Part One
Basic Physics of Chaos and Synchronization in Lasers

Optical Communication with Chaotic Lasers: Applications of Nonlinear Dynamics and Synchronization, First Edition.
Atsushi Uchida.

1 Introduction

The topics of this book widely cover both basic sciences and engineering applications by using lasers and chaos. The basic concepts of chaos, lasers, and synchronization are described in the first part of this book. The second part of this book deals with the engineering applications with chaotic lasers for information–communication technologies, such as optical chaos communication, secure key distribution, and random number generation. The bridge between basic scientific researches and their engineering applications to optical communications are treated in this book.

The history of research activities of laser and chaos is summarized in Table 1.1. Since the laser was invented in 1960 and the concept of chaos was found in 1963, these two major research fields were developing individually. In 1975, a milestone work was published on the findings of the connection between laser and chaos. In the 1980s, there were enormous research activities for the experimental observation of chaotic laser dynamics and the proposal of laser models that were used to explain the experimental results, from the fundamental physics point of view. Two important methodologies were proposed in 1990, that is, control and synchronization of chaos, which led to engineering applications of chaotic lasers such as stabilization of laser output and optical secure communication. There were many research reports on control and synchronization of chaos in various laser systems in the 1990s, and researchers tended to utilize chaotic lasers for engineering applications, rather than just for the studies on fundamental physics. Two important experimental demonstrations of optical communication with chaotic lasers were reported in 1998. Since then, many researchers have concentrated on working on the implementation of optical communication systems with chaotic lasers in the 2000s, including two major European projects, for the purpose of secure communication. High-performance demonstrations were reported in the field experiments of commercial optical-fiber networks, where bit rates of 10 Gb/s with low bit-error rate (BER) were achieved over 100 km fiber transmission. Meanwhile, other promising applications with chaotic lasers were demonstrated experimentally, that is, chaotic lidar and a random number generator, which could be research seeds for the next decade.

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Table 1.1 History of research activities of laser and chaos (milestone studies).

Year	Research activity	References
1960	Invention of laser	(Maiman, 1960)
1963	Discovery of chaos	(Lorenz, 1963)
1975	Connection between laser and chaos	(Haken, 1975)
1979	Proposal of Ikeda chaos	(Ikeda, 1979)
1980	Proposal of Lang-Kobayashi equations for semiconductor laser with optical feedback	(Lang and Kobayashi, 1980)
1984	Classification of chaotic laser models	(Arecchi *et al.*, 1984)
1980's∼	Intensive works on the observation of chaos in lasers	(Gioggia and Abraham, 1983) (Weiss *et al.*, 1983)
1990	Proposal of control of chaos	(Ott *et al.*, 1990)
1990	Proposal of synchronization of chaos	(Pecora and Carroll, 1990)
1992	First experimental observation of controlling chaos in lasers	(Roy *et al.*, 1992)
1993	First demonstration of chaos communication (circuit)	(Cuomo and Oppenheim, 1993)
1994	First experimental observation of synchronization of chaos in lasers	(Roy and Thornburg, 1994 (Sugawara *et al.*, 1994)
1990's∼	Intensive works on control and synchronization of chaos in lasers	(Meucci *et al.*, 1994) (Fischer *et al.*, 2000)
1994	Proposal of optical communication with chaotic lasers (numerical simulation)	(Colet and Roy, 1994) (Mirasso *et al.*, 1996)
1998	First experimental demonstration of optical communication with chaotic lasers	(VanWiggeren and Roy, 1998a) (Goedgebuer *et al.*, 1998)
2000's∼	Intensive works on optical communication with chaotic lasers	(Tang and Liu, 2001c) (Kusumoto and Ohtsubo, 2002)
2004	First demonstration of chaotic lidar and radar	(Lin and Liu, 2004a, 2004b)
2005	First demonstration of field experiment on optical communication with chaotic lasers	(Argyris *et al.*, 2005)
2008	Proposal of photonic integrated circuit for optical communication with chaotic laser	(Argyris *et al.*, 2008)
2008	First experimental demonstration of random number generator with chaotic lasers	(Uchida *et al.*, 2008a)
2010	High-performance demonstration of optical chaos communication with chaotic lasers (10 Gb/s)	(Lavrov *et al.*, 2010) (Argyris *et al.*, 2010c)
2010's∼	What is next?	

This book overviews all the research activities related to laser and chaos in the recent half-century. The fundamental knowledge of chaotic laser dynamics is enormous thanks to the efforts of basic researchers in this field. This book describes chaotic temporal dynamics in various laser systems (Chapters 2–4), and techniques for synchronization of chaos (Chapters 5 and 6), which are key components for the implementation of optical chaos communication. The detailed descriptions of optical

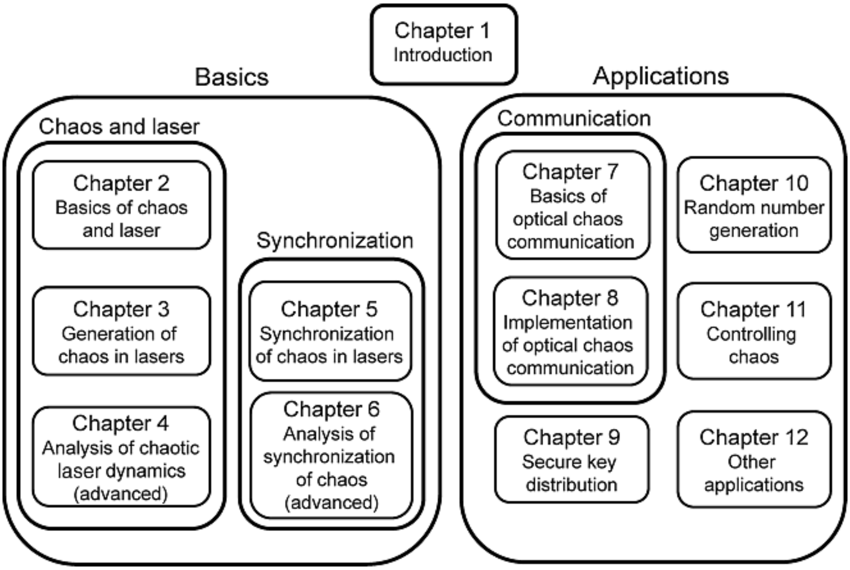

Figure 1.1 Contents of all the chapters in this book. It is recommended for readers to independently select and read the chapters that are interesting to them.

communication with chaotic lasers are found in this book (Chapters 7 and 8), which is one of the main goals of this book. Other possible and promising applications are described, such as secure key distribution, random number generation, dynamical memory, and chaotic lidar and radar (Chapters 9–12).

The contents of all the chapters in this book are summarized in Figure 1.1. The first part of the book corresponds to the description of the basic sciences of lasers, chaos, and synchronization (Chapters 2–6), and the second part describes their engineering applications (Chapters 7–12). All the chapters are written simply and comprehensively for beginners to avoid complicated mathematical formula. Mathematical formula is summarized in an Appendix of each chapter if necessary. For example, several mathematical models for various laser systems are described in the Appendices of Chapters 3 and 5. Chapters 4 and 6 are specially designed for advanced learners and are written with complicated mathematical formula and difficult technical terms, so beginners may skip Chapters 4 and 6. Readers may start reading any of the chapters based on their interests, since each chapter has been organized independently.

Source codes of the C programming language are listed in the Glossary for numerical simulations of laser models, and readers may try to do numerical simulations by themselves with chaotic laser models. This would be a good exercise for students and would be a good start of research work for researchers. The acronyms of technical terms and units are also listed in the Glossary.

1.1 Lasers and Chaos

1.1.1 Lasers

The term of “LASER” is an abbreviation of light amplification by stimulated emission of radiation. The laser is artificial light that has opened up a variety of applications in many scientific and engineering fields. The laser can be considered as one of the most important inventions in the twentieth century. The important characteristic of the laser is “coherence”, which implies that the photons oscillate similarly in phase. Coherent light sources have outstanding characteristics compared with natural (incoherent) light: bright intensity output, high photon energy, good directionality, single wavelength, and narrow spectrum bandwidth in order to enable interference. This “clean” light has brought new technologies such as optical communication, compact disk systems, precise measurement, material processing, medical application, remote sensing, and so on. Most laser devices have nonlinear effects in their laser media, and inherent instabilities are sometimes unavoidable (see Chapter 2).

1.1.2 Chaos

From the beginning of laser history, instabilities of laser output are inevitable due to its inherent nonlinearity (Maiman, 1960; Maiman *et al.*, 1961), even though many efforts have been made to stabilize laser output for many engineering applications. Most lasers including semiconductor, fiber, solid-state, and gas lasers produce temporal and spatial instabilities of laser output at certain operating conditions or with an additional external perturbation. It has been known that these instabilities can be derived from a deterministic rule of laser dynamics, which can be described by using a set of differential rate equations. These types of instabilities have been known as “deterministic chaos” and can be distinguished from instabilities due to stochastic or quantum noise.

The term “chaos” is generally used to describe disturbance or turbulence in many situations. One of the most acceptable definitions of chaos in science is the instabilities derived from a “deterministic” rule. The term “chaos” has been used to describe fluctuations or time-varying (or space-varying) irregular phenomena that are governed by a deterministic rule, which can be described by using a set of mathematical equations (see Chapter 2). Chaos is a counterintuitive concept in the sense that one may find a mathematical rule in irregular fluctuations of complex dynamics.

One of the important characteristics of deterministic chaos is known as “sensitive dependence on initial conditions” (Lorenz, 1963). If two chaotic temporal sequences start from very close but slightly different initial conditions, the two sequences behave similarly at the beginning, however, they start to diverge exponentially in time and never show the same behavior again. This characteristic can be quantitatively measured by using the maximum Lyapunov exponent, and the existence of the positive maximum Lyapunov exponent is a proof of deterministic chaos. A tiny error of the initial conditions makes chaotic irregular sequences unpredictable. This fact implies that chaos is unpredictable for a long-term duration due to the sensitive dependence on initial conditions, although chaos is predictable for a short-term period due to the existence of deterministic rules.

1.1.3
Connection Between Lasers and Chaos

The connection between lasers and chaos was made in 1975 (Haken, 1975), where it was shown that a set of nonlinear differential equations from Maxwell–Bloch equations for a laser model resembles the Lorenz equations that are the basic model for deterministic chaos (see Chapter 2). Since then, many observations have been reported in experiments and numerical simulation in the 1980s (see Chapter 3). It was found that most lasers produce chaotic fluctuations of laser intensity. In the 1990s two major techniques for harnessing chaos were proposed: synchronization of chaos and control of chaos. Many attempts using these two techniques have been demonstrated in the literature. These research activities have led to engineering applications of optical communications with synchronized chaotic lasers.

One of the most well-known examples of chaotic laser output is a semiconductor laser with optical feedback. Semiconductor lasers have been widely used for optical compact disk and optical communication systems. A small fraction of the laser output is invariably fed back into the semiconductor laser because of the reflection occurring at the disk surface or the edge of optical fiber. Figure 1.2a shows a simple model of a semiconductor laser with optical self-feedback for the generation of chaos. A laser beam is reflected from an external mirror and fed back into the laser cavity. The optical intensity in the laser cavity is perturbed by the self-feedback light that affects the interaction between photons and the carrier density in the laser medium. A chaotic temporal waveform of laser output intensity can be observed experimentally, as shown in Figure 1.2b. Fast intensity fluctuation can be found of the order of

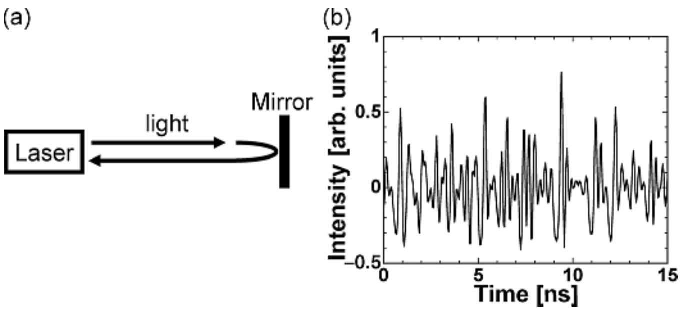

Figure 1.2 (a) Model of semiconductor laser with optical self-feedback. (b) Example of experimentally observed temporal waveform of laser output intensity in semiconductor laser with optical feedback.

GHz (the period of fluctuation is in the order of nanoseconds, 10^{-9} s), which corresponds to the relaxation oscillation frequency of the semiconductor laser (see Chapter 4).

Regardless of the observation of rich dynamics in chaotic laser output, many researchers did not pay attention to the instability of laser output. Most laser engineers avoided irregular fluctuations of laser output and considered that lasers were supposed to be stable, single mode (single wavelength), and have a narrow optical spectrum for use in engineering applications. Irregular pulsations of laser output need to be controlled and eliminated in order to generate stable output or periodic pulsations for large-intensity spikes, known as Q-switching pulses. On the other hand, the characteristics of rich temporal dynamics of laser output are suitable as a universal nonlinear dynamical model from the viewpoint of nonlinear dynamics in the interdisciplinary research fields. In addition, constructive combination between a laser and chaos could open up novel engineering applications, such as optical chaos communication, secure key distribution, and random number generation, which will be treated in detail in this book.

1.1.4 Chaos and Noise

One of the simplest questions is how to find chaos in laser intensity dynamics. What characteristics distinguish deterministic chaos from stochastic noise? The important feature of chaos is the fact that chaos is completely governed by a deterministic rule, which is described by a set of nonlinear equations without any stochastic terms. The determinism of chaos can distinguish irregular dynamics from noise. The instability of chaos results from inherent nonlinear interaction in dynamical systems, but not from stochastic noise.

By contrast, noise is defined as an irregular temporal waveform generated from a stochastic process that is based on a statistical law, but not on a deterministic rule. A simple example is throwing a dice. Each process of throwing a dice may not be deterministic, however, a statistical law may be evident after the repetition of many trials. Irregular temporal waveforms of noise are not completely deterministic, and cannot be described as a set of nonlinear equations. Instead, the behavior of noise can be sometimes described by a set of differential equations driven by a sequence of random numbers as a stochastic term (e.g., Langevin equations, see Chapter 2).

1.2 Synchronization of Chaos and Optical Communication

1.2.1 Synchronization of Chaos

"Synchronization" indicates temporal behaviors with a certain relationship. The same behavior of two temporal oscillations can be found for identical synchronization.

It has been known that two pendulums fixed on a common wall oscillate with the same frequency and phase (Pikovsky *et al.*, 2001). Synchronization of periodic oscillators is commonly used in many engineering applications of communications.

A question is whether chaotic temporal behaviors can be synchronized. Synchronization of chaos is a counterintuitive concept, since chaos has a strong dependence on initial conditions, which indicate that two nearby trajectories start to diverge exponentially in time and never reach the same state. The concept of synchronization of chaos thus contradicts the basic characteristics of chaos. It has been found that chaotic systems can be synchronized under certain conditions. A drive dynamical system itself may have unstable temporal waveforms, however, a response dynamical system with respect to the coupling signal from the drive system can become stable and the response system is able to follow the unstable temporal waveforms of the drive system. The change in the susceptibility to the coupling signal is the essence to obtain synchronization of chaos (see Chapter 5).

To achieve identical synchronization of chaos, similar hardware with similar parameter settings are required for coupled chaotic laser systems. This symmetry results in an identical synchronous solution of the coupled laser systems, and identical synchronization can be achieved. In addition, the identical synchronous solution must be stable. The stability of the synchronization depends on the particular dynamical system, but regions of stability in parameter space have been observed in various laser systems (see Chapter 6).

A remarkable demonstration of chaos synchronization was carried out and led to possible applications of secret communications (Pecora and Carroll, 1990). In optical systems, synchronization of chaos in coupled lasers has been investigated numerically and experimentally (Winful and Rahman, 1990; Roy and Thornburg, 1994; Sugawara *et al.*, 1994). Synchronization of chaos in one-way coupled optical systems and a potential application to optical secret communication have been studied (see Chapter 5).

1.2.2
Optical Communication with Synchronized Chaotic Lasers

Synchronization of chaos leads to an important application to optical chaos communications (VanWiggeren and Roy, 1998a; Goedgebuer *et al.*, 1998). Standard optical communication utilizes optical periodic carrier for encoding and decoding the message and there is no consideration for security in hardware level. Optical chaos communication leads to an additional layer of security or privacy in optical communication by using chaotic temporal carriers. These hardware-dependent optical communication systems have been developed and there have been international projects to implement optical chaos communication systems in real-world optical networks (OCCULT, 2001; PICASSO, 2006).

The basic concept of chaos communication is shown in Figure 1.3. A message is concealed in a chaos carrier in a transmitter, and the mixed signal consisting of the chaos and the message is sent to a receiver laser. The technique of synchronization of

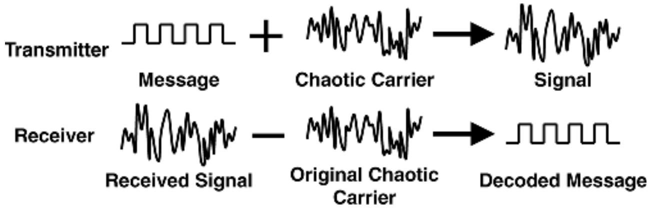

Figure 1.3 Schematics of the concept of optical communication with synchronized chaotic lasers.

chaos is used to reproduce the original chaos carrier in the receiver. The message can be recovered by subtracting the synchronized chaos carrier from the transmission signal. The quality of chaos synchronization strongly affects the degree of the recovered message signal (see Chapter 7).

Let us look at the procedure of the chaos communication in detail. The chaotic system in the transmitter produces a chaotic carrier to mask the message signal. The encoded signal is sent to the receiver and it is used for both decoding the message and achieving synchronization of chaos. In the receiver, a similar chaotic system can reproduce a nearly identical chaotic carrier by adjusting with a set of the static parameter values that are considered as static keys to be shared beforehand. If the receiver succeeds in synchronizing chaos by using the chaotic hardware and the static keys, it can reproduce a similar chaotic carrier and can succeed in decoding the original message.

The main purpose of chaos communication is to hide the "existence" of a message by a chaotic carrier waveform, which is known as steganography, compared with the technique of hiding the "meaning" of the message, known as cryptography (see Chapter 7). One of the most important techniques in chaos communication is to share the same chaotic carrier between the distant users by using synchronization of chaos. To achieve synchronization of chaos, similar hardware systems as well as similar parameter values are required in the transmitter and the receiver. The tolerance of synchronization against parameter mismatch is one of the measures for the level of privacy in chaos communication, that is, narrow parameter regions for achieving synchronization result in more privacy since it is difficult for eavesdroppers to achieve synchronization of chaos.

Recent advances on practical implementation of optical chaos communications will be described in this book (see Chapter 8), including the demonstration of chaos communication in commercial optical-fiber networks at 2.5-10 Gb/s over a 100-km distance with low BER (Argyris *et al.*, 2005; Argyris *et al.*, 2010c; Lavrov *et al.*, 2010). Several advanced techniques including photonic integrated circuits and forward error correction have been implemented in chaos communications.

For chaos communications, it is necessary for two legitimate users to share a common secret key prior to the communication process. The system parameters provide a private key because the two communicating lasers must have nearly identical parameters, otherwise synchronization is failed. The distribution of a private key is the main weakness of any secure communication system, which is known as a secure key distribution problem (Maurer, 1993). Chaotic lasers could be useful for the physical implementation of secure key distribution based on information theory. The architecture based on chaotic lasers offers large key-establishing rates at long communication ranges. In addition, optical transmitters and receivers used for conventional optical communication systems, including erbium-doped fiber amplifiers (EDFAs) and dispersion compensation fibers (DCFs), can be used for chaos-based secure key distribution without using specially designed hardware. Several schemes for chaos-based secure key distribution systems have been proposed and demonstrated as a new way of secure key distribution (see Chapter 9).

1.3 Random Number Generation with Chaotic Lasers and Other Applications

1.3.1 Random Number Generation

Another promising application with chaotic lasers is random number generation (see Chapter 10). The output of chaotic lasers provides fast temporal dynamics of chaos with large spectral bandwidth. Typical bandwidth of semiconductor lasers is a few GHz, which is determined by the relaxation oscillation frequency. The speed of lasers is advantageous for the applications of physical random number generation. The combination of the characteristics of the complexity in chaos and large bandwidth in lasers could open up a new research field of fast physical random number generation. The output of chaotic devices could be both unpredictable as well as statistically random because they generate large-amplitude random signals from microscopic noise by nonlinear amplification and mixing mechanisms.

The concept of random number generation is shown in Figure 1.4. A chaotic signal of laser output is detected by a photodetector and converted to a binary digital signal

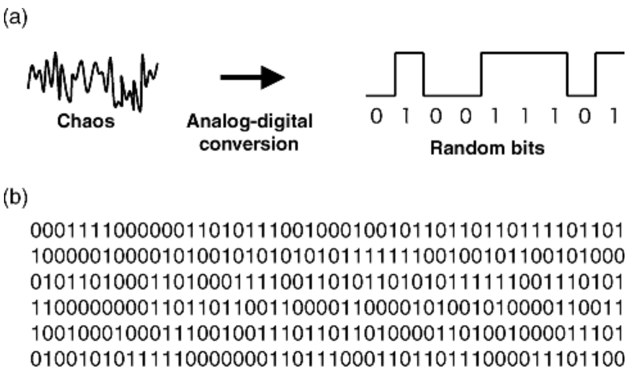

Figure 1.4 (a) Schematics of the concept of random number generation with chaotic lasers and (b) the sequence of generated digital bits.

by an analog-to-digital converter (ADC). The ADC converts the input analog signal into a binary digital signal by comparing with the threshold voltage. The output binary random signal is a stream of bits as shown in Figure 1.4a (with the format of nonreturn to zero (NRZ)) and Figure 1.4b (as bits “0” or “1”).

Random number generation with chaotic lasers has been intensively investigated since the first demonstration was published in 2008 (Uchida *et al.*, 2008a). Many schemes have been demonstrated and random-bit generation rates from 1.7 to 400 Gb/s have been reported with verified randomness (see Chapter 10).

1.3.2
Controlling Chaos and Other Applications

Another direction of the research with chaotic lasers is controlling chaos, which is a natural way for engineering applications. Chaos needs to be avoided in many engineering systems, and the research on control and stabilization of chaos has strong motivation in nonlinear dynamical systems that are used for engineering applications. The research field on controlling chaos grew rapidly during the 1990s, where a chaotic temporal signal can be stabilized onto an unstable periodic orbit in a chaotic attractor based on control theory (Ott *et al.*, 1990). The aim of chaos control is to obtain a stable or periodic temporal waveform by adding a small external perturbation. The techniques for controlling chaos have been applied to many interdisciplinary research fields, because there have been many dynamical systems that are required to stabilize chaotic instabilities and fluctuations (see Chapter 11).

Other promising applications with chaotic lasers have been proposed. Remote sensing applications with chaotic lasers have been reported as chaotic lider and radar systems. Blind signal separation using independent component analysis has been applied to chaotic temporal waveforms of laser output for the purpose of multiplexing communications. In addition, fractal optics have been used for wireless optical communication applications as a chaos mirror (see Chapter 12).

1.4
Research Directions for Engineering Applications with Chaotic Lasers

The research directions with chaotic lasers for engineering applications are summarized in Figure 1.5. There are three main research directions treated in this book, related to the concepts of how to harness chaos. For chaos communication applications, the characteristics of chaos are used in a straightforward way, that is, the determinism of chaos results in synchronization ability of chaos, and the middle degrees of complexity are suitable to hide a message signal. By contrast, for the applications of random number generation, the randomness of chaos needs to be maximized and determinism of chaos needs to be eliminated by converting analog chaos signals to binary signals. The important technique is how to extract and distill the randomness from deterministic chaos for this application. The research on

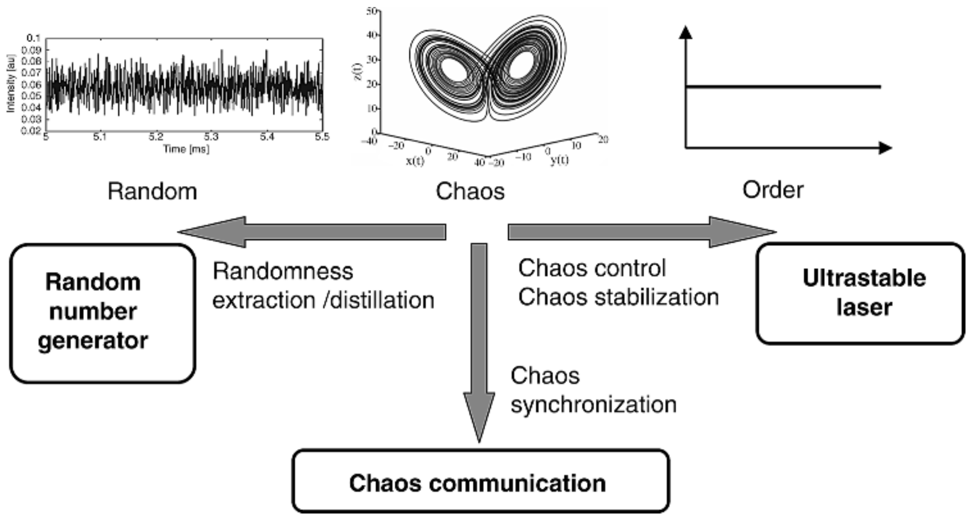

Figure 1.5 Three research directions for engineering applications with chaotic lasers.

random number generation requires a new engineering approach of chaos for maximizing the randomness of chaos.

By contrast, control and stabilization of chaos is a technique to completely avoid complexity and instability of chaos. To design and establish ultrastable lasers, chaos-control techniques may be useful for the suppression of chaotic instabilities. The features of deterministic chaos including unstable periodic orbits can be utilized for controlling chaos. The research on controlling chaos is the opposite direction of the research on random number generation, depending on minimizing or maximizing the complexity of chaos, respectively.

1.5 Outline of This Book

This book consists of two main parts: basics and applications. The first part corresponds to the description of basic physics of chaos and laser. Based on chaos and laser theories, the generation techniques of chaos in lasers are described in detail. Synchronization of chaos is also described and discussed, which is the basis for optical chaos communication. Chapters 2–6 cover the first part for the basics.

The second part of this book describes applications of chaotic lasers to optical communication and information technologies. The basic concept and recent advances of optical chaos communication systems are described in detail with experimental and numerical results from the literature. Two related topics to chaos communications are also described: secure key distribution based on information theory and random number generation with chaotic lasers. The use of chaotic lasers for fast physical random number generators has attracted increased interest, and this topic is overviewed in detail. Other possible applications of chaotic lasers are described, including controlling chaos, remote sensing, multiplexing communications with chaotic codes, and a chaos mirror for wireless communications. Chapters 7–12 cover the second part for the applications.

Readers do not need to read the text from Chapters 2–12 in sequence, instead, they can select and read chapters that are interesting to them. The contents of the chapters in this book are already summarized in Figure 1.1. The contents of each chapter will be described in more detail in the following.

1.5.1 Contents of Chapters: First Parts for the Basics

The first part of this book corresponds to the basics for chaos, laser, and synchronization, described from Chapters 2–6.

In Chapter 2, the basics of chaos and laser are introduced and explained. First the history of the observation of chaotic intensity fluctuations in lasers is described. The basic theories for chaos and laser are described separately. The connection between chaos and laser is made, and a basic single-mode laser model consisting of the variables of electric field, population inversion, and polarization of matter is described. The classification based on the decay rate of the laser model is explained with a rate-equation model.

In Chapter 3, the generation techniques of chaos in lasers are classified and overviewed with experimental and numerical examples from the literature. Most commercial lasers can be destabilized and produce chaotic output intensity by using the following techniques: feedback, coupling, modulation, and addition of nonlinear devices. These techniques are described in detail for various laser media, such as semiconductor lasers, fiber lasers, solid-state lasers, and gas lasers. The chaotic dynamics generated by using a passive optical system with time-delayed feedback is also described in electro-optic systems. The techniques for distinguishing between chaos and noise are discussed. Readers can learn a variety of chaotic temporal dynamics and bifurcations from many examples in various laser systems in Chapter 3.

Chapter 4 is specially designed for the readers who would like to learn analytical, numerical, and experimental techniques for the study on nonlinear dynamics in lasers. Standard techniques of nonlinear analysis in chaotic laser systems are introduced and explained in detail. A semiconductor laser with time-delayed optical feedback is used as an example of the analysis of chaotic laser dynamics. Experimental, analytical, and numerical results are shown. Experimental observations with temporal waveforms and RF spectra of chaotic laser intensities can be found in detail. For analytical techniques, Lang–Kobayashi equations are introduced and steady-state solutions are obtained. Linear stability analysis is also performed, which is a powerful tool to determine whether laser instability results from inherent nonlinearity of a deterministic model. For numerical simulation, temporal waveforms are generated from the Lang–Kobayashi equations. The maximum Lyapunov exponent, which is a reliable measure for the existence of chaos, is numerically calculated from the linearized equations. Other measures for complexity of chaos are introduced and calculated, such as the Lyapunov spectrum, Kolmogorov–Sinai entropy and Kaplan–Yorke dimension. Finally, Lang–Kobayashi equations with gain saturation are introduced to be consistent with experimental observations. The description in Chapter 4 may be very technical, but it would be useful for the readers who intend to start studying and working on chaotic laser dynamics.

In Chapter 5, synchronization of chaos in lasers is overviewed. The concept of synchronization of chaos is introduced and explained. The history of synchronization of chaos is described in electronic circuits and laser systems. Several coupling schemes for synchronization of chaos are explained, and synchronization is classified into two types: identical synchronization and generalized synchronization. These two types of synchronization can be distinguished by observing the delay time between two synchronized chaotic temporal waveforms in lasers with time-delayed feedback. Experimental and numerical examples of synchronization of chaos are also overviewed in various types of lasers, such as semiconductor lasers, electro-optic systems, fiber lasers, solid-state lasers, and gas lasers from the literature. Other types of synchronization such as phase synchronization and generalized synchronization are also described. Finally, a new concept of consistency is introduced and discussed, which indicates the reproducibility of a driven laser with respect to a repeated input signal.

In Chapter 6, standard techniques for the analysis of synchronization of chaos are introduced and explained in detail. Chapter 6 is specially designed for the readers who would like to learn experimental and numerical techniques for synchronization of chaos by using a realistic laser model. Synchronization of chaos is observed in two unidirectionally coupled semiconductor lasers with time-delayed optical feedback. Both experimental and numerical results are shown. For numerical simulation, coupled Lang–Kobayashi equations are introduced as a model for synchronization of chaos. Two types of synchronization (identical and generalized synchronizations) are observed and distinguished clearly in both the experiment and numerical simulation. The conditions for achieving synchronization of chaos are systematically investigated in wide parameter regions. Linear stability analysis is performed and the maximum conditional Lyapunov exponent is obtained, which is a good indicator of the stability of synchronous solutions. Generalized synchronization with low correlation is also investigated by using auxiliary system approach, where a chaotic output of a drive laser is injected into two response lasers that have identical parameter values but different initial conditions. High-quality synchronization of chaos between the two response lasers is obtained, while the correlation between the drive and each of the response lasers is relatively low for generalized synchronization. Chapter 6 may be very technical (as well as Chapter 4), but it would be very useful for the readers who intend to start studying and working on the analysis of synchronization of chaotic lasers.

1.5.2
Contents of Chapters: Second Part for the Applications

The second part of this book corresponds to engineering applications of chaotic lasers, described from Chapters 7–12.

In Chapter 7, the basic concept of optical chaos communication with chaotic lasers is introduced and summarized. The history of secret communication is described, and the technique of steganography leads to an invention of chaos communication. The concept and characteristics of chaos communication are discussed. Several encoding and decoding methods are introduced and characterized. The basic techniques for the evaluation of optical communication systems are also introduced.

Chapter 8 is considered as the main chapter for this book's main title, namely optical communication with chaotic lasers. The history of chaos communication is first described, and examples of optical chaos communication systems are introduced from the literature with different encoding/decoding techniques and laser configurations. Recent advances on optical chaos comunications are also described, including 10 Gb/s transmission over a 100-km distance with low BER. The privacy of chaos communication is also discussed in detail and several attempts of the attacks to chaos communication systems are described. Photonic integrated circuits have been invented for the implementation to real-world communication systems. Other encoding and decoding techniques are introduced, such as polarization encoding, spatiotemporal encoding, and multiplexing communications. Finally, new perspectives of optical chaos communication to biological communication systems are described. Readers will find a comprehensive overview in the field of optical chaos communication in Chapter 8.

In Chapter 9, secure key distribution based on information-theoretic security with chaotic lasers is described. Chaotic lasers can be useful for secure key distribution, and this topic results from the research activities in optical chaos communication. The importance of secure key distribution and the concept of information-theoretic security are described. Examples of the implementation of secure key distribution based on information-theoretic security with chaotic lasers or optical noise are described in detail from the literature.

In Chapter 10, a novel engineering application of random number generation with chaotic lasers is described and discussed. The use of chaotic lasers for random number generation was proposed in 2008 (Uchida *et al.*, 2008a), and intense research activities have been reported. The combination of chaos (complexity) and laser (fast oscillation with large bandwidth) results in a new research field of fast physical random number generation. The needs for fast physical random number generators are discussed and the types of random number generators are classified. Examples of random number generators with chaotic lasers are described in detail from the literature. Different methods for random-bit extraction from chaotic signals are proposed and discussed. The application of random number generation to quantum key distribution is also introduced. Numerical evaluation for nondeterminism of generated random bits is performed with a laser model. For comparison, conventional physical and pseudorandom number generators are introduced and discussed.

Finally, statistical evaluation of random numbers is described in detail. This chapter overviews the recent development of random number generation and would be informative for the readers who intend to start studying and working on random number generation.

In Chapter 11, controlling chaos is described. Unstable periodic orbits are used to control and stabilize chaos by using a perturbation with feedback. Nonfeedback control of chaos is also described and discussed. Several schemes of controlling chaos in lasers and their engineering applications are described from the literature.

In Chapter 12, other applications with chaotic lasers are introduced and described. Remote sensing with chaotic lasers is a promising application such as chaotic radar and lidar for precise detection. Blind-source separation of chaotic signals by using independent component analysis is described for mulplexing communication systems. Finally, fractal patterns in regular-polyhedral mirror–ball structures are observed and a chaos mirror is invented for the application of wireless optical communication.

Readers can learn the potential of the interdisciplinary research field between chaos and laser that would result in new findings in science and novel engineering applications.