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The State of the Art in Quantum Communications

1.1. Quantum mechanics as a generalized probability theory

We have borrowed part of Michel Bitbol's analysis of quantum mechanics analyzed as a probability theory. In his pamphlet, he says [BIT 98]:

"The argument that I will defend here makes two propositions. First, quantum mechanics is not just a physical theory that uses probability calculation; it is itself a generalized form of probability calculation, coupled with a probabilistic evaluation process via the set use of symmetries. Secondly, quantum mechanics does not merely have a predictive function like other physical theories; it is a formalization of the possibility conditions of any prediction focused on phenomena whose circumstances of detection are also production conditions".

In this spirit, we begin by quickly showing the architecture of standard quantum mechanics:

1) The formal core of this theory is a vector space defined on a set of complex numbers and provided with a scalar product otherwise known as Hilbert space.

2) Special operators are defined on this space, called "observables", which, through their "proper values", provide a list of the possible results of a measuring operation.

3) A vector of the Hilbert space, called a state vector, is linked to each preparation (that is to say, to the fixing of conditions that are a prerequisite for measuring).

4) By applying the Born rule to this state vector, we obtain a function that assigns probabilities to the results of any measurement carried out as a result of the preparation.

5) A variable space–time interval and diverse physical circumstances can separate the end of the operation from the measurement preparation and operation; we take account of this via an evolution equation for the state vectors.

Here, I would like to emphasize the major difference between the probability functions of classic probability theory, and those obtained from state vectors in quantum mechanics by applying the Born rule. The classic probability functions link a number between 0 and 1 to each "event" in the widest sense, defined by Kolmogorov as a sub-set of elementary events. The set of these event sub-sets includes the empty set and the comprehensive set and it is provided with a Boolean algebra structure by the union and intersection operations. In other words, the classic probability functions are defined on a Boolean algebra. On the other hand, when we take account of the Hilbert space properties, the quantum probability functions are not defined on a Boolean algebra; they are defined on different and richer structures called "orthoalgebras".

This structural disparity between the classic and quantum probability functions explains why it is not sufficient to assume that quantum mechanics uses probability theory. Quantum mechanics itself consists of a new and enlarged form of probability theory [BLI 29]. The new circuits are a conjunction of quantum theory and probability theory, as shown in Figure 1.1.

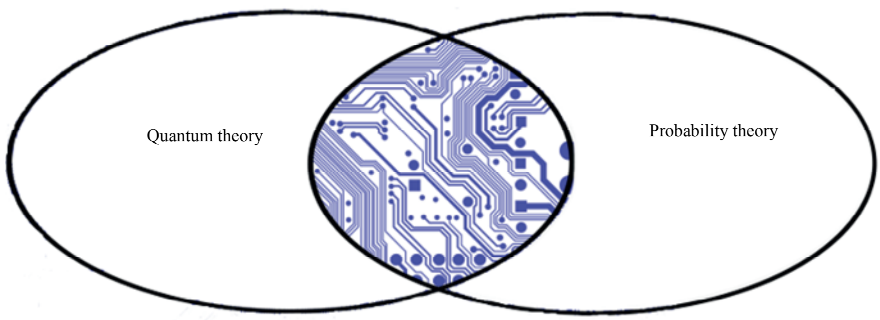

Figure 1.1. *Connection between quantum theory and probability theory*

The following references advance the same vision [THA 15, FER 08, FER 09] and [FER 11].

Starting from this analysis, it seems important to us to examine quantum communications on the basis of the probabilities and concepts related to this theory: covariance, correlation [FUR 12] inference and random processes, while including some concepts specific to quantum mechanics: contextuality [DZH 14], non-locality [RAB 14], paradoxes such as Schrodinger's cat, the Einstein, Podolsky and Rosen paradox, Bell inequalities [KRE 14, RAS 15] and decoherence [KOK 11]. Subsequently, we will try to create a synthesis between proponents of determinism and randomness.

Standard quantum mechanics undeniably violates the notion of separability that we have normally considered valid under classical physics. In relating the phenomenon of non-separability to the all-important concept of potentiality, we effectively create a coherent picture of correlations between the spatially-separated entangled enigmatic systems. Moreover, we support the idea that the generalized phenomenon of quantum non-separability involves contextuality, which, in turn, results in a relational, structural design of quantum objects, considered to carry dispositional properties [KAR 07].

1.2. Contextuality

Quantum computers promise enormous advantages over their classic counterparts, but the source of their power in quantum IT remains inaccessible. Here, we show a remarkable equivalence between the appearance of contextuality and the possibility of universal quantum calculation via the magic state that we call distillation, which is the main model for creating a quantum computer with tolerance to breakdown. Furthermore, this connection suggests a unifying paradigm for quantum IT resources: the non-locality of quantum mechanics is a particular type of contextuality, and non-locality is already known to be an essential resource for realizing the advantages of quantum communication. In addition to clarifying these fundamental questions, this work sets out the resource framework for quantum calculation, which has a number of practical applications, such as characterizing efficiency and compromising between distinct theoretical and experimental schema to reach a robust quantum calculation, and to place limits for the classic simulation of quantum algorithms [HOW 14].

1.3. Indeterminism and contextuality

These two historic remarks, one on the link between the concept of probability and the secondary concept of quality, and the other on calculating probabilities, designed as an instrument of predictive control for our situation of entanglement in the network of natural relationships, will now help us to unravel two interpretative nodes of quantum physics, each relying on indeterminism.

The first involves the notion, very widely known from Heisenberg's founding work of around 1927–1930, of an uncontrollable disturbance that the measuring agent is supposed to exercise on the microscopic subject measured. It is interesting to note that this "disturbance" was assigned a double role by its creators.

On the one hand, underlined by Bohr at the end of the 1920s, uncontrollable disturbance is the reason why the quantum phenomenon is indivisible, that is to say it is impossible to distinguish what in the phenomenon results from the object and what results from the measurement agent. The disturbance would explain, in other words, taken this time from Heisenberg, that quantum physics causes the mode of secondary qualities to become generalized, with their inevitable reference to the context in which they are manifested, to the detriment of that of the primary intrinsic qualities. But, on the other hand, according to an article from 1927 in which Heisenberg shows the relationships known as "uncertainty" relationships for the first time [HEI 27], the disturbance also accounts for indeterminism in quantum physics. The incompressible and uncontrollable disturbance created by the measuring agent is what prevents the two groups of variables that make up a particle's initial state from being known completely; consequently, Heisenberg concludes, the principle of causality, which links an initial state and a final state, links them in a way which is binding and remains inapplicable in quantum physics. The model of the "disturbance" also enables a direct relationship between contextuality and indeterminism to be shown, since the disturbance results in the phenomena's contextuality as much as in indeterminism for their subject. Later on, at the beginning of the 1950s, Paulette Destouches-Février demonstrated much more rigorously a theorem according to which any predictive theory relying on phenomena defined in relation to experimental contexts, some of which are mutually incompatible, is "essentially indeterminist" [BER 49].

1.4. Contextuality and hidden variables

The question of knowing if quantum phenomena can be explained by classic models with hidden variables has been the subject of lengthy debate. In 1964, Bell showed that certain types of classic models cannot explain the predictions of quantum mechanics for specific states of distant particles, and certain types of hidden variable models have been experimentally excluded. An intuitive characteristic of classic models is non-contextuality: the property that any measurement has a value independent of the other compatible measurements carried out at the same time. However, a theorem drawn up by Kochen, Specker and Bell shows that non-contextuality is in conflict with quantum mechanics. The conflict lies in the structure of the theory, which is independent of the properties of the special states. The question of knowing if the Kochen–Specker theorem could be tested experimentally has been discussed. The first tests for quantum contextuality have been suggested recently and undertaken with photons and neutrons. Here, we carry out an experiment with trapped ions, which shows a conflict between state independence and non-contextuality [KIR 09].

1.5. Non-locality and contextuality

We use the mathematical language of beam theory to give a unified treatment of non-locality and contextuality, in a framework that generalizes the familiar probability tables used in non-locality theory for arbitrary measurements: this includes Kochen–Specker configurations. We show that contextuality and non-locality, a particular case, correspond exactly to obstacles to the existence of global sections. We describe a linear, algebraic approach for calculating these obstacles, which permits a systematic treatment of non-locality and contextuality. We distinguish an adequate hierarchy of no-go theorem forces, and we show that the three main examples, taken from Bell, Hardy and Greenberger, and Horne and Zeilinger, respectively, occupy higher levels of this hierarchy. A general correspondence is shown between the existence of variable, local, hidden implementations using negative probabilities, and "no signaling"; this depends on a result showing the linear sub-spaces generated by the non-contextual and "no signaling" models. The maximal non-locality is generalized to maximal contextuality; it is characterized in purely qualitative terms, with Kochen–Specker results as generic. These models are independent proofs of maximal contextuality, and a new combinatorial state is given; it generalizes the "proofs of parity" much discussed in literature. This shows that quantum mechanics obeys "no signalling" families of commuting observables that are represented as a tensorial product of different factors [ABR 11].

Quantum contextuality is one of the fundamental notions in quantum mechanics. It has been shown that some tests of the Kochen–Specker theorem, such as those based on rays, can be converted into a non-state independent contextuality inequality. This question, namely if a proof of the Kochen–Specker theorem can always be converted into a non-contextuality inequality, remains open. In Yu's article, there is an answer to this question. The author shows that all types of proofs of the Kochen–Specker theorem, based on rays, or any other observable, can always be converted into independent non-contextuality inequalities. Furthermore, a constructive proof also provides a general approach for determining an inequality independent of state non-contextuality from a demonstration of the Kochen–Specker theorem [YU 15].

The Bell inequality (for space), Kochen–Specker inequality (for contextuality) and Leggett–Garg inequality (for time), are based on plausible, classic but entirely distinct hypotheses. For each of these inequalities, realization is equivalent to a joint probability distribution for all the observables in the experiment. This involves a joint distribution for all the pairs of observables, and this stands whether we know if they commute or not. This indifference is the basis for unifying the inequalities above in the general context of correlation inequalities. When the physical scenario is such that the correlated pairs are all compatible, the resulting correlation is of the "no signaling" type; it can be local or have multiple particles, corresponding to contextuality or to Bell inequalities. If the pairs are incompatible, the resulting correlation corresponds to the Leggett–Garg inequalities. If quantum mechanics violates all these inequalities, this will suggest a direct link between the theory's local, spatial and temporal properties [DAS 13].

Winter [WIN 14] gives an experimental test of the Bell-Kochen–Specker theorem following Meyer, Kent and Clifton's demonstrations, which ensures that predictions using quantum mechanics are indistinguishable from the non-contextual model.

In theoretical physics, a no-go theorem is a theorem that affirms that a certain situation is not physically possible. More specifically, this term describes results of quantum mechanics such as the Bell theorem and the Kochen–Specker theorem and governs the types of hidden variables admissible, which attempt to explain the apparent randomness of quantum mechanics as being a determinism involving hidden states.

In quantum information theory, non-communication theory is a result that gives conditions under which the instantaneous transfer of information between two observers is impossible.

1.6. Bell states

In the course of the last few decades, substantial theoretical and experimental progress has been made in understanding the quantum nature of physical phenomena, which is the basis of current and future technologies. Quantum correlations such as the entanglement of the states of composite systems and the phenomenon of quantum discord, which are linked to other aspects of quantum correlations, quantum contextuality and, linked to these phenomena, uncertainty relations for variables and combined entropies, such as Shannon and Reyi entropies, and inequalities for spin states, such as Bell inequalities, reflect the quantum properties of micro- and macro-systems. The mathematical methods needed to describe all the quantum phenomena mentioned above were also the subject of intense study at the end of the last century, and at the start of the new century.

Another new direction in elaborating the mathematical approach to quantum physics is tomography, which offers a new vision of quantum states. In the tomographic image of quantum mechanics, the states are identified with equitable conditional probability distributions, which contain the same information on states as the wave function or the density matrix. The tomographic approach's mathematical methods are based on the study of the quantization schema's star product (the associative product). The tomographic star product provides an additional understanding of the associative product, which is linked to the existence of specific pairs of operators called quantifiers and dequantifiers [MAN 13].

1.7. Violation of the Leggett–Garg inequality

By weakly measuring the polarization of a photon between two strong polarization measurements, the author experimentally studies the correlation between the appearance of abnormal values in weak quantum measurements [GOG 11]. A quantitative formulation of the latter concept is expressed in terms of a L-G (Leggett–Garg) inequality for the results of subsequent measurements of an individual quantum system. We experimentally violate the Leggett–Garg inequality over several measurements. Moreover, we demonstrate experimentally that there is a correlation between obtaining unexpected weak values and violating the Leggett–Garg inequality [GOG 11].

Assano and coauthors interpret the Leggett–Garg inequality as a contextual probabilistic inequality in which the collected data are combined in experiments in three different contexts.

In the original version of this inequality, the contexts have a temporal nature, they are represented by three pairs (t_1, t_2), (t_2, t_3), (t_3, t_4) with $t_1<t_2<t_3<t_4$. They generalize the Leggett–Garg conditions of macroscopic realism and non-invasive measurability in a general contextual framework. Their formulation is developed in purely probabilistic terms, the existence of a context independent of a probability distribution (two-dimensional) and the possibility of reconstructing marginal probability distributions from P. They determine an inequality analogous to L-G, which they call the contextual L-G, and as a quanticity test, they use statistical data collected in a series of experiments in recognizing ambiguous figures. In the experimental study, the figure under examination is Schröder's stairs, which is shown with rotations from different angles, the contexts are coded by dynamic rotations in three directions: clockwise,

anti-clockwise, and in a random direction. The data demonstrate a violation of the contextual L-G inequality for the combinations of contexts mentioned above [ASA 14].

1.8. Violation of the Bell inequality

Non-local correlations between spatially separated systems have been discussed broadly in the context of the Einstein, Podolsky and Rosen paradox (EPR) and Bell inequalities. Many ideas and experiments intended to test the hidden-variable theories and violation of the Bell inequalities have been mentioned, usually these photons consist of correlation, although recently an experiment was carried out with Be^+ (Beryllium ions). Nevertheless, there is considerable benefit in showing that these correlations (resulting from quantum entanglement) are not just a particularity of photons. Here, we measure the correlations between simple neutrons' two degrees of freedom (including spatial and spin components); this removes the requirement for a source of neutrons in entangled pairs, which present a considerable technical challenge. An inequality equivalent to Bell is introduced to clarify the correlations that can supervene between observables of independent degrees of freedom. We demonstrate the violation of this Bell inequality as follows: the measured value is 2.051 ± 0.019, clearly higher than the value of 2 predicted by the classic hidden variable theories [HAS 03].

Experimental situations in which quantum effects are observed pose a fundamental question to be taken into consideration: this is the compatibility between the description of phenomena and the objective reality hypothesis. This work tackles Bohm's ontological interpretation of quantum mechanics, concentrating on the use of the term "trajectory" and the difficulties associated with connecting it to a real (objective) trajectory. The conclusion is that the realistic interpretation applied to Bohm trajectories is very debatable [BOS 13].

Bohm gives an interpretation of quantum theory in terms of hidden variables [BOH 52]. Another, equally interesting article on the significance of electromagnetic potentials in quantum theory is given by Aharonov and Bohm. In this article, the authors discuss some useful properties of electromagnetic potentials in the quantum domain [AHA 59].

1.9. EPR paradox

This is a major article known as the EPR paradox. The authors stipulate that in a complete theory, there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of making a prediction with certainty without disturbing the system. In quantum mechanics, for the case of two physical quantities described by non-commuting operators, knowing one precedes knowing the other. Therefore, either the description of the reality given by the wave function in quantum mechanics is not complete or these two quantities cannot simultaneously have reality [EIN 35]. An experiment showing the EPR paradox has been carried out by Birgit Dopfer [DOP 98] and is shown on Figure 1.2.

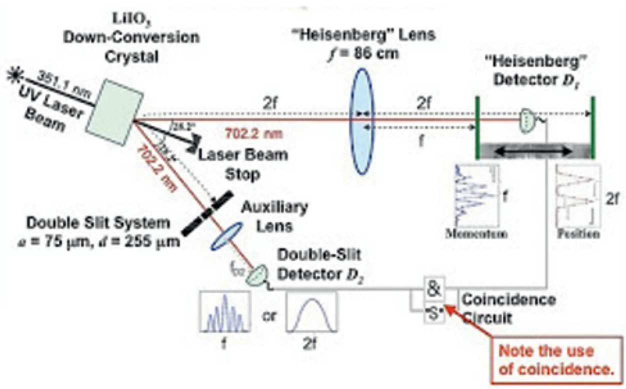

Figure 1.2. *Einstein-Podolsky-Rosen experiment according to [DOP 98]. For a color version of the figure, see www.iste.co.uk/benslama/quantum.zip*

The EPR paradox, an abbreviation of Einstein-Podolsky-Rosen, is a thought experiment developed by Albert Einstein, Boris Podolsky and Nathan Rosen; its first goal was to refute the Copenhagen interpretation of quantum physics. The Copenhagen interpretation is opposed to the existence of any state of a quantum system before any measurement. In effect, there is no proof that this state exists before it is observed, and assuming it does raises certain contradictions.

Indeed, if two particles are transmitted and a conservation relationship exists between one of their properties (for example, the sum of their spins should be null, that is to say there should be quantum entanglement of the state of these two-particle system), knowing the state of the first after measuring it tells us the state of the second particle before a measurement is later made on it, whereas – according to the Copenhagen interpretation – the measured value is determined randomly at the moment of measurement. If the measurement on the first particle has given “+”, and the first particle is thus now in state “+”, the measurement on the second will always give “–”.

One of the problems is that this latter particle can, at the moment of measurement, be located at as large a distance as desired in the observable universe from the first. The world line that links the two events, “measurement on particle 1” and, “measurement on particle 2” in space–time can be a space curve, and the second particle therefore absolutely cannot, in the latter instance, “be informed” in any way whatsoever, of the state that the first was in after measurement. How can we believe, in these conditions, that the state in which the second particle is found after measurement was not determined from the start, in contradiction with the Copenhagen representation?

This paradox was developed by Albert Einstein and two of his collaborators, Boris Podolsky and Nathan Rosen, to expose what appeared to be a contradiction in quantum mechanics, or at least a contradiction of at least one with the three following hypotheses:

1) It is impossible for a signal to exceed speed c (relativistic causality).

2) Quantum mechanics is complete and describes reality entirely (no hidden local variable).

3) The two distant particles form two entities that can be considered independently of one another, each being localized in space–time (locality).

The EPR argument, as presented in 1935, is based on the following reasoning.

First of all, we must remember that the uncertainty principle states that it is impossible simultaneously to know the precise value of two physical quantities called incompatibles (typically, the speed and position of a particle). The more precisely one quantity is measured, the less determinate the measurement of the other.

The EPR draws two mutually exclusive statements from this principle:

1) The description of reality given by quantum mechanics is not complete.

2) The two incompatible physical quantities do not simultaneously have an objective reality.

The Copenhagen interpretation reaches the conclusion that (2) is true and (1) is false, so the EPR intends to demonstrate that (1) is true and (2) is false.

To do this, they refine a thought experiment that leads to the simultaneous determination of two non-commutable physical quantities, and so lead to the conclusion that (2) is false and consequently (the two statements being mutually exclusive) (1) is true.

To demonstrate that (2) is false, it is vital to define precisely what the notion of the “reality” of a physical quantity is (for example, the “position”). EPR reveals a sufficient “reality”:

If, without disturbing a system’s state at all, it is possible to predict the value of a physical quantity of this system with certainty (with a probability equal to 1), then there is an element of reality corresponding to this physical quantity.

The thought experiment suggested in 1935 is quite complex, but can be described more simply without changing its meaning (see Figure 1.3).

If P_1 and P_2 are two photons entangled in such a way as to have a total angular moment equal to zero (anti-correlated spins), then the two non-commutable physical quantities used in the reasoning are: (1) The spin measured in a direction Sx (2) The spin measured in another direction Sz.

If P_1 is measured along to Sx, then – without disturbing P_2 (the locality principle is assumed) the measurement of P_2 is necessarily known along this axis (the opposing axis). Similarly, if P_2 is measured along to Sz, then – without disturbing P_1, the measurement of P_1 is necessarily known according to this axis (the opposing axis again).

Therefore, measuring P_1 along one axis and P_2 along the other enables the value of two physical quantities to be predicted with certainty. These two quantities therefore possess an objective reality, and consequently (2) is false and (1) is true.

This is the paradox initially created by EPR.

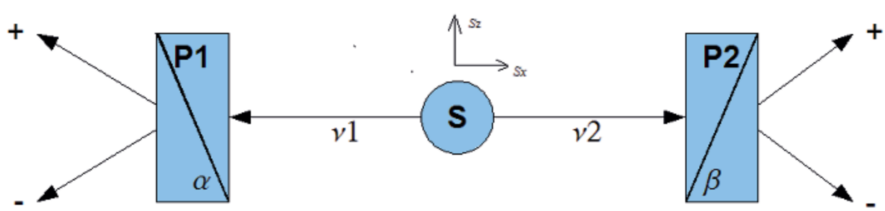

Figure 1.3. *Pictorial explanation of the EPR paradox*