

LEARNING MODULE

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The Term Structure and Interest Rate Dynamics

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LEARNING OUTCOMES

Mastery	The candidate should be able to:
☐	describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve
☐	describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping
☐	describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management
☐	describe the strategy of rolling down the yield curve
☐	explain the swap rate curve and why and how market participants use it in valuation
☐	calculate and interpret the swap spread for a given maturity
☐	describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk
☐	explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve
☐	explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks
☐	explain the maturity structure of yield volatilities and their effect on price volatility
☐	explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes

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SPOT RATES, FORWARD RATES, AND THE FORWARD RATE MODEL

- ☐ describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve
- ☐ describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping

Interest rates are both a barometer of the economy and an instrument for its control. The term structure of interest rates—market interest rates at various maturities—is a vital input into the valuation of many financial products. The quantification of interest rate risk is of critical importance to risk managers. Understanding the determinants of interest rates, and thus the drivers of bond returns, is imperative for fixed-income market participants. Here, we explore the tools necessary to understand the term structure and interest rate dynamics—that is, the process by which bond yields and prices evolve over time.

Section 1 explains how spot (or current) rates and forward rates, which are set today for a period starting in the future, are related, as well as how their relationship influences yield curve shape. Section 2 builds upon this foundation to show how forward rates impact the yield-to-maturity and expected bond returns. Section 3 explains how these concepts are put into practice by active fixed-income portfolio managers.

The swap curve is the term structure of interest rates derived from a periodic exchange of payments based on fixed rates versus short-term market reference rates rather than default-risk-free government bonds. Sections 4 and 5 describe the swap curve and its relationship to government yields, known as the swap spread, and explains their use in valuation.

Section 6 describes traditional theories of the term structure of interest rates. These theories outline several qualitative perspectives on economic forces that may affect the shape of the term structure.

Section 7 describes yield curve factor models. The focus is a popular three-factor term structure model in which the yield curve changes are described in terms of three independent movements: level, steepness, and curvature. These factors can be extracted from the variance–covariance matrix of historical interest rate movements.

Section 8 builds on the factor model and describes how to manage the risk of changing rates over different maturities. Section 9 concludes with a discussion of key variables known to influence interest rates, the development of interest rate views based on forecasts of those variables, and common trades tailored to capitalize on an interest rate view. A summary of key points concludes the reading.

Spot Rates and Forward Rates

We first explain the relationships among spot rates, forward rates, yield-to-maturity, expected and realized returns on bonds, and the shape of the yield curve. We then discuss the assumptions made about forward rates in active bond portfolio management.

The price of a risk-free single-unit payment (e.g., \$1, €1, or £1) after N periods is called the **discount factor** with maturity N , denoted by DF_N . The yield-to-maturity of the payment is called a **spot rate**, denoted by Z_N . That is,

$$DF_N = \frac{1}{(1 + Z_N)^N} \quad (1)$$

The N -period discount factor, DF_N , and the N -period spot rate, z_N , for a range of maturities in years $N > 0$ are called the **discount function** and the **spot yield curve** (or, more simply, **spot curve**), respectively. This spot curve represents the term structure of interest rates. Note that the discount function completely identifies the spot curve and vice versa, because both contain the same set of information about the time value of money.

The spot curve shows, for various maturities, the annualized return on an option-free and default-risk-free **zero-coupon bond** (**zero** for short) with a single payment at maturity. For this reason, spot rates are also referred to as zero-coupon yields or zero rates. The spot rate as a yield concept avoids the need for a reinvestment rate assumption for coupon-paying securities.

As Equation 1 suggests, the spot curve is a benchmark for the time value of money received on a future date as determined by the market supply and demand for funds. It is viewed as the most basic term structure of interest rates because no reinvestment risk is involved; the stated yield equals the actual realized return if the zero is held to maturity. Thus, the yield on a zero-coupon bond maturing in year T is regarded as the most accurate representation of the T -year interest rate.

A **forward rate** is an interest rate determined today for a loan that will be initiated in a future period. The set of forward rates for loans of different maturities with the same future start date is called the **forward curve**. Forward rates and forward curves can be mathematically derived from the current spot curve.

Denote the forward rate of a loan initiated A periods from today with tenor (further maturity) of B periods by $f_{A,B-A}$. Consider a forward contract in which one party, the buyer, commits to pay another party, the seller, a forward contract price $f_{A,B-A}$ at time A for a zero-coupon bond with maturity $B - A$ and unit principal. Because this is an agreement to do something in the future, no money is exchanged at contract initiation. At A , the buyer will pay the seller the contracted forward price and will receive from the seller at time B a payment defined here as a single currency unit.

The **forward pricing model** describes the valuation of forward contracts. The no-arbitrage principle, which simply states that tradable securities with identical cash flow payments must have the same price, may be used to derive the model as shown in Equation 2:

$$DF_B = DF_A \times F_{A,B-A} \quad (2)$$

The discount factors DF_A and DF_B represent the respective prices for period A and a longer period B needed to derive the forward price, $F_{A,B-A}$, a contract which starts in the future at time A and ends at time B . To understand the reasoning behind Equation 2, consider two alternative investments: (1) buying a two-year zero-coupon bond at a cost of $DF_2 = 0.93$ and (2) entering into a one-year forward contract to purchase a one-year zero-coupon bond for $DF_1 = 0.95$. Because the payoffs in two years are the same and the initial costs of the investments must be equal, the no-arbitrage forward price $F_{1,1}$ must equal $0.93/0.95$, or 0.9789 . Otherwise, any trader could sell the overvalued investment and buy the undervalued investment with the proceeds to generate risk-free profits with zero net investment.

Example 1 should help confirm your understanding of discount factors and forward prices. Please note that the solutions in the examples that follow may be rounded to two or four decimal places.

EXAMPLE 1

Spot and Forward Prices and Rates (1)

Consider a two-year loan beginning in one year ($A = 1$, $B = 3$). The one-year spot rate is $z_1 = z_A = 7\% = 0.07$. The three-year spot rate is $z_3 = z_B = 9\% = 0.09$.

1. Calculate the one-year discount factor: $DF_A = DF_1$.

Solution:

Using Equation 1,

$$DF_1 = \frac{1}{(1 + 0.07)^1} = 0.9346$$

2. Calculate the three-year discount factor: $DF_B = DF_3$.

Solution:

$$DF_3 = \frac{1}{(1 + 0.09)^3} = 0.7722$$

3. Calculate the forward price of a two-year bond to be issued in one year:

$$F_{A,B-A} = F_{1,2}.$$

Solution:

Using Equation 2,

$$0.7722 = 0.9346 \times F_{1,2}.$$

$$F_{1,2} = 0.7722 \div 0.9346 = 0.8262.$$

4. Interpret your answer to Problem 3.

Solution:

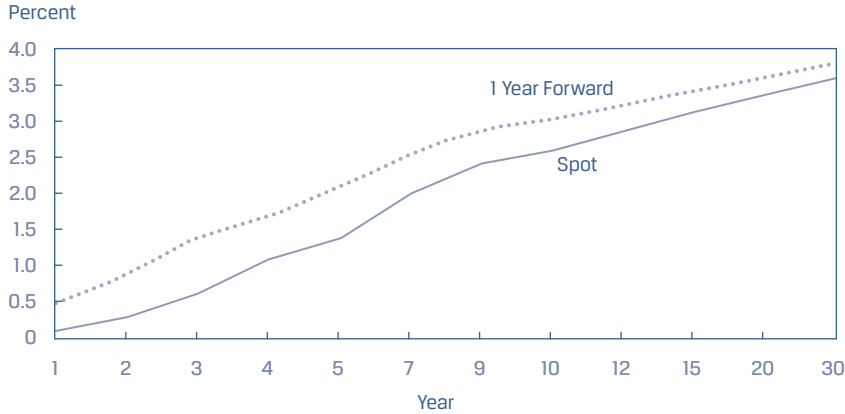
The forward contract price of $F_{1,2} = 0.8262$ is the price agreed on today, to be paid one year from today for a bond with a two-year maturity and a risk-free unit-principal payment (e.g., \$1, €1, or £1) at maturity in three years. As shown in the solution to 3, it is calculated as the three-year discount factor, $DF_3 = 0.7722$, divided by the one-year discount factor, $DF_1 = 0.9346$.

The Forward Rate Model

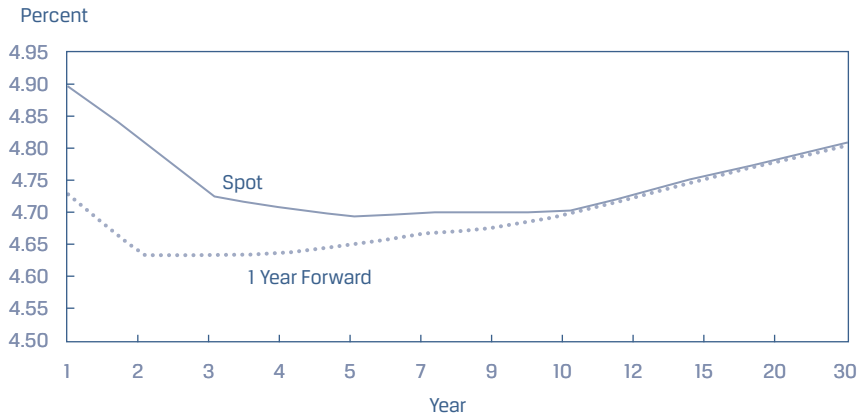
This section uses the forward rate model to establish that forward rates are above spot rates when the spot curve is upward sloping and below spot rates when the spot curve slopes downward. Exhibit 1 shows these spot versus forward relationships for the US Treasury yield curve in July 2013 versus December 2006, respectively. As we illustrate later, the relationship between spot and forward rates is important for future rate expectations as well as valuing fixed-income instruments.

Exhibit 1: Spot and Forward Curves

A. Spot vs. Forward US Treasury Yields, July 2013



B. Spot vs. Forward US Treasury Yields, December 2006



In contrast to the forward price $F_{A,B-A}$, the forward rate $f_{A,B-A}$ is the discount rate for a risk-free unit-principal payment (e.g., \$1, €1, or £1) B periods from today, valued at time A , such that the present value equals the forward contract price, $DF_{A,B-A}$. Then, by definition,

$$DF_{A,B-A} = \frac{1}{(1 + F_{A,B-A})^{B-A}} \quad (3)$$

By substituting Equation 1 and Equation 3 into Equation 2, the forward pricing model can be expressed in terms of rates as noted by Equation 4, which is the **forward rate model**:

$$(1 + z_B)^B = (1 + z_A)^A (1 + f_{A,B-A})^{B-A} \quad (4)$$

Thus, the spot rate for B periods, which is z_B , and the spot rate for A periods, which is z_A , imply a value for the $(B-A)$ -period forward rate at A , $f_{A,B-A}$. Equation 4 is important because it shows how forward rates may be extrapolated from spot rates—that is, they are implicit in the spot rates at any given point in time.

Equation 4 suggests two ways to interpret forward rates. For example, suppose $f_{7,1}$, the rate agreed on today for a one-year loan to be made seven years from today, is 3%. Then 3% is the

- reinvestment rate that would make an investor indifferent between buying an eight-year zero-coupon bond or investing in a seven-year zero-coupon bond and at maturity reinvesting the proceeds for one year. In this sense, the forward rate can be viewed as a type of breakeven interest rate.
- one-year rate that can be locked in today by buying an eight-year zero-coupon bond rather than investing in a seven-year zero-coupon bond and, when it matures, reinvesting the proceeds in a zero-coupon instrument that matures in one year. In this sense, the forward rate can be viewed as a rate that can be locked in by extending maturity by one year.

Example 2 addresses forward rates and the relationship between spot and forward rates.

EXAMPLE 2

Spot and Forward Prices and Rates (2)

The spot rates for three hypothetical zero-coupon bonds (zeros) with maturities of one, two, and three years are given in the following table.

Maturity (T)	1	2	3
Spot rates	$z_1 = 9\%$	$z_2 = 10\%$	$z_3 = 11\%$

1. Calculate the forward rate for a one-year zero issued one year from today, $f_{1,1}$.

Solution:

$f_{1,1}$ is calculated as follows (using Equation 4):

$$\begin{aligned} (1 + z_2)^2 &= (1 + z_1)^1 (1 + f_{1,1})^1 \\ (1 + 0.10)^2 &= (1 + 0.09)^1 (1 + f_{1,1})^1 \\ f_{1,1} &= \frac{(1.10)^2}{1.09} - 1 = 11.01\% \end{aligned}$$

2. Calculate the forward rate for a one-year zero issued two years from today, $f_{2,1}$.

Solution:

$f(2,1)$ is calculated as follows:

$$\begin{aligned} (1 + z_3)^3 &= (1 + z_2)^2 (1 + f_{2,1})^1 \\ (1 + 0.11)^3 &= (1 + 0.10)^2 (1 + f_{2,1})^1 \\ f_{2,1} &= \frac{(1.11)^3}{(1.10)^2} - 1 = 13.03\% \end{aligned}$$

3. Calculate the forward rate for a two-year zero issued one year from today, $f_{1,2}$.

Solution:

$f_{1,2}$ is calculated as follows:

$$\begin{aligned}(1 + z_3)^3 &= (1 + z_1)^1 (1 + f_{1,2})^2 \\ (1 + 0.11)^3 &= (1 + 0.09)^1 (1 + f_{1,2})^2 \\ f_{1,2} &= \sqrt[2]{\frac{(1.11)^3}{(1.09)^1}} - 1 = 12.01\%\end{aligned}$$

4. Based on your answers to 1 and 2, describe the relationship between the spot rates and the implied one-year forward rates.

Solution:

The upward-sloping zero-coupon yield curve is associated with an upward-sloping forward curve (a series of increasing one-year forward rates because 13.03% is greater than 11.01%). This dynamic is explained further in the following discussion.

The relationship between spot rates and one-period forward rates may be demonstrated using the forward rate model and successive substitution, resulting in Equation 5 and Equation 6:

$$(1 + z_T)^T = (1 + z_1) (1 + f_{1,1}) (1 + f_{2,1}) (1 + f_{3,1}) \dots (1 + f_{T-1,1}) \quad (5)$$

$$z_T = \left\{ (1 + z_1) (1 + f_{1,1}) (1 + f_{2,1}) (1 + f_{3,1}) \dots (1 + f_{T-1,1}) \right\}^{\frac{1}{T}} - 1 \quad (6)$$

Equation 6 shows that the spot rate for a security with a maturity of $T > 1$ can be expressed as a geometric mean of the spot rate for a security with a maturity of $T = 1$ and a series of $T - 1$ forward rates.

Equation 6 is critical for active fixed-income portfolio managers. Although the question of whether forward rates are unbiased estimators of market consensus expectations remains open to debate, implied forward rates are generally the best available and most accessible proxy for market expectations of future spot rates. If an active trader can identify a series of short-term bonds whose actual returns exceed today's quoted forward rates, then the total return over her investment horizon would exceed the return on a maturity-matching, buy-and-hold strategy if the yield curve were to remain relatively stable. Later, we will apply this concept to dynamic hedging strategies and the local expectations theory.

Example 3 and Example 4 explore the relationship between spot and forward rates.

EXAMPLE 3

Spot and Forward Prices and Rates (3)

1. Given the data and conclusions for z_1 , $f_{1,1}$, and $f_{2,1}$ from Example 2:

$$z_1 = 9\%$$

$$f_{1,1} = 11.01\%$$

$$f_{2,1} = 13.03\%$$

Show that the two-year spot rate of $z_2 = 10\%$ and the three-year spot rate of $z_3 = 11\%$ are geometric averages of the one-year spot rate and the forward rates.

Solution:

Using Equation 5,

$$(1 + z_2)^2 = (1 + z_1) (1 + f_{1,1})$$

$$z_2 = \sqrt[2]{(1 + 0.09) (1 + 0.1101)} - 1 \approx 10\%$$

$$(1 + z_3)^3 = (1 + z_1) (1 + f_{1,1}) (1 + f_{2,1})$$

$$z_3 = \sqrt[3]{(1 + 0.09) (1 + 0.1101) (1 + 0.1303)} - 1 \approx 11\%$$

We can now consolidate our knowledge of spot and forward rates to explain important relationships between the spot and forward rate curves. The forward rate model (Equation 4) can also be expressed as Equation 7.

$$\left\{ \frac{1 + z_B}{1 + z_A} \right\}^{\frac{A}{B-A}} (1 + z_B) = 1 + f_{A,B-A} \quad (7)$$

To illustrate, suppose $A = 1$, $B = 5$, $z_1 = 2\%$, and $z_5 = 3\%$; the left-hand side of Equation 7 is

$$\left(\frac{1.03}{1.02} \right)^{\frac{1}{4}} (1.03) = (1.0024) (1.03) = 1.0325,$$

so $f_{1,4} = 3.25\%$. Given that the yield curve is upward sloping—so, $z_B > z_A$ —Equation 7 implies that the forward rate from A to B is greater than the long-term spot rate: $f_{A,B-A} > z_B$. This is the case in our example, because $3.25\% > 3.00\%$. Conversely, when the yield curve is downward sloping, then $z_B < z_A$ and the forward rate from A to B is lower than the long-term spot rate: $f_{A,B-A} < z_B$. Equation 7 also shows that if the spot curve is flat, all one-period forward rates equal the spot rate. For an upward-sloping yield curve— $z_B > z_A$ —the forward rate rises as time periods increase. For a downward-sloping yield curve— $z_B < z_A$ —the forward rate declines as time periods increase.

EXAMPLE 4

Spot and Forward Prices and Rates (4)

Given the spot rates $z_1 = 9\%$, $z_2 = 10\%$, and $z_3 = 11\%$, as in Example 2 and Example 3:

1. Determine whether the forward rate $f_{1,2}$ is greater than or less than the long-term rate, z_3 .

Solution:

The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$, or in general, $z_B > z_A$. Thus, the forward rate will be greater than the long-term rate, or $f_{A,B-A} > z_B$. Note from Example 2 that $f_{1,2} = 12.01\% > z_3 = 11\%$.

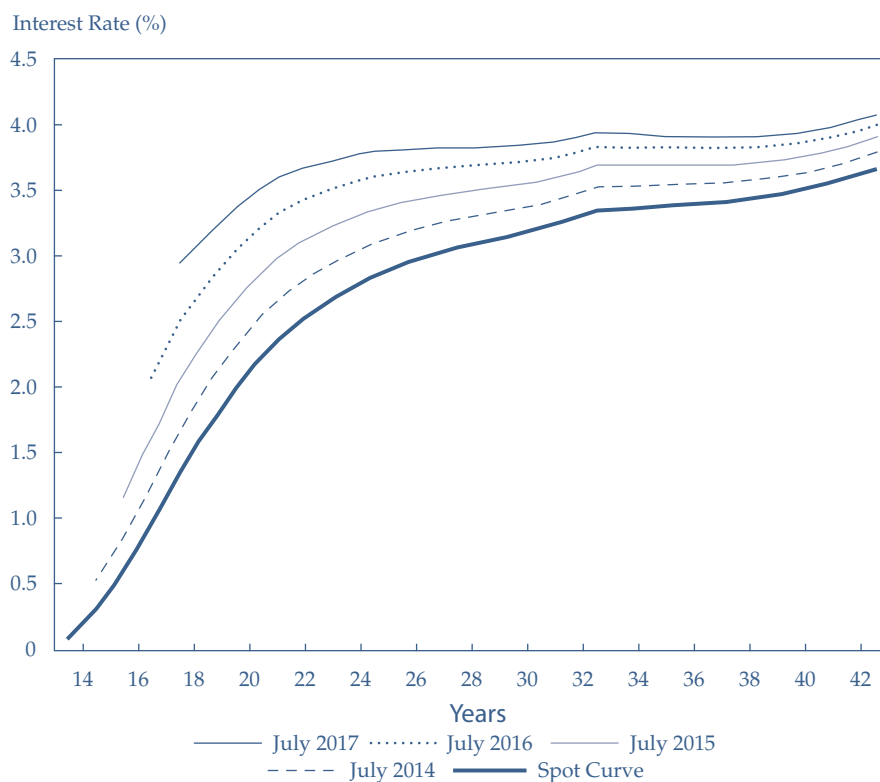
2. Determine whether forward rates rise or fall as the initiation date, A , for the forward rate is later.

Solution:

The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$. Thus, the forward rates will rise with increasing A . This relationship was shown in Example 2, in which $f_{1,1} = 11.01\%$ and $f_{2,1} = 13.03\%$.

These relationships are illustrated in Exhibit 2 and Exhibit 3 as an extension of Exhibit 1. The spot rates for US Treasuries as of 31 July 2013 constructed using interpolation are the lowest, as shown in the table following the exhibit. Note that the spot curve is upward sloping. The forward curves for the end of July 2014, 2015, 2016, and 2017 are also presented in Exhibit 2. Because the yield curve is upward sloping, these forward curves are all above the spot curve and become successively higher and steeper as the forward period increases, the highest of which is that for July 2017.

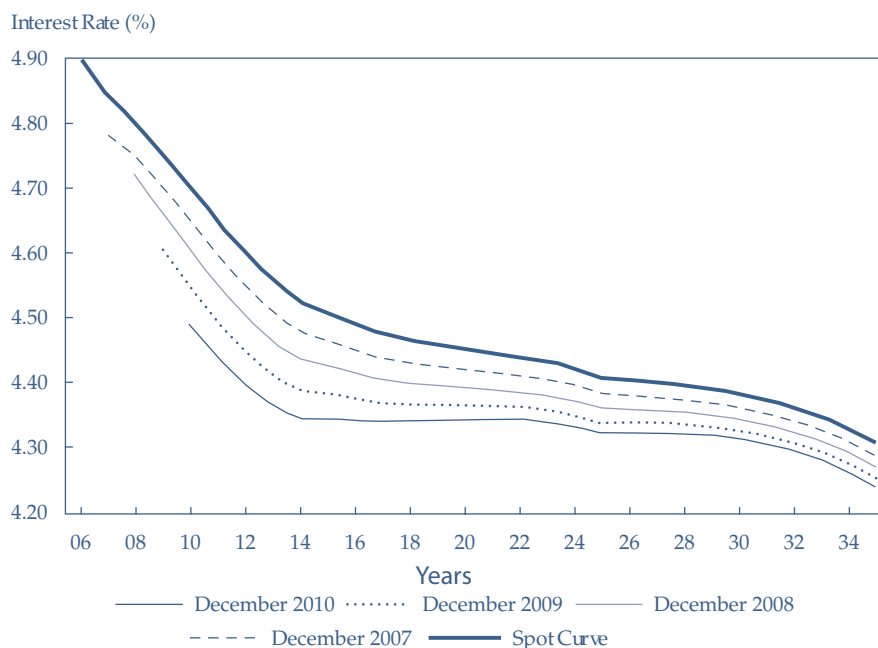
Exhibit 2: Historical Example: Upward-Sloping Spot Curve vs. Forward Curves, 31 July 2013



Maturity (years)	1	2	3	5	7	10	20	30
Spot rate (%)	0.11	0.33	0.61	1.37	2.00	2.61	3.35	3.66

Exhibit 3 shows the opposite case of a downward sloping spot curve based on US Treasury rates as of 31 December 2006. This data also uses interpolation and is somewhat modified to make the yield curve more downward sloping for illustrative purposes. The spot curve and forward curves for the end of December 2007, 2008, 2009, and 2010 are presented in Exhibit 3.

Exhibit 3: Historical Example: Downward-Sloping Spot Curve vs. Forward Curves, 31 December 2006 (modified for illustrative purposes)



Maturity (years)	1	2	3	5	7	10	20	30
Spot rate (%)	4.90	4.82	4.74	4.70	4.60	4.51	4.41	4.31

The highest curve is the spot yield curve, and it is downward sloping. The forward curves are below the spot curve, with longer forward periods associated with lower forward curves, the lowest of which is dated December 2010.

An important point that can be inferred from Exhibit 2 and Exhibit 3 is that forward rates do not extend beyond the longest maturity on today's yield curve. For example, if yields reach a 30-year maturity on today's yield curve, then a three-year forward model will extend just 27 years. Similarly, four years hence, the longest-maturity forward rate would be $f_{4,26}$.

In summary, when the spot curve slopes upward, the forward curve will lie above the spot curve. Conversely, when the spot curve slopes downward, the forward curve will lie below the spot curve. This dynamic reflects the basic mathematical truth that when an average is rising (falling), the marginal data point must be above (below) the average. In this case, the spot curve represents an average over an entire period and the forward rates represent the marginal changes between future periods.

We have thus far discussed the spot curve and the forward curve. Another curve important in practice is the government par curve. The **par curve** represents the yields to maturity on coupon-paying government bonds, priced at par, over a range of maturities. In practice, recently issued ("on the run") bonds are most often used to create the par curve, because these securities are most liquid and typically priced at or close to par.

The par curve is important for valuation in that it can be used to construct a zero-coupon yield curve. The process considers a coupon-paying bond as a portfolio of zero-coupon bonds. The zero-coupon rates are determined by using the par yields and solving for the zero-coupon rates one by one, from the shortest to longest maturities using a forward substitution process known as **bootstrapping**.

WHAT IS BOOTSTRAPPING?

Because the practical details of deriving the zero-coupon yield are beyond the scope of this reading, the concept of bootstrapping may be best shown using a numerical illustration. Suppose the following yields are observed for annual coupon sovereign debt:

Par Rates:

One-year par rate = 5%, two-year par rate = 5.97%, three-year par rate = 6.91%, four-year par rate = 7.81%. From these data, we can bootstrap zero-coupon rates.

Zero-Coupon Rates:

Given annual coupons, the one-year zero-coupon rate equals the one-year par rate because it has one cash flow, whereas two-year and longer maturity bonds have coupon payments prior to maturity.

The derivation of zero-coupon rates begins with the two-year maturity. The two-year zero-coupon rate is determined by using $z_1 = 5\%$ and solving for z_2 in the following equation for one monetary unit of current market value:

$$1 = \frac{0.0597}{(1.05)} + \frac{1 + 0.0597}{(1 + z_2)^2}$$

In the equation, 0.0597 and 1.0597 represent payments from interest and principal and interest, respectively, per unit of principal value. The equation implies that $z_2 = 6\%$. We have bootstrapped the two-year spot rate. Continuing with forward substitution, the three-year zero-coupon rate can be bootstrapped by solving for z_3 using the known values of the one-year and two-year spot rates of 5% and 6%:

$$1 = \frac{0.0691}{(1.05)} + \frac{0.0691}{(1.06)^2} + \frac{1 + 0.0691}{(1 + z_3)^3}$$

Thus, $z_3 = 7\%$. Finally, we solve for the four-year zero-coupon rate, z_4 :

$$1 = \frac{0.0781}{(1.05)} + \frac{0.0781}{(1.06)^2} + \frac{0.0781}{(1.07)^3} + \frac{1 + 0.0781}{(1 + z_4)^4}$$

In summary, $z_1 = 5\%$, $z_2 = 6\%$, $z_3 = 7\%$, and $z_4 = 8\%$.

In the preceding discussion, we considered an upward-sloping (spot) yield curve (Exhibit 2) and an inverted or downward-sloping (spot) yield curve (Exhibit 3). In developed markets, yield curves are most commonly upward sloping with diminishing marginal increases in yield for identical changes in maturity; that is, the yield curve “flattens” at longer maturities. Because nominal yields incorporate a premium for expected inflation, an upward-sloping yield curve is generally interpreted as reflecting a market expectation of rising or at least stable future inflation (associated with relatively strong economic growth). The existence of risk premiums (e.g., for the greater interest rate risk of longer-maturity bonds) also contributes to a positive slope.

An inverted yield curve (Exhibit 3) is less common. Such a term structure may reflect a market expectation of declining future inflation rates (because a nominal yield incorporates a premium for expected inflation) from a relatively high current level. Expectations of an economic slowdown may be one reason to anticipate a decline in inflation, and a downward-sloping yield curve is frequently observed before recessions. A flat yield curve typically occurs briefly in the transition from an upward-sloping to

a downward-sloping yield curve, or vice versa. A humped yield curve, which is relatively rare, occurs when intermediate-term interest rates are higher than short- and long-term rates.

2

YTM IN RELATION TO SPOT AND FORWARD RATES



describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management

Yield-to-maturity (YTM) is perhaps the most familiar pricing concept in bond markets. In this section, we clarify how it is related to spot rates and a bond's expected and realized returns.

How is the yield-to-maturity related to spot rates? In bond markets, most bonds outstanding have coupon payments and many have various options, such as a call provision. The YTM of these bonds with maturity T would not be the same as the spot rate at T but should be mathematically related to the spot curve. Because the principle of no arbitrage shows that a bond's value is the sum of the present values of payments discounted by their corresponding spot rates, the YTM of the bond should be some weighted average of spot rates used in the valuation of the bond.

Example 5 addresses the relationship between spot rates and YTM.

EXAMPLE 5

Spot Rate and Yield-to-Maturity

Recall from earlier examples the spot rates $z_1 = 9\%$, $z_2 = 10\%$, and $z_3 = 11\%$. Let y_T be the YTM.

1. Calculate the price of a two-year annual coupon bond using the spot rates. Assume the coupon rate is 6% and the face value is \$1,000. Next, state the formula for determining the price of the bond in terms of its YTM. Is z_2 greater than or less than y_2 ? Why?

Solution:

Using the spot rates,

$$\text{Price} = \frac{\$60}{(1 + 0.09)^1} + \frac{\$1,060}{(1 + 0.10)^2} = \$931.08$$

Using the YTM,

$$\text{Price} = \frac{\$60}{(1 + y_2)} + \frac{\$1,060}{(1 + y_2)^2} = \$931.08$$

Note that y_2 is used to discount both the first- and second-year cash flows. Because the bond can have only one price, it follows that $z_1 < y_2 < z_2$ because y_2 is a weighted average of z_1 and z_2 and the yield curve is upward sloping. Using a calculator, one can calculate the YTM as $y_2 = 9.97\%$, which is less than $z_2 = 10\%$ and greater than $z_1 = 9\%$, just as we would expect. Note that y_2 is much closer to z_2 than to z_1 because the bond's largest cash flow occurs in Year 2, thereby giving z_2 a greater weight than z_1 in the determination of y_2 .

2. Calculate the price of a three-year annual coupon-paying bond using the spot rates. Assume the coupon rate is 5% and the face value is £100. Next, write a formula for determining the price of the bond using the YTM. Is z_3 greater or less than y_3 ? Why?

Solution:

Using the spot rates,

$$\text{Price} = \frac{£5}{(1+0.09)^1} + \frac{£5}{(1+0.10)^2} + \frac{£105}{(1+0.11)^3} = £85.49$$

Using the yield-to-maturity,

$$\text{Price} = \frac{£5}{(1+y_3)} + \frac{£5}{(1+y_3)^2} + \frac{£105}{(1+y_3)^3} = £85.49.$$

Note that y_3 is used to discount all three cash flows. Because the bond can have only one price, y_3 must be a weighted average of z_1 , z_2 , and z_3 . Given that the yield curve is upward sloping in this example, $y_3 < z_3$. Using a calculator to compute YTM, $y_3 = 10.93\%$, which is less than $z_3 = 11\%$ and greater than $z_1 = 9\%$ —just as we would expect, because the weighted YTM must lie between the highest and lowest spot rates. Note that y_3 is much closer to z_3 than it is to z_2 or z_1 because the bond's largest cash flow occurs in Year 3, thereby giving z_3 a greater weight than z_1 and z_2 in the determination of y_3 .

Investors can expect to earn the yield-to-maturity on a bond only under extremely restrictive assumptions. The YTM is the expected rate of return for a bond held to maturity, assuming that all promised coupon and principal payments are made in full when due and that coupons are reinvested at the original YTM. As interest rates change, the reinvestment of coupons at the original YTM is unlikely. The YTM can provide a poor estimate of expected return if (1) interest rates are volatile, (2) the yield curve is sloped either upward or downward, (3) there is significant risk of default, or (4) the bond has one or more embedded options (e.g., put, call, or conversion). If either (1) or (2) is the case, reinvestment of coupons would not be expected to be at the assumed rate (YTM). Case 3 implies that actual cash flows may differ from those assumed in the YTM calculation, and in Case 4, the exercise of an embedded option would result in a holding period shorter than the bond's original maturity.

The realized return is the actual bond return during an investor's holding period. It is based on actual reinvestment rates and the yield curve at the end of the holding period. If we had perfect foresight, the expected bond return would equal the realized bond return.

To illustrate these concepts, assume that $z_1 = 5\%$, $z_2 = 6\%$, $z_3 = 7\%$, $z_4 = 8\%$, and $z_5 = 9\%$. Consider a five-year annual coupon bond with a coupon rate of 10%. The forward rates extrapolated from the spot rates are $f_{1,1} = 7.0\%$, $f_{2,1} = 9.0\%$, $f_{3,1} = 11.1\%$, and $f_{4,1} = 13.1\%$. The price, determined as a percentage of par, is 105.43.

The yield-to-maturity of 8.62% can be determined by solving

$$105.43 = \frac{10}{(1+y_5)} + \frac{10}{(1+y_5)^2} + \frac{10}{(1+y_5)^3} + \frac{10}{(1+y_5)^4} + \frac{110}{(1+y_5)^5}$$

The yield-to-maturity of 8.62% is the bond's expected return assuming no default, a holding period of five years, and a reinvestment rate of 8.62%. But what if the forward rates are assumed to be the future spot rates?

Using the forward rates as the expected reinvestment rates results in the following expected cash flow at the end of Year 5:

$$10(1+0.07)(1+0.09)(1+0.111)(1+0.131) + 10(1+0.09)(1+0.111)(1+0.131) + 10(1+0.111)(1+0.131) + 10(1+0.131) + 110 \approx 162.22$$

Therefore, the expected bond return is $(162.22 - 105.43)/105.43 = 53.87\%$ and the expected annualized rate of return is 9.00% [solve $(1 + x)^5 = 1 + 0.5387$].

From this example, we can see that the expected rate of return is not equal to the YTM even if we make the generally unrealistic assumption that the forward rates are the future spot rates. The YTM is generally a realistic estimate of expected return only if the yield curve is flat. Note that in the foregoing formula, all cash flows were discounted at 8.62% regardless of maturity.

Example 6 will reinforce your understanding of various yield and return concepts.

EXAMPLE 6

Yield and Return Concepts

1. When the spot curve is upward sloping, the forward curve:

- A. lies above the spot curve.
- B. lies below the spot curve.
- C. is coincident with the spot curve.

Solution:

A is correct. Points on a spot curve can be viewed as an average of single-period rates over given maturities, whereas forward rates reflect the marginal changes between future periods.

2. Which of the following statements concerning the YTM of a default-risk-free bond is *most* accurate? The YTM of such a bond:

- A. equals the expected return on the bond if the bond is held to maturity.
- B. can be viewed as a weighted average of the spot rates applying to its cash flows.
- C. will be closer to the realized return if the spot curve is upward sloping rather than flat through the life of the bond.

Solution:

B is correct. The YTM is the discount rate that, when applied to a bond's promised cash flows, equates those cash flows to the bond's market price and the fact that the market price should reflect discounting promised cash flows at appropriate spot rates.

3. When the spot curve is downward sloping, a later initiation date results in a forward curve that is:

- A. closer to the spot curve.
- B. a greater distance above the spot curve.
- C. a greater distance below the spot curve.

Solution:

C is correct. This answer follows from the forward rate model as expressed in Equation 6. If the spot curve is downward sloping (upward sloping), a later initiation date will result in a forward curve that is a greater distance below (above) the spot curve. See Exhibit 2 and Exhibit 3.

Yield Curve Movement and the Forward Curve

This section establishes several important results concerning forward prices and the spot yield curve to demonstrate the relevance of the forward curve to active bond investors.

The forward contract price remains unchanged as long as future spot rates evolve as predicted by today's forward curve. If a trader expects the future spot rate to be below what is predicted by the prevailing forward rate, the forward contract value is expected to increase and the trader would buy the forward contract. Conversely, if the trader expects the future spot rate to be above that predicted by the existing forward rate, then the forward contract value is expected to decrease and the trader would sell the forward contract.

Using the forward pricing model defined by Equation 2, we can determine the forward contract price that delivers a $(B - A)$ -period-maturity bond at time A , $F_{A,B-A}$, using Equation 8 (which is Equation 2 solved for the forward price):

$$F_{A,B-A} = \frac{DF_B}{DF_A} \quad (8)$$

Now suppose that after t periods, the new discount function for some maturity time T period, denoted as DF_T^{new} , is the same as the forward discount function implied by today's discount function, as shown by Equation 9.

$$DF_T^{new} = \frac{DF_{t+T}}{DF_t} \quad (9)$$

Next, after a lapse of t periods, the time to expiration of the contract is $A - t$, and the forward contract price at time t is $F_{A-t,B-A}^{new}$. Equation 8 can be rewritten as Equation 10:

$$F_{A-t,B-A}^{new} = \frac{DF_{B-t}^{new}}{DF_{A-t}^{new}} \quad (10)$$

Substituting Equation 9 into Equation 10 and adjusting for the lapse of time t results in Equation 11:

$$F_{A-t,B-A}^{new} = \frac{DF_{B-t}^{new}}{DF_{A-t}^{new}} = \frac{\frac{DF_B}{DF_t}}{\frac{DF_A}{DF_t}} = \frac{DF_B}{DF_A} = F_{A,B-A} \quad (11)$$

Equation 11 shows that the forward contract price remains unchanged as long as future spot rates are equal to what is predicted by today's forward curve. Therefore, a change in the forward price is the result of a deviation of the spot curve from what is predicted by today's forward curve.

To make these observations concrete, consider a flat yield curve for which the interest rate is 4%. Using Equation 1, the discount factors for the one-year, two-year, and three-year terms are, to four decimal places, as follows:

$$DF_1 = \frac{1}{(1 + 0.04)} = 0.9615$$

$$DF_2 = \frac{1}{(1 + 0.04)^2} = 0.9246$$

$$DF_3 = \frac{1}{(1 + 0.04)^3} = 0.8890$$

Therefore, using Equation 8, the forward contract price that delivers a one-year bond at Year 2 is

$$F_{2,1} = \frac{DF_3}{DF_2} = \frac{0.8890}{0.9246} = 0.9615.$$

Suppose the future discount function at Year 1 is the same as the forward discount function implied by the Year 0 spot curve. The lapse of time is $t = 1$. Using Equation 9, the discount factors for the one-year and two-year terms one year from today are as follows:

$$DF_1^{new} = \frac{DF_2}{DF_1} = \frac{0.9246}{0.9615} = 0.9615$$

$$DF_2^{new} = \frac{DF_3}{DF_1} = \frac{0.8890}{0.9615} = 0.9246$$

Using Equation 10, the price of the forward contract one year from today is

$$F_{2,1}^{new} = \frac{DF_2^{new}}{DF_1^{new}} = \frac{0.9246}{0.9615} = 0.9615.$$

The price of the forward contract is unchanged. This will be the case as long as future discount functions are the same as those based on today's forward curve.

From this numerical example, we can see that if the spot rate curve is unchanged, then each bond "rolls down" the curve and earns the current one-period spot rate and subsequent forward rates. Specifically, when one year passes, a three-year bond will return $(0.9246 - 0.8890)/0.8890 = 4\%$, which is equal to the spot rate. Furthermore, if another year passes, the bond will return $(0.9615 - 0.9246)/0.9246 = 4\%$, which is equal to the implied forward rate for a one-year security one year from today.

3

ACTIVE BOND PORTFOLIO MANAGEMENT



describe the strategy of rolling down the yield curve

One way that active bond portfolio managers attempt to outperform the bond market's return is by anticipating changes in interest rates relative to the projected evolution of spot rates reflected in today's forward curves.

The forward rate model (Equation 4) provides insight into these issues. By rearranging terms in Equation 4 and setting the time horizon to one period, $A = 1$, we obtain

$$\frac{(1 + z_B)^B}{(1 + f_{A,B-A})^{B-A}} = (1 + z_A)^A. \quad (12)$$

The numerator of the left-hand side of Equation 12 is for a bond with an initial maturity of B periods and a remaining maturity of $B - A$ periods after A periods pass. Suppose the prevailing spot yield curve after one period ($A = 1$) is the current forward curve; then, Equation 12 shows that the total return on the bond is the one-period risk-free rate. The following sidebar shows that returns on bonds of varying tenor over a one-year period always equal the one-year rate (the risk-free rate over the one-year period) if the spot rates evolve as implied by the current forward curve at the end of the first year.

WHEN SPOT RATES EVOLVE AS IMPLIED BY THE CURRENT FORWARD CURVE

As in earlier examples, assume the following:

$$z_1 = 9\%$$

$$z_2 = 10\%$$

$$z_3 = 11\%$$

$$f_{1,1} = 11.01\%$$

$$f_{1,2} = 12.01\%$$

If the spot curve one year from today reflects the current forward curve, the return on a zero-coupon bond for the one-year holding period is 9%, regardless of the bond's maturity. The following computations assume a par amount of 100 and represent the percentage change in price. Given the rounding of price and the forward rates to the nearest hundredth, the returns all approximate 9%. With no rounding, however, all answers would be precisely 9%.

The return of the one-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 91.74 and is worth the par amount of 100 at maturity.

$$\left(100 \div \frac{100}{1+z_1}\right) - 1 = \left(100 \div \frac{100}{1+0.09}\right) - 1 = \frac{100}{91.74} - 1 = 9\%.$$

The return of the two-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 82.64. One year from today, the two-year bond has a remaining maturity of one year. Its price one year from today is 90.08, determined as the par amount divided by 1 plus the forward rate for a one-year bond issued one year from today.

$$\left(\frac{100}{(1+f_{1,1})} \div \frac{100}{(1+z_2)^2}\right) - 1 = \left(\frac{100}{(1+0.1101)} \div \frac{100}{(1+0.10)^2}\right) - 1 = \frac{90.08}{82.64} - 1 = 9\%$$

The return of the three-year zero-coupon bond over the one-year holding period is 9%. The bond is purchased at a price of 73.12. One year from today, the three-year bond has a remaining maturity of two years. Its price one year from today of 79.71 reflects the forward rate for a two-year bond issued one year from today.

$$\left(\frac{100}{(1+f_{1,2})^2} \div \frac{100}{(1+z_3)^3}\right) - 1 = \left(\frac{100}{(1+0.1201)^2} \div \frac{100}{(1+0.11)^3}\right) - 1 = \frac{79.71}{73.12} - 1 \approx 9\%$$

This numerical example shows that the return of a bond over a one-year period is always the one-year rate (the risk-free rate over the one period) if the spot rates evolve as implied by the current forward curve.

But if the spot curve one year from today differs from today's forward curve, the returns on each bond for the one-year holding period will not all be 9%. To show that the returns on the two-year and three-year bonds over the one-year holding period are not 9%, we assume that the spot rate curve at Year 1 is flat with yields of 10% for all maturities.

The return on a one-year zero-coupon bond over the one-year holding period is

$$\left(100 \div \frac{100}{1+0.09}\right) - 1 = 9\%.$$

The return on a two-year zero-coupon bond over the one-year holding period is

$$\left(\frac{100}{1+0.10} \div \frac{100}{(1+0.10)^2} \right) - 1 = 10\%.$$

The return on a three-year zero-coupon bond over the one-year holding period is

$$\left(\frac{100}{(1+0.10)^2} \div \frac{100}{(1+0.11)^3} \right) - 1 = 13.03\%.$$

The bond returns are 9%, 10%, and 13.03%. The returns on the two-year and three-year bonds differ from the one-year risk-free interest rate of 9%.

Equation 12 provides a total return investor with a means to evaluate the cheapness or expensiveness of a bond of a certain maturity. If any of the investor's expected future spot rates is below a quoted forward rate for the same maturity, then (all else being equal) the investor would perceive the bond to be undervalued, in the sense that the market is effectively discounting the bond's payments at a higher rate than the investor and the bond's market price is below the intrinsic value perceived by the investor.

Another example will reinforce the point that if a portfolio manager's projected spot curve is above (below) the forward curve and his expectation turns out to be true, the return will be less (more) than the one-period risk-free interest rate.

For the sake of simplicity, assume a flat yield curve of 8% and that a trader holds a three-year bond paying an 8% annual coupon. Assuming a par value of 100, the current market price is also 100. If today's forward curve turns out to be the spot curve one year from today, the trader will earn an 8% return.

If the trader projects that the spot curve one year from today is above today's forward curve—for example, a flat yield curve of 9%—the trader's expected rate of return is 6.24%, which is less than 8%:

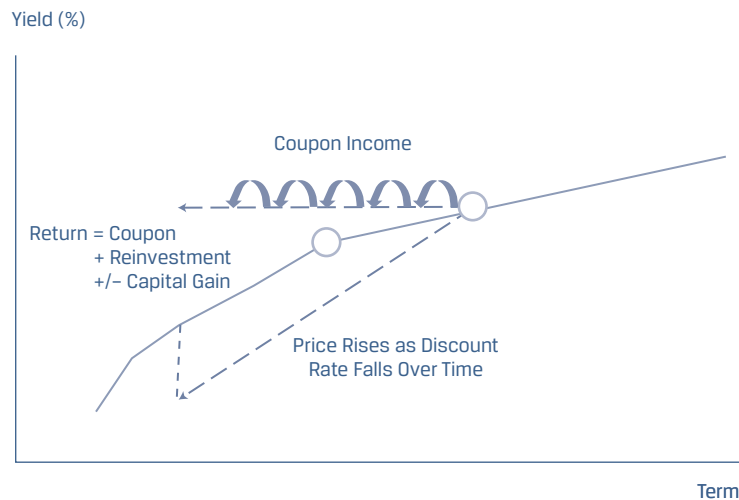
$$\frac{8 + \frac{8}{1+0.09} + \frac{108}{(1+0.09)^2}}{100} - 1 = 6.24\%$$

If the trader predicts a flat yield curve of 7%, the trader's expected return is 9.81%, which is greater than 8%:

$$\frac{8 + \frac{8}{1+0.07} + \frac{108}{(1+0.07)^2}}{100} - 1 = 9.81\%$$

As the gap between the projected future spot rate and the forward rate widens, so too will the difference between the trader's expected return and the original YTM of 8%.

This logic is the basis for a popular yield curve trade called **rolling down the yield curve**, also referred to as riding the yield curve. As we have noted, when a yield curve is upward sloping, the forward curve is always above the current spot curve. If the trader expects the yield curve to remain static over an investment horizon, then buying bonds with a maturity longer than the investment horizon would provide a total return greater than the return on a maturity-matching strategy. The bond's total return will depend on the spread between the forward rate and the spot rate as well as the maturity of the bond. The longer the bond's maturity, the more sensitive its total return is to the spread. This strategy is shown in Exhibit 4.

Exhibit 4: Rolling Down the Yield Curve

The return on a yield curve rolldown strategy may be demonstrated using a simple example. As stated earlier, the investment return on a fixed-rate (non-defaulted and non-callable) bond return may be defined as follows:

Bond return = Receipt of promised coupons (and principal)

+ Reinvestment of coupon payments (12)

+/- Capital gain/Loss on sale prior to maturity

Say we observe one-, three-, four-, five- and six-year spot rates on annual coupon bonds trading at par of 2%, 4%, 5%, 6%, and 7%, respectively. An investor with a five-year maturity target decides to forgo a matched-maturity 6% five-year bond in favor of the 7%, six-year bond given her expectation of an unchanged yield curve over the next two years. We can compare the annualized return over two years for both bonds, assuming unchanged yields, as follows.

The 6% five-year bond purchased for 100 returns 117.67 in two years $[(6 \times 1.02) + 6 + 105.55]$, which consists of the first year's coupon reinvested at the one-year rate, the second annual coupon, and the capital gain on the sale of the 6% bond with three years to maturity at an unchanged three-year yield of 4% $[105.55 = 6/1.04 + 6/(1.04)^2 + 106/(1.04)^3]$. The annualized rate of return is 8.476% [solve for r , where $(117.67/100) = (1 + r)^2$].

The 7% six-year bond purchased at par returns 121.23 in two years $[(7 \times 1.02) + 7 + 107.09]$ with an annualized return of 1 bond 10.10%. The excess return of nearly 2% results from both higher coupon income than the five-year matched maturity bond as well as a larger capital gain on the sale of the 7% bond with four years to maturity at an unchanged four-year yield of 5% $[107.09 = 7/1.05 + 7/(1.05)^2 + 7/(1.05)^3 + 107/(1.05)^4]$.

In the years following the 2008 financial crisis, many central banks acted to keep short-term interest rates very low. As a result, yield curves subsequently had a steep upward slope (see Exhibit 2). For active fixed-income managers, this situation provided an incentive to access short-term funding and invest in long-term bonds. This is just one form of a carry trade, referred to as a maturity spread carry trade, and is subject to significant interest rate risk, such as an unexpected increase in future spot rates

(e.g., as a result of a spike in inflation). The maturity spread carry trade, in which the trader borrows short term and lends long term in the same currency, is common in an upward-sloping yield curve environment.

In summary, when the yield curve slopes upward, as a bond approaches maturity or “rolls down the yield curve,” it is valued at successively lower yields and higher prices. Using this strategy, a bond can be held for a period of time as it appreciates in price and then sold before maturity to realize a higher return. As long as interest rates remain stable and the yield curve retains an upward slope, this strategy can continuously add to the total return of a bond portfolio.

Example 7 addresses how the preceding analysis relates to active bond portfolio management.

EXAMPLE 7

Active Bond Portfolio Management

1. The “rolling down the yield curve” strategy is executed by buying bonds whose maturities are:
 - A. equal to the investor’s investment horizon.
 - B. longer than the investor’s investment horizon.
 - C. shorter than the investor’s investment horizon.

Solution:

B is correct. A bond with a longer maturity than the investor’s investment horizon is purchased but then sold prior to maturity at the end of the investment horizon. If the yield curve is upward sloping and yields do not change, the bond will be valued at successively lower yields and higher prices over time. The bond’s total return will exceed that of a bond whose maturity is equal to the investment horizon.

2. A bond will be overvalued if the expected spot rate is:
 - A. equal to the current forward rate.
 - B. lower than the current forward rate.
 - C. higher than the current forward rate.

Solution:

C is correct. If the expected discount rate is higher than the forward rate, then the bond will be overvalued. The expected price of the bond is lower than the price obtained from discounting using the forward rate.

3. Assume a flat yield curve of 6%. A three-year £100 bond is issued at par paying an annual coupon of 6%. What is the portfolio manager’s expected return if he predicts that the yield curve one year from today will be a flat 7%?
 - A. 4.19%
 - B. 6.00%
 - C. 8.83%

Solution:

A is correct. Expected return will be less than the current YTM of 6% if yields increase to 7%. The expected return of 4.19% is computed as follows:

$$\frac{6 + \frac{6}{1+0.07} + \frac{106}{(1+0.07)^2}}{100} - 1 \approx 4.19\%$$

4. A forward contract price will increase if:

- A. future spot rates evolve as predicted by current forward rates.
- B. future spot rates are lower than what is predicted by current forward rates.
- C. future spot rates are higher than what is predicted by current forward rates.

Solution:

B is correct. The forward rate model can be used to show that a change in the forward contract price requires a deviation of the spot curve from that predicted by today's forward curve. If the future spot rate is lower than what is predicted by the prevailing forward rate, the forward contract price will increase because it is discounted at an interest rate that is lower than the originally anticipated rate.

THE SWAP RATE CURVE

4



explain the swap rate curve and why and how market participants use it in valuation

Earlier, we described the spot rate curve of default-risk-free bonds as a measure of the time value of money. The swap rate curve, or swap curve for short, is another important representation of the time value of money used in fixed-income markets. Here we will discuss how the swap curve is used in valuation, where the spread of swap rates over government benchmark rates is a proxy for perceived credit risk relative to risk-free debt.

Swap Rate Curve

Interest rate swaps are an integral part of the fixed-income market. These derivative contracts usually involve the net exchange, or swap, of fixed-rate for floating-rate interest payments, and these contracts are an essential tool for investors who use them to hedge, speculate on, or otherwise modify risk. The fixed and floating payments are determined by multiplying the respective rate by a principal (or notional) amount for each interest period over the swap maturity. The rate for the fixed leg of an interest rate swap is known as the **swap rate**. The swap rate is analogous to the YTM on a government bond, which as we saw earlier may be derived from zero rates using bootstrapping. The key difference between the swap rate and the government bond rate is that the swap rate is derived using short-term lending rates rather than default-risk-free rates. Swap floating rates historically referenced short-term survey-based interest rates, such as three- or six-month US dollar Libor (London Interbank Offered Rate) and are slated to transition to transaction-based market reference rates (MRR) based on secured overnight funding transactions. The yield curve of swap rates is called the **swap rate curve** or, more simply, the **swap curve**. Because it is based on so-called **par swaps**, in which the fixed rate is set so that no money is exchanged at contract

initiation—the present values of the fixed-rate and benchmark floating-rate legs being equal—the swap curve is a type of par curve. When we refer to the “par curve” here, however, the reference is to the government par yield curve.

The swap market is a highly liquid market for two reasons. First, unlike bonds, a swap does not have multiple borrowers or lenders, only counterparties who exchange cash flows. Such arrangements offer significant flexibility and customization in the swap contract’s design. Second, swaps provide one of the most efficient ways to hedge interest rate risk. The Bank for International Settlements (BIS) estimates that the notional amount outstanding on interest rate swaps was nearly \$350 trillion as of June 2020.

Many countries do not have a liquid government bond market with maturities longer than one year. The swap curve is a necessary market benchmark for interest rates in these countries. In countries where the private sector is much bigger than the public sector, the swap curve is a far more relevant measure of the time value of money than is the government’s cost of borrowing.

Swaps are frequently used as a benchmark in Europe, whereas in Asia, the swap markets and the government bond markets have developed in parallel, and both are used in valuation in credit and loan markets.

Why Do Market Participants Use Swap Rates When Valuing Bonds?

Government spot curves and swap rate curves are the chief reference curves in fixed-income valuation. The choice between them can depend on multiple factors, including the relative liquidity of these two markets. In the United States, where there is both an active Treasury security market and a swap market, the choice of a benchmark for the time value of money often depends on the interest rate exposure profile of the institution using the benchmark. On one hand, wholesale banks frequently use the swap curve to value assets and liabilities because they hedge their balance sheet with swaps. On the other hand, retail banks with little exposure to the swap market are more likely to use the government spot curve as their benchmark.

Let us illustrate how a financial institution uses the swap market for its internal operations. Consider the case of a bank raising funds using a certificate of deposit (CD). Assume the bank can borrow \$10 million in the form of a CD that bears interest of 1.5% for a two-year term. Another \$10 million CD offers 1.70% for a three-year term. The bank can arrange two swaps: (1) The bank receives 1.50% fixed and pays MRR minus 10 bps with a two-year term and a notional amount of \$10 million, and (2) the bank receives 1.70% fixed and pays MRR minus 15 bps with a three-year term and a notional amount of \$10 million. After issuing the two CDs and committing to the two swaps, the bank has raised \$20 million with an annual funding cost for the first two years of MRR minus 12.5 bps applied to the total notional amount of \$20 million. The fixed interest payments received from the counterparty to the swap are paid to the CD investors; in effect, fixed-rate liabilities have been converted to floating-rate liabilities. The margins on the floating rates become the standard by which value is measured in assessing the bank’s total funding cost.

By using the swap curve as a benchmark for the time value of money, the investor can adjust the swap spread so that the swap will be fairly priced given the spread. Conversely, given a swap spread, the investor can determine a fair price for the bond. We will use the swap spread in the following section to determine the value of a bond.

How Do Market Participants Use the Swap Curve in Valuation?

Although benchmark swap rates are quoted for specific maturities, swap contracts may be customized by two parties in the over-the-counter market. The fixed payment can be specified by an amortization schedule or involve a coupon with non-standard payment dates. In this section, we will focus on zero-coupon bonds. The yields on these bonds determine the swap curve, which, in turn, can be used to determine bond values.

Each forward date has an associated discount factor that represents the value today of a unit payment that one would hypothetically receive on the forward date expressed as a decimal fraction. For example, if we expect to receive ₩10,000 (10,000 South Korean won) in one year and the current price of the security is ₩9,259.30, then the discount factor for one year will be 0.92593 ($= ₩9,259.30/₩10,000$). Note that the rate associated with this discount factor is $1/0.92593 - 1 \approx 8.00\%$.

To price a swap using current market rates, as mentioned we must solve for a constant fixed rate that sets the present value of fixed-leg payments equal to the present value of floating-leg payments over the life of the swap. Once established, the fixed cash flows are specified by the coupon rate set at the time of the original agreement. Pricing the floating leg is more complex than pricing the fixed leg because, by definition, its cash flows change with future changes in interest rates. The forward rate for each floating payment date is calculated by using the forward curves.

Let s_T stand for the T -period swap rate. Because the value of a swap at origination is set to zero, the swap rates must satisfy Equation 13. Note that the swap rates can be determined from the spot rates and the spot rates can be determined from the swap rates.

$$\sum_{t=1}^T \frac{s_T}{(1+z_t)^t} + \frac{1}{(1+z_T)^T} = 1 \quad (13)$$

The right-hand side of Equation 13 is the value of the floating leg, which is always 1 at origination. The swap rate is determined by equating the value of the fixed leg, on the left-hand side, to the value of the floating leg.

Example 8 addresses the relationship between the swap rate curve and spot curve.

EXAMPLE 8

Determining the Swap Rate Curve

Suppose a government spot curve implies the following discount factors:

$$DF_1 = 0.9524$$

$$DF_2 = 0.8900$$

$$DF_3 = 0.8163$$

$$DF_4 = 0.7350$$

1. Given this information, determine the swap rate curve.

Solution:

Recall from Equation 1 that $DF_N = \frac{1}{(1+Z_N)^N}$. Therefore,

$$z_N = \left(\frac{1}{DF_N} \right)^{1/N} - 1$$

$$z_1 = \left(\frac{1}{0.9524} \right)^{1/1} - 1 = 5.00\%$$

$$z_2 = \left(\frac{1}{0.8900} \right)^{1/2} - 1 = 6.00\%$$

$$z_3 = \left(\frac{1}{0.8163} \right)^{1/3} - 1 = 7.00\%$$

$$z_4 = \left(\frac{1}{0.7350} \right)^{1/4} - 1 = 8.00\%$$

Using Equation 12, for $N = 1$,

$$\frac{s_1}{(1+z_1)} + \frac{1}{(1+z_1)} = \frac{s_1+1}{(1+0.05)} = 1$$

Therefore, $s_1 = 5\%$.

For $T = 2$,

$$\frac{s_2}{(1+z_1)} + \frac{s_2}{(1+z_2)^2} + \frac{1}{(1+z_2)^2} = \frac{s_2}{(1+0.05)} + \frac{s_2+1}{(1+0.06)^2} = 1$$

Therefore, $s_2 = 5.97\%$.

For $T = 3$,

$$\begin{aligned} & \frac{s_3}{(1+z_1)} + \frac{s_3}{(1+z_2)^2} + \frac{s_3}{(1+z_3)^3} + \frac{1}{(1+z_3)^3} \\ &= \frac{s_3}{(1+0.05)} + \frac{s_3}{(1+0.06)^2} + \frac{s_3}{(1+0.07)^3} + \frac{1}{(1+0.07)^3} = 1 \end{aligned}$$

Therefore, $s_3 = 6.91\%$.

For $T = 4$,

$$\begin{aligned} & \frac{s_4}{(1+z_1)} + \frac{s_4}{(1+z_2)^2} + \frac{s_4}{(1+z_3)^3} + \frac{s_4}{(1+z_4)^4} + \frac{1}{(1+z_4)^4} \\ &= \frac{s_4}{(1+0.05)} + \frac{s_4}{(1+0.06)^2} + \frac{s_4}{(1+0.07)^3} + \frac{s_4}{(1+0.08)^4} + \frac{1}{(1+0.08)^4} = 1 \end{aligned}$$

Therefore, $s_4 = 7.81\%$.

Note that the swap rates, spot rates, and discount factors are all mathematically linked together. Having access to data for one of the series allows you to calculate the other two.

5

THE SWAP SPREAD AND SPREADS AS A PRICE QUOTATION CONVENTION

- ☐ calculate and interpret the swap spread for a given maturity
- ☐ describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk

The swap spread is a popular way to indicate credit spreads in a market. The **swap spread** is defined as the spread paid by the fixed-rate payer of an interest rate swap over the rate of the “on-the-run” (most recently issued) government security with the same maturity as the swap. The spread captures the yield premium required for credit relative to the benchmark government bond. Because swap rates are built from market rates for short-term risky debt, this spread is a barometer of the market’s

perceived credit risk relative to default-risk-free rates. This spread typically widens countercyclically, exhibiting greater values during recessions and lower values during economic expansions.

The term “swap spread” is sometimes also used as a reference to a bond’s basis point spread over the interest rate swap curve and is a measure of the credit and/or liquidity risk of a bond. Here, a swap spread is an excess yield of swap rates over the yields on government bonds, and we use the terms I-spread, ISPRD, or interpolated spread to refer to bond yields net of the swap rates of the same maturities. In its simplest form, the I-spread can be measured as the difference between the yield-to-maturity of the bond and the swap rate given by a straight-line interpolation of the swap curve.

Often, fixed-income prices will be quoted as a swap rate plus (or minus) a spread, for which the yield is simply the yield on an equal-maturity government bond plus the swap spread. For example, if the fixed rate of a five-year fixed-for-float MRR swap is 2.00% and the five-year Treasury is yielding 1.70%, the swap spread is $2.00\% - 1.70\% = 0.30\%$, or 30 bps.

For euro-denominated swaps, the government yield used as a benchmark is most frequently Bunds (German government bonds) with the same maturity. Gilts (UK government bonds) are used as a benchmark in the United Kingdom.

Although the Libor swap curve is being phased out, it has historically been considered to reflect the default risk of A1/A+ rated commercial banks. The transition from Libor to MRR based on secured overnight funding rates will increase the influence of demand and supply conditions in government debt markets on swap rates. Another reason for the popularity of the swap market is that it is led by major financial institutions rather than controlled by governments, so swap rates are more comparable across different countries. The swap market also has more maturities with which to construct a yield curve than do government bond markets. Historically, cash or deposit rates such as Libor have been used for short-maturity yields; interest rate futures such as Eurodollar futures contracts have maturities of up to a year; and swap rates extend to maturities of up to 50 years in US dollars or euro. As the market transitions from Libor, the concept of this spread will be consistent with whichever market-based alternative to Libor emerges.

HISTORY OF THE US SWAP SPREAD SINCE 2008

The fact that governments generally pay less than private entities do in order to borrow suggests that swap spreads should always be positive. However, the 30-year Treasury swap spread turned negative following the collapse of Lehman Brothers Holdings Inc. in September 2008. Strong demand for duration combined with tighter liquidity and greater counterparty risk were widely cited as reasons for this phenomenon. For the period shown, the 30-year Treasury swap spread hit a record low (−62 bps intramonth) during November 2008. The 30-year Treasury swap spread was at or above zero for more than a year before becoming negative once again (see Exhibit 5). A recent study by the Federal Reserve Bank of New York (Boyarchenko, Gupta, Steele, and Yen, 2018) suggests that negative swap spreads have persisted because of increased regulatory capital requirements among swap dealers following the financial crisis.

Exhibit 5: US Swap Spread, January 2008–May 2020 (monthly data)

To illustrate the use of the swap spread in fixed-income pricing, consider a US\$1 million investment in GE Capital (GECC) notes with a coupon rate of 1 5/8% (1.625%) that matures on 2 July 2024. Coupons are paid semiannually. The evaluation date is 12 July 2021, so the remaining maturity is 2.97 years [= 2 + (350/360)]. The Treasury rates for two-year and three-year maturities are 0.525% and 0.588%, respectively. By simple interpolation between these two rates, the US Treasury rate for 2.97 years is 0.586% [= 0.525% + (350/360)(0.588% - 0.525%)]. If the swap spread for the same maturity is 0.918%, then the yield-to-maturity on the bond (assuming the *i*-spread is zero for this case) is 1.504% (= 0.918% + 0.586%). Given the yield-to-maturity, the invoice price (price including accrued interest) for US\$1 million face value is as follows:

$$\begin{aligned} & \frac{1,000,000 \left(\frac{0.01625}{2} \right)}{\left(1 + \frac{0.01504}{2} \right)^{\left(1 - \frac{10}{180} \right)}} + \frac{1,000,000 \left(\frac{0.01625}{2} \right)}{\left(1 + \frac{0.01504}{2} \right)^{\left(2 - \frac{10}{180} \right)}} + \dots + \\ & \frac{1,000,000 \left(\frac{0.01625}{2} \right)}{\left(1 + \frac{0.01504}{2} \right)^{\left(6 - \frac{10}{180} \right)}} + \frac{1,000,000}{\left(1 + \frac{0.01504}{2} \right)^{\left(6 - \frac{10}{180} \right)}} = \text{US\$1,003,954.12} \end{aligned}$$

The left-hand side sums the present values of the semiannual coupon payments and the final principal payment of US\$1,000,000. The accrued interest rate amount is US\$451.39 [= 1,000,000 × (0.01625/2)(10/180)]. Therefore, the clean price (price not including accrued interest) is US\$1,003,502.73 (= 1,003,954.12 - 451.39).

The swap spread helps an investor to identify the time value, credit, and liquidity components of a bond's YTM. If the bond is default free, then the swap spread could provide an indication of the bond's liquidity, or it could provide evidence of market mispricing. The higher the swap spread, the higher the return that investors require for credit and/or liquidity risks. Another approach introduced in an earlier reading is to calculate a constant yield spread over a government (or interest rate swap) spot curve instead. This spread is known as the zero volatility spread (Z-spread) of a bond over the benchmark rate.

Spreads as a Price Quotation Convention

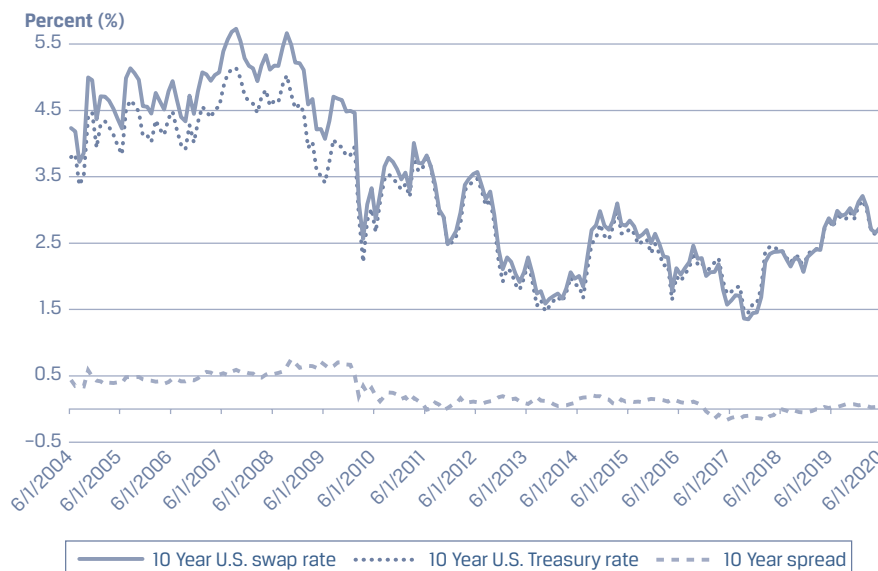
Treasury curves and swap curves represent different benchmarks for fixed-income valuation. It is therefore important to distinguish between a bond price quote that uses the bond yield net of a benchmark Treasury yield and one that uses a swap rate.

The Treasury rate can differ from the swap rate for the same term for several reasons. Unlike the cash flows from US Treasury bonds, the cash flows from swaps are subject to greater default risk. Market liquidity for specific maturities may differ. For example, some parts of the term structure of interest rates may be more actively traded with swaps than with Treasury bonds. Finally, arbitrage between these two markets cannot be perfectly executed.

Swap spreads to the Treasury rate (as opposed to **I-spreads**, which are bond rates net of the swap rates of the same maturities) are simply the differences between swap rates and government bond yields of a particular maturity. One problem in defining swap spreads is that, for example, a 10-year swap matures in exactly 10 years, whereas this condition is true for a 10-year government bond only at the time of issuance. By convention, therefore, the 10-year swap spread is defined as the difference between the 10-year swap rate and the 10-year on-the-run government bond. Swap spreads of other maturities are defined similarly.

The curves in Exhibit 6 show the relationship between 10-year Treasury notes and 10-year swap rates. The 10-year swap spread is the 10-year swap rate less the 10-year Treasury note yield. Although positive swap spreads reflecting the difference between MRR-based rates and default-risk-free US government yields were historically the norm, these spreads have narrowed to zero or negative levels since the 2008 financial crisis because of higher swap dealer capital requirements and leverage constraints.

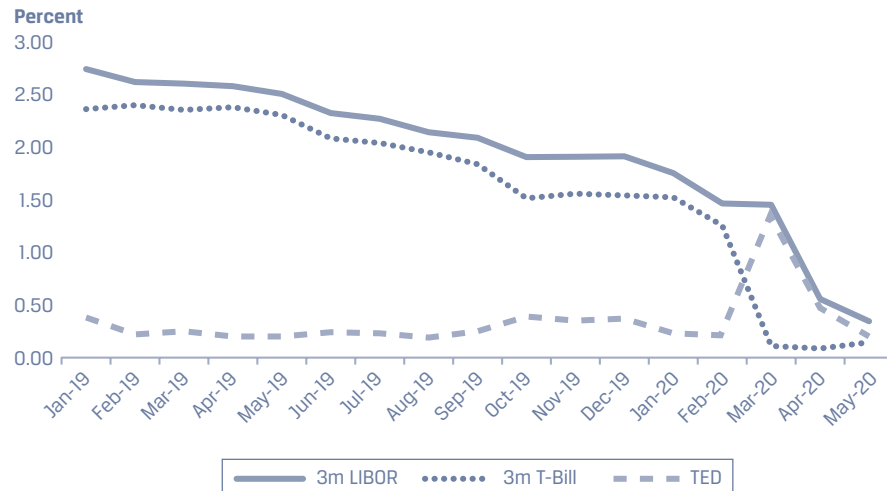
Exhibit 6: 10-Year US Swap Rate vs. 10-Year US Treasury Rate



Market participants often use interest rate spreads between short-term government and risky rates as a barometer to evaluate relative credit and liquidity risk. For example, the difference between MRR and the yield on a Treasury bill of the same maturity, or **TED spread**, has historically been a key indicator of perceived credit and liquidity risk. TED is an acronym formed from an abbreviation for the US T-bill (T) and the

ticker symbol for the MRR-based Eurodollar futures contract (ED). Exhibit 7 shows the historical TED spread. An increase in the TED spread signals greater perceived credit and liquidity risk, as occurred in early 2020 amid market turmoil related to the COVID-19 pandemic.

Exhibit 7: TED Spread, January 2019–May 2020 (end-of-month data)



Another popular measure of such risk is the MRR–OIS spread, formerly the **Libor–OIS spread**, which is the difference between MRR and the **overnight indexed swap (OIS) rate**. An OIS is an interest rate swap in which the periodic floating rate of the swap equals the geometric average of a daily unsecured overnight rate (or overnight index rate). The index rate is typically the rate for overnight unsecured lending between banks, such as the federal funds rate for US dollars or Eonia (Euro OverNight Index Average) for euros. As market participants transition away from survey-based Libor to alternative benchmarks based on actual transaction data, the **secured overnight financing rate (SOFR)**, or overnight cash borrowing rate collateralized by US Treasuries, has gained prominence and is expected to replace Libor in the future. A barometer of the US Treasury repurchase (or repo) market, SOFR is a daily volume-weighted index of all qualified repo market transactions and is influenced by supply and demand conditions in secured funding markets. The shift to overnight secured funding benchmarks extends globally—for example, the secured European Short-Term Rate (ESTR) has been recommended to replace Eonia, and the Canadian Overnight Repo Rate Average (CORRA) is proposed to replace the survey-based unsecured Canadian Dollar Offered Rate (CDOR).

6

TRADITIONAL THEORIES OF THE TERM STRUCTURE OF INTEREST RATES



explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve

This section presents four traditional theories of the underlying economic factors that affect the shape of the yield curve.

Expectations Theory

One branch of traditional term structure theory focuses on interpreting term structure shape in terms of investors' expectations. Historically, the first such theory is known as the **unbiased expectations theory**, also called **pure expectations theory**. It says that the forward rate is an unbiased predictor of the future spot rate; its broadest interpretation is that bonds of any maturity are perfect substitutes for one another. For example, buying a bond with a maturity of five years and holding it for three years has the same expected return as buying a three-year bond or buying a series of three one-year bonds.

The predictions of the unbiased expectations theory are consistent with the assumption of risk neutrality. In a risk-neutral world, investors are unaffected by uncertainty and risk premiums do not exist. Every security is risk free and yields the risk-free rate for that particular maturity. Although such an assumption leads to interesting results, it clearly is in conflict with the large body of evidence showing that investors are risk averse.

A theory that is similar but more rigorous than the unbiased expectations theory is the **local expectations theory**. Rather than asserting that every maturity strategy has the same expected return over a given investment horizon, this theory instead contends that the expected return for every bond over short periods is the risk-free rate. This conclusion results from an assumed no-arbitrage condition in which bond pricing does not allow for traders to earn arbitrage profits.

The primary way that the local expectations theory differs from the unbiased expectations theory is that it can be extended to a world characterized by risk. Although the theory requires that risk premiums be nonexistent for very short holding periods, no such restrictions are placed on longer-term investments. Thus, the theory is applicable to both risk-free as well as risky bonds.

Although the local expectations theory is economically appealing, it is often observed that short-holding-period returns on long-dated bonds in fact exceed those on short-dated bonds. The need for liquidity and the ability to hedge risk essentially ensure that the demand for short-term securities will exceed that for long-term securities. Thus, both the yields and the actual returns for short-dated securities are typically lower than those for long-dated securities.

Liquidity Preference Theory

Whereas expectations theories leave no room for risk aversion, liquidity preference theory attempts to account for it. **Liquidity preference theory** asserts that **liquidity premiums** exist to compensate investors for the added interest rate risk they face when lending long term and that these premiums increase with maturity. Thus, given an expectation of unchanging short-term spot rates, liquidity preference theory predicts an upward-sloping yield curve. The forward rate provides an estimate of the expected spot rate that is biased upward by the amount of the liquidity premium, which invalidates the unbiased expectations theory. The liquidity premium for each consecutive future period should be no smaller than that for the prior period.

For example, the US Treasury offers bonds that mature in 30 years. Most investors, however, have shorter investment horizons than 30 years. For investors to hold these bonds, they would demand a higher return for taking the risk that the yield curve changes and that they must sell the bond prior to maturity at an uncertain price. That incrementally higher return is the liquidity premium. Note that this premium is not

to be confused with a yield premium for the lack of liquidity that thinly traded bonds may bear. Rather, it is a premium applying to all long-term bonds, including those with deep markets.

Liquidity preference theory fails to offer a complete explanation of the term structure. Rather, it simply argues for the existence of liquidity premiums. For example, a downward-sloping yield curve could still be consistent with the existence of liquidity premiums if one of the factors underlying the shape of the curve is an expectation of deflation (i.e., a negative rate of inflation resulting from monetary or fiscal policy actions). Expectations of sharply declining spot rates may also result in a downward-sloping yield curve if the expected decline in interest rates is severe enough to offset the effect of the liquidity premiums.

In summary, liquidity preference theory claims that lenders require a liquidity premium as an incentive to lend long term. Thus, forward rates derived from the current yield curve provide an upwardly biased estimate of expected future spot rates. Although downward-sloping or hump-shaped yield curves may sometimes occur, the existence of liquidity premiums implies that the yield curve will typically be upward sloping.

Segmented Markets Theory

Unlike expectations theory and liquidity preference theory, **segmented markets theory** allows for lender and borrower preferences to influence the shape of the yield curve. The result is that yields are not a reflection of expected spot rates or liquidity premiums. Rather, they are solely a function of the supply and demand for funds of a particular maturity. That is, each maturity sector can be thought of as a segmented market in which yield is determined independently from the yields that prevail in other maturity segments.

The theory is consistent with a world in which asset/liability management constraints exist, either regulatory or self-imposed. In such a world, investors might restrict their investment activity to a maturity sector that provides the best match for the maturity of their liabilities. Doing so avoids the risks associated with an asset/liability mismatch.

For example, because life insurers sell long-term liabilities against themselves in the form of life insurance contracts, they tend to be most active as buyers in the long end of the bond market. Similarly, because the liabilities of pension plans are long term, they typically invest in long-term securities. Why would they invest short term given that those returns might decline while the cost of their liabilities stays fixed? In contrast, money market funds would be limited to investing in debt with maturity of one year or less, in general.

In summary, the segmented markets theory assumes that market participants are either unwilling or unable to invest in anything other than securities of their preferred maturity. It follows that the yield of securities of a particular maturity is determined entirely by the supply and demand for funds of that particular maturity.

Preferred Habitat Theory

The **preferred habitat theory** is similar to the segmented markets theory in proposing that many borrowers and lenders have strong preferences for particular maturities, but it does not assert that yields at different maturities are determined independently of each other.

The theory contends, however, that if the expected additional returns to be gained become large enough, institutions will be willing to deviate from their preferred maturities or habitats. For example, if the expected returns on longer-term securities exceed those on short-term securities by a large enough margin, an intermediate-term bond fund might lengthen the maturities of their assets. And if the excess returns expected

from buying short-term securities become large enough, life insurance companies might stop limiting themselves to long-term securities and place a larger part of their portfolios in shorter-term investments.

The preferred habitat theory is based on the realistic notion that agents and institutions will accept additional risk in return for additional expected returns. In accepting elements of both the segmented markets theory and the unbiased expectations theory, yet rejecting their extreme polar positions, the preferred habitat theory moves closer to explaining real-world phenomena. In this theory, both market expectations and the institutional factors emphasized in the segmented markets theory influence the term structure of interest rates.

PREFERRED HABITAT AND QE

The term “quantitative easing” (QE) refers to an unconventional monetary policy used by central banks to increase the supply of money in an economy when central bank and/or interbank interest rates are already close to zero. The first of several QE efforts by the US Federal Reserve began in late 2008, following the establishment of a near-zero target range for the federal funds rate. Since then, the Federal Reserve has greatly expanded its holdings of long-term securities via a series of asset purchase programs, with the goal of putting downward pressure on long-term interest rates and thereby making financial conditions even more accommodative. Exhibit 8 presents information regarding the securities held by the Federal Reserve on 20 September 2007 (when all securities held by the Fed were US Treasury issuance) and on 29 October 2014 (when the Federal Reserve ended its third round of QE).

Exhibit 8: Securities Held by the US Federal Reserve

(US\$ billions)	20 Sep 2007	29 Oct 2014
Securities held outright	780	4,219
US Treasury	780	2,462
Bills	267	0
Notes and bonds, nominal	472	2,347
Notes and bonds, inflation indexed	36	115
Inflation compensation	5	16
Federal agency	0	40
Mortgage-backed securities	0	1,718

As Exhibit 8 shows, the Federal Reserve’s security holdings on 20 September 2007 consisted entirely of US Treasury securities, and about 34% of those holdings were short term in the form of T-bills. On 29 October 2014, only about 58% of the Federal Reserve’s security holdings were Treasury securities, and none were T-bills. Furthermore, the Federal Reserve held well over US\$1.7 trillion of mortgage-backed securities (MBS), which accounted for 41% of all securities held.

Prior to the QE efforts, the yield on MBS was typically in the 5%–6% range. It declined to less than 2% by the end of 2012. Concepts related to preferred habitat theory could possibly help explain that drop in yield.

The purchase of MBS by the Federal Reserve reduced the supply of these securities that was available for private purchase. Assuming that many MBS investors are either unwilling or unable to withdraw from the MBS market because of their comparative experience and expertise in managing interest rate and repayment risks of MBS versus option-free bonds, MBS investing institutions

would have a “preferred habitat” in the MBS market. If they were unable to meet investor demand without bidding more aggressively, these buyers would drive down yields on MBS.

The Federal Reserve’s purchase of MBS also resulted in a reduction in MBS yields. If a homeowner prepays on a mortgage, the payment is sent to MBS investors on a pro rata basis. Although investors are uncertain about when such a prepayment will be received, prepayment is more likely in a declining interest rate environment.

Use Example 9 to test your understanding of traditional term structure theories.

EXAMPLE 9

Traditional Term Structure Theories

1. Many fixed-income portfolio managers are limited in or prohibited from high-yield bond investments. When a bond is downgraded from an investment-grade to a high-yield (junk) rating, it is referred to as a *fallen angel*. Because of restrictions, many pension funds sell fallen angels when they are downgraded from investment grade to high yield (junk). This coordinated selling action often results in depressed prices and attractive yields for the fallen angels. Which of the following reasons best explains why fallen angel yields often exceed otherwise identical bonds?
 - A. The preferred habitat theory
 - B. The segmented markets theory
 - C. The local expectations theory

Solution:

B is correct. Market segmentation in this example results from the requirement that some fixed-income fund managers are prohibited or limited in their capacity to hold high-yield bonds. The segmentation results in selling pressure on fallen angels that depresses their prices.

2. The term structure theory in which investors can be induced by relatively attractive yields to hold debt securities whose maturities do not match their investment horizon is *best* described as the:
 - A. preferred habitat theory.
 - B. segmented markets theory.
 - C. unbiased expectations theory.

Solution:

A is correct. Preferred habitat theory asserts that investors are willing to deviate from their preferred maturities if yield differentials encourage the switch. Segmented markets theory is more rigid than preferred habitat in that asset/liability management constraints force investors to buy securities whose horizons match those of their liabilities. The unbiased expectations theory makes no assumptions about maturity preferences. Rather, it contends that forward rates are unbiased predictors of future spot rates.

3. The unbiased expectations theory assumes investors are:
 - A. risk averse.

- B. risk neutral.
- C. risk seeking.

Solution:

B is correct. The unbiased expectations theory asserts that different maturity strategies, such as rollover, maturity matching, and riding the yield curve, have the same expected return. By definition, a risk-neutral party is indifferent about choices with equal expected payoffs, even if one choice is riskier. Thus, the predictions of the theory are consistent with the existence of risk-neutral investors.

4. Market evidence shows that forward rates are:

- A. unbiased predictors of future spot rates.
- B. upwardly biased predictors of future spot rates.
- C. downwardly biased predictors of future spot rates.

Solution:

B is correct. The existence of a liquidity premium ensures that the forward rate is an upwardly biased estimate of the future spot rate. Market evidence clearly shows that liquidity premiums exist, and this evidence effectively refutes the predictions of the unbiased expectations theory.

5. Market evidence shows that short holding-period returns on short-maturity bonds *most* often are:

- A. less than those on long-maturity bonds.
- B. about equal to those on long-maturity bonds.
- C. greater than those on long-maturity bonds.

Solution:

A is correct. Although the local expectations theory predicts that the short-run return for all bonds will equal the risk-free rate, most of the evidence refutes that claim. Returns from long-dated bonds are generally higher than those from short-dated bonds, even over relatively short investment horizons. This market evidence is consistent with the risk-expected return trade-off that is central to finance and the uncertainty surrounding future spot rates.

YIELD CURVE FACTOR MODELS

7



explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks

The effect of yield volatilities on price is an important consideration in fixed-income investment, particularly for risk management and portfolio evaluation. In this section, we describe measuring and managing the interest rate risk of bonds.

A Bond's Exposure to Yield Curve Movement

Shaping risk is defined as the sensitivity of a bond's price to the changing shape of the yield curve. The yield curve's shape changes continually, and yield curve shifts are rarely parallel. For active bond management, a bond investor may want to base trades on a forecasted yield curve shape or may want to hedge the yield curve risk on a bond portfolio using swaps. Shaping risk also affects the value of many options, which is very important because many fixed-income instruments have embedded options.

Exhibit 9 and Exhibit 10 show historical yield curve movements for US and European swap rates from March 2006 until March 2020. The exhibits show the considerable swap yield curve changes over time. In both cases, the pre-financial-crisis March 2006 yield curves represent the highest swap yields and those from March 2020 (amid the COVID-19 pandemic-related market turmoil) the lowest. In the United States, however, the end of quantitative easing and tighter monetary policy resulted in a rebound in swap yields prior to 2020, whereas in Europe, yields remained low or negative because of continued accommodative monetary policy. Note that the vertical axis values of the three exhibits differ, and the horizontal axis is not to scale.

Exhibit 9: Historical US Swap Yield Curve Movements

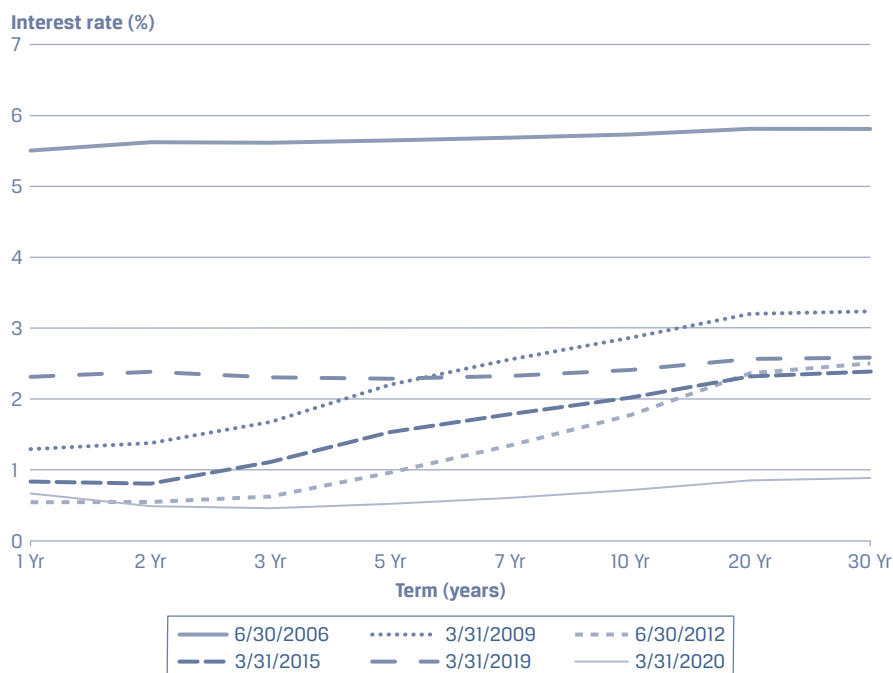
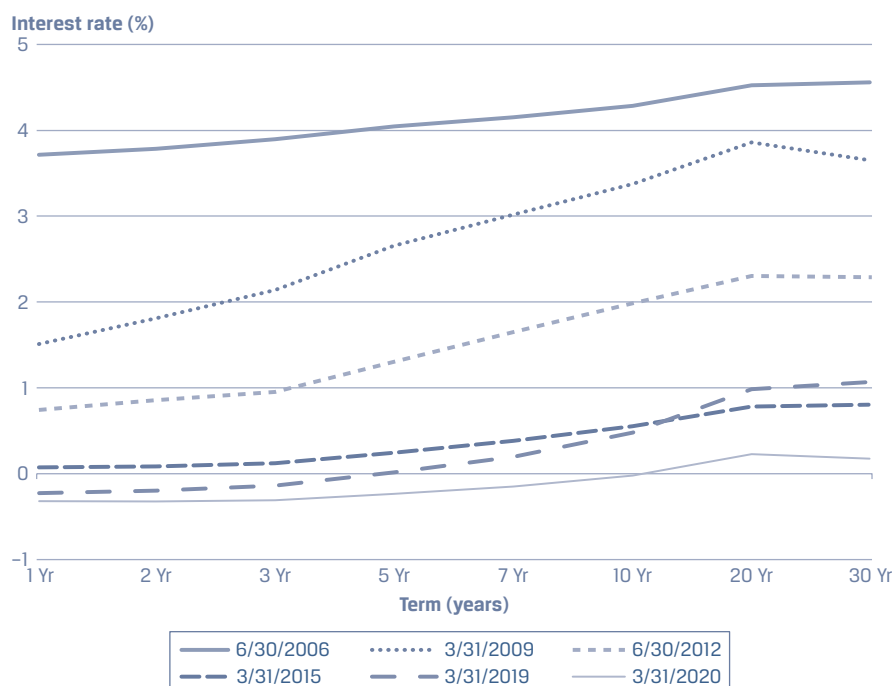
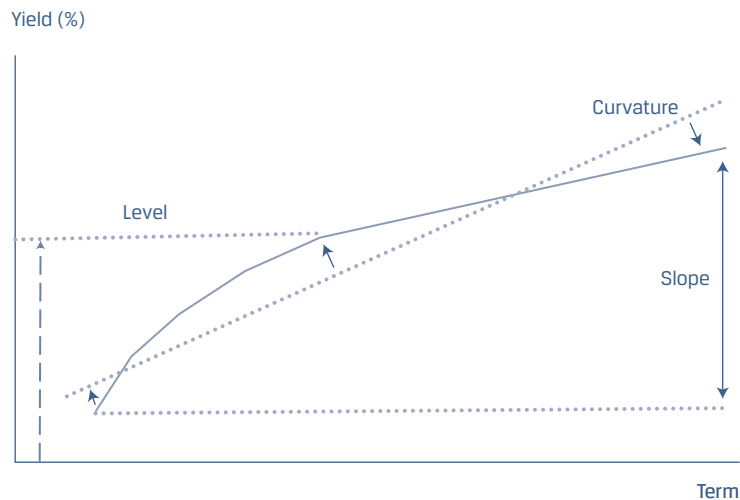


Exhibit 10: Historical European Swap Yield Curve Movements

Factors Affecting the Shape of the Yield Curve

The previous section showed that the yield curve can take nearly any shape. The challenge for a fixed-income manager is to implement a process to manage the yield curve shape risk in her portfolio. One approach is to find a model that reduces most of the possible yield curve movements to a probabilistic combination of a few standardized yield curve movements. This section presents one of the best-known yield curve factor models.

A **yield curve factor model** is defined as a model or a description of yield curve movements that can be considered realistic when compared with historical data. Research has led to models that can describe these movements with some accuracy. One specific yield curve factor model is the three-factor model of Litterman and Scheinkman (1991), who found that yield curve movements are historically well described by a combination of three independent movements, which they interpreted as **level**, **steepness**, and **curvature**. The level movement refers to an upward or downward shift in the yield curve. The steepness movement refers to a non-parallel shift in the yield curve when either short-term rates change more than long-term rates or long-term rates change more than short-term rates. The curvature movement is a reference to movement in three segments of the yield curve: The short-term and long-term segments rise while the middle-term segment falls, or vice versa. Exhibit 11 illustrates these factors.

Exhibit 11: Primary Yield Curve Factors: Level, Slope, and Curvature

In practice, the level movement factor explains most of the total changes in swap and bond market yields. This factor may be interpreted as a reflection of parallel yield curve moves in which rates move in the same direction and by a similar order of magnitude. The steepness factor addresses the shape of the curve, with short-term yields typically moving more than long-term yields. These changes take place over time and therefore explain less of the total variance in rates than the level factor. Finally, the third factor, curvature, tends to have a negative impact on intermediate yields and a positive impact on short- and long-term yields. This variable explaining the “twist” in the yield curve has the smallest impact of the three.

8

THE MATURITY STRUCTURE OF YIELD CURVE VOLATILITIES



explain the maturity structure of yield volatilities and their effect on price volatility

Yield Volatility

Quantifying interest rate volatilities is important for fixed income managers for at least two reasons. First, most fixed-income instruments and derivatives have embedded options. Option values, and hence the values of the fixed-income instrument, crucially depend on the level of interest rate volatilities. Second, fixed-income interest rate risk management is clearly an important part of any management process, and such risk management includes controlling the impact of interest rate volatilities on the instrument’s price volatility.

The term structure of interest rate volatilities is a representation of the yield volatility of a zero-coupon bond for every maturity of security. This volatility curve (or “vol”) or volatility term structure measures yield curve risk.

Interest rate volatility is not the same for all interest rates along the yield curve. On the basis of the typical assumption of a lognormal model, the uncertainty of an interest rate is measured by the annualized standard deviation of the proportional change in a bond yield over a specified interval. For example, if the interval is a one-month period, then the specified interval equals 1/12 years. This measure, called interest rate volatility, is denoted $\sigma(t, T)$, which is the volatility of the rate for a security with maturity T at time t . The term structure of volatilities is given by Equation 14:

$$\sigma(t, T) = \frac{\sigma[\Delta r(t, T) / r(t, T)]}{\sqrt{\Delta t}} \quad (14)$$

In Exhibit 12, to illustrate a term structure of volatility, the data series is deliberately chosen to end before the 2008 financial crisis, which was associated with some unusual volatility magnitudes.

Exhibit 12: Historical Example: US Treasuries, August 2005–December 2007

Maturity (years)	0.25	0.50	1	2	3	5	7	10	20	30
$\sigma(t, T)$	0.3515	0.3173	0.2964	0.2713	0.2577	0.2154	0.1885	0.1621	0.1332	0.1169

For example, the 35.15% standard deviation for the three-month T-bill in Exhibit 12 is based on a monthly standard deviation of $0.1015 = 10.15\%$, which annualizes as

$$0.1015 \div \sqrt{\frac{1}{12}} = 0.3515 = 35.15\%.$$

The volatility term structure typically shows that short-term rates are more volatile than long-term rates. That said, long-term bond *prices* tend to vary more than short-term bond prices given the impact of duration. Research indicates that short-term volatility is most strongly linked to uncertainty regarding monetary policy, whereas long-term volatility is most strongly linked to uncertainty regarding the real economy and inflation. Furthermore, most of the co-movement between short-term and long-term volatilities appears to depend on the ever-changing correlations among these three determinants (monetary policy, the real economy, and inflation). During the period of August 2005–December 2007, long-term volatility was lower than short-term volatility, falling from 35.15% for the 0.25-year rate to 11.69% for the 30-year rate.

Managing Yield Curve Risks Using Key Rate Duration

Yield curve risk—the risk to portfolio value arising from unanticipated changes in the yield curve—can be managed on the basis of several measures of sensitivity to yield curve movements. Management of yield curve risk involves changing the identified exposures to desired values by trades in security or derivative markets (the details fall under the rubric of fixed-income portfolio management and thus are outside the scope of this reading).

One available measure of yield curve sensitivity is effective duration, which measures the sensitivity of a bond's price to a small parallel shift in a benchmark yield curve. Another is based on **key rate duration**, which measures a bond's sensitivity to a small change in a benchmark yield curve at a specific maturity segment. Using one of these last two measures allows identification and management of “shaping risk”—that is, sensitivity to changes in the shape of the benchmark yield curve—in addition to the risk associated with parallel yield curve changes, which is addressed adequately by effective duration.

To make the discussion more concrete, consider a portfolio of 1-year, 5-year, and 10-year zero-coupon bonds with \$100 value in each position; total portfolio value is therefore \$300. Also consider the hypothetical set of factor movements shown in the following table:

Year	1	5	10
Parallel	1	1	1
Steepness	-1	0	1
Curvature	1	0	1

In the table, a parallel movement or shift means that all the rates shift by an equal amount—in this case, by a unit of 1. A steepness movement means that the yield curve steepens with the long rate shifting up by one unit and the short rate shifting down by one unit. A curvature movement means that both the short rate and the long rate shift up by one unit, whereas the medium-term rate remains unchanged. These movements need to be defined, as they are here, such that none of the movements can be a linear combination of the other two movements. Next, we address the calculation of the various yield curve sensitivity measures.

Because the bonds are zero-coupon bonds, each bond's effective duration is the same as its maturity. The portfolio's effective duration is the weighted sum of the effective duration of each bond position; for this equally weighted portfolio, effective duration is $0.333(1 + 5 + 10) = 5.333$.

To calculate key rate durations, consider various yield curve movements. First, suppose that the one-year rate changes by 100 bps while the other rates remain the same; the sensitivity of the portfolio to that shift is $1/[(300)(0.01)] = 0.3333$. We conclude that the key rate duration of the portfolio ($KeyDur_{Full}$) to the one-year rate, denoted $KeyDur_1$, is 0.3333. Likewise, the key rate durations of the portfolio to the 5-year rate, $KeyDur_5$, and the 10-year rate, $KeyDur_{10}$, are 1.6667 and 3.3333, respectively. Note that the sum of the key rate durations is 5.333, which is the same as the effective duration of the portfolio. This fact can be explained intuitively. Key rate duration measures the portfolio risk exposure to each key rate. If all the key rates move by the same amount, then the yield curve has made a parallel shift, and as a result, the proportional change in value has to be consistent with effective duration. The related model for yield curve risk based on key rate durations ($KeyDur$) is as follows:

$$\begin{aligned} KeyDur_{Full} = \% \Delta P &= \left(\frac{\Delta P}{P} \right) \approx -KeyDur_1 \Delta z_1 - KeyDur_5 \Delta z_5 - KeyDur_{10} \Delta z_{10} \\ &= -0.3333 \Delta z_1 - 1.6667 \Delta z_5 - 3.3333 \Delta z_{10} \end{aligned} \quad (15)$$

Next, we can calculate a measure based on the decomposition of yield curve movements into parallel, steepness, and curvature movements, as described earlier. Define D_L , D_S , and D_C as the sensitivities of portfolio value to small changes in the level, steepness, and curvature factors, respectively. Based on this factor model, Equation 16 shows the proportional change in portfolio value that would result from a small change in the level factor (Δx_L), the steepness factor (Δx_S), and the curvature factor (Δx_C).

$$\begin{aligned} KeyDur_{Full} = \% \Delta P \\ &= \left(\frac{\Delta P}{P} \right) \approx -KeyDur_L \Delta x_L - KeyDur_S \Delta x_S - KeyDur_C \Delta x_C \end{aligned} \quad (16)$$

Because $KeyDur_L$ is by definition sensitivity to a parallel shift, the proportional change in the portfolio value per unit shift (the line for a parallel movement in the table) is $5.3333 = (1 + 5 + 10)/[(300)(0.01)]$. The sensitivity for steepness movement can be calculated as follows (see the line for steepness movement in the table). When the steepness makes an upward shift of 100 bps, it would result in a downward shift of 100

bps for the 1-year rate, resulting in a gain of \$1, and an upward shift for the 10-year rate, resulting in a loss of \$10. The change in value is therefore $(1 - 10) \cdot \text{KeyDur}_5$ is the negative of the proportional change in price per unit change in this movement and in this case is $3.0 = -(1 - 10)/[(300)(0.01)]$. Considering the line for curvature movement in the table, $\text{KeyDur}_C = 3.6667 = (1 + 10)/[(300)(0.01)]$. Thus, for our hypothetical bond portfolio, we can analyze the portfolio's yield curve risk using the following equation:

$$\text{KeyDur}_{Full} = \% \Delta P = \left(\frac{\Delta P}{P} \right) \approx -5.3333 \Delta x_L - 3.0 \Delta x_S - 3.6667 \Delta x_C$$

For example, if $\Delta x_L = -0.0050$, $\Delta x_S = 0.002$, and $\Delta x_C = 0.001$, the predicted change in portfolio value would be +1.7%. It can be shown that key rate durations are directly related to level, steepness, and curvature in this example and that one set of sensitivities can be derived from the other. One can use the numerical example to verify that relation by decomposing changes in the term structure into level, slope, and curvature factors:

$$\text{KeyDur}_L = \text{KeyDur}_1 + \text{KeyDur}_5 + \text{KeyDur}_{10}$$

$$\text{KeyDur}_S = -\text{KeyDur}_1 + \text{KeyDur}_{10}$$

$$\text{KeyDur}_C = \text{KeyDur}_1 + \text{KeyDur}_{10}$$

Example 10 reviews concepts from this section and the preceding sections.

EXAMPLE 10

Term Structure Dynamics

1. The most important factor in explaining changes in the yield curve has been found to be:

- A. level.
- B. curvature.
- C. steepness.

Solution:

A is correct. Research shows that upward and downward shifts in the yield curve explain more than 75% of the total change in the yield curve.

2. A movement of the yield curve in which the short rate decreases by 150 bps and the long rate decreases by 50 bps would *best* be described as a:

- A. flattening of the yield curve resulting from changes in level and steepness.
- B. steepening of the yield curve resulting from changes in level and steepness.
- C. steepening of the yield curve resulting from changes in steepness and curvature.

Solution:

B is correct. Both the short-term and long-term rates have declined, indicating a change in the level of the yield curve. Short-term rates have declined more than long-term rates, indicating a change in the steepness of the yield curve.

3. The yield curve starts off flat, and then intermediate-maturity yields decrease by 10 bps while short- and long-maturity yields remain constant. This movement is *best* described as involving a change in:

- A. level only.
- B. curvature only.
- C. level and curvature.

Solution:

B is correct. The curve starts off flat, with identical short, intermediate, and long rates. Both the short-term and long-term rates remained constant, indicating no change in the level of the yield curve. Intermediate rates decreased, however, resulting in curvature.

4. Typically, short-term interest rates:

- A. are less volatile than long-term interest rates.
- B. are more volatile than long-term interest rates.
- C. have about the same volatility as long-term interest rates.

Solution:

B is correct. A possible explanation is that expectations for long-term inflation and real economic activity affecting longer-term interest rates are slower to change than those related to shorter-term interest rates.

5. Suppose for a given portfolio that key rate changes are considered to be changes in the yield on 1-year, 5-year, and 10-year securities. Estimated key rate durations are $KeyDur_1 = 0.50$, $KeyDur_2 = 0.70$, and $KeyDur_3 = 0.90$. What is the percentage change in the value of the portfolio if a parallel shift in the yield curve results in all yields declining by 50 bps?

- A. -1.05%.
- B. +1.05%.
- C. +2.10%.

Solution:

B is correct. A decline in interest rates would lead to an increase in bond portfolio value: $-0.50(-0.005) - 0.70(-0.005) - 0.90(-0.005) = 0.0105 = 1.05\%$.

9

DEVELOPING INTEREST RATE VIEWS USING MACROECONOMIC VARIABLES



explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes

Interest rate dynamics such as changes in spot versus forward rates and the level, steepness, and curvature of the yield curve are influenced by key economic variables and market events. Implied forward rates serve as market-neutral reference points for fixed income traders. As we illustrated earlier, if today's forward rates are realized in

the future, then bond values will simply roll down the yield curve. In practice, active fixed-income market participants establish their own views on future interest rate developments and then position their portfolios in order to capitalize on differences between their own rate view and the market consensus. If their forecast is accurate, the portfolio generates greater returns than it would have otherwise.

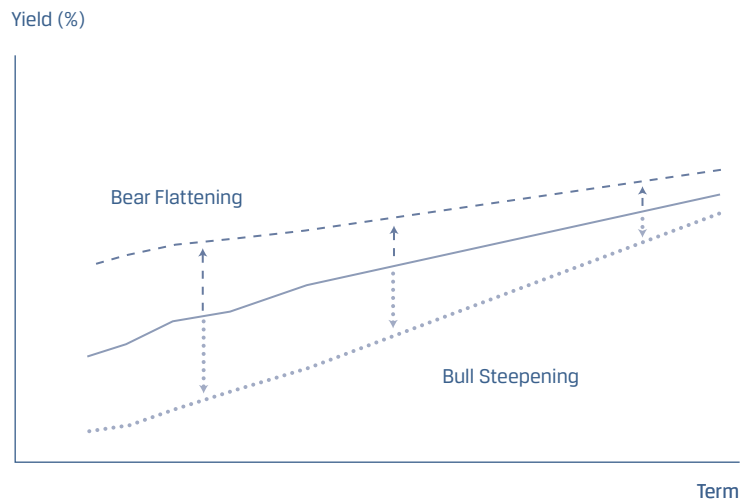
This section reviews the key drivers of interest rates before moving on to establishing views and positioning fixed-income portfolios to capitalize on a specific interest rate view.

The term **bond risk premium** refers to the expected excess return of a default-free long-term bond less that of an equivalent short-term bond or the one-period risk-free rate. This premium is also referred to as the term (or duration) premium, and it is usually measured using government bonds to capture uncertainty of default-free rates, whereas credit, liquidity, and other risks may increase the overall risk premium for a specific bond. Unlike *ex post* observed historical returns, the bond risk premium is a forward-looking expectation and must be estimated.

Several macroeconomic factors influence bond pricing and required returns such as inflation, economic growth, and monetary policy, among others.

Research shows that although inflation, GDP, and monetary policy explain most of the variance of bond yields, short- and intermediate-term bond yields are driven mostly by monetary policy, whereas long-term rate volatility is mostly linked to uncertainty regarding the real economy and inflation. Monetary policy explains about two-thirds of short- and intermediate-term bond yield variation, with the remaining third roughly equally attributable to economic growth and factors including inflation. In contrast, inflation explains nearly two-thirds of long-term yield variation, and the remaining third is largely attributable to monetary policy.

Monetary policy impacts the bond risk premium. Central banks such as the European Central Bank control the money supply and influence interest rates through policy tools in order to achieve stable prices and sustainable economic growth. During economic expansions, monetary authorities raise benchmark rates to help control inflation. This action is often consistent with **bearish flattening**, or short-term bond yields rising more than long-term bond yields, resulting in a flatter yield curve. During economic recessions or anticipated recessions, the monetary authority cuts benchmark rates to help stimulate economic activity. The lowering of interest rates is associated with **bullish steepening**, in which short-term rates fall by more than long-term yields, resulting in a steeper term structure. These monetary policy actions lead to procyclical short-term interest rate changes. Exhibit 13 shows these two yield curve changes.

Exhibit 13: Examples of Yield Curve Flattening and Steepening

In recent years, central banks have increasingly used their balance sheets for large-scale asset purchases. For example, the Federal Reserve has bought large quantities of US Treasury bonds and mortgage-backed securities. The intended purpose is to stimulate economic activity by increasing the money supply through benchmark bond purchases and driving down the bond risk premium, encouraging capital allocation to incrementally higher-risk assets. Asset purchases impact the term structure by raising demand in a range of maturity segments.

Other factors that influence bond prices, yields, and the bond risk premium include fiscal policy, the maturity structure of debt, and investor demand.

Benchmark government bonds are the means by which nations fund their cumulative (current and past) budget deficits. Greater deficits require more borrowing, which influences both bond supply and required yield. Thus, fiscal supply-side effects affect bond prices and yields by increasing (decreasing) yields when budget deficits rise (fall). In the late 1990s, market participants believed the US government would run fiscal surpluses, leading to a reduction in government bond supply as the Treasury stopped issuing new 30-year bonds for four years. The expected reduction in supply drove long-maturity Treasury yields lower.

Longer government debt maturity structures predict greater excess bond returns. This is effectively a segmented market factor, wherein the greater supply of bonds of long-term maturity increases the yield in that market segment.

Domestic investor demand is a key driver of bond prices, especially among pension funds and insurance companies that use long-dated government bonds to match expected future liabilities. Greater domestic investor demand increases prices and reduces the bond risk premium.

Non-domestic investor demand influences government bond prices and may result either from holding reserves or from actions associated with currency exchange rate management. Non-domestic flows significantly influence bond prices because inflows (outflows) bid up (down) bond prices, lowering (raising) the bond risk premium.

During highly uncertain market periods, investors flock to government bonds in what is termed a **flight to quality**. This term refers to investors' selling off higher-risk asset classes such as stocks and commodities in favor of default-risk-free government bonds. A flight to quality is often associated with **bullish flattening**, in which the yield curve flattens as long-term rates fall by more than short-term rates.

Fixed-income trades based on interest rate forecasts can take a variety of forms, often using bond futures contracts to avoid significant portfolio turnover. Remember that any interest rate view must be evaluated relative to the current short rate and forward curve, because they reflect returns earned by investors rolling down the curve under the current set of implied forward rates.

Investors expecting interest rates to fall will generally extend portfolio duration relative to a benchmark to take advantage of bond price increases from falling rates, whereas investors expecting higher rates will shorten portfolio duration to reduce exposure to falling bond prices.

To capitalize on a steeper curve under which long-term rates rise relative to short-term rates, traders will short long-term bonds and purchase short-term bonds. If on the other hand a trader forecasts curve flattening, whereby short-term rates rise relative to long-term rates, she may capitalize on this trend by purchasing long-term bonds and selling short-term bonds short. In both the expected steepening and flattening trades, the position may be designed as duration neutral in order to insulate from changes in the level of the term structure. Fixed-income investors with long-only investment mandates may alternate between portfolios concentrated in a single maturity, known as a **bullet portfolio**, and those with similar duration that combine short and long maturities, known as a **barbell portfolio**. For example, an investor may seek to capitalize on an expected bullish flattening of the yield curve by shifting from a bullet to a barbell position.

EXAMPLE 11

Building a Rate View Based On Economic Forecasts and Monetary Policy

Morgan Salaz is a fixed income analyst responsible for advising fixed income clients about bond trading opportunities. In the current recessionary environment, the level of government bond yields is low and the term structure is nearly flat. Salaz's firm forecasts that after a brief recession, economic growth will return quickly during the coming 12 months.

1. Which of the following changes to the yield curve is consistent with Salaz's expectation of increasing economic growth over the coming year?

- A. Decrease in the level
- B. Decrease in the term spread of long-term rates over short-term rates
- C. Increase in the term spread of long-term rates over short-term rates

Solution:

C is correct. Economic growth forecasts impact long-term rates. The view that economic growth will return to robust levels is consistent with a shift to a positively sloped term structure.

2. Salaz also expects the Federal Reserve to decrease asset purchases of long-term bonds as the economic recovery continues. Which of the following scenarios is consistent with this view? The reduced asset purchases will likely:

- A. amplify the effect of increased economic activity on the term spread.
- B. dampen the effect of increased economic activity on the term spread.

C. have no effect on the term spread.

Solution:

A is correct. Reduced asset purchases constitute a negative shift in demand for longer-term bonds, which raises their yields. The reduced asset purchases of long-maturity bonds would add to the effect of greater economic activity, both of which will increase the term spread.

SUMMARY

- The spot rate for a given maturity can be expressed as a geometric average of the short-term rate and a series of forward rates.
- Forward rates are above (below) spot rates when the spot curve is upward (downward) sloping, whereas forward rates are equal to spot rates when the spot curve is flat.
- If forward rates are realized, then all bonds, regardless of maturity, will have the same one-period realized return, which is the first-period spot rate.
- If the spot rate curve is upward sloping and is unchanged, then each bond “rolls down” the curve and earns the forward rate that rolls out of its pricing (i.e., an N -period zero-coupon bond earns the N -period forward rate as it rolls down to be a $N - 1$ period security). This dynamic implies an expected return in excess of short-maturity bonds (i.e., a **term premium**) for longer-maturity bonds if the yield curve is upward sloping.
- Active bond portfolio management is consistent with the expectation that today’s forward curve does not accurately reflect future spot rates.
- The swap curve provides another measure of the time value of money.
- Swaps are an essential tool frequently used by investors to hedge, take a position in, or otherwise modify interest rate risk.
- Bond quote conventions often use measures of spreads. Those quoted spreads can be used to determine a bond’s price.
- Swap curves and Treasury curves can differ because of differences in their credit exposures, liquidity, and other supply/demand factors.
- Market participants often use interest rate spreads between short-term government and risky rates as a barometer to evaluate relative credit and liquidity risk.
- The local expectations theory, liquidity preference theory, segmented markets theory, and preferred habitat theory provide traditional explanations for the shape of the yield curve.
- Historical yield curve movements suggest that they can be explained by a linear combination of three principal movements: level, steepness, and curvature.
- The volatility term structure can be measured using historical data and depicts yield curve risk.
- The sensitivity of a bond value to yield curve changes may make use of effective duration, key rate durations, or sensitivities to parallel, steepness, and curvature movements. Using key rate durations or sensitivities to parallel, steepness, and curvature movements allows one to measure and manage shaping risk.

- The term bond risk premium refers to the expected excess return of a default-free long-term bond less that of an equivalent short-term bond or the one-period risk-free rate
- Several macroeconomic factors influence bond pricing and required returns such as inflation, economic growth, and monetary policy, among others.
- During highly uncertain market periods, investors flock to government bonds in a flight to quality that is often associated with bullish flattening, in which long-term rates fall by more than short-term rates.
- Investors expecting rates to fall will generally extend (shorten) portfolio duration to take advantage of expected bond price increases (decreases)
- When investors expect a steeper (flatter) curve under which long-term rates rise (fall) relative to short-term rates, they will sell (buy) long-term bonds and purchase (sell) short-term bonds.

REFERENCES

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PRACTICE PROBLEMS

1. Given spot rates for one-, two-, and three-year zero coupon bonds, how many forward rates can be calculated?
2. Give two interpretations for the following forward rate: The two-year forward rate one year from now is 2%.
3. Describe the relationship between forward rates and spot rates if the yield curve is flat.
4. Which forward rate cannot be computed from the one-, two-, three-, and four-year spot rates? The rate for a:
 - A. one-year loan beginning in two years
 - B. two-year loan beginning in two years
 - C. three-year loan beginning in two years
5. Consider spot rates for three zero-coupon bonds: $z(1) = 3\%$, $z(2) = 4\%$, and $z(3) = 5\%$. Which statement is correct? The forward rate for a one-year loan beginning in one year will be:
 - A. less than the forward rate for a one-year loan beginning in two years.
 - B. greater than the forward rate for a two-year loan beginning in one year.
 - C. greater than the forward rate for a one-year loan beginning in two years.
6. If one-period forward rates are decreasing with maturity, the yield curve is *most likely*:
 - A. flat.
 - B. upward sloping.
 - C. downward sloping.

The following information relates to questions 7-17

A one-year zero-coupon bond yields 4.0%. The two- and three-year zero-coupon bonds yield 5.0% and 6.0%, respectively.

7. The rate for a one-year loan beginning in one year is *closest* to:
 - A. 4.5%.
 - B. 5.0%.
 - C. 6.0%.
8. The forward rate for a two-year loan beginning in one year is *closest* to:
 - A. 5.0%.

- B. 6.0%.
 - C. 7.0%.
9. The forward rate for a one-year loan beginning in two years is *closest* to:
- A. 6.0%.
 - B. 7.0%.
 - C. 8.0%.
10. The five-year spot rate is not provided here; however, the forward price for a two-year zero-coupon bond beginning in three years is known to be 0.8479. The price today of a five-year zero-coupon bond is *closest* to:
- A. 0.7119.
 - B. 0.7835.
 - C. 0.9524.
11. The one-year spot rate z_1 is 4%, the forward rate for a one-year loan beginning in one year is 6%, and the forward rate for a one-year loan beginning in two years is 8%. Which of the following rates is *closest* to the three-year spot rate?
- A. 4.0%
 - B. 6.0%
 - C. 8.0%
12. The one-year spot rate z_1 is 5%, and the forward price for a one-year zero-coupon bond beginning in one year is 0.9346. The spot price of a two-year zero-coupon bond is *closest* to:
- A. 0.87.
 - B. 0.89.
 - C. 0.93.
13. In a typical interest rate swap contract, the swap rate is *best* described as the interest rate for the:
- A. fixed-rate leg of the swap.
 - B. floating-rate leg of the swap.
 - C. difference between the fixed and floating legs of the swap.
14. A two-year fixed-for-floating MRR swap is 1.00%, and the two-year US Treasury bond is yielding 0.63%. The swap spread is *closest* to:
- A. 37 bps.
 - B. 100 bps.
 - C. 163 bps.
15. The swap spread is quoted as 50 bps. If the five-year US Treasury bond is yielding

- 2%, the rate paid by the fixed payer in a five-year interest rate swap is *closest* to:
- A. 0.50%.
 - B. 1.50%.
 - C. 2.50%.
16. If the three-month T-bill rate drops and MRR remains the same, the relevant TED spread:
- A. increases.
 - B. decreases.
 - C. does not change.
17. Given the yield curve for US Treasury zero-coupon bonds, which spread is *most* helpful pricing a corporate bond? The:
- A. Z-spread.
 - B. TED spread.
 - C. MRR–OIS spread, formerly the Libor–OIS spread.
-

The following information relates to questions 18–24

Jane Nguyen is a senior bond trader for an investment bank, and Chris Alexander is a junior bond trader at the bank. Nguyen is responsible for her own trading activities and also for providing assignments to Alexander that will develop his skills and create profitable trade ideas. Exhibit 1 presents the current par and spot rates.

Exhibit 1: Current Par and Spot Rates		
Maturity	Par Rate	Spot Rate
One year	2.50%	2.50%
Two years	2.99%	3.00%
Three years	3.48%	3.50%
Four years	3.95%	4.00%
Five years	4.37%	

Note: Par and spot rates are based on annual-coupon sovereign bonds.

Nguyen gives Alexander two assignments that involve researching various questions:

- Assignment 1 What is the yield-to-maturity of the option-free, default-risk-free bond presented in Exhibit 2? Assume that the bond is held to maturity, and use the rates shown in Exhibit 1.

Exhibit 2: Selected Data for \$1,000 Par Bond

Bond Name	Maturity (T)	Coupon
Bond Z	Three years	6.00%

Note: Terms are today for a T -year loan.

Assignment 2 Assuming that the projected spot curve two years from today will be below the current forward curve, is Bond Z fairly valued, undervalued, or overvalued?

After completing his assignments, Alexander asks about Nguyen's current trading activities. Nguyen states that she has a two-year investment horizon and will purchase Bond Z as part of a strategy to ride the yield curve. Exhibit 1 shows Nguyen's yield curve assumptions implied by the spot rates.

18. Based on Exhibit 1, the five-year spot rate is *closest to*:
 - A. 4.40%.
 - B. 4.45%.
 - C. 4.50%.
19. Based on Exhibit 1, the market is *most likely* expecting:
 - A. deflation.
 - B. inflation.
 - C. no risk premiums.
20. Based on Exhibit 1, the forward rate of a one-year loan beginning in three years is *closest to*:
 - A. 4.17%.
 - B. 4.50%.
 - C. 5.51%.
21. Based on Exhibit 1, which of the following forward rates can be computed?
 - A. A one-year loan beginning in five years
 - B. A three-year loan beginning in three years
 - C. A four-year loan beginning in one year
22. For Assignment 1, the yield-to-maturity for Bond Z is *closest to* the:
 - A. one-year spot rate.
 - B. two-year spot rate.
 - C. three-year spot rate.
23. For Assignment 2, Alexander should conclude that Bond Z is currently:
 - A. undervalued.

- B. fairly valued.
 - C. overvalued.
24. By choosing to buy Bond Z, Nguyen is *most likely* making which of the following assumptions?
- A. Bond Z will be held to maturity.
 - B. The three-year forward curve is above the spot curve.
 - C. Future spot rates do not accurately reflect future inflation.

The following information relates to questions 25-29

Laura Mathews recently hired Robert Smith, an investment adviser at Shire Gate Advisers, to assist her in investing. Mathews states that her investment time horizon is short, approximately two years or less. Smith gathers information on spot rates for on-the-run annual-coupon government securities and swap spreads, as presented in Exhibit 1. Shire Gate Advisers recently published a report for its clients stating its belief that, based on the weakness in the financial markets, interest rates will remain stable, the yield curve will not change its level or shape for the next two years, and swap spreads will also remain unchanged.

Exhibit 1: Government Spot Rates and Swap Spreads

	Maturity (years)			
	1	2	3	4
Government spot rate	2.25%	2.70%	3.30%	4.05%
Swap spread	0.25%	0.30%	0.45%	0.70%

Smith decides to examine the following three investment options for Mathews:

- Investment 1: Buy a government security that would have an annualized return that is nearly risk free. Smith is considering two possible implementations: a two-year investment or a combination of two one-year investments.
- Investment 2: Buy a four-year, zero-coupon corporate bond and then sell it after two years. Smith illustrates the returns from this strategy using the swap rate as a proxy for corporate yields.
- Investment 3: Buy a lower-quality, two-year corporate bond with a coupon rate of 4.15% and a Z-spread of 65 bps.

When Smith meets with Mathews to present these choices, Mathews tells him that she is somewhat confused by the various spread measures. She is curious to know whether there is one spread measure that could be used as a good indicator of the risk and liquidity of money market securities during the recent past.

25. In his presentation of Investment 1, Smith could show that under the no-arbitrage principle, the forward price of a one-year government bond to be

issued in one year is *closest* to:

- A. 0.9662.
- B. 0.9694.
- C. 0.9780.

26. In presenting Investment 1, using Shire Gate Advisers' interest rate outlook, Smith could show that riding the yield curve provides a total return that is *most likely*:

- A. lower than the return on a maturity-matching strategy.
- B. equal to the return on a maturity-matching strategy.
- C. higher than the return on a maturity-matching strategy.

27. In presenting Investment 2, Smith should show an annual return *closest* to:

- A. 4.31%.
- B. 5.42%.
- C. 6.53%.

28. The bond in Investment 3 is *most likely* trading at a price of:

- A. 100.97.
- B. 101.54.
- C. 104.09.

29. The *most* appropriate response to Mathews question regarding a spread measure is the:

- A. Z-spread.
- B. TED spread.
- C. MRR–OIS spread, formerly the Libor–OIS spread.

The following information relates to questions 30-40

Liz Tyo is a fund manager for an actively managed global fixed-income fund that buys bonds issued in Countries A, B, and C. She and her assistant are preparing the quarterly markets update. Tyo begins the meeting by distributing the daily rates sheet, which includes the current government spot rates for Countries A, B, and C as shown in Exhibit 1

Exhibit 1: Today's Government Spot Rates

Maturity	Country A	Country B	Country C
One year	0.40%	−0.22%	14.00%
Two years	0.70	−0.20	12.40
Three years	1.00	−0.12	11.80
Four years	1.30	−0.02	11.00
Five years	1.50	0.13	10.70

Tyo asks her assistant how these spot rates were obtained. The assistant replies, “Spot rates are determined through the process of bootstrapping. It entails backward substitution using par yields to solve for zero-coupon rates one by one, in order from latest to earliest maturities.”

Tyo then provides a review of the fund's performance during the last year and comments, “The choice of an appropriate benchmark depends on the country's characteristics. For example, although Countries A and B have both an active government bond market and a swap market, Country C's private sector is much bigger than its public sector, and its government bond market lacks liquidity.”

Tyo further points out, “The fund's results were mixed; returns did not benefit from taking on additional risk. We are especially monitoring the riskiness of the corporate bond holdings. For example, our largest holdings consist of three four-year corporate bonds (Bonds 1, 2, and 3) with identical maturities, coupon rates, and other contract terms. These bonds have Z-spreads of 0.55%, 1.52%, and 1.76%, respectively.”

Tyo continues, “We also look at risk in terms of the swap spread. We considered historical three-year swap spreads for Country B, which reflect that market's credit and liquidity risks, at three different points in time.” Tyo provides the information in Exhibit 2.

Exhibit 2: Selected Historical Three-Year Rates for Country B

Period	Government Bond Yield (%)	Fixed-for-Floating MRR Swap (%)
1 month ago	−0.10	0.16
6 months ago	−0.08	0.01
12 months ago	−0.07	0.71

Tyo then suggests that the firm was able to add return by riding the yield curve. The fund plans to continue to use this strategy but only in markets with an attractive yield curve for this strategy.

She moves on to present her market views on the respective yield curves for a five-year investment horizon.

Country A: “The government yield curve has changed little in terms of its level and shape during the last few years, and I expect this trend to continue. We assume that future spot rates reflect the current forward curve for all maturities.”

Country B: “Because of recent economic trends, I expect a reversal in the slope of the current yield curve. We assume that future spot rates will be higher than current forward rates for all maturities.”

Country C: “To improve liquidity, Country C’s central bank is expected to intervene, leading to a reversal in the slope of the existing yield curve. We assume that future spot rates will be lower than today’s forward rates for all maturities.”

Tyo’s assistant asks, “Assuming investors require liquidity premiums, how can a yield curve slope downward? What does this imply about forward rates?”

Tyo answers, “Even if investors require compensation for holding longer-term bonds, the yield curve can slope downward—for example, if there is an expectation of severe deflation. Regarding forward rates, it can be helpful to understand yield curve dynamics by calculating implied forward rates. To see what I mean, we can use Exhibit 1 to calculate the forward rate for a two-year Country C loan beginning in three years.”

30. Did Tyo’s assistant accurately describe the process of bootstrapping?
 - A. Yes
 - B. No, with respect to par yields
 - C. No, with respect to backward substitution
31. The swap curve is a better benchmark than the government spot curve for:
 - A. Country A.
 - B. Country B.
 - C. Country C.
32. Based on Exhibit 2, the implied credit and liquidity risks as indicated by the historical three-year swap spreads for Country B were the lowest:
 - A. 1 month ago.
 - B. 6 months ago.
 - C. 12 months ago.
33. Based on Exhibit 1 and Tyo’s expectations, which country’s term structure is currently best for traders seeking to ride the yield curve?
 - A. Country A
 - B. Country B
 - C. Country C
34. Based on Exhibit 1 and assuming Tyo’s market views on yield curve changes are realized, the forward curve of which country will lie below its spot curve?
 - A. Country A
 - B. Country B
 - C. Country C
35. Based on Exhibit 1 and Tyo’s expectations for the yield curves, Tyo *most likely* perceives the bonds of which country to be fairly valued?
 - A. Country A

- B. Country B
 - C. Country C
36. With respect to their discussion of yield curves, Tyo and her assistant are *most likely* discussing which term structure theory?
- A. Pure expectations theory
 - B. Local expectations theory
 - C. Liquidity preference theory
37. Tyo's assistant should calculate a forward rate *closest* to:
- A. 9.07%.
 - B. 9.58%.
 - C. 9.97%.
38. During economic expansions, monetary authorities raise benchmark rates to help control inflation. This action is *most* often consistent with:
- A. bearish flattening.
 - B. bullish steepening.
 - C. bearish steepening.
39. When government budget deficits fall, fiscal supply-side effects are *most* likely to result in:
- A. higher bond yields.
 - B. a steeper yield curve.
 - C. lower bond yields.
40. A flight to quality is most often associated with:
- A. a general rise in the level of interest rates.
 - B. bullish flattening.
 - C. bearish flattening.
-

The following information relates to questions 41-42

41. Define the yield-to-maturity for a coupon bond.
42. Is it possible for a coupon bond to earn less than the yield-to-maturity if held to maturity?
-

43. If a bond trader believes that current forward rates overstate future spot rates, how might she profit from that conclusion?
44. Explain the strategy of rolling down the yield curve.
45. What are the advantages of using the swap curve as a benchmark of interest rates relative to a government bond yield curve?
46. What is the TED spread, and what type of risk does it measure?
47. What is the SOFR rate, and which market conditions does it reflect?

The following information relates to questions 48-51

Rowan Madison is a junior analyst at Cardinal Capital. Sage Winter, a senior portfolio manager and Madison's supervisor, meets with Madison to discuss interest rates and review two bond positions in the firm's fixed-income portfolio.

Winter begins the meeting by asking Madison to state her views on the term structure of interest rates. Madison responds:

"Yields are a reflection of expected spot rates and risk premiums. Investors demand risk premiums for holding long-term bonds, and these risk premiums increase with maturity."

Winter tells Madison that, based on recent changes in spreads, she is concerned about a perceived increase in counterparty risk in the economy and its effect on the portfolio. Madison asks Winter:

"Which spread measure should we use to assess changes in counterparty risk in the economy?"

Winter is also worried about the effect of yield volatility on the portfolio. She asks Madison to identify the economic factors that affect short-term and long-term rate volatility. Madison responds:

"Short-term rate volatility is mostly linked to uncertainty regarding monetary policy, whereas long-term rate volatility is mostly linked to uncertainty regarding the real economy and inflation."

Finally, Winter asks Madison to analyze the interest rate risk portfolio positions in a 5-year and a 20-year bond. Winter requests that the analysis be based on level, slope, and curvature as term structure factors. Madison presents her analysis in Exhibit 1.

Exhibit 1: Three-Factor Model of Term Structure

Factor	Time to Maturity (years)	
	5	20
Level	-0.4352%	-0.5128%
Steepness	-0.0515%	-0.3015%
Curvature	0.3963%	0.5227%

Note: Entries indicate how yields would change for a one standard deviation increase in a factor.

Winter asks Madison to perform two analyses:

- Analysis 1: Calculate the expected change in yield on the 20-year bond resulting from a two-standard-deviation increase in the steepness factor.
- Analysis 2: Calculate the expected change in yield on the five-year bond resulting from a one-standard-deviation decrease in the level factor and a one-standard-deviation decrease in the curvature factor.

48. Madison's views on the term structure of interest rates are *most* consistent with the:
- A. local expectations theory.
 - B. segmented markets theory.
 - C. liquidity preference theory.
49. Is Madison's response regarding the factors that affect short-term and long-term rate volatility correct?
- A. Yes
 - B. No, she is incorrect regarding factors linked to long-term rate volatility
 - C. No, she is incorrect regarding factors linked to short-term rate volatility
50. Based on Exhibit 1, the results of Analysis 1 should show the yield on the 20-year bond decreasing by:
- A. 0.3015%.
 - B. 0.6030%.
 - C. 0.8946%.
51. Based on Exhibit 1, the results of Analysis 2 should show the yield on the five-year bond:
- A. decreasing by 0.8315%.
 - B. decreasing by 0.0389%.
 - C. increasing by 0.0389%.
-
52. According to the local expectations theory, what would be the difference in the one-month total return if an investor purchased a five-year zero-coupon bond versus a two-year zero-coupon bond?
53. Compare the segmented market and the preferred habitat term structure theories.

The following information relates to questions 54-56

54. List the three factors that have empirically been observed to affect Treasury security returns and explain how each of these factors affects returns on Treasury securities.
 55. What has been observed to be the most important factor in affecting Treasury returns?
 56. Which measures of yield curve risk can measure shaping risk?
-

SOLUTIONS

1. Three forward rates can be calculated from the one-, two- and three-year spot rates. The rate on a one-year loan that begins at the end of Year 1 can be calculated using the one- and two-year spot rates; in the following equation, one would solve for $f_{1,1}$:

$$[1 + z_2]^2 = [1 + z_1][1 + f_{1,1}]$$

The rate on a one-year loan that starts at the end of Year 2 can be calculated from the two- and three-year spot rates. In the following equation, one would solve for $f_{2,1}$:

$$[1 + z_3]^3 = [1 + z_2]^2[1 + f_{2,1}]$$

Additionally, the rate on a two-year loan that begins at the end of Year 1 can be computed from the one- and three-year spot rates. In the following equation, one would solve for $f_{1,2}$:

$$[1 + z_3]^3 = [1 + z_1][1 + f_{1,2}]^2$$

2. For the two-year forward rate one year from now of 2%, the two interpretations are as follows:
- 2% is the rate that will make an investor indifferent between buying a three-year zero-coupon bond or investing in a one-year zero-coupon bond and, when it matures, reinvesting in a zero-coupon bond that matures in two years.
 - 2% is the rate that can be locked in today by buying a three-year zero-coupon bond rather than investing in a one-year zero-coupon bond and, when it matures, reinvesting in a zero-coupon bond that matures in two years.
3. A flat yield curve implies that all spot interest rates are the same. When the spot rate is the same for every maturity, successive applications of the forward rate model will show that all the forward rates will also be the same and equal to the spot rate.
4. C is correct. There is no spot rate information to provide rates for a loan that terminates in five years. That is $f_{2,3}$ is calculated as follows:

$$f_{2,3} = \sqrt[3]{\frac{[1 + z_5]^5}{[1 + z_2]^2}} - 1$$

This equation indicates that in order to calculate the rate for a three-year loan beginning at the end of two years, one needs the five-year spot rate, z_5 , and the two-year spot rate, z_2 . However, z_5 is not provided.

5. A is correct. The forward rate for a one-year loan beginning in one year, $f_{1,1}$, is $1.04^2/1.03 - 1 = 5\%$. The rate for a one-year loan beginning in two years, $f_{2,1}$, is $1.05^3/1.04^2 - 1 = 7\%$. This confirms that an upward-sloping yield curve is consistent with an upward-sloping forward curve.
6. C is correct. If one-period forward rates are decreasing with maturity, then the forward curve is downward sloping. This turn implies a downward-sloping yield curve where longer-term spot rates z_{B-A} are less than shorter-term spot rates z_A .

7. C is correct. From the forward rate model, we have

$$[1 + z_2]^2 = [1 + z_1]^1 [1 + f_{1,1}]^1$$

Using the one- and two-year spot rates, we have

$$(1 + 0.05)^2 = (1 + 0.04)^1 [1 + f_{1,1}]^1, \text{ so } \frac{(1 + 0.05)^2}{(1 + 0.04)^1} - 1 = f_{1,1} = 6.010\%.$$

8. C is correct. From the forward rate model,

$$[1 + z_3]^3 = [1 + z_1]^1 [1 + f_{1,2}]^2$$

Using the one- and three-year spot rates, we find

$$(1 + 0.06)^3 = (1 + 0.04)^1 [1 + f_{1,2}]^2, \text{ so } \sqrt{\frac{(1 + 0.06)^3}{(1 + 0.04)^1}} - 1 = f_{1,2} = 7.014\%.$$

9. C is correct. From the forward rate model,

$$[1 + z_3]^3 = [1 + z_2]^2 [1 + f_{2,1}]^1$$

Using the two- and three-year spot rates, we find

$$(1 + 0.06)^3 = (1 + 0.05)^2 [1 + f_{2,1}]^1, \text{ so } \frac{(1 + 0.06)^3}{(1 + 0.05)^2} - 1 = f_{2,1} = 8.029\%.$$

10. A is correct. We can convert spot rates to spot prices to find $DF_3 = \frac{1}{(1.06)^3} = 0.8396$.

The forward pricing model can be used to find the price of the five-year zero as $DF_5 = DF_A \times F_{A,B-A}$, so $DF_5 = DF_3 F_{3,2} = 0.8396 \times 0.8479 = 0.7119$.

11. B is correct. Applying the forward rate model, we find

$$[1 + z_3]^3 = [1 + z_1]^1 [1 + f_{1,1}]^1 [1 + f_{2,1}]^1$$

$$\text{So } [1 + z_3]^3 = (1 + 0.04)^1 (1 + 0.06)^1 (1 + 0.08)^1,$$

$$\sqrt[3]{1.1906} - 1$$

$$= z_3 = 5.987\%.$$

12. B is correct. We can convert spot rates to spot prices and use the forward pricing model, so we have $DF_1 = \frac{1}{(1.05)^1} = 0.9524$.

The forward pricing model is

$$DF_2 = DF_1 \times F_{1,1}, \text{ so } DF_2 = DF_1 F_{1,1} = 0.9524 \times 0.9346 = 0.8901.$$

13. A is correct. The swap rate is the interest rate for the fixed-rate leg of an interest rate swap.

14. A is correct. The swap spread = $1.00\% - 0.63\% = 0.37\%$, or 37 bps.

15. C is correct. The fixed leg of the five-year fixed-for-floating swap will be equal to the five-year Treasury rate plus the swap spread: $2.0\% + 0.5\% = 2.5\%$.

16. A is correct. The TED spread is the difference between the three-month MRR and the three-month Treasury bill rate. If the T-bill rate falls and MRR does not change, the TED spread will increase.

17. A is correct. The Z spread is the single rate that, when added to the rates of the spot yield curve, will provide the correct discount rates to price a particular risky

bond.

18. B is correct. The five-year spot rate is determined by using forward substitution and using the known values of the one-year, two-year, three-year, and four-year spot rates, as follows:

$$1 = \frac{0.0437}{1.025} + \frac{0.0437}{(1.03)^2} + \frac{0.0437}{(1.035)^3} + \frac{0.0437}{(1.04)^4} + \frac{1 + 0.0437}{(1 + z_5)^5}$$

$$z_5 = \sqrt[5]{\frac{1.0437}{0.8394}} - 1 = 4.453\%$$

19. B is correct. The spot rates imply an upward-sloping yield curve, $z_3 > z_2 > z_1$. Because nominal yields incorporate a premium for expected inflation, an upward-sloping yield curve is generally interpreted as reflecting a market expectation of increasing, or at least level, future inflation (associated with relatively strong economic growth).

20. C is correct. A one-year loan beginning in three years, or $f_{3,1}$, is calculated as follows:

$$[1 + z_{3+1}]^{3+1} = [1 + z_3]^3 [1 + f_{3,1}]^1$$

$$[1.04]^4 = [1.035]^3 [1 + f_{3,1}]$$

$$f_{3,1} = \frac{(1.04)^4}{(1.035)^3} - 1 = 5.514\%$$

21. C is correct. Exhibit 1 provides five years of par rates, from which the spot rates for z_1, z_2, z_3, z_4 , and z_5 can be derived. Thus the forward rate $f_{1,4}$ can be calculated as follows:

$$f_{1,4} = \sqrt[4]{\frac{[1 + z_5]^5}{[1 + z_1]^1}} - 1$$

22. C is correct. The yield-to-maturity, y_3 , of Bond Z should be a weighted average of the spot rates used in the valuation of the bond. Because the bond's largest cash flow occurs in Year 3, z_3 will have a greater weight than z_1 and z_2 in determining y_3 .

Using the spot rates:

$$\text{Price} = \frac{\$60}{(1.025)^1} + \frac{\$60}{(1.030)^2} + \frac{\$1,060}{(1.035)^3} = \$1,071.16$$

Using the yield-to-maturity:

$$\text{Price} = \frac{\$60}{[1 + y(3)]^1} + \frac{\$60}{[1 + y(3)]^2} + \frac{\$1,060}{[1 + y(3)]^3} = \$1,071.16$$

The computed result is $y_3 = 3.46\%$, which is closest to the three-year spot rate of 3.50%.

23. A is correct. Alexander projects that the spot curve two years from today will be below the current forward curve, which implies that her expected future spot rates beyond two years will be lower than the quoted forward rates. Alexander would perceive Bond Z to be undervalued in the sense that the market is effectively discounting the bond's payments at a higher rate than she would, and the bond's market price is below her estimate of intrinsic value.

24. B is correct. Nguyen's strategy is to ride the yield curve, which is appropriate when the yield curve is upward sloping. The yield curve implied by Exhibit 1 is upward sloping, which implies that the three-year forward curve is above the current spot curve. When the yield curve slopes upward, as a bond approaches maturity or "rolls down the yield curve," the bond is valued at successively lower yields and higher prices.

25. B is correct. The forward pricing model is based on the no-arbitrage principle and is used to calculate a bond's forward price based on the spot yield curve. The spot curve is constructed by using annualized rates from option-free and default-risk-free zero-coupon bonds.

Equation 2: $DF_B = DF_A \times F_{A,B-A}$; we need to solve for $F_{1,1}$.

$$DF_1 = 1/(1 + 0.0225)^1 \text{ and } DF_2 = 1/(1 + 0.0270)^2,$$

$$F_{1,1} = DF_2/DF_1 = 0.9481/0.9780 = 0.9694.$$

26. C is correct. When the spot curve is upward sloping and its level and shape are expected to remain constant over an investment horizon (Shire Gate Advisers' view), buying bonds with a maturity longer than the investment horizon (i.e., riding the yield curve) will provide a total return greater than the return on a maturity-matching strategy.

27. C is correct. The swap spread is a common way to indicate credit spreads in a market. The four-year swap rate (fixed leg of an interest rate swap) can be used as an indication of the four-year corporate yield. Riding the yield curve by purchasing a four-year zero-coupon bond with a yield of 4.75% {i.e., 4.05% + 0.70%, $[P_4 = 100/(1 + 0.0475)^4 = 83.058]$ } and then selling it when it becomes a two-year zero-coupon bond with a yield of 3.00% {i.e., 2.70% + 0.30%, $[P_2 = 100/(1 + 0.0300)^2 = 94.260]$ } produces an annual return of 6.53%: $(94.260/83.058)^{0.5} - 1.0 = 0.0653$.

28. B is correct. The Z-spread is the constant basis point spread that is added to the default-free spot curve to price a risky bond. A Z-spread of 65 bps for a particular bond would imply adding a fixed spread of 65 bps to maturities along the spot curve to correctly price the bond. Therefore, for the two-year bond, $z_1 = 2.90\%$ (i.e., 2.25% + 0.65%), $z_2 = 3.35\%$ (i.e., 2.70% + 0.65%), and the price of the bond with an annual coupon of 4.15% is as follows:

$$P = 4.15/(1 + 0.029)^1 + 4.15/(1 + 0.0335)^2 + 100/(1 + 0.0335)^2,$$

$$P = 101.54.$$

29. C is correct. The MRR–OIS spread is considered an indicator of the risk and liquidity of money market securities. This spread measures the difference between MRR and the OIS rate.

30. C is correct. The assistant states that bootstrapping entails *backward* substitution using par yields to solve for zero-coupon rates one by one, in order from latest to earliest maturities. Bootstrapping entails *forward* substitution, however, using par yields to solve for zero-coupon rates one by one, in order from earliest to latest maturities.

31. C is correct. Country C's private sector is much bigger than the public sector, and the government bond market in Country C currently lacks liquidity. Under such circumstances, the swap curve is a more relevant benchmark for interest rates.

32. B is correct. The historical three-year swap spread for Country B was the lowest six months ago. Swap spread is defined as the spread paid by the fixed-rate payer of an interest rate swap over the rate of the “on the run” (most recently issued) government bond security with the same maturity as the swap. The lower (higher) the swap spread, the lower (higher) the return that investors require for credit and/or liquidity risks.

The fixed rate of the three-year fixed-for-floating Libor swap was 0.01% six months ago, and the three-year government bond yield was -0.08% six months ago. Thus the swap spread six months ago was $0.01\% - (-0.08\%) = 0.09\%$.

One month ago, the fixed rate of the three-year fixed-for-floating Libor swap was 0.16%, and the three-year government bond yield was -0.10% . Thus the swap spread one month ago was $0.16\% - (-0.10\%) = 0.26\%$.

Twelve months ago, the fixed rate of the three-year fixed-for-floating Libor swap was 0.71%, and the three-year government bond yield was -0.07% . Thus, the swap spread 12 months ago was $0.71\% - (-0.07\%) = 0.78\%$.

33. A is correct. Country A's yield curve is upward sloping—a condition for the strategy—and more so than Country B's.
34. B is correct. The yield curve for Country B is currently upward sloping, but Tyo expects a reversal in the slope of the current yield curve. This means she expects the resulting yield curve for Country B to slope downward, which implies that the resulting forward curve would lie below the spot yield curve. The forward curve lies below the spot curve in scenarios in which the spot curve is downward sloping; the forward curve lies above the spot curve in scenarios in which the spot curve is upward sloping.
- A is incorrect because the yield curve for Country A is currently upward sloping and Tyo expects that the yield curve will maintain its shape and level. That expectation implies that the resulting forward curve would be above the spot yield curve.
- C is incorrect because the yield curve for Country C is currently downward sloping and Tyo expects a reversal in the slope of the current yield curve. She thus expects the resulting yield curve for Country C to slope upward, which implies that the resulting forward curve would be above the spot yield curve.

35. A is correct. Tyo's projected spot curve assumes that future spot rates reflect, or will be equal to, the current forward rates for all respective maturities. This assumption implies that the bonds for Country A are fairly valued because the market is effectively discounting the bond's payments at spot rates that match those projected by Tyo.
- B and C are incorrect because Tyo's projected spot curves for the two countries do not match the current forward rates for all respective maturities. In the case of Country B, she expects future spot rates to be higher (than the current forward rates that the market is using to discount the bond's payments). For Country C, she expects future spot rates to be lower (than the current forward rates). Hence, she perceives the Country B bond to be currently overvalued and the Country C bond to be undervalued.

36. C is correct. Liquidity preference theory suggests that liquidity premiums exist to compensate investors for the added interest rate risk that they face when lending long term and that these premiums increase with maturity. Tyo and her assistant are assuming that liquidity premiums exist.

37. A is correct. From the forward rate model, $f_{3,2}$, is found as follows:

$$[1 + z_5]^5 = [1 + z_3]^3[1 + f_{3,2}]^2$$

Using the three-year and five-year spot rates, we find

$$(1 + 0.107)^5 = (1 + 0.118)^3[1 + f_{3,2}]^2, \text{ so}$$

$$\sqrt[2]{\frac{(1 + 0.107)^5}{(1 + 0.118)^3}} - 1 = f_{3,2} = 9.07\%$$

38. A is correct. This action is most often consistent with bearish flattening, or short-term bond yields rising more than long-term bond yields resulting in a flatter yield curve.
39. C is correct. When government budget deficits fall, fiscal supply-side effects are most likely to result in lower bond yields.
40. B is correct. A flight to quality is most often associated with bullish flattening, in which the yield curve flattens as long term rates fall by more than short-term rates.
41. The yield-to-maturity of a coupon bond is the expected rate of return on a bond if the bond is held to maturity, there is no default, and the bond and all coupons are reinvested at the original yield-to-maturity.
42. Yes, it is possible. For example, if reinvestment rates for the future coupons are lower than the initial yield-to-maturity, a bondholder may experience lower realized returns.
43. If forward rates are higher than expected future spot rates, the market price of the bond will be lower than the intrinsic value. This dynamic occurs because, everything else held constant, the market is currently discounting the bonds cash flows at a higher rate than the investor's expected future spot rates. The investor can capitalize on this scenario by purchasing the undervalued bond. If expected future spot rates are realized, then bond prices should rise, thus generating gains for the investor.
44. The strategy of rolling down the yield curve is one in which a bond trader attempts to generate a total return over a given investment horizon that exceeds the return to bond with maturity matched to the horizon. The strategy involves buying a bond with maturity more distant than the investment horizon. Assuming an upward-sloping yield curve, if the yield curve does not change level or shape, as the bond approaches maturity (or rolls down the yield curve) it will be priced at successively lower yields. So as long as the bond is held for a period less than maturity, it should generate higher returns because of price gains.
45. Some countries do not have active government bond markets with trading at all maturities. For those countries without a liquid government bond market but with an active swap market, there are typically more points available to construct a swap curve than a government bond yield curve. For those markets, the swap curve may be a superior benchmark.
46. The TED spread is the difference between MRR and the US T-bill rate of matching maturity. It is an indicator of perceived credit and liquidity risk. In particular, because sovereign debt instruments are typically the benchmark for the lowest default risk instruments in a given market, and loans between banks (often at MRR) have some counterparty risk, the TED spread is considered to at least in part reflect default (or counterparty) risk in the banking sector.

47. The secured overnight financing rate (SOFR), or overnight cash borrowing rate collateralized by US Treasuries, is a barometer of the US Treasury repurchase (or repo) market. SOFR is a volume-weighted index of all qualified repo market transactions on a given day and is influenced by supply and demand conditions in secured funding markets.
48. C is correct. Liquidity preference theory asserts that investors demand a risk premium, in the form of a liquidity premium, to compensate them for the added interest rate risk they face when buying long-maturity bonds. The theory also states that the liquidity premium increases with maturity.
49. A is correct. Madison's response is correct; research indicates that short-term rate volatility is mostly linked to uncertainty regarding monetary policy, whereas long-term rate volatility is mostly linked to uncertainty regarding the real economy and inflation.
50. B is correct. Because the factors in Exhibit 1 have been standardized to have unit standard deviations, a two-standard-deviation increase in the steepness factor will lead to the yield on the 20-year bond decreasing by 0.6030%, calculated as follows:
- $$\text{Change in 20-year bond yield} = -0.3015\% \times 2 = -0.6030\%$$
51. C is correct. Because the factors in Exhibit 1 have been standardized to have unit standard deviations, a one-standard-deviation decrease in both the level factor and the curvature factor will lead to the yield on the five-year bond increasing by 0.0389%, calculated as follows:
- $$\text{Change in five-year bond yield} = 0.4352\% - 0.3963\% = 0.0389\%$$
52. The local expectations theory asserts that the total return over a one-month horizon for a five-year zero-coupon bond would be the same as for a two-year zero-coupon bond.
53. Both theories attempt to explain the shape of any yield curve in terms of supply and demand for bonds. In segmented market theory, bond market participants are limited to purchase of maturities that match the timing of their liabilities. In the preferred habitat theory, participants have a preferred maturity for asset purchases, but they may deviate from it if they feel returns in other maturities offer sufficient compensation for leaving their preferred maturity segment.
54. Studies have shown that three factors affect Treasury returns: (1) changes in the level of the yield curve, (2) changes in the slope of the yield curve, and (3) changes in the curvature of the yield curve. Changes in the level refer to upward or downward shifts in the yield curve. For example, an upward shift in the yield curve is likely to result in lower returns across all maturities. Changes in the slope of the yield curve relate to the steepness of the yield curve. Thus, if the yield curve steepens, higher returns for short-maturity bonds and lower returns for long-maturity bonds will likely occur. An example of a change in the curvature of the yield curve is a situation where rates fall at the short and long end of the yield curve while rising for intermediate maturities. In this situation, returns on short and long maturities are likely to rise while declining for intermediate-maturity bonds.
55. Empirically, the most important factor is the change in the level of interest rates.
56. Key rate durations and a measure based on sensitivities to level, slope, and curvature movements can address shaping risk, but effective duration cannot.

