1
Introduction

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Light travels fast. Photons are its swift messengers. Designing an optical cavity is the appropriate answer to the necessity of enhancing the interaction time between photons and matter, through the recirculation of light amidst an optical medium of bounded size. The wave nature of light allows these multiple passes to interfere, and buildup at resonant wavelengths or frequencies. As a result, the nonlinear response of materials can be more easily awakened in optical cavities than in transmission experiments, as much as nonlinear effects can be sharply revealed through the microscopic wavelength ruler that is naturally embedded into a resonant optical cavity.

The science of photonics is diffusing continuously into all major sectors of the industry, from manufacturing and communications to medicine and environmental monitoring. Consequently, there is an increasing demand to develop efficient, compact, and cost-effective optical sources or devices to perform a wide range of tasks, which include high-speed spectroscopy and sensing, material processing, communication and data processing, and metrology. The optical cavity, either in its active – laser – or passive form, is becoming more and more an applicative node. The tremendous progress made in the field of semiconductor laser sources in the past decades makes a striking illustration. Currently, a wide range of other optical cavity designs show great promises, such as microresonators and fiber lasers. Thus, it is important to consider, in a newly unified way, the broad range of fast and ultrafast dynamics that now make a coherent conceptual frame for light generation and processing. This is the scope of this book, definitely grounded on fundamental considerations and also highlighting major applications.

The recirculation of light in the optical cavity produces a feedback mechanism, whose association with nonlinearity is conducive to the appearance of bistable behaviors. Thus, bistability is a generic feature of nonlinear optical cavities. As the optical nonlinearity can be of dispersive or dissipative nature – or rigorously, a combination of the two – various types of optical bistability can, accordingly, manifest [1]. Bistability is a fascinating property that naturally translates into the notion of information storage and manipulation. However, due to the scale of optical wavelengths, the density of optically stored information cannot compete with the memory density implemented in modern CMOS technology.

Thus, applicative motivations clearly shift toward the realization of optical buffer memories or arbitrary long delay lines, all-optical switches, and other advanced optical processing units. Recent technological breakthroughs in optical cavities, from microresonators to ultra-long fiber cavities, have entitled the exploration of nonlinear optical dynamics over unprecedented spatial and temporal orders of magnitude.

Indeed, the key advantage of optical technology lays in its potentially ultrahigh bandwidth, or equivalently, ultrafast transmission. As soon as the material nonlinearity response time is in the femtosecond to the picosecond range, as it is the case, for instance, with Kerr nonlinearity in most glass materials, the effects of nonlinearity and dispersion can combine to shape up short-to-ultrashort optical pulses. When the previous combination reaches a balance amid a single optical pulse, the latter becomes an optical soliton, which subsequently maintains its specific temporal waveform during propagation [2–3].

However, dealing with the arbitrarily long propagation distances that are accumulated through the succession of cavity round-trips, losses cannot be avoided. Several strategies can be implemented to compensate for these losses. In general, a coherent driving field can inject the cavity, either continuously or synchronously. Or, the cavity may include an incoherently pumped gain medium that makes it a laser system. In all cases, to maintain a given light state in the optical cavity, a balance between gain and losses should be reached. As soon as nonlinearity is involved in this dissipative balance, stable pulsed light states pertain to the general class of dissipative solitons [4]. The qualifier "dissipative" has to be understood in the broad dynamical sense attributed by Nicolis and Prigogine [5], and not as a mere synonym of a "lossy" system. Instead of remaining deceptive, dissipative systems, through the perpetual and bidirectional exchange of energy with their environment, can manifest conditions for self-organization. They have become during the past decade a fascinating area of research [4, 6]. Dissipative solitons are confined wave-packets of light whose existence and stability crucially depend on the energy balance. The dissipative soliton paradigm is underlying most of the chapters of the present volume, encompassing a myriad of possible dynamics, from stationary to pulsating and chaotic pulsed regimes.

Conceptually, the simplest nonlinear optical cavity scheme may correspond to a single transverse mode, coherently driven, cavity incorporating an ultrafast Kerr medium. It can then be considered as "the hydrogen atom" of nonlinear optical cavity dynamics. This statement obviously stands as long as polarization dynamics, thermal, and additional higher-order effects are not involved. In its distributed form, the optical cavity can be scaled from a compact microring to a kilometer-long fiber ring. Interestingly, going beyond the mean-field equation describing optical bistability, the stability analysis unveils the – parameter dependent – possibility of reaching various modulated and pulsating regimes. This underpins the existence of well-localized temporal cavity solitons, which can be considered as individual, addressable optical bits of information that travel round the optical cavity. Coen and Erkintalo discuss this topic extensively in Chapter 2, in correspondence with fiber ring cavity experiments.

For high-finesse-driven resonators, the Lugiato–Lefever equation (LLE) is a central dynamical model [7, 8]. It is used in several chapters, and chapter authors were allowed to retain their own derivation and notations for the reader's convenience. The solitary wave solutions of the LLE have been called *cavity solitons*, and they belong to the general class of dissipative solitons, as explained previously and reflected in Lugiato and Lefever's [7] seminal 1987 paper. Interestingly, the latter original study considered the spatiotemporal dynamics of optical cavities with high Fresnel numbers, namely, the possibility to form transverse patterns and spots – spatial cavity solitons. This readily increases the complexity of the nonlinear optical cavity system: not only from the theoretical point of view, where complex two-dimensional transverse structures, such as vortices, can form without having an equivalent in the purely temporal case but also – and mostly – because it is extremely difficult to realize a homogeneous wide-aperture cavity. Then, losing the translational symmetry, the spatial cavity solitons are set to move transversally, sensing the gradients of their local environments, until they become pinned by defects.

In active media, laser light can be generated with versatile underlying dynamics, from stationary to pulsating and chaotic ones. Emphasizing on ultrafast dynamics, the vital arena for the information technology, we find the soliton as a common conceptual keyword, thriving into its modern developments with the closely related denominations of dissipative solitons and cavity solitons.

Broad-area vertical-cavity surface emitting lasers (VCSELs) make an advantageous technological platform to experiment cavity solitons [9]. They benefit from the compactness and high nonlinearity of grown semiconductor structures. It is also possible to combine their optical gain with a particular feedback mechanism, to circumvent the necessity of an external coherent driving transverse wave. Such a design allows the observation of specific cavity solitons, namely, laser cavity solitons. As can be expected, the use of a semiconductor material increases the complexity of the underlying theoretical description, involving a set of coupled nonlinear equations for the carrier density along with the intracavity optical field. It is also particularly interesting to study the interactions between neighboring laser cavity solitons, and their coherence properties. All these points are addressed in Chapter 3, by Ackemann *et al.*, maturing a thorough expertise about complex spatiotemporal dynamics. A challenging perspective is to find experimental conditions where cavity solitons localized both transversally and in the time domain, namely, laser light bullets, could manifest. Until now, stable light bullets that can be manipulated remain the holy grail of nonlinear optics [10, 11].

Semiconductor laser cavities are characterized by a short gain relaxation time of the order of a nanosecond or less. By extending the cavity length, with a delay line in air, one may shift the laser dynamical regime from class B to class A. Now, let us consider a single transverse-mode semiconductor laser. When the gain relaxation lifetime becomes much shorter than the cavity round-trip time, and by using a suitable saturable absorber, it is possible to modelock the extended semiconductor laser cavity with multiple independent pulses. Because these pulses do not interact, when sufficiently separated, they can be considered as "localized structures," a particular class of dissipative solitons, making a laser analog of the temporal cavity solitons described in Chapter 1. Similarly, these laser temporal cavity solitons can be manipulated as independent information bits. This is the main subject of Chapter 4, by Barland *et al.*

In general, any extended optical cavity will experimentally feature some amount of spatial inhomogeneity and drift. These two additional effects can lead to a wide range of bifurcations, including excitable behavior. Interestingly, an analogy can be made between the transverse spatial case, and the longitudinal temporal case, when the spatial inhomogeneity is replaced by the periodic injection of driving pulses. When the injection period does not precisely coincide with the cavity fundamental repetition frequency, an equivalent drift source also appears. These fundamental explorations of nonlinear cavity dynamics, relevant for systems ranging from broad-area VCSELs to microresonators and frequency comb generation, are developed in Chapter 5 by Parra-Rivas *et al.*

Frequency combs establish a modern workhorse for high-precision spectral measurements that can be performed in a record time [12–14]. Driven by the perspective of developing compact, efficient, and reliable frequency comb generators, the research on microresonators has expanded tremendously during the past 10 years. One has to realize the major gaps, technological as well as conceptual ones, needed to progress from the observation of optical bistability [15], usually dominated by slow thermal nonlinearities, to the efficient cascading of four-wave mixing, exploiting the ultrafast Kerr nonlinearity of the optical material [16], and nowadays, to the study and control of the coherence properties of the generated frequency combs.

It is remarkable that the latest research trends have brought closer several scientific communities that used to thrive quite separately. This is particularly true concerning the communities of optical microresonators, of nonlinear dynamics, and of mode-locked lasers. As the reader may have guessed, this recent movement represents a major motivation to encompass, as much as possible, the joined expertise into a single book volume. Also, it is quite conspicuous that the concept of a dissipative soliton represents a major thread linking all these current researches. The cross-fertilization between various areas is also represented by the alternation of spectral and temporal pictures, or temporal and spatial pictures, that are put in vivid correspondence.

Chapters 6 and 7 successively unveil the major developments in the area of microresonators designed for frequency comb generation. Chapter 6, by Herr *et al.*, unfolds the recent history of the progresses made, from experimental and fundamental points of view, which lead to the mastering of frequency comb generation in specifically shaped, ultrahigh quality factor, crystalline microresonators. The focus is then on the particular experimental conditions and parameters that allow the multiple frequency lines to lock in order to promote the formation of temporal dissipative solitons that circulate round the laser-driven microresonator. Despite being based on completely different technological platforms, with around five orders of magnitude difference between their respective sizes, driven microresonators and fiber ring cavities behave similarly, as attested by the common use of the LLE to reveal the most salient temporal dynamics. Naturally, this huge dimension gap initially made scientific languages to develop relatively apart: whereas it is natural to describe the microresonator dynamics in terms of spectral modes that are well resolved with any standard optical spectrum analyzer, the real-time observation of pulsed structures is challenging, due to their high repetition rate. The picture is opposite for long fiber cavities, which also feature a much lower finesse: the tightly packed, weakly modulated, spectral modes remain generally unresolved, but the low cavity repetition rate makes the direct observation of the temporal structures in the making a natural scope, despite the gigahertz bandwidth limitation of the analyzing electronics. After recalling the equivalence between the spectral and temporal formalisms, Chapter 7 by Coillet *et al*. takes on the solid grounds of the temporal modeling of driven-microresonator dynamics to explore the wide range of dynamical properties of frequency combs, backed by comparisons with experimental results, which includes Turing patterns, bright and dark solitons, as well as higher-order pulsations and chaotic states. The latter are shown to support the transient formation of optical rogue waves, which are extreme-wave events ubiquitous in highly dimensional nonlinear systems [17].

Although the technological platforms of fiber cavities and microresonators may seem a world apart, there is an original research at their crossroad, which is represented by microfiber, nanofiber, and microcoil resonators. As a prerequisite, this has implied during the past decade, the transformation of artisanship skills to automated procedures, in order to routinely produce high-quality submicron-diameter optical fibers drawn from optical fibers of standard diameters. That said, microfibers represent an easily accessible and particularly versatile technological platform [18], which is able to test a wide range of concepts of integrated nonlinear optics, such as frequency conversion, pulse shaping, and nonlinear switching. Chapter 8, by Abdul Khudus *et al.*, reviews all these prospects in the light of both experimental and numerical explorations.

The crossroad between microresonators and fiber lasers represents a different recent conjunction. Although it is known that both platforms can be used separately to generate optical frequency combs, it would be highly desirable to combine the technological connectivity and the dynamical self-organization that are naturally available in fiber lasers with the multi-gigahertz frequency range spacing of comb lines that are intrinsic to microresonators. Pasquazi *et al*. relate this endeavor in Chapter 9. The authors explain the advantages and constraints inherent to the association of a long and a short cavity, aimed at processing distinct tasks for the establishment of a high-harmonic laser mode locking, and emphasize on the stability features of the demonstrated pulsed sources.

For passively mode-locked fiber lasers, harmonic mode locking is just one among the myriads of multipulse dynamics that can be experimented. Owing to the general efficiency of fiber lasers, to the scalability of their pulsed dynamics, and to the availability of intense pump modules, the number of ultrashort pulses that can coexist in a fiber laser cavity can be varied from a few to thousands. In contrast to the situation in passive-driven resonators (Chapter 2) and to some specific class-A lasers (Chapter 4), multiple pulses in mode-locked fiber lasers always interact in a manifested way, which can become dramatic for some cavity parameter settings. This topic is thoroughly developed in Chapter 10 by Sanchez *et al.*, and explained in the light of dissipative soliton dynamics. Remarkable collective behaviors include soliton crystals, which could represent an alternative way to obtain stable frequency combs and high-harmonic mode locking, and complex soliton dynamics such as soliton rain and dissipative optical rogue waves (DRWs). A bridge is established among DRWs, chaotic pulse bunching, and noise-like pulse emission, which are denominations of chaotic dynamics well represented in the arena of short-pulse fiber laser dynamics.

In Chapter 11, Chang and Akhmediev build a noteworthy bridge between rogue waves and another category of short-pulse chaotic dynamics: exploding solitons. Exploding solitons are remarkable states linked to the existence of strange attractors [19]. In addition, chaotic dynamics in highly-dimensional nonlinear systems are conducive to rogue wave formations [17]. The authors having a foremost expertise in both topics quite naturally anticipate and demonstrate numerically the existence of a significant overlap between DRW and exploding soliton dynamics, using a passively mode-locked laser propagation model.

The area of short-pulse fiber lasers is extremely dynamic [20–22]. The recent dissipative soliton paradigm [23] applied to mode-locked lasers has allowed relaxing several of the previously implied cavity design constraints, notably concerning the sign of the chromatic dispersion, and the amount of acceptable losses of the fiber laser cavity. Such renewed freedom has promoted a lot of creative cavity design, including all-normal-dispersion chirped-pulsed lasers [24], and soliton–similariton mode-locked lasers [25] as prominent illustrations. One clear objective is now to obtain optimized pulse features – in terms of pulse energy, optical bandwidth, or pulse duration – out of all-fibered laser oscillators. The analysis of the current pulse energy limitations of mode-locked fiber lasers, and the possibilities to circumvent them, are explored in detail in Chapter 12 by Babin *et al*.

Beyond representing a complication for experiments and modeling alike, polarization introduces a smart degree of freedom to the dynamics, which can dispel some fundamental impossibility. For example, we all know that, by using polarization components and a nonreciprocal element such as a Faraday rotator, the principle of reversible ray tracing does not hold any longer, which is very useful to design the optical isolators and circulators that are ubiquitous in laser and optical communication technology. Involving polarization and vector systems, in general, the dynamical domains multiply. For instance, using birefringent fibers, parameters can be found to trigger modulation instability (MI) in the normal dispersion regime, through simple propagation experiments [26, 27], whereas scalar MI necessitates an anomalous dispersion regime. In the light of the previous chapters, there is a recurrent observation: as soon as the space of parameters expands, new dynamical breaches appear, such as MI in driven, normally dispersive resonators – by virtue of the detuning extra degree of freedom [28], and bright dissipative soliton pulses in normally dispersive laser cavity – by virtue of the dissipative terms [29].

Involving polarization dynamics can have far-reaching consequences for laser systems. Indeed, for instance, dispersive (crossed-Kerr effect) and/or dissipative (crossed gain saturation) contributions can initiate short-pulse dynamics in the absence of known (scalar) saturable absorber in the laser cavity [30]. To address this topic, the most instructive route begins with a journey through vectorial solid-state lasers, where the piecewise cavity design allows controlling and modeling well all the anisotropies. This is presented by Brunel *et al.*, in Chapter 13. Although disconcertingly simple linear physics is sufficient to understand the onset of self-pulsing at specific beat frequencies, the pulse shaping definitely involves the presence of crossed nonlinearities. Subsequently combinations of anisotropy, saturable absorption, and frequency-shifted feedback lead to various illustrations of synchronization mechanisms among the vector cavity modes.

The modeling of vector fiber ring lasers is more challenging, as the fiber anisotropies are not uniformly distributed, and not precisely known, in the experiment. However, vector pattern formations and complex self-pulsing dynamics are found to abound in fiber laser experiments, even without any explicit saturable absorber element, with an underlying perspective of chaos synchronization [31]. Wabnitz *et al.*, in Chapter 14, present the recent exploration of these vector dynamics, and put forward universal distributed vector equations as phenomenological models allowing to picture qualitatively well the experimentally observed dynamics.

Egorov and Lederer, in Chapter 15, review the nonlinear dynamics of a markedly different vector laser system: the strong coupling between excitons and photons in a driven quantum-well semiconductor microcavity. Exciton-polaritons are half-light, half-matter quantum quasi-particles that have attracted considerable attention during the past few years [32, 33]. Numerical investigations demonstrate how the nonlinear coupling can form stable localized exciton-polaritons collective states, namely, cavity polariton solitons, which can be of different types and spatial dimensionality.

Almost all nonlinear dynamical systems involved in this volume are of quasi-infinite dimensionality – being modeled by partial derivative equations. This explains the great diversity of the nonlinear dynamics that can be found in these systems, from localized solitons to pattern formation, and from stationary to pulsating and chaotic evolutions. However, when considering practical applications, it may become vital to develop efficient and reliable automated procedures of surveillance and control of these complex dynamical systems. This is the focus of Chapter 16, by Kutz *et al.*, where essential dimensionality-reduction methods, sparse representation, and data-driven machine learning strategies are presented.

To conclude this introductory chapter, I am particularly grateful to all the eminent specialists, representing various aspects of nonlinear optical cavity dynamics, who have contributed to this volume. They have allowed me to gather key contributions that illustrate the diversity of dynamics as well as the strong analogies and cross-fertilization between topics that used to thrive more independently, spanning from mode-locked lasers to driven cavities, miniature and microcavities for frequency comb generation, to spatially extended cavities and spatiotemporal cavity solitons. Considering also the vital soaring of photonics in our industries and its growing impact on our societies, with the potential of nonlinear optical cavity dynamics in particular to perform ultrafast light engineering and analysis, I hope that this monograph will serve as a valuable reference guide for photonics researchers and graduate students as well as a source of inspiration for photonics engineers.

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