1 THE PURPOSE AND SCOPE OF THEORY OF STRUCTURES

Without doubt, many are convinced that the calculations should determine the dimensions unequivocally and conclusively. However, in the light of the impossibility of taking into account all secondary circumstances, every calculation constitutes only a basis for the design engineer, who thus has to grapple with those secondary circumstances...

A totally simple form of calculation alone is therefore possible and sufficient.

Robert MAILLART (1938)

1.1 General

Theory of structures is a subdiscipline of applied mechanics which is configured to suit the needs of civil and structural engineers. The purpose of theory of structures is to present systematically the knowledge about the behaviour of structures at rest, to expand that knowledge and to prepare it for practical applications. It forms the basis for the design of every new structure and the examination of every existing one.

The terms and methods used in the theory of structures enable the engineer to adopt a uniform approach not tied to any particular type of construction (concrete, steel, composite, timber or masonry). With the advent of the computer in the third quarter of the 20th century, this approach gradually became *structural mechanics*, the discipline to which theory of structures belongs today.

At the heart of every theory of structures exercise there is a *structural model*, which is obtained through isolation and idealisation and takes into account the geometry of the structure, the properties of the construction materials and the possible actions. Determining the action effects, i. e. the structure's responses to the actions, is carried out with the help of *analytical models* that link the governing force and deformation variables via equilibrium and compatibility conditions plus constitutive equations.

1.2 The basis of theory of structures

Structural behaviour is expressed in the form of *internal* and *external force* and *deformation variables* (loads and stresses plus displacements and strains). Static relationships (equilibrium conditions and static boundary conditions, see chapter 5) link the force variables, kinematic relationships (kinematic relationships and boundary conditions, see chapter 6) link the deformation variables, and constitutive relationships (see chapter 7) link the internal force and deformation variables. The most general statements within the scope of theory of structures are obtained when the internal and external force and deformation variables are rigorously associated in the form of *work-associated variables* (see chapter 8) [1].

Statics is based on three fundamental principles of mechanics. According to the *principle of virtual work*, a (statically admissible) force state (equilibrium set of force variables) fulfilling the static relationships in conjunction with a (kinematically admissible) deformation state (compatibility set of deformation variables) fulfilling the kinematic relationships does not perform any work. Added to this are the *reaction principle* (for every force there is a equal and opposite reaction with the same direction of action) and the *free-body principle* (every part removed from a system in equilibrium undergoing compatible deformation is itself in equilibrium and undergoes compatible deformation).

Looking beyond its link with mechanics, theory of structures has a special significance for *structural engineering* (see chapters 3 and 4). It is a tool for assessing the stability, strength and stiffness of a structure that either exists or is being designed. This application of theory of structures manifests itself in specific methods developed for ascertaining structural behaviour in general and (numerical) treatment in individual cases.

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1.3 Methods of theory of structures

The principle of virtual work can be expressed as the principle of virtual deformations or the principle of virtual forces. The systematic application of these two principles leads to a series of *dual* kinematic or static *methods*. On the kinematic side it is important to mention LAND's method for determining influence lines (section 12.3), the displacement method for solving statically indeterminate framed structures (chapter 17 and section 19.3) and the kinematic method of limit analysis (sections 21.3 and 21.7). On the static side we have the work theorem for determining single deformations (section 14.2), the force method for solving statically indeterminate framed structures (chapter 16 and section 19.2) and the static method of limit analysis (sections 21.3 and 21.7).

Assuming linear elastic behaviour and small deformations leads to linear statics, in which all the force and deformation variables may be superposed. This possibility of superposition is used extensively in theory of structures, especially in the force and displacement methods. Introducing unknown force or deformation variables and superposing their effects on those of external actions results in sets of linear equations for the unknowns.

However, the *superposition law* no longer applies in the case of non-linear materials problems (chapters 20 and 21) and non-linear geometrical problems (chapter 22). In such instances an (incremental) *iterative procedure* is generally necessary. Errors caused by simplifications at the beginning are evaluated step by step and successively reduced through appropriate corrections.

Analogies can often be used to make complex situations more accessible, or to reduce them to simpler, known situations. Examples of this are the membrane analogy (section 13.4.2) and the sand hill analogy (section 21.4.4) for dealing with elastic or plastic torsion problems, and MOHR's analogy for determining deformation diagrams (section 15.3.2). Combined warping and pure torsion problems (section 13.4.4) can be approached in a similar way to combined shear and bending problems (section 18.5.2) or bending problems in beams with tension (section 18.9). Edge disturbance problems in cylindrical shells (sections 18.7.4 and 26.5) can be reduced to the theory of beams on elastic foundation (section 18.4.4); this theory is also useful for approximating edge disturbance problems in spherical (section 26.7.3) and other shells (section 26.7.4). Furthermore, plates (chapter 23) can be idealised as plane trusses, slabs (chapter 24) as grillages, and folded plates (chapter 25) and shells (chapter 26) as space trusses or spatial frameworks

The development of powerful numerical methods has led to the methods of *graphical statics* (section 10.1) gradually losing the importance they had in the past. However, graphical aids still represent an unbeatable way of illustrating the flow of the forces in structures, e. g. with thrust lines (section 5.3.2, Figs. 17.19 and 21.7) or truss models (section 23.4.2). They represent an indispensable foundation for conceptual design (section 3.2) and the detailing of structural members and their connections.

The development of numerical methods has also brought about a change in the significance of *experimental statics*. From the 1920s through to the 1970s, loading tests on scale models made from celluloid, acrylic sheet and other materials were central to understanding the elastic loadbearing behaviour of complex structures. Such tests are no longer significant today. What continues to be important, however, is scientific testing to verify theoretical models, primarily in conjunction with non-linear phenomena, new materials or forms of construction and accidental actions. In structural design, physical models are not only useful for form-finding and detailing, but also very helpful when assessing the quality of the structural behaviour of the design. During the dimensioning, tests are a sensible backup if, for example, there are no appropriate analytical models available or a large number of identical structural members is required.

Finally, specific measurements during and after execution enable extremely valuable comparisons with the predicted behaviour of a structure – a source of experience that is all too often neglected.

In the *numerical methods* of theory of structures, it is the *finite element method* (FEM) that plays the leading role (section 19.3). These days FEM is the basis of almost all structural calculations. Users have extremely powerful tools at their disposal in the shape of appropriate modern computer programs. But to be able to deploy such programs responsibly, designers should at least understand the basics of the algorithms on which they are based. First and foremost, however, the engineer's knowledge of theory of structures should enable him or her to check the computer output critically. The crucial thing here is the ability to be able to approximate complex issues by reducing them to simple, understandable problems. Adequate training in the classical methods of theory of structures, which this book aims at, will supply the foundation for that ability.

1.4 Statics and structural dynamics

When it comes to dynamic problems, the principle of virtual work has to be formulated taking into account *inertial forces* (proportional to acceleration): the motion in a system is such that at any point in time the internal, external and inertial forces are in equilibrium. Appropriate additional terms in the equilibrium conditions turn them into *equations of motion*, and can be included, for example, within the scope of the finite element method by way of local and global *mass matrices*. Instead of a set of linear equations, this leads to a set of simultaneous ordinary second-order differential equations for the (time-dependent) node displacement parameters. Assuming constant coefficients, the differential equations can be decoupled according to the method of *modal analysis*. The associated eigenvalue problem leads to a solution in the form of superposed *natural vibrations*.

Generally, damping forces must also be taken into account in the equations of motion. In order that the differential equations remain linear, it is usual to assume that these forces are proportional to velocity. And so that a modal analysis remains possible with decoupled natural vibrations, we use a so-called *modal damping* for simplicity.

Structural dynamics is essentially readily accessible via statics. However, adding the dimension of time makes a more in-depth examination necessary so that dynamic processes become just as familiar as static phenomena. In the end, engineers prepared to make the effort obtain a broader view of theory of structures.

1.5 Theory of structures and structural engineering

For *structural engineering*, theory of structures is an ancillary discipline, like materials science. The knowledge and experience of practising design engineers in this and other relevant special subjects, e. g. geotechnics and construction technology, must be adequate for the complexity and significance of the jobs to which they are assigned. Furthermore, appropriate practical experience with the respective types of construction is an essential requirement for managing the design and execution of construction projects.

Theory of structures plays a role in all phases of conventional project development, from the preliminary design and tender design to the detail design, but in different ways, to suit the particular phase. Whereas for the conceptual design rough structural calculations are adequate, the subsequent phases require analyses of structural safety and serviceability that can be verified by others – and not just for the final condition of the structure, but especially for critical conditions during construction.

Besides new-build projects, the conservation and often the deconstruction of structures also throw up their share of interesting theory of structures problems. Frequently such tasks are far more demanding than those of new structures because fewer, if any, standards are available to help the engineer, and appraising the current condition of a structure is often difficult and associated with considerable uncertainties. The development of appropriate structural and actions models in such cases can be extremely tricky yet fascinating.

Looking beyond the immediate uses of structural design, there are various applications that can be handled with the methods of theory of structures, especially in mechanical engineering, shipbuilding and automotive manufacture, aerospace engineering, too. We are thus part of the great interdisciplinary field of *structural mechanics*.