{BA} \qquad [1.9]$$

When solving a problem, to write that every force has an equal and opposite reaction is to write something new with regard to the fundamental dynamics principle.

These principles have been rewritten in various different forms, which lead to equations that are often much more directly applicable. A few of these equations are given in the following sections.

1.1.2. *Momentum theorem*

We can rewrite the fundamental dynamics principle by noting that mass is invariable:

$$m\frac{d\vec{V}}{dt}=\frac{d}{dt}m\vec{V}=\frac{d\vec{p}}{dt} \qquad [1.10]$$

A momentum vector has also been introduced:

$$\vec{p}=m\vec{V} \qquad [1.11]$$

And the fundamental dynamics principle is also found to be rewritten in terms of momentum:

$$m\frac{d\vec{p}}{dt}=\vec{F} \qquad [1.12]$$

In the course of mechanics, it is demonstrated that this equation applies in material points to the center of a group's mass, whether it is continuous or discontinuous and alterable or otherwise. m is therefore replaced by the total mass of the system's points and $\vec{F}$ then represents the resultant of the forces applied to these points. This is what constitutes the center of mass theorem.

NOTE: It goes without saying that we do not intend to write a "digest" here on the course of fluid mechanics.

It would be impossible to attempt to reproduce a complete mechanics course. However, we must insist upon the consequences of these principles which will be directly applied when establishing fluid mechanics theorems. We will build upon the mechanics of point power, and if the reader deems it necessary, they can refer to a dedicated textbook to study system mechanics, which constitutes a more complex domain. Furthermore, in the appendix, we can find a reminder of fluid mechanics equations for a continuous fluid system. This script will be used when demonstrating Euler's first theorem.

We observe that while mass becomes variable with speed, it is this expression that remains valid in particular mechanics. This is also the case for relativist dynamics.

1.1.3. *Kinetic energy theorem*

Forced movement implies work. Here we will give mechanics an energetic dimension. The work of a force $\vec{F}$ when applied to a material point during a time dt provides calculated work from the force and this point's small movement $d\,\vec{r}$:

$$dW = \vec{F}.d\,\vec{r} \qquad [1.13]$$

$d\,\vec{r}$ is a small vector, which indicates not only the small distance traveled, but also the carrying line of this movement or direction, and the movement's route. It is linked to speed by:

$$d\,\vec{r} = \vec{V}\,dt \qquad [1.14]$$

Remember the dynamic relation:

$$\vec{F} = m\frac{d\vec{V}}{dt} \qquad [1.15]$$

The work is written as:

$$dW = m\frac{d\vec{V}}{dt}.\vec{V}\,dt \qquad [1.16]$$

It can be observed that

$$\frac{dV^2}{dt} = \frac{d\vec{V}.\vec{V}}{dt} = 2.\vec{V}.\frac{d\vec{V}}{dt} \qquad [1.17]$$

Finally, it becomes:

$$dW = \frac{m}{2}\frac{dV^2}{dt}\,dt = \frac{dmV^2}{2} \qquad [1.18]$$

The work performed has helped to increase the quantity $\frac{mV^2}{2}$ carried by the material point. This is how kinetic energy appears:

$$E_C = \frac{mV^2}{2} \qquad [1.19]$$

1.1.4. *Forces deriving from a potential*

In a frame of reference $Oxyz$, where Oz is vertical, the force of gravity $\vec{F}_G$ applied to a mass of $m = 1\ kg$ will have the following components:

$$F_{Gx} = 0 \qquad [1.20.a]$$

$$F_{Gy} = 0 \qquad [1.20.b]$$

$$F_{Gz} = -g \qquad [1.20.c]$$

Furthermore, the operating gradient is defined by associating the vector $\vec{grad}\,f$ with a function $f(x, y\,z)$ by:

$$\left(\vec{grad}\,f\right)_x = \frac{\partial\phi}{\partial x} \qquad [1.21.a]$$

$$\left(\vec{grad}\, f\right)_y = \frac{\partial \phi}{\partial y} \qquad [1.21.b]$$

$$\left(\vec{grad}\, f\right)_z = \frac{\partial \phi}{\partial z} \qquad [1.21.c]$$

Therefore, $\vec{F}_G$ can be written in the form of a gradient:

$$\vec{F}_G = -\vec{grad}\, \phi_G \qquad [1.22]$$

which, by definition, implies the following about the gradient:

$$F_{Gx} = -\frac{\partial \phi_G}{\partial x} = 0 \qquad [1.23.a]$$

$$F_{Gy} = -\frac{\partial \phi_G}{\partial x} = 0 \qquad [1.23.b]$$

$$F_{Gz} = -\frac{\partial \phi_G}{\partial x} = -g \qquad [1.23.c]$$

By identifying:

$$\phi_G = gz + Cte \qquad [1.24]$$

Therefore, it can be said that $\vec{F}_G$ is derived from the potential ϕ_G. It is worth at least being aware of this.

In general terms, it is said that a force $\vec{F}$ is derived from a potential $\phi(x, y, z)$ when

$$\vec{F} = -\vec{grad}\, \phi \qquad [1.25]$$

This property is not universal: in particular, friction forces or electromagnetic forces are not derived from a potential.

1.1.5. *Conserving the energy of a material point*

The work performed by a force derived from a potential during a time period of dt is written as:

$$dW = \vec{F}.d\vec{r} = -\vec{grad}\,\phi.d\vec{r} \qquad [1.26]$$

By developing the scalar product, this can be rewritten in the Cartesian form:

$$dW = -\vec{grad}\,\phi.d\vec{r} = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz \qquad [1.27]$$

The exact total differential is seen to appear ϕ on the time dt, meaning the variation $d\phi$ between the starting point at t and the arrival point at $t + dt$:

$$dW = -\,d\phi \qquad [1.28]$$

By coupling the equations together, we obtain:

$$dW = dE_C = -\,d\phi \qquad [1.29]$$

which can be rewritten as:

$$dE_C + d\phi = 0 \qquad [1.30]$$

Thus, the total energy appears:

$$E_T = E_C + \phi \qquad [1.31]$$

Sum of the kinetic energy and the potential energy, which is conserved when the material point is moving.

NOTE: Remember that when part or all of the forces is or are not derived from a potential, the mechanical energy of the material point is not conserved. The mechanical work of the forces which is not derived from a potential is generally transformed into another form of energy. Thus, friction transforms mechanical energy into thermal energy. This enters into the domain of thermodynamics. The mechanical energy (work) is no longer conserved, but the first principle applies to the two forms of energy: work and heat.

These relations for the material point recalled here have been extended into finite volumes of matter. Curious readers may refer to more elaborate mechanical courses. The aim of this chapter lies in the need for the readers to place themselves within the framework of a basic general culture of mechanics.

All of the notions that have been recalled here will be useful when we begin interpreting Bernoulli's theorem.

1.2. The "fluid state"

The term "fluid state" refers here to the way in which all of the states of matter used to be understood: solid, liquid, gas and plasma, a classification that has been recognized more recently.

In this group, fluid mechanics applies to the last three of these "states".

Solid mechanics deals with alterable and unalterable elastic solids with a blurred boundary and a few creep or pasty rheology problems.

NOTE: It is important not to confuse this expression, which can be traced back to the oldest "fluid state", with the notion of "state in thermodynamics", which relates to a set of thermodynamic variables which we will discuss later.

When approached from the mechanics perspective, this "fluid state" prompts us to:

a) define this state in terms of its nature, its physical qualities and its movements;

b) describe the forces that can be applied to a fluid: what are they and how are they written?

1.2.1. *Fluid properties*

1.2.1.1. *The first property of fluid is its continuity*

Physically, continuity signifies that fluid density, regardless of how small it may be, contains matter. This allows a density to be defined, like the ratio of a small fluid density dm to the small volume $d\omega$ that it occupies:

$$\rho = \frac{dm}{d\omega} \qquad [1.32]$$

For those who like mathematics, we observe that physical continuity connects a notion of continuity for the mass occupying a given volume. This mass $dm(d\,\omega)$ also has a derivative called density. In mathematical terms, the expression is:

$$\rho \text{ exists such that: } \rho = \lim_{d\,Vol \to 0} \frac{dm}{d\,\omega} \qquad [1.33]$$

NOTE: Herein lies a paradox. The mechanical engineer attributes this continuity property to fluid. We know that at the smallest scale of physics, matter is not continuous. Moreover, if there was no fluid discontinuity at the molecular level, we would not be able to determine its essential properties: possible compressibility, existence of pressure and temperature, thermal conduction and matter diffusivity when mixed.

A paradox is merely a poorly asked question. There are at least six or seven orders of magnitude (powers of 10) between the molecular phenomena and the mechanics of a fluids physicist. Admittedly, continuity is just a modeling tool, but it is robust. At the pipeline level, everything happens "as if" the fluid was continuous.

1.2.1.2. *Compressibility*

Density has been defined as a local property. There are many cases where this value of ρ is constant in all fluids. Therefore, it can be said that fluid is incompressible. This will be our definition of incompressibility here. Incompressible is synonymous with $\rho = Cte$.

There are other cases where the density varies from one fluid point to another. Therefore, it can be said that fluid is compressible. This situation is mainly concerned with gas. But the compressibility of liquids may cause certain problems: there is writing on static fluids at the deepest pits of the Pacific Ocean and there are acoustics in liquids (without fluid compressibility, there is no possibility of sound being disseminated).

Determining density, according to its parameters, relates to thermodynamics. All of a fluid state's thermodynamic variables are linked in its equation of state. Subsequently, for a gas, we will need this equation of state. As the equation for so-called perfect gases is the one that is commonly used, we will use it too.

This equation links the three thermodynamic variables: pressure p, molar volume M_{mol} or density p, absolute temperature T, expressed in Kelvin.

This is most often written for a mole (remember that the mole is defined by the number of molecules it contains, namely the Avogadro number $N = 6,022.10^{23}$):

$$pV_{mol} = RT \quad [1.34]$$

Here R is the universal constant of perfect gases, the value of which is $R = 8,3144621 J.mol^{-1}.K^{-1}$. In light of the level of precision of the models, for the examples used in this book, we will choose $R = 8,31 J.mol^{-1}.K^{-1}$. (Some authors use $R = 8,315 J.mol^{-1}.K^{-1}$.)

In mechanics, where the approach is more based on mass, an alternative expression is preferred, which directly uses density as a thermodynamic variable. Therefore, the fluid's molar mass M needs to be brought in.

Noting that

$$\rho = \frac{M}{V_{mol}} \quad [1.35]$$

We obtain the following state equation:

$$pV_{mol} = \frac{pM}{\rho} = RT \quad [1.36]$$

$$pV_{mol} = rT \quad [1.37]$$

with $r = \frac{R}{M}$ [1.38]

NOTE: When the ideal-gas law i is written under this form, r is no longer a universal constant. It depends on the nature of the fluid.

NOTE: They are called perfect gases because the equation is simple. No gas is intrinsically perfect. This equation is verified by all low-pressure gases. This relation was brought about by the works of Boyle, Mariotte and Charles in earlier times.

"Low-pressure" is a relative expression which may be translated as "any pressure lower than 100 bars in a generous approximation, or 10 bars if one prefers to be pedantic". When looking at compressible fluid problems, we will see that this is a highly acceptable hypothesis.

No fluid is intrinsically incompressible. Contrastingly, a gas can be attributed with an incompressibility property. Incompressible is a synonym of $\rho = Cte$, as is written above. If a flow's conditions are such that ρ varies very little, then $\rho = Cte$ is physically pertinent. Furthermore, we will also see the evolution of pressure strongly coupled with speed. For flows with a relatively weak speed, the pressures vary relatively little and density can easily be deemed a constant. This considerably simplifies the analysis process.

NOTE: For a gas in which the speed scale that establishes a barrier between "incompressible" flows and a "strongly coupled compressible" flow is defined based on the speed of sound in the fluid. Once again, it is in fact the flow that is either "incompressible" or "compressible".

1.2.2. *Forces applied to a fluid*

This section will respond to various questions: how do forces applied to a finite volume of fluid occur? What are these forces and how are they written?

1.2.2.1. *Surface forces, volume forces*

Let us begin with a fluid domain D contained within a closed surface S.

The "exterior" of this domain D will apply two types of forces:

Remote forces which in principle have an application point at all points in the domain D. These are *volume forces*. As a general rule, they are written per mass unit, $\vec{F}_V$.

There are *contact forces* between the external fluid of D and the internal fluid of D. These forces are localized all over the surface S. These are *surface forces*.

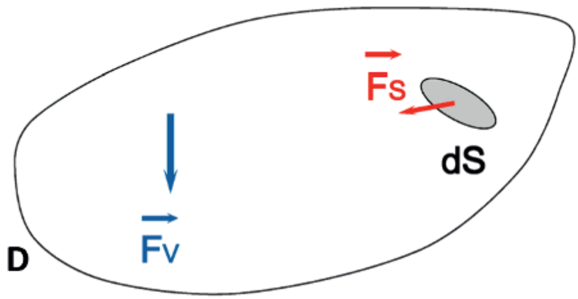

Figure 1.1. *Surface forces, volume forces*

1.2.2.2. *Volume force scripts*

a) As a general rule, volume forces are written per mass unit, $\vec{F}_V$.

These forces are expressed per volume unit as $\rho \vec{F}_V$, and the forces applied to a small basic volume $d\omega$ are written as:

$$d\vec{F} = \rho \vec{F}_V d\omega \tag{1.39}$$

b) These volume forces may be "remote" forces, which results in a force field. There are different origins at play here: forces of gravity, which are the most frequent, electrostatic forces and electromagnetic forces. Another type of force will occur when the reference frame, where the problem's equations are written, is no longer inertial (or Galilean).

It is now known that inertia forces appear.

NOTE: Remember that in Newtonian mechanics, an inertial or Galilean frame (they will be used as synonyms here) is a reference frame in a uniform straight-lined movement (meaning in inertial movement, in the sense of Newton's first law) with respect to an absolute frame. In an inertial frame, Newton's second law, along with the absolute frame, applies.

c) The same as for some inertia forces; some remote forces can be derived from a potential.

In this case, a function ϕ_V will be defined as:

$$\vec{F}_V = -\,\vec{grad}\,\phi_V \tag{1.40}$$

In general, a potential is defined by force nature. If all volume forces are derived from a potential, then the resulting volume forces will also be derived from a potential. This potential function will be the sum at each point in the potential's space of each force. Warning: if just one of the volume forces is not derived from a potential, then the result will not derive from a potential.

Forces of gravity and electrostatic forces are derived from a potential. Electromagnetic forces are not derived from a potential (they are derived from a "vector potential"). Inertia forces, which result from an accelerated translation or a uniform rotation, may be derived from a potential. Examples of this will be given in Chapter 2.

d) Note on calculating inertia forces: Generally, when it comes to a frame with a translation given by a vector $\overrightarrow{OO'}(t)$ and a rotation defined by the rotation vector $\vec{\Omega}$, the inertia force scripts take a complex form. Although we will not be using this expression in all of its complexity here, we will remind the reader of it anyhow.

For a material point of mass m, the inertia force will be:

$$\vec{F}_{inert} = -m\vec{\Gamma}_{rel} \qquad [1.41]$$

$\vec{\Gamma}_{rel}$ is calculated based on the different relative movements of the point and the frames.

By appointing $O'x'y',z'$, which is the non-inertial frame where we will solve the problem, where Oxy,z is the Galilean frame against which the frame $O'x'y',z'$ moves and $\vec{r}'$ is the position vector of our material point written in the frame $O'x'y',z'$, we express $\vec{\Gamma}_{rel}$:

$$\vec{\Gamma}_{rel} = \frac{d^2 OO'}{dt^2} + 2\vec{\Omega} \wedge \frac{d\vec{r}'}{dt} + \vec{\Omega} \wedge \left(\vec{\Omega} \wedge \vec{r}'\right) + \frac{d\vec{\Omega}}{dt} \wedge \vec{r}' \qquad [1.42]$$

1.2.2.3. *Surface force scripts*

We will discuss two approaches here:

a basic approach, which is sufficient for grasping various problems, and with which all readers of this book should be immediately familiar;

a more complete approach, which we will particularly need for the chapter dedicated to boundary layers, where more complex formulation is required.

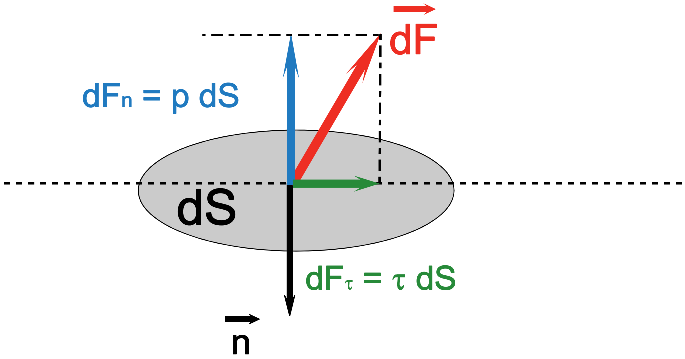

Figure 1.2. *Surface forces: normal forces, tangential forces*

a) Simplified approach

Remember that surface forces are applied from the exterior to the fluid contained within the field, by the fluid immediately in contact with the "internal" fluid at the "S level".

That being so, dS is an elementary surface of this surface S and $\vec{n}$ the unitary normal vector to dS .

NOTE: Remember that this unitary vector is carried by the normal force to dS and has a norm equal to 1. Conventionally, this unitary vector is always directed toward the exterior of D , whose purpose is to satisfy the integral vectorial relations.

Two components can generally be distinguished in the surface force $d\vec{F}$ applied to a surface dS:

a normal component carried by $\vec{n}$ and directed toward the interior, $d\vec{F}_n$;

a tangential component $d\vec{F}_\tau$, which is perpendicular to $\vec{n}$, and a tangent to dS.

These two components have an intensity which is proportional to dS. Thus, two finite parameters are defined, the *pressure* p and the *tangential stress* τ such as:

$$dF_n = p\, dS \qquad [1.43.a]$$

$$dF_\tau = \tau\, dS \qquad [1.43.b]$$

NOTE: The definition of the given pressure must be carefully accepted. In certain instances, the normal component of the volume forces contains terms resulting from the viscosity that is, in principle, reserved for the tangential forces in the previous script. Nevertheless, this will be a pertinent vision for the majority of the applications expanded upon in this book.

b) These forces are a result of the molecular nature of fluid matter

The pressure forces, which are normal to S, result from an exchange of momentum, due to the collision of molecules from the internal and external fluids. This collision is localized to the previously defined S interface. Should a gas interact with a solid wall, such as in a piston for example, which will be familiar to the thermodynamicist, the molecule shocks determine the pressure. Boltzmann modeled this type of mechanics and, in so doing, was able to theoretically establish the law of perfect gases.

It can be demonstrated that the pressure in a fluid point is isotropic. It does not depend on the orientation of the surface dS given by $\vec{n}$.

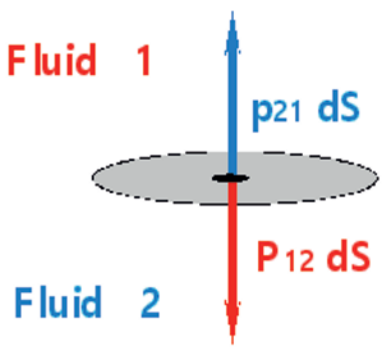

Figure 1.3. *On the interface between the two fluids, the pressure is continuous*

It also demonstrates that the pressure is continuous on the interface between the two fluids 1 and 2. That is, p_{12} is the pressure applied by fluid 1 onto fluid 2, at the level of an elementary surface dS, and p_{21} the pressure applied by fluid 2 onto fluid 1. At the level of dS, the law of action and reaction implies that the action of 1 on 2 is equal and opposite to the action of 2 on 1, in terms of intensity:

$$p_{12}\, dS = p_{21}\, dS\,; \quad p_{12} = p_{21} \qquad [1.44]$$

Therefore, the pressure is continuous throughout the boundary between fluid 1 and fluid 2.

The tangential forces result from the so-called viscosity phenomenon. This phenomenon was brought to light by the Couette flow experience in its most basic form. This experience, which we will not describe here, allows us to demonstrate that for a flow that is parallel to a flat solid plate, where the speed $u(y)$ varies in a linear way with the distance y to the wall, the tangential stress τ applied by this wall onto the fluid is "most frequently" given by:

$$\tau = \mu \frac{du(y)}{dy} \qquad [1.45]$$

$\dfrac{du(y)}{dy}$ is very often appointed by the "speed gradient", which is a misnomer. The correct term is shearing.

NOTE: "Speed gradient" is a misnomer because $u(y)$ is a vector component. We will see that the speed can be derived from a potential, but only when there is zero viscosity! In kinematics, an operator $\vec{V}.\vec{grad}\,\vec{V}$ will appear, which is only a means of facilitating the script.

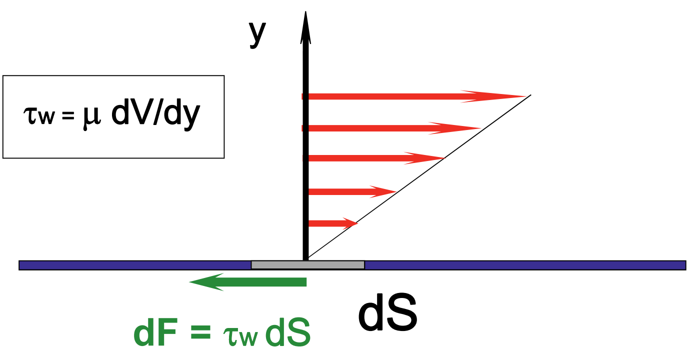

Figure 1.4. *Shearing and stress: Newton's law*

Rheology is the name given to the study of the relationship between stress and shearing.

The linear relationship between stress and shearing [1.45] is fulfilled by the majority of fluid currents, gases or liquids (water, oils, etc.). Therefore, it can be said that fluids fulfill Newton's law or that the fluid is "Newtonian".

μ is therefore defined as dynamic viscosity. It is important to retain this adjective as kinematic viscosity is also defined as the ratio of dynamic viscosity to density. The range of this definition will be discussed further on:

$$\nu = \frac{\mu}{\rho} \qquad [1.46]$$

"Pure" fluids are generally Newtonian. Once the fluid is "charged", meaning when it becomes a solid particle carrier, the rheological behavior becomes more complex and the linear relationship between stress and shearing becomes invalid.

Different models have led to expressions of these elements, some of which are more complex and some less complex (some have been found to be comprised of three lines of equations).

In this instance, we will use a form proposed by Oswald-De Waele:

$$\tau = k\left[\frac{du(y)}{dy}\right]^n \qquad [1.47]$$

where k is a viscosity coefficient.

Examples include non-Newtonian fluids, both of this type and others.

Some fluids are "memorized", such as the so-called Bingham fluids, which can be modeled by:

$$\tau = \tau_0 + k\frac{du(y)}{dy} \qquad [1.48]$$

where τ_0 is a residual stress.

Some fluids have a rheology that varies in time: these are called thixotropic fluids.

NOTE: In practice, the most common non-Newtonian fluids are blood (which contains approximately 45% of solid extract), gels and products (purées, soups) made in the agri-foodstuffs industry. A fluid's memory can also be experienced when we turn a spoon around in a good traditional soup.

c) Scripts developed from tangential stresses. Stresses tensor

This is indeed an oversimplified description. Writing the script correctly requires the stresses tensor σ_{ij} to be defined.

NOTE: The σ_{ij} are actually components of $d\vec{F}_S$ that relate to the three privileged directions, written as i. This indicates that the tension on any surface can be expressed according to the known tensions for the three privileged surfaces dS.

Therefore, the force $d\vec{F}_S$ will be written based on a tension $\vec{\tau}$ such as $d\vec{F}_S = \vec{\tau}\, dS$.

The ith component of $\vec{\tau}$, τ_i will be written as:

$$\tau_i = \sigma_{ij}\, n_j \qquad [1.49]$$

σ_{ij} depends on the fluid's rheology. For a Newtonian fluid, the linearity between stresses and shearing leads to the expression:

$$\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \eta \; div \vec{V} \qquad [1.50]$$

where μ is the dynamic viscosity that we have already seen, which is an essential parameter, and η is the so-called "volume" viscosity, which may be linked to μ.

1.2.2.4. *Surface forces and usual units*

It is recommended that all digital applications are performed in the MKS (meter, kilogram, second) system or the SI system. When using any formula, the parameters must be expressed in SI. We will follow this rule for all of the examples dealt with in this work.

The major parameters in fluid mechanics are always pressure and viscosity. For this reason, beyond legal units, alternative forms have been used to indicate these parameters. We should know all of the different expressions of these parameters in different technical fields.

a) Pressure units.

A pressure is the ratio of a force on a surface. In Newtonian terms, it can be expressed as a meter squared, written as $N.m^{-2}$. A specific unit, the *Pascal,* has been defined and written as Pa .

The pascal represents a very weak pressure. Expressed in water height (see Chapter 2), it is of a $1Pa \approx 0{,}1mm\,CE$ nature. Here, CE signifies a water column. Other units have been defined for the technical dialogue. By using a barometer, we can give a pressure value in liquid "height".

Thus, a *standardized atmosphere* corresponds to a "mercury" pressure of $h = 76\,cm$. Knowing that the mercury density is $\rho = 13600\,kg.m^{-3}$, the pressure in Pascal units will be:

$$1atm = \rho gh = 13600*9{,}81*0{,}76 = 1{,}013.10^{5}\ Pa \qquad [1.51]$$

The atmosphere emerges at practically $100\ 000\,Pa$. In industrial practice, 1.3% seems to be unchanged, and in this way, the *bar* is defined by:

$$1bar = 10^{5}\ Pa \qquad [1.52]$$

We can incidentally find some old units that are not commonly used nowadays. The reader may find it useful to remind themselves of these by way of reading an older publication.

The CGS (centimeter, gram, second) system has defined some other units. The force unit is the dyne, $1dyne = 10^{-5}N$. The result is a pressure unit, the dyne per centimeter squared or barye:

$$1\ barye = \frac{10^{-5}N}{10^{-4}} = 0{,}1Pa \qquad [1.53]$$

We can also cite another old unit, the pièze (pz), which is inherited from the MTS (meter, ton, second) system: $1pz = 10^{3}\ Pa$. We will also notice the hectopièze, which was the pressure unit still used by furnace manufacturers in the middle of the 20th Century. $1\ hpz = 1\ bar$.

Lastly, it should be noted that the millibar appeared in certain domestic barometers.

NOTE: We note that on a good quality barometer, the mercury centimeter scale must lag behind the millibar scale.

b) Viscosity units

There is more than just historical interest in having a knowledge of viscosity units in MKSA (SI) and CGS systems, considering that we can still come across data (particularly in handbooks) that stems from this system. Furthermore, when a viscosity is given, it is rarely indicated whether the viscosity is dynamic or kinematic. Nothing more than the unit is given.

Dynamic viscosity has the following dimension: $ML^{-1}T^{-1}$.

Kinematic viscosity has the following dimension: L^2T^{-1}.

Historically, the names were given in the CGS system.

The dynamic viscosity unit is the *Poise*: $1\ poise = 1\ kg.cm^{-1}.s^{-1}$ 𝕜.

The kinematic viscosity unit is *stokes*: $1\ stk = 1\ cm^2.s^{-1}$.

The names in MKSA (SI) system were derived from the CGS system.

The dynamic viscosity unit is the *Poiseuille*: $1\ Pl = 1\ kg.m^{-1}.s^{-1}$.

The kinematic viscosity unit is *myriastokes*: $1\ myriastokes = 1 m^2.s^{-1}$.

In principle, myriastokes should appear in an official document. Therefore, it is necessary to know it. Although it is not always known, “Myria” is the significant prefix 10^4. The myriastokes is not very commonly used, as $m^2.s^{-1}$ tends to be the preferred usage.

It is important to know the following conversion:

$$1 poiseuille = 10\ poises; \quad 1\ Pl = 10\ ps$$

There is a very important sub-multiple of the poise, that is, the centipoise.

In fact, it is $1 cps = 10^{-3}\ Pl$, which is the order of magnitude of water viscosity.

1.2.2.5. *Perfect fluids. Real fluids*

Studying the aforementioned surface forces at this stage allows us to establish a deep insight into fluid dynamics.

In the simplified presentation, we will break down the normal and tangential components.

There are then three types of situations involved:

In fluid statics, there are no tangential components on the surface forces. Only pressure forces occur.

In fluid dynamics, we find two different cases:

1) The tangential components either do not exist or are negligible. In this instance, they would be perfect fluid dynamics.

2) The tangential components need to be taken into account. In this instance, they must be real fluid dynamics.

Just as accounting for the fluid's compressibility depends on the problem in question, there is no intrinsically perfect or real fluid. We have seen that viscosity forces are linked to the speed "gradients". If these "gradients" are weak, then so are the viscosity forces. *Nevertheless, perfect fluid is associated with the notion of zero viscosity*, even though they are never intrinsically associated!

NOTE: It is not the fluid that is perfect, but the problem. The term "perfection" is just the expression of the physicist's satisfaction with such a simplified problem.

1.3. How to broach a question in fluid mechanics

1.3.1. *The different approaches of fluid mechanics*

Three approaches may be considered for a fluid mechanics problem:

1) *The "table corner" solution,* which is the most basic and the fastest, not necessarily the least formative for the budding or not-quite-so budding physicist.

2) *The complete equation,* the simplification of these equations depending on the proposed problem and the analytical approach. This approach will be adopted when dealing with problems further on.

3) *The digital approach.* This will not be exempt from a prior simplification of the equations in question, whether the calculation means are technically limited or the modeling is vital, as in the case of turbulence.

1.3.2. *Strategies for arriving at a reasoned solution*

1.3.2.1. *The project of this book : give methods*

This book is first and foremost addressed to students, readers who wish to learn, and as such are subject to assessment. We would like to dissuade this type of reader from approaching this work as they would a recipe book.

The examples to be dealt with herein are formalized and complete, in keeping with the "academic" spirit. In principle, they have a solution and all of the elements (data, tables) are provided to avoid wasting time on external research.

NOTE: This method could be criticized within the framework of a certain pedagogy, but research efficiency with regard to time management is also a relevant strategy.

This publication is also aimed at professionals called upon to resolve concrete problems in a professional space, which the student is also destined to do.

In real life, problems may be incomplete or poorly asked. There is often much data that remains to be found.

The following lines attempt to provide an analysis grid that will help readers tackle any problem. Having no intention to revolutionize pedagogical concepts, the methodology presented here results from common sense. It is also implicit to any exceptionally gifted student, to whom there is no need to explain this.

Quite the contrary, once this methodology has been integrated, it will become unconscious and will instead constitute a simple task of reasoning for the young (or not quite so young) fluid mechanical engineer.

1.3.2.2. *What to do when faced with a problem*

Before doing anything else, *know the physical situation.* This reflective activity can be aligned with the plan of this publication.

Then, identify the principles to be written and the laws of the course that are applicable to this situation, including the declensions. This step will enable us to avoid multiple scripts of the same physics law, as well as scripts of inadequate laws.

Recognizing the physical situation is generally a simple operation resulting from common sense.

NOTE: For those using this book, the division of this work by chapter has already been anticipated. Therefore, Any practitioner faced with a problem will be able to refer to a chapter in the following book that is best able to help them.

a) you are faced with an *immobile* fluid: you are dealing with fluid *statics* (Chapter 2).

b) You are given Eulerian characteristics of a flow and you need to find this flow's structure: You are dealing with fluid *kinematics* (Chapter 3).

c) *The fluid flows*.

Is the fluid *compressible or incompressible*: is it a gas or a liquid? Which types of pressure are at play? What are the speeds at play in the flow? If it is compressible, then we must continue with the procedure. Otherwise, we can go directly to the chapter dedicated to compressible flows.

Is it perfect, is it real: what can we say about its viscosity? What do we know about the flow?

If there is zero viscosity and there are no notable speed "gradients", then we have access to the simple solution for perfect fluids dynamics (Chapter 4).

Or, we could be dealing with a pipeline, or something closer to a significant wall. In this instance, we are in the field of real fluids.

So then, what is our objective? To calculate the loss of energy in a pipeline: using well-delineated methods to calculate charge losses will suffice (Chapter 5). Or, is the problem more complex than this? In this case, we need to take a closer look at the flow's structure and understand all types of boundary layers (external, internal and jets).

Perhaps a global approach regarding a system's thrusts (Euler's theorem application) will suffice. In such a case, a chapter dedicated to thrust and propulsion will help us.

Beyond that, in each type of situation, we examine the data we have about the problem and before doing anything else, we ask ourselves what we are looking for.

With regard to writing equations, there is one absolute rule which must always reign: all scripts relate to a principle. We must remain conscious of what we are writing.

The numerous examples that are processed in the following chapters are aimed at helping the reader.

1.4. Conclusion

This introductory chapter has put fluid mechanics back into the general framework of the initial concepts for all kinds of mechanics.

Matter continuously requires a particular approach that led us to specifically formulate the forces applied to matter in this state.

We also wanted to give the reader a larger strategical framework to solve problems, whether they occur in an academic realm or in a more open industrial realm.

We must now return to how these principles are implemented. To do this, in the following chapters, we will need to divide problems according to the analytical framework detailed above. The chapters are divided into the logical segments imposed by this reflection: fluid statics, fluid kinematics, perfect fluid dynamics, real fluid dynamics, broached from various angles; the technical approach to charge loss, the global approach to thrusting, a more analytical approach to flows at borders. After this, we will be able to concentrate on the specifics of compressible flows and then use a digital approach to broach the complexity of flows.