1
Review of Classical Theories

A compact review of classical theories is presented, including the classical and statistical mechanics and electromagnetism. These theories are inherently intertwined with quantum mechanics and provide the general background from which to understand the quantum mechanics in a proper perspective.

1.1
Harmonic Oscillator

The harmonic oscillator (HO) is one of the simplest, yet ubiquitous dynamical systems appearing in a variety of physical and chemical systems such as electromagnetic waves and molecules. The HO is a particle attached to a spring, executing oscillatory motion. When the spring is compressed or stretched, the spring provides a restoring force for putting the particle back to the equilibrium position (Figure 1.1). In the process, an oscillatory motion ensues, and the motion represents a variety of important natural phenomena such as molecular vibrations and electromagnetic waves.

Newton's equation of motion of the HO reads as

$$m\ddot{x} = -kx \tag{1.1}$$

where m is the mass of the oscillator, x the displacement from the equilibrium position, and k the spring constant. The double dots denote the second-order differentiation with respect to time, and $-kx$ is Hook's restoring force. The equation can be put into a form

$$\ddot{x} + \omega^2 x = 0, \quad \omega^2 \equiv \frac{k}{m} \tag{1.2}$$

where ω is the characteristic frequency. Trigonometric functions, for example, $\sin \omega t$, $\cos \omega t$ are well-known solutions of Eq. (1.2). When the oscillator is pulled by x_0 and gently released, for instance, the displacement $x(t)$ and the velocity $v(t)$ are given by

$$x(t) = x_0 \cos \omega t, \quad v(t) \equiv \dot{x}(t) = -\omega x_0 \sin \omega t \tag{1.3}$$

and $x(t)$, $v(t)$ oscillate in time in quadrature (Figure 1.2) with the period $T = 2\pi/\omega$.

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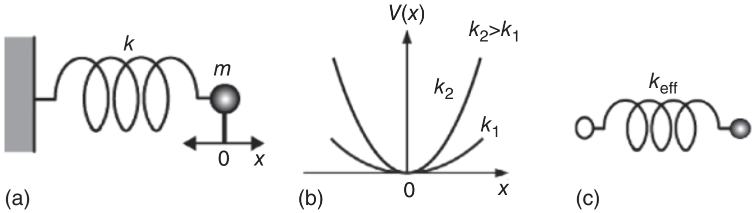

Figure 1.1 The harmonic oscillator, a particle of mass m attached to a spring with the spring constant k (a); the potential energy of HO (b); a diatomic molecule as represented by two atoms coupled via an effective spring constant (c).

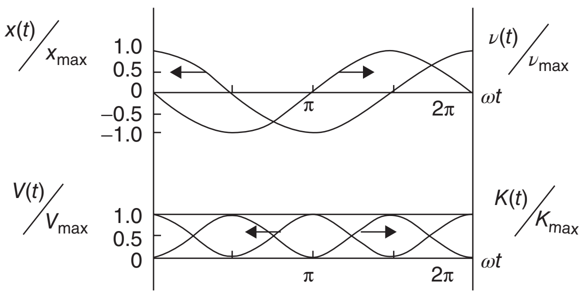

Figure 1.2 The displacement x, velocity v, and kinetic K and potential V energies versus ωt, all scaled with respective maximum values. The total energy $K + V$ is constant in time, and HO is a conservative system.

The potential energy of the HO is obtained by integrating the work done for displacing the HO from the equilibrium position to x against the restoring force:

$$V(x) \equiv -\int_0^x dx\,(-kx) = \frac{1}{2}kx^2 \tag{1.4}$$

The total energy is often denoted by Hamiltonian H and is expressed in terms of the linear momentum p_x and the displacement x as

$$H \equiv K + V = \frac{p_x^2}{2m} + \frac{1}{2}kx^2 \tag{1.5}$$

Given H, Hamilton's equations of motion read as

$$\dot{x} \equiv \frac{\partial H}{\partial p_x} = \frac{p_x}{m} \quad \dot{p}_x \equiv -\frac{\partial H}{\partial x} = -kx \tag{1.6}$$

The pair of equations in (Eq. (1.6)), when combined, reduces to Newton's equation of motion, and the variables x, p_x are known as *canonically conjugate variables*. The essence of classical mechanics is to solve the equation of motion and to precisely specify the position and momentum of a particle or a system of particles.

1.2
Boltzmann Distribution Function

The properties of macroscopic quantities are derived from the dynamics of an ensemble of microscopic objects such as electrons, holes, atoms, and molecules. Statistical mechanics describes such an ensemble of particles by means of the distribution function, $f\,(\boldsymbol{r}, \boldsymbol{v}, t)$. The function represents the probability of finding the particles in the phase space volume element $\boldsymbol{drdv}$ at $\boldsymbol{r}$, $\boldsymbol{v}$, and t. Thus, when multiplied by density n of the particle $f\,(\boldsymbol{r}, \boldsymbol{v}, t)\,\boldsymbol{drdv}$ represents the number of particles in the volume element at t.

The change in time of $f\,(\boldsymbol{r}, \boldsymbol{v}, t)$ is given from the chain rule by

$$\frac{df\left(\underline{r}, \underline{v}, t\right)}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \cdots + \frac{\partial f}{\partial v_x}\frac{\partial v_x}{\partial t} + \cdots = \frac{\partial f}{\partial t} + \underline{v}\cdot\nabla f + \underline{a}\cdot\nabla_v f \tag{1.7a}$$

where the operators

$$\nabla \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}, \quad \nabla_v \equiv \hat{x}\frac{\partial}{\partial v_x} + \hat{y}\frac{\partial}{\partial v_y} + \hat{z}\frac{\partial}{\partial v_z} \tag{1.7b}$$

are the gradient operators with respect to $\boldsymbol{r}$, $\boldsymbol{v}$, and $\boldsymbol{a}$ is the acceleration. The distribution function also changes in time due to collisions by which the particles are pushed out of or pulled into the volume element. Hence, the transport equation is given by

$$\frac{\partial f}{\partial t} + \underline{v}\cdot\nabla f + \frac{\underline{F}}{m}\cdot\nabla_v f = \left.\frac{\delta f}{\delta t}\right|_{\text{coll}}, \quad \underline{a} = \frac{\underline{F}}{m} \tag{1.8}$$

with $\boldsymbol{F}$ denoting the force.

Equilibrium

In the thermodynamic equilibrium, the distribution function f_0 is independent of time, that is, $(\partial/\partial t)f_0 = 0$, and the collision term should also be put to zero. This is because every process is balanced by its inverse process in equilibrium (detailed balancing). Consequently, the number of particles pushed out of and pulled into the phase space volume element due to collision is the same. Thus, the one-dimensional transport equation in equilibrium is given from Eq. (1.8) by

$$v_x \cdot \frac{\partial f_0}{\partial x} - \frac{1}{m}\frac{\partial\varphi}{\partial x}\frac{\partial f_0}{\partial v_x} = 0, \quad F_x \equiv -\frac{\partial\varphi}{\partial x} \tag{1.9}$$

where the force has been expressed in terms of the potential φ.

We may look for the solution in the form

$$f_0\left(x, v_x\right) = N\mathrm{e}^{-E(x)/k_BT}, \quad E\left(x\right) = \frac{mv_x^2}{2k_BT} + \varphi\left(x\right) \tag{1.10}$$

where N is the constant of integration and k_B the Boltzmann constant having the value $1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ or $8.617 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$, and $E(x)$ is the total energy at x, consisting of kinetic and potential energies. By inserting Eq. (1.10) into Eq. (1.9)

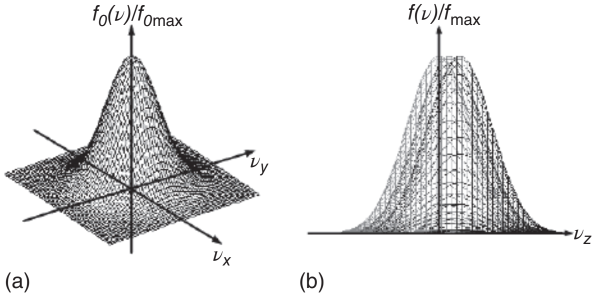

Figure 1.3 The distribution function of an ensemble of free particles in equilibrium (a) and under an electric field in the z-direction (b), all scaled with the maximum values; $f_0(\boldsymbol{v})$ is symmetric in $\boldsymbol{v}$, while $f(v)$ is not in the z-direction.

and carrying out the differentiation with respect to x and v_x, we find that Eq. (1.10) is indeed the solution. Also N can be used for normalizing $f_0(x, v_x)$. For a system of free particles in which $\varphi = 0$, the normalized equilibrium distribution function is given by

$$f_0\left(v_x\right) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} e^{-mv_x^2/2k_B T} \tag{1.11}$$

where N has been found from the normalization condition,

$$N\int_{-\infty}^{\infty} dv_x e^{-mv_x^2/2k_B T} = N\left(\frac{2\pi k_B T}{m}\right)^{1/2} = 1$$

Naturally, $f_0(v_x)$ can be generalized to three dimensions as

$$f_0\left(\underline{v}\right) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_B T}, \quad v^2 = v_x^2 + v_y^2 + v_z^2 \tag{1.12}$$

The function f_0 is the celebrated Boltzmann distribution function for a system of free particles, and the exponential factor appearing therein is called the *Boltzmann probability factor*. Clearly, $f_0(\boldsymbol{v})$ is symmetric in $\boldsymbol{v}$ and represents the fact that there is no preferred direction, a well-known property of the equilibrium (Figure 1.3).

Equipartition Theorem

In equilibrium, the probability of a particle moving from left to right is the same as that of moving from right to left (Eq. (1.11)). Therefore, the average velocity is zero, but the average value of v_x^2 is not zero and can be found as

$$\begin{aligned}\left\langle v_x^2\right\rangle &\equiv \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_{-\infty}^{\infty} dv_x v_x^2 e^{-mv_x^2/2k_B T} \int_{-\infty}^{\infty} dv_y e^{-mv_y^2/2k_B T} \int_{-\infty}^{\infty} dv_z e^{-mv_z^2/2k_B T} \\ &= \frac{k_B T}{m}\end{aligned} \tag{1.13}$$

By inspection, we can write

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{k_B T}{m} \tag{1.14}$$

Hence, the total average kinetic equation is given by

$$\frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \left(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \right) = \frac{3}{2} k_B T \tag{1.15}$$

which represents the equipartition theorem, namely, that the average kinetic energy of a free particle is equally divided into x-, y-, and z-directions, respectively, in equilibrium.

Nonequilibrium Distribution Function

Let us next consider an ensemble of electrons uniformly distributed in space and subjected to an electric field in the z-direction, $\hat{z}\mathrm{E}_0$. In this case, f is independent of $\boldsymbol{r}$ and at the steady state $\partial f/\partial t = 0$; hence, Eq. (1.8) reads in relaxation approach as

$$\frac{(-q\mathrm{E}_0)}{m_n}\frac{\partial f}{\partial v_z} = -\frac{f - f_0}{\tau}; \qquad \left.\frac{\delta f}{\delta t}\right|_{\mathrm{coll}} = -\frac{f - f_0}{\tau} \tag{1.16}$$

where $-q\mathrm{E}_0$ is the force acting on an electron with charge $-q$ and mass m_n. The collision term used describes the system relaxing back to the equilibrium in a time scale determined by τ called the *longitudinal relaxation time*, and f_0 and f are the equilibrium and nonequilibrium distribution functions, respectively. Let us assume for simplicity that f does not depart very much from f_0, that is, $f - f_0 \ll f, f_0$. In this case, we can find f iteratively by putting $f = f_0$ on the left-hand side, obtaining

$$f \approx f_0 + \frac{q\mathrm{E}_0 \tau}{m_n}\frac{\partial f_0}{\partial v_z} = f_0 \left(1 - \frac{q\mathrm{E}_0 \tau v_z}{k_B T} \right) \tag{1.17}$$

where Eq. (1.12) has been used for f_0. Clearly, f is asymmetric in v_z due to the electric field applied, while symmetric in v_x, v_y as shown in Figure 1.3.

Mobility and Conductivity

Once f is found, the physical quantities of interest can be specified explicitly. For example, consider the average velocity of electrons. As f is still symmetric with respect to v_x, v_y, $\langle v_x \rangle = \langle v_y \rangle = 0$ but $\langle v_z \rangle$ is not zero and is given by

$$\langle v_z \rangle \equiv \frac{\displaystyle\int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z v_z \; f_0 \left(1 - \frac{q\mathrm{E}_0 \tau(v) v_z}{k_B T} \right)}{\displaystyle\int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \; f_0 \left(1 - \frac{q\mathrm{E}_0 \tau(v) v_z}{k_B T} \right)} = -\frac{q\mathrm{E}_0}{m_n} \langle \tau_n \rangle \tag{1.18a}$$

where

$$\langle \tau_n \rangle \equiv \frac{m_n}{k_B T} \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z v_z^2 \tau(v) f \tag{1.18b}$$

denotes the effective relaxation time. Note in Eq. (1.18) that the first integral in the numerator and the second integral in the denominator vanish because the integrands therein are odd functions of v_z. This renders the denominator equal to unity because f_0 is a normalized distribution function (Eq. (1.12)). The relaxation time depends in general on the velocity $\boldsymbol{v}$ and has been averaged over.

The average velocity $\langle v_z \rangle$ derived in Eq. (1.18) represents the drift velocity with which all electrons move uniformly on top of their random thermal motion. The drift velocity is driven by E_0 exerting force on the electrons and can be viewed as the output of E_0:

$$v_{dn} \equiv \langle v_z \rangle = -\frac{q\mathrm{E}_0 \langle \tau_n \rangle}{m_n} \equiv -\mu_n \mathrm{E}_0, \quad \mu_n \equiv \frac{q \langle \tau_n \rangle}{m_n} \tag{1.19}$$

The response function μ_n connecting the input field and the output drift velocity is called the *mobility*. The current density of electrons due to drift is therefore given from Eq. (1.19) by

$$J_\mathrm{D} \equiv -q \sum_{j=1}^{n} (\underline{v}_{jth} + v_{dn}) = -\sigma_n \mathrm{E}_0, \quad \sigma_n \equiv q\mu_n n \tag{1.20}$$

where n is the electron density, and the random thermal velocities v_{jth} sum up to zero. The quantity σ_n connecting E_0 to J_D is known as the *conductivity*. The mobility μ_n and conductivity σ_n are the key transport coefficients.

1.3 Maxwell's Equations and EM Waves

Maxwell's equations are the foundations of the electromagnetism and are summarized as follows. When the charge and current density ρ and $\boldsymbol{J}$ are spatially distributed and vary in time, the electric $\mathbf{E}(\boldsymbol{r},t)$ and magnetic $\boldsymbol{B}(\boldsymbol{r},t)$ fields are generated and coupled to each other according to Maxwell's equations:

$$\nabla \times \underline{\mathrm{E}} = -\frac{\partial \underline{B}}{\partial t} \tag{1.21}$$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} \tag{1.22}$$

$$\nabla \cdot \underline{\mathrm{E}} = \frac{\rho}{\varepsilon} \tag{1.23}$$

$$\nabla \cdot \underline{B} = 0 \tag{1.24}$$

The displacement vectors $\boldsymbol{D}$ and $\boldsymbol{B}$ are correlated to $\mathbf{E}$ and the magnetic field intensity $\boldsymbol{H}$ via the permittivity ε and the permeability μ of the medium as

$$\underline{D} = \varepsilon \underline{\mathrm{E}}, \quad \underline{B} = \mu \underline{H} \tag{1.25}$$

The addition of the continuity or charge conservation equation renders Maxwell's equations self-contained:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{J} = 0 \tag{1.26}$$

As well known, Eq. (1.21) is Faraday's law of induction, specifying $\boldsymbol{B}(\boldsymbol{r},t)$ as the source of generating **E**, while Eq. (1.22) is Ampere's circuital law describing $\boldsymbol{J}$ as the source for generating $\boldsymbol{B}$. Also Eq. (1.23) represents Coulomb's law and Eq. (1.24) is the theoretical statement of the fact that no magnetic monopole has been observed. Ampere's circuital law, Eq. (1.22), was complemented by Maxwell, who introduced $\partial \underline{D}/\partial t$, called the *displacement current*. The modification was necessitated by the fact that the curl of any vector, $\nabla \times \underline{A}$, should be solenoidal, that is, $\nabla \cdot \nabla \times \underline{A} \equiv 0$, as can be readily verified. With $\boldsymbol{D}$ thus introduced, the requirement that $\boldsymbol{H}$ in Eq. (1.22) is solenoidal is satisfied, because the divergence operation on the right-hand side of Eq. (1.22) reduces the equation to the continuity equation to become zero. Maxwell's equations are rooted in the observed laws of nature and have successfully undergone the test of time and have been the source of unceasing applications.

Wave Equation

The electric and magnetic fields **E** and $\boldsymbol{H}$ coupled inherently via the two laws Eqs. (1.21) and (1.22) can be decoupled and examined separately. Thus, consider a medium free of charge ρ and $\boldsymbol{J}$. Then, the curl operations on both sides of Eq. (1.21) lead to

$$\nabla \times \nabla \times \underline{\mathrm{E}} \equiv \left[\nabla\nabla \cdot -\nabla^2\right] \underline{\mathrm{E}} = -\nabla^2 \underline{\mathrm{E}}; \quad \nabla \cdot \underline{\mathrm{E}} \propto \rho = 0 \tag{1.27a}$$

$$\nabla \times \left(-\frac{\partial \underline{B}}{\partial t}\right) = -\mu \frac{\partial}{\partial t}\left(\underline{J} + \varepsilon \frac{\partial}{\partial t}\underline{\mathrm{E}}\right) = -\mu\varepsilon \frac{\partial^2}{\partial t^2}\underline{\mathrm{E}}, \quad \underline{J} = 0 \tag{1.27b}$$

where a vector identity and Ampere's law have been used in Eqs. (1.27a) and (1.27b), respectively. Hence, by equating Eqs. (1.27a) and (1.27b), there results the wave equation:

$$\nabla^2 \underline{\mathrm{E}} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\underline{\mathrm{E}} = 0, \quad \frac{1}{v^2} \equiv \mu\varepsilon = \mu_0\varepsilon_0\mu_r\varepsilon_r = \frac{1}{(c/n)^2} \tag{1.28}$$

Here, v is the velocity of light in the medium in which $\mu_r = 1$ and is specified in terms of the velocity of light in the vacuum $1/\mu_0\varepsilon_0$ and the index of refraction n via $\varepsilon_r = n^2$,with ε_r denoting the dielectric constant. Clearly, $\boldsymbol{D}$ is indispensable in bringing out the wave nature of the electromagnetic field. We can likewise derive the identical wave equation for $\boldsymbol{H}$.

Plane Waves and Wave Packets

A typical solution of the wave equation (1.28) is the plane wave

$$\underline{\mathrm{E}}(z,t) = \hat{x}\mathrm{E}_0 e^{-i(\omega t - kz)}, \quad \omega = \frac{k}{\sqrt{\mu\varepsilon}} \tag{1.29}$$

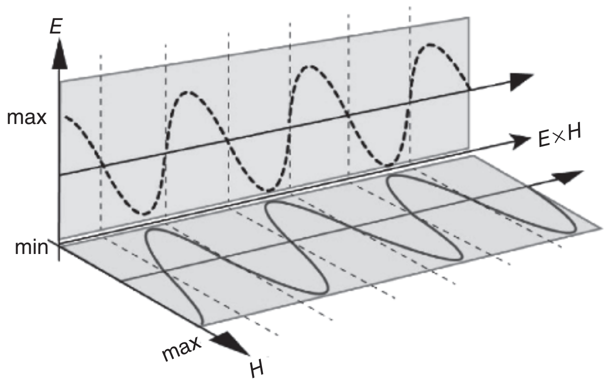

Figure 1.4 Spatial profiles of electric and magnetic fields traveling in the z-direction. Also shown is the Poynting vector, accompanying the propagation with the power.

propagating in the z-direction, for example, with the wave vector $k = 2\pi/\lambda$ obeying the dispersion relation as given in Eq. (1.29). The amplitude $\mathbf{E_0}$ has to be taken perpendicular to $\boldsymbol{k}$, say in the x-direction so that $\nabla \cdot \underline{\mathrm{E}} \propto \hat{z} \cdot \hat{x} = 0$ in accordance with Coulomb's law. In this case, the $\boldsymbol{H}$-field is obtained from Eqs. (1.21) and (1.29) as

$$\underline{H} = \hat{y}\sqrt{\varepsilon/\mu}\mathrm{E}_0 e^{-i(\omega t - kz)}, \quad \hat{y} = \hat{z} \times \hat{x} \tag{1.30}$$

Therefore, $\mathbf{E}$, $\boldsymbol{H}$, and $\boldsymbol{k}$ are mutually perpendicular, and the complex Pointing vector $\mathrm{E} \times H^*$ represents the power flow in the z-direction, as shown in Figure 1.4.

Wave Packets

The wave equation (1.28) is linear, so that the linear superposition of plane waves is also the solution:

$$\underline{\mathrm{E}}(z,t) = \mathrm{Re}\sum_n \underline{\mathrm{E}}_n e^{-i(\omega_n t - k_n z)} = \mathrm{Re}\int_{-\infty}^{\infty} dk\underline{\mathrm{E}}(k)\, e^{-i(\omega t - kz)} \tag{1.31}$$

The wave packet can be put into a compact form by Taylor expanding ω at k_0:

$$\omega(k) = \omega(k_0) + v_g(k - k_0) + \alpha(k - k_0)^2 + \cdots; \quad v_g \equiv \frac{\partial\omega(k_0)}{\partial k} \tag{1.32}$$

In a linear medium $\alpha = 0$, $v_g = c/n$, and by using Eq. (1.32), we can express Eq. (1.31) as

$$\underline{\mathrm{E}}(z,t) = \mathrm{Re}e^{-i(\omega_0 t - k_0 z)}\int_{-\infty}^{\infty} dk\underline{\mathrm{E}}(k)\, e^{i(z - v_g t)(k - k_0)} \tag{1.33}$$

and represent the wave packet in terms of two components: (i) the mode function oscillating with the carrier frequency ω_0 and propagating with the phase velocity ω_0/k_0 and (ii) the envelope contributed by superposed plane waves

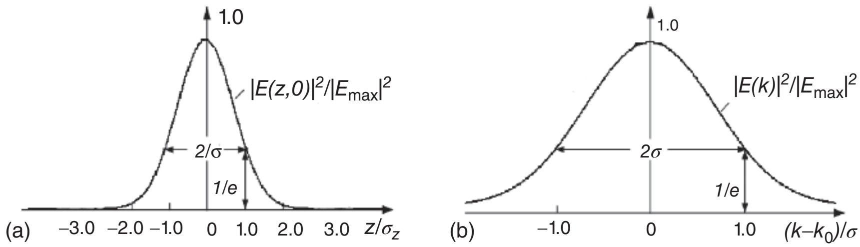

Figure 1.5 Spatial profile of the field intensity in the z-direction (a) and power spectrum versus the wave vector (b).

and propagating with the group velocity v_g. For the Gaussian spectral density centered at k_0,

$$\underline{\mathrm{E}}(k) = \frac{\underline{\mathrm{E}}_0 e^{-(k-k_0)^2/2\sigma^2}}{\sqrt{2\pi\sigma}} \tag{1.34}$$

the integration of Eq. (1.33) yields

$$\underline{\mathrm{E}}(z,t) = \mathrm{Re}\underline{\mathrm{E}}_0 e^{-i(\omega_0 t - k_0 z)} e^{[-\sigma^2(z-v_g t)^2/2]} \tag{1.35}$$

The wave packet in this case consists of a Gaussian envelope propagating with the group velocity v_g, while the mode function rapidly oscillates within the envelope and propagates with the phase velocity ω_0/k_0.

Shown in Figure 1.5 are the spatial profile of the wave packet Eq. (1.35) at $t = 0$ and the power spectrum. The bandwidth of the power spectrum Δk is often defined by the width between two $1/e$ points from its peak, that is, $\Delta k = 2\sigma$. The spatial extent of the intensity envelope is likewise specified by $\Delta z = 2/\sigma = 4/\Delta k$. Given Δk, the frequency band width is given from the dispersion relation by $\Delta\omega = v_g \Delta k = 2v_g\sigma$. Finally, the time duration of the wave packet is given by $\Delta t = \Delta z/v_g = 4/\Delta\omega$. Therefore, the wave packet is characterized by the basic relation

$$\Delta z \propto \frac{1}{\Delta k}, \quad \Delta t \propto \frac{1}{\Delta\omega} \tag{1.36}$$

where the proportionality constants are of the order of unity and depends on the dispersion relation occurring in the power spectrum. The relationship (Eq. (1.36)) is of fundamental importance in quantum mechanics and is followed up in due course.

The Interference

The interference effect is a signature of the wave and was demonstrated by Young with his classic double-slit experiment as shown in Figure 1.6. In this experiment, two plane waves emanating from a distant source are passed through two slits. The two beams are detected on a screen L distance away from the slits. At a point

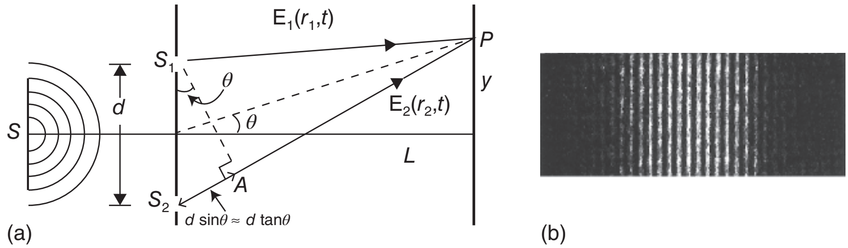

Figure 1.6 (a) Young's double-slit experimental scheme and (b) the observed fringe pattern.

P on the screen, the total field registered consists of the two plane waves:

$$\underline{E}\left(\underline{r}, t\right) = \sum_{j=1}^{2} \mathrm{Re}\underline{E}_0 e^{-i\left(\omega t - \underline{k}_j \cdot \underline{r}_j\right)} \tag{1.37}$$

The detected time-averaged intensity is thus given by

$$I = \left\langle \left(\underline{E}_1 + \underline{E}_2\right) \cdot \left(\underline{E}_1^* + \underline{E}_2^*\right) \right\rangle_t = \left|E_1\right|^2 + \left|\underline{E}_2\right|^2 + \left(\underline{E}_1 \cdot \underline{E}_2^* + \underline{E}_2 \cdot \underline{E}_1^*\right) \tag{1.38}$$

and consists of two background and interference terms, respectively. Naturally, the latter two terms depend on the difference in optical paths the two beams have traversed before reaching P. The resulting phase difference is given in the far-field approximation by $kd \sin\theta$ (Figure 1.6), and therefore I reads as

$$I = 2\left|\underline{E}_0\right|^2 (1 + \cos\varphi), \quad \varphi = kd \sin\theta \simeq \left(\frac{2\pi}{\lambda}\right) d \left(\frac{y}{L}\right) \tag{1.39}$$

where d and y are the space between two slits and the height of P on the screen, respectively. For $L \gg y$, $\sin\theta \simeq \tan\theta \simeq y/L$. Obviously, the interference term adds to or subtracts from the background, depending on the relative phase between the two beams. The maximum and minimum intensities are attained for $\varphi = 2n\pi$ and $\varphi = 2\pi\,(n + 1/2)$, respectively, with n denoting an integer. Therefore, bright and dark strips appear at $y_n = (\lambda L/d)\, n$ and $y_n = (\lambda L/d)\,(n + 1/2)$, respectively.

Problems

1.1 The H_2 molecule consists of two protons coupled via an effective spring with the spring constant k. The 1D Hamiltonian is given by (Figure 1.7)

$$H = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} k\left(x_1 - x_2\right)^2$$

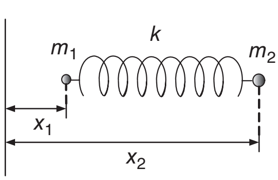

Figure 1.7 Two particles coupled via a spring with spring constant k.

(a) Introduce the center of mass and relative coordinates as

$$X = x_1 + x_2, \quad x = x_1 - x_2$$

and express the Hamiltonian in terms of X and x and interpret the result.

(b) Write down the equations of motion for the center of mass X and relative displacement x and interpret the equations of motion.

1.2 Find the thermal velocity of (a) electron, (b) proton, (c) H_2 molecule, and (d) particle of mass 1 g at $T = 10$, 300, and 1000 K.

1.3 (a) Show that the electric field given in Eq. (1.29) is the solution of the wave equation, provided ω, k satisfy the dispersion relation, $\omega^2 = v^2k^2$ with k denoting the wave vector.

(b) Show that the magnetic field intensity H given in Eq. (1.30) and **E** in Eq. (1.29) satisfy Faraday's law of induction and Ampere's circuital law in a medium free of charge and current.

(c) Derive the wave equation of ***H***.

1.4 Given the wave packet Eq. (1.35), find variance of $\left|\underline{\mathrm{E}}(z,t)\right|^2$ at $t = 0$

$$(\Delta z)^2 = \left\langle (z - \langle z \rangle)^2 \right\rangle; \quad \langle a \rangle \equiv \int_{-\infty}^{\infty} dza|\mathrm{E}(z,0)|^2$$

1.5 By using the relations

$$\underline{\hat{x}} \cdot \underline{\hat{x}} = \underline{\hat{y}} \cdot \underline{\hat{y}} = \underline{\hat{z}} \cdot \underline{\hat{z}} = 1, \quad \underline{\hat{x}} \cdot \underline{\hat{y}} = \underline{\hat{y}} \cdot \underline{\hat{z}} = \underline{\hat{z}} \cdot \underline{\hat{x}} = 0,$$
$$\underline{\hat{x}} \times \underline{\hat{y}} = \underline{\hat{z}}, \underline{\hat{y}} \times \underline{\hat{z}} = \underline{\hat{x}}, \quad \underline{\hat{z}} \times \underline{\hat{x}} = \underline{\hat{y}}$$

show that all vectors are solenoidal, that is, $\nabla \cdot \nabla \times \underline{A} \equiv 0$.

1.6 By combining Eqs. (1.23), (1.25), and (1.26), show that ***H*** in Eq. (1.22) is solenoidal.

Suggested Readings

1. D. M. Kim, Introductory Quantum Mechanics for Semiconductor Nanotechnology, Wiley-VCH, 2010.
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