1

Background and Essentials

1 What is the photon energy range corresponding to the UV radiation band?
Answer: 10 nm–400 nm corresponds to 124 eV–3.1 eV.

Solution:
The quantum energy k of any electromagnetic photon is given in keV by

$$k = h\nu = \frac{hc}{\lambda} = \frac{12.3982\ \text{keV Å}}{\lambda} = \frac{1.23982\ \text{keV nm}}{\lambda}$$

where 1 Å(Angstrom) = 10^{-10} m, Planck's constant is $h = 6.62607 \times 10^{-34}$ J s $= 4.13561 \times 10^{-18}$ keV s (note that 1.6022×10^{-16} J = 1 keV), and the velocity of light in vacuum is $c = 2.99792 \times 10^{8}$ m/s $= 2.99792 \times 10^{18}$ Å/s $= 2.99792 \times 10^{17}$ nm s^{-1}.
Therefore for the UV radiation, which is in the range of 10 nm–400 nm, the equation yields 124 eV–3.1 eV.

2 For a kinetic energy of 100 MeV, calculate the velocity β, for (a) electrons, (b) protons, and (c) alpha particles. The corresponding rest energies are given in the Data Tables.
Answer: (a) 0.9999; (b) 0.4282; (c) 0.2271

Solution:
We can apply either of the relations

$$\beta^2 = \frac{\tau(\tau+2)}{(\tau+1)^2}, \quad \text{with } \tau = E/m_0c^2$$

or

$$\beta^2 = \frac{E(E+2m_0c^2)}{(E+m_0c^2)^2}$$

From the Data Tables, the rest energies are $m_ec^2 = 0.51099$ MeV, $m_pc^2 = 938.272$ MeV, and $m_\alpha c^2 = 3727.38$ MeV. These yield
(a) Electrons: 0.9999
(b) Protons: 0.4282
(c) Alpha particles: 0.2271

Fundamentals of Ionizing Radiation Dosimetry: Solutions to Exercises, First Edition.
Pedro Andreo, David T. Burns, Alan E. Nahum, and Jan Seuntjens.

3 Conversely given a value of $\beta = 0.95$, calculate the corresponding kinetic energies of electrons, protons, and α particles.
Answer: (a) 1.1255 MeV; (b) 2066.6 MeV; (c) 8209.86 MeV

Solution:
The relation between the kinetic energy and the speed (β) is

$$E = \frac{m_0 c^2 \beta^2}{2\sqrt{1-\beta^2}}$$

Using the rest energies from the previous exercise, we get
(a) Electrons: 1.1255 MeV
(b) Protons: 2066.6 MeV
(c) α-particles: 8209.86 MeV

4 The result of a given process is derived as the product of several independent quantities, $Q = \prod q_i$. The type A and B uncertainties of each q_i, $(u_A, u_B)_i$, given as a relative standard uncertainty, are (0.1, 0.5), (0.01, 0.1), (0.02, 0.4), and (0.3, 0.19). Determine the combined standard uncertainty of Q.
Answer: $u_c(Q) = 0.75$

Solution:
Use the *law of propagation of uncertainty* twice: first for each of the respective types of uncertainty to yield the overall u_A and u_B types,

$$u_A = \sqrt{\sum_i u_{A_i}^2}, \quad u_B = \sqrt{\sum_i u_{B_i}^2}$$

and then for the combination of these two to yield $u_c(Q)$.
Hence

Quantity	Rel standard uncertainty	
	$(u_A)_i$	$(u_B)_i$
q_1	0.10	0.50
q_2	0.01	0.10
q_3	0.02	0.40
q_4	0.30	0.19
Combined	$u_A = 0.32$	$u_B = 0.68$

resulting in a combined uncertainty

$$u_c(Q) = \sqrt{u_A^2 + u_B^2} = 0.75$$

5 Given the following set of data (75.4, 79.7, 75.0, 77.0, 78.4), with standard uncertainties (0.95, 0.5, 0.2, 1.2, 0.8), (a) determine the non-weighted and weighted means and the corresponding type A uncertainties. (b) Determine the Birge ratio for the data and comment on the uncertainty estimates of the data.
Answer: $\bar{x} = 77.1$, $s_{\bar{x}} = 0.89$; $\bar{x}_w = 75.8$, $s_{\bar{x}_w} = 0.18$; $R_{Birge} = 2.2$

Solution:

(a) Requires the straightforward application of Eqs. (1.41)–(1.46), where the different terms are

i	x_i	$(x_i - \bar{x})^2$	s_i	$w_i(= 1/s_i^2)$	$w_i\ x_i$	$w_i(x_i - \bar{x}_w)^2$
1	75.4	2.89	0.95	1.11	83.55	0.18
2	79.7	6.76	0.50	4.00	318.80	60.79
3	75.0	4.41	0.20	25.00	1875.00	16.07
4	77.0	0.01	1.20	0.69	53.47	1.00
5	78.4	1.69	0.80	1.56	122.50	10.55
n	$\sum_i x_i$	$\sum_i (x_i - \bar{x})^2$		$\sum_i w_i$	$\sum_i w_i\ x_i$	$\sum_i w_i(x_i - \bar{x}_w)^2$
5	385.5	15.8		32.36	2453.32	88.58
Eqs	(1.41)	(1.43)		(1.45)	(1.44)	(1.46) num
	$\bar{x}$	$s(\bar{x})$		$s(\bar{x}_w)_{\text{int}}$	$\bar{x}_w$	$s(\bar{x}_w)_{\text{ext}}$
	77.10	0.89		0.18	75.80	0.83

(b) The Birge ratio is given by

$$R_{\text{Birge}} = \frac{s(\bar{x}_w)_{\text{int}}}{s(\bar{x}_w)_{\text{ext}}} = 2.2$$

$R_{\text{Birge}} = 2.2$ is a sign that some uncertainties have been under/over estimated. We typically think that we can make estimates at, say, the 20% level. A Birge significantly greater than 1.2 or 1.3 is a reasonable sign of under/overestimation. However, one proviso is the balance of uncertainties. One huge under/overestimate can make Birge large even if other uncertainties are properly estimated, especially for small data sets. This could be the case with data #3, where $s = 0.20$ might be an underestimation.

6 Using the half-width of the set of data in the previous exercise, estimate the type B uncertainty assuming rectangular, triangular, and Gaussian (with $k = 2$) distributions. Which of the three is considered to be more conservative?

Answer: $u_{B\ rect} = 1.36$, $u_{B\ trian} = 0.96$, $u_{B\ Gauss} = 1.18$; the 95% Gaussian is more conservative.

Solution:

The half-width of the set of data, $[-L, +L]$, is determined as

$$L = \frac{\max(x_i) - \min(x_i)}{2} = \frac{79.7 - 75.0}{2} = 2.35$$

Hence

$$u_{\text{B,rect}} = \frac{L}{\sqrt{3}} = \frac{2.35}{1.73} = 1.36$$

$$u_{\mathrm{B,trian}} = \frac{L}{\sqrt{6}} = \frac{2.35}{2.45} = 0.96$$

$$u_{\mathrm{B,95\%}} = \frac{L}{2} = \frac{2.35}{2} = 1.18$$

The rectangular distribution is a special case, because in general for most data sets there is a higher probability that the true value lies nearer to the middle than at the extremes. This leaves the triangular and Gaussian ($k = 2 \rightarrow 95\%$) distributions being conceptually similar, with the 95% Gaussian being more conservative.