1

Building Blocks and Interactions

Our job in physics is to see things simply, to understand a great many complicated phenomena in a unified way, in terms of a few simple principles.

S. Weinberg, *Nobel Lecture, 1979.*

1.1 What Are the Nuclei Made Of?

The standard textbook statement reads “Nuclei consist of *protons* and *neutrons*.” For the major part of what we are going to discuss in the course, this simple notion is approximately true. Indeed, complex nuclei in the Universe were mostly “cooked” in stars by the processes of consecutive addition of neutrons and protons and their mutual transformations. It is relatively easy to extract these particles back from the nuclei since the *separation energy* per particle is typically only 6 – 8 MeV, less than 1% of the mass of the proton (p) or neutron (n) that is of order ~ 1 GeV $= 10^3$ MeV.

In many nuclei with an abnormal ratio between the proton and neutron numbers, the separation energy is even significantly lower than the value mentioned earlier. And still the statement of our first sentence has a limited range of validity. In general, the answer to the question of nuclear constituents depends on the kind of phenomena we are interested in. Various experimental studies emphasize different aspects of nuclear structure. Different patterns can be resolved at different *energy scales* by specifically adjusted experimental tools.

The *nuclear forces* that keep the nucleus together are induced through exchange by mediating quanta – *mesons*, similar to how the *electromagnetic* interactions are generated by the exchange of *photons*. Roughly speaking, at energies small compared to the masses of particles that are capable of serving as mediators of nuclear forces, the nucleus indeed looks as an object composed of protons and neutrons. (Note that such a range of energies does not exist in the case of electromagnetic interactions carried by *massless* photons.)

Protons and neutrons have very similar nuclear properties, and they are called by a unifying term “*nucleons*” (N). The mesons of the lightest family, *pions* ($\pi^{+,0,-}$), are approximately seven times lighter than the nucleons. Corresponding energies $E < m_\pi c^2 \approx 140$ MeV define a domain of *low-energy* nuclear physics where the nucleus can be considered as being made of *nonrelativistic* nucleons. In this domain, mesons are *virtual* particles hidden in nucleon – nucleon interactions, and they rarely appear

Physics of Atomic Nuclei, First Edition. Vladimir Zelevinsky and Alexander Volya.

as autonomous entities. Here, we have in mind the *excitation energy* of a nucleus with respect to its *ground state* rather than the total mass or *binding energy* of a complex nucleus or energy of the beams used by experimentalists for inducing nuclear reactions and producing excited nuclei.

The area of low-energy nuclear physics where the main new results of the near future are to be obtained is related to *radioactive beam facilities*. The nucleus appears here as a self-bound *many-body* nucleonic system with its own intrinsic degrees of freedom, *single-particle* and *collective*. Such systems, along with complex atoms and molecules, micro- and nanodevices of condensed matter physics, and artificial systems, such as cold atoms in traps or future quantum computers, are called *mesoscopic*. They occupy an intermediate place between macroscopic and microscopic worlds. The peculiarity of this class of physical systems is twofold: they are sufficiently large to reveal generic *statistical* regularities; at the same time they are sufficiently small to allow physicists to study, theoretically and experimentally, individual *quantum states*. This area is the subject of our main interest.

At higher energies, one can directly see mesons and intrinsic excitations of nucleons – *nucleon resonances*, as well as heavier relatives of nucleons containing *strange* quarks – *hyperons*. Relativistic effects become more and more important. This is the region of *intermediate energy* nuclear physics that can be characterized by energies up to few gigaelectronvolts. One of the most effective tools for studying this field is electron scattering (e.g., at Thomas Jefferson National Accelerator Facility) with the wavelength short enough for resolving individual nucleons and some features of their intrinsic structure inside the nucleus.

Finally, at even higher energies and in processes with high momentum transfer, it is possible to resolve deep constituents of nucleons and mesons: *quarks* and *gluons*. Physicists expect that at densities several times higher than the normal nuclear density, the nucleons melt forming a quark–gluon soup. Such a phase of hot and dense nuclear matter supposedly existed at the early stages of the Universe and can be recreated in relativistic heavy ion collisions. Important results in this direction were obtained at the Super Proton Synchrotron (CERN) and at Relativistic Heavy Ion Collider (RHIC, Brookhaven); they are being reinforced by research work at the Large Hadron Collider (LHC) also at CERN. Such experiments detect collision products of heavy nuclei, as gold on gold, at very high energies up to 200 GeV – 10 TeV per nucleon. This area can be called *high-energy* nuclear physics. The energy of 10 TeV ($1\ \text{TeV} = 10^3\ \text{MeV}$) used in proton accelerators is in fact *macroscopic* being equal to 16 erg; in cosmic rays, rare events were detected at energy of the order 10^8 erg.

As a whole, nuclear physics covers a huge energy range from electronvolts (an upper limit for the hypothetical neutrino mass that might be measured in nuclear beta-decay and energy of slow neutrons used in many applications) and kiloelectronvolts (gamma-ray energies in low-energy nuclear transitions and energies of astrophysical nuclear reactions) to gigaelectronvolts and teraelectronvolts (RHIC and LHC). Although we start here with basic features of a more broad picture, our topics are taken mostly from low-energy physics. The main theoretical tools will be nonrelativistic quantum mechanics and quantum statistics. Nevertheless, even at this stage, some relativistic effects are to be included. As a whole, nuclear structure and reactions give an exceedingly rich material for learning how quantum mechanics works in the real world and for understanding mesoscopic physics in general.

1.2 Proton and Neutron

We start with a short description of main nuclear constituents, nucleons. Their properties will be taken as an empirical input to the physics of nuclei. A free proton and a free neutron have nearly equal masses. In convenient energy units (1 MeV = 1.602×10^{-6} erg = 1.602×10^{-13} J),

$$M_p c^2 = 938.272 \text{ MeV}, \quad M_n c^2 = 939.565 \text{ MeV}, \tag{1.1}$$

so that their relative mass difference is $\delta M/M_p = 0.14\%$. Since the free neutron is slightly heavier, it turns out to be unstable and undergoes *beta-decay* into the proton, electron, and electron antineutrino (Section 1.4). The *half-life* of the neutron is $t_{1/2} \approx 615$ s; the *mean* lifetime τ that appears in the *exponential decay* law,

$$N(t) = N(0)e^{-t/\tau}, \tag{1.2}$$

is $\tau = t_{1/2}/\ln 2 = 881.5$ s. The exact lifetime is currently being debated. The number quoted is a result of averaging of several conflicting experimental data. Experiments with neutron beams [1] and with ultracold neutrons in a magnetic bottle [2] produce results that differ by as much as 8 s. Nuclei can be stable against neutron decay only because the nuclear forces prefer an optimal ratio between the proton and neutron numbers so that the spontaneous neutron decay inside the nucleus may be energetically forbidden. Moreover, nuclei with proton excess may undergo beta-decay in the opposite direction transforming protons into neutrons.

Joint requirements of relativity and quantum theory put limitations to a notion of a free particle. For a particle of mass m, an attempt to localize its wave packet within a time interval Δt gets meaningless when the corresponding energy uncertainty $\Delta E \sim \hbar/\Delta t$ becomes comparable to the mass (1.1). For the nucleons, the characteristic time is

$$\Delta t_N \sim \frac{\hbar}{Mc^2} \approx 0.7 \times 10^{-24} \text{ s}. \tag{1.3}$$

At times shorter than Δt_N due to the large energy uncertainty, the nucleon wave function loses its *single-particle* nature and acquires components with particle – antiparticle pairs. The minimum localization length is of the order of

$$\lambda_N \sim c\Delta t_N \sim \frac{\hbar}{Mc} \approx 2 \times 10^{-14} \text{ cm}. \tag{1.4}$$

For any particle of mass m, the `Compton wavelength` $\hbar/mc$, as in Eq. (1.4), defines the limit of spatial localization when it is still possible to keep the single-particle character of the description.

The nucleons have *spin* 1/2. Therefore, they are *fermions*, particles that obey `Fermi statistics`, that is, many-body wave functions have to be *antisymmetric* with respect to interchange of *all* (space and spin) variables of any pair of *identical* nucleons. Because of the similarity between the proton and the neutron with respect to strong (nuclear) forces, it will be possible and useful to introduce higher, *isobaric*, symmetry and approximately consider them as different charge states of the nucleon. It is necessary to mention that relativistic theory introduces *antiparticles* with the same masses and the same lifetimes as the corresponding particles. Antiprotons and antineutrons have been discovered and studied. The striking predominance of particles and absence of antiparticles in the observable Universe is still not fully understood.

1.3 Strong Interactions

Nucleons take part in all known interactions. The main features of the nuclear world are shaped by *strong* interactions that are responsible for nuclear forces and stability of matter. Usually all particles participating in strong interactions are called *hadrons*. Nucleons are the lightest fermions among hadrons. The *mesons* mediating strong interactions are hadrons with an integer spin. Particles with an integer spin are *bosons*. According to current concepts, low-energy nuclear forces are manifestations of interactions at a more fundamental level between the constituents of hadrons, quarks, and gluons. These interactions are described by the *quantum chromodynamics* (QCD).

As far as we know, there are two additive quantities, *charges*, which are absolutely conserved in all observed processes, *electric* charge and *baryonic* charge. Only hadrons can have a nonzero baryonic charge B. In this case, they are called *baryons*; mesons are hadrons with $B = 0$. The proton and the neutron have the same baryonic charge that is taken to be equal to +1; this is a signature of the isobaric symmetry. If the baryonic charge is strictly conserved, the lightest baryon – proton – has to be stable. Experiments searching for proton decay did not find such events. All charges of antiparticles are of the same magnitude and of the opposite sign relative to those of corresponding particles.

Neutrons carry no electric charge and protons have an elementary electric charge $+e$, Thus, a nucleus with the electric charge Ze and baryonic charge B is treated, within the above-discussed limitations of low energy, as built of Z protons and N neutrons that comprise the mass number $A = Z + N = B$. These two quantum numbers, Z and B, are sufficient to characterize a nuclear species, *nuclide*. However, standard excessive notations, when a nuclide is designated as ${}^{A}_{Z}Y_N$, with Y being a chemical symbol of the element (of course, equivalent to the knowledge of Z), do no harm. The nuclides with the same A but different Z (different proton–neutron composition) form a set of *isobars*, those with the same Z but different N are *isotopes* of the same *chemical element*, while the nuclides with the same N and different Z are *isotones*.

On the nucleon level, "strong" interactions are indeed strong: for a pair of nucleons at a distance of the order of the nucleon Compton wavelength (1.4), typical interaction energy is of the order of Mc^2. However, these interactions have a relatively *short range* $r_0 \simeq (1 \div 2)$ fm $\simeq 10^{-13}$ cm and rapidly die away at $r > r_0$. The shortest range of interactions still keeping identity of the nucleons corresponds to the size of the distribution of quark–gluon matter inside the nucleons, $\simeq 0.8$ fm. At short distances, the interaction is similar to the *hard-core repulsion*. At larger distances the interaction is *attractive*, this allows the nucleus to be self-bound and determines a typical interparticle distance inside the nucleus.

The longest range of nuclear forces can be estimated with the aid of the uncertainty relation used in Eq. (1.4). Assume that the interaction is mediated by field quanta being emitted by one particle and absorbed by another particle (Figure 1.1). Let a quantum of the intermediate field have mass m. The creation of a virtual particle of mass m is associated with the energy uncertainty $\Delta E \sim mc^2$. This determines the lifetime $\Delta t \sim \hbar/mc^2$ of such a fluctuation and the maximum possible propagation distance

$$\Delta R \sim c\Delta t \sim \frac{\hbar}{mc}. \tag{1.5}$$

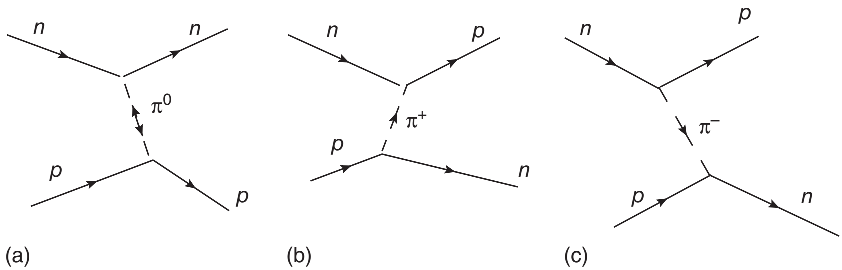

Figure 1.1 Meson exchange as a mechanism of the nucleon interaction.

Hence, the range of nuclear forces is defined by the Compton wavelength of the *lightest* particle that can mediate the strong interaction. The lightest meson is the neutral pion π^0 with $mc^2 = 135$ MeV that corresponds to $\hbar/mc = 1.46$ fm. (It is useful to know that $\hbar c = 197.327$ MeV fm.) The shortest time relevant for nuclear processes is determined by the lifetime of such fluctuations or by the flight time across the size of the range of nuclear forces, 1 fm = 10^{-13} cm, at the speed of light, $\Delta t \sim 1$ fm$/c = 0.3 \times 10^{-23}$ s. Of course, typical times of flight increase for slower particles and larger nuclear sizes; the latter can reach 10^{-12} cm for the heaviest nuclei.

1.4 Electromagnetic Interactions and Charge Distribution

Nucleons also interact with *electromagnetic* fields. Protons have positive electric charge $+e$ equal in magnitude and opposite in sign to the charge of electrons. Therefore, protons repel each other via Coulomb forces. Due to the long-range nature of the electric force (the electrostatic potential $\sim 1/r$), it prevails at large distances beyond the radius of the strong force. However, at distances $r \sim \lambda_N$, the Coulomb energy

$$\frac{e^2}{\lambda_N} \sim \frac{e^2}{\hbar c} Mc^2 \equiv \alpha Mc^2 \tag{1.6}$$

is small compared to Mc^2. The dimensionless *fine structure constant*

$$\alpha = \frac{e^2}{\hbar c} \approx 1/137 \tag{1.7}$$

is characteristic for the relative strength of electromagnetic interactions. Its smallness allows one to take into account these interactions in a regular *perturbative* manner prescribed by *quantum electrodynamics* (QED), which is the most precise branch of modern theoretical physics. Due to the relative weakness of electromagnetic forces, the best *nondemolishing* probes of nuclear structure and charge distribution rely on electromagnetic interactions (electron scattering, muonic atoms, etc.). Electron scattering from nuclear targets reveals that charge and currents inside the nucleons can be characterized by the nucleon *charge radius* ~ 0.8 fm. It is harder to get precise quantitative information on the distribution of nuclear matter.

Problem 1.1 Estimate the energy shift of the ground atomic state for the electron and the negative muon as a function of the nuclear size.

Solution

For a nucleus of finite size, the electrostatic potential $\varphi(\mathbf{r})$ acting onto atomic electrons is different from the potential $\varphi_0(\mathbf{r}) = Ze/r$ of the point charge Ze. The difference

$$\delta\varphi(\mathbf{r}) = \varphi(\mathbf{r}) - \frac{Ze}{r} \tag{1.8}$$

can be considered as a source of perturbation that induces the shift of energy levels of atomic electrons

$$\delta E_e = \int d^3r\ \delta\varphi(\mathbf{r})\rho_e(\mathbf{r}). \tag{1.9}$$

The shift can be measured by the displacement of atomic spectral lines. Here $\rho_e(\mathbf{r})$ is the unperturbed electron charge density

$$\rho_e(\mathbf{r}) = -e|\psi_e(\mathbf{r})|^2 \tag{1.10}$$

expressed in terms of the electron wave function $\psi_e(\mathbf{r})$. For s-electrons the wave function is real and spherically symmetric so that the integral in (1.9) contains only the isotropic (monopole) component of the nuclear electrostatic potential. Outside the nuclear volume, according to the Gauss theorem, this component is the same as for the point-like charge, so that the integrand in (1.9) vanishes outside the nucleus. Inside the nucleus, the electron s-wave function is practically a constant $\psi_e(0)$, and this value can be taken outside the integral leading to

$$\delta E_e = -e|\psi_e(0)|^2 \int d^3r \left(\varphi(r) - \frac{Ze}{r}\right). \tag{1.11}$$

Using the explicit expression for the potential inside the uniformly charged sphere of radius R with the total charge Ze,

$$\varphi(r) = \frac{Ze}{2R}\left(3 - \frac{r^2}{R^2}\right), \quad r \le R, \tag{1.12}$$

and calculating the integral in Eq. (1.11), we come to

$$\delta E_e = \frac{2\pi}{5}|\psi_e(0)|^2 Ze^2R^2. \tag{1.13}$$

It is customary to introduce the mean square charge radius of the nucleus via its charge density $\rho_{\rm ch}(\mathbf{r})$,

$$\overline{r^2_{\rm ch}} = \frac{1}{Ze}\int d^3r\ \rho_{\rm ch}(\mathbf{r})r^2. \tag{1.14}$$

For a nucleus as a uniformly charged sphere of radius R,

$$\overline{r^2_{\rm ch}} = \frac{3}{5}R^2, \tag{1.15}$$

so that our result (1.13) can be written as

$$\delta E_e = \frac{2\pi}{3}|\psi_e(0)|^2 Ze^2\overline{r^2_{\rm ch}}. \tag{1.16}$$

It is easy to see that in this form the result is quite general and applicable to any spherically symmetric charge distribution inside the nucleus. To show this, we can use the identity with the Laplace operator

$$\nabla^2 r^2 = 6 \tag{1.17}$$

and to rewrite the integral in (1.11) with the aid of the twofold integration by parts as

$$I \equiv \int d^3r \left(\varphi - \frac{Ze}{r}\right)\frac{\nabla^2 r^2}{6} = \frac{1}{6}\int d^3r\, r^2\nabla^2\left(\varphi - \frac{Ze}{r}\right). \tag{1.18}$$

Here the integrated parts disappear because at the surface the potential φ is continuously matched to that of the point charge. Now, we have

$$\nabla^2\frac{Ze}{r} = -4\pi Ze\delta(\mathbf{r}), \tag{1.19}$$

so that, being multiplied by r^2, this term vanishes, and

$$\nabla^2\varphi = -4\pi\rho_{\rm ch}(\mathbf{r}) \tag{1.20}$$

for any charge distribution. As a result, we obtain

$$I = -\frac{2\pi}{3}\int d^3r r^2\rho_{\rm ch}(\mathbf{r}) = -\frac{2\pi Ze}{3}\overline{r^2_{\rm ch}}, \tag{1.21}$$

which leads to the general expression (1.16).

In a hydrogen-like atom with the nuclear charge Ze, an unperturbed energy of the electron 1s-state $E_e = Ze^2/2a$, where $a = a_B/Z$ is the radius of the orbit, and a_B is the Bohr radius (see Eq. (1.33)). In this case,

$$|\psi_e(0)|^2 = \frac{1}{\pi a^3}, \tag{1.22}$$

and the energy shift is determined by the ratio of nuclear and atomic sizes squared,

$$\frac{\delta E_e}{E_e} = \frac{4\overline{r^2_{\rm ch}}}{3a^2} = \frac{4Z^2\overline{r^2_{\rm ch}}}{3a_B^2}. \tag{1.23}$$

In complex atoms, because of the screening by other electrons, $|\psi_e(0)|^2$ increases $\sim Z$ rather than $\sim Z^3$ in (1.22). But the shift of levels is still measurable. Usually one is interested in the relative difference of electron terms for the isotopes of the same chemical element. This *isotopic shift* reveals the change of the nuclear charge distribution induced by the change of the neutron number. Large changes are seen if the adjacent isotopes with nuclear spin $J = 0$ have very different shapes. As the deformation axis is uniformly distributed in space, the deformation of the nucleus with $J = 0$ gives rise to an apparent increase in the mean square radius.

The effect calculated earlier is absent for $p, d,\ldots$ and higher orbital electron states because then the radial electron wave function is vanishingly small inside the nucleus. The deformed nucleus has nonzero higher *multipole moments* of the charge distribution, most frequently the quadrupole moment Q. Then the perturbation of the potential is also deformed and can be felt by the nonspherical electron states. The order of magnitude of the quadrupole effect is $\sim eQ/a^3$, which again contains the ratio of nuclear and atomic radii squared. However, in such cases, one needs to consider the *quadrupole hyperfine structure* of atomic spectra.

The effect is much stronger for *muons* being, according to (1.23), enhanced roughly by $(m_\mu/m_e)^2 \approx 4\times 10^4$. In heavy nuclei, the radius of the lowest muon orbit is already inside the nucleus so that the distortion of the hydrogen-like muon wave function is large and the perturbative approach fails. The energy shifts of states in muon atomic spectra (in the X-ray range) provide one of the best ways of measuring the nuclear charge radius. One can use even heavier negatively charged particles, such as pions, kaons, or antiprotons. But in those cases, the strong interactions are to be taken into account; they allow nuclear absorption of pions and kaons and annihilation of antiprotons.

Problem 1.2 A *charge form-factor* is a Fourier image of the spatial charge distribution density $\rho(\mathbf{r})$. Calculate the charge form-factor $F_{\rm ch}(\mathbf{q})$ as a function of the wave vector $\mathbf{q}$ for the neutral hydrogen atom in its ground state assuming the point-like proton. How this form-factor is changed by a finite size of the proton? At what values of the momentum transfer q, one can detect this change?

Solution

The electron wave function in the ground state of the hydrogen atom is spherically symmetric [in accordance with (1.22)],

$$\psi(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B}, \quad a_B = \frac{\hbar^2}{me^2}. \tag{1.24}$$

The atomic charge density consists of nuclear and electronic parts,

$$\rho(\mathbf{r}) = \rho_p(\mathbf{r}) + \rho_e(\mathbf{r}) = \rho_p(\mathbf{r}) - e|\psi(\mathbf{r})|^2. \tag{1.25}$$

The electron contribution to the atomic form-factor [QP, II, 3.4] is

$$F_e(\mathbf{q}) = -\frac{e}{\pi a_B^3} \int d^3r\, e^{i(\mathbf{q}\cdot\mathbf{r}) - 2r/a_B}. \tag{1.26}$$

A straightforward evaluation of the integral (it depends only on the magnitude q of the vector $\mathbf{q}$) gives

$$F_e(q) = -\frac{e}{[1 + (qa_B/2)^2]^2}. \tag{1.27}$$

As it should be, $F_e(0) = -e$, the total charge of the electron cloud. When the wavelength of the probe becomes much smaller than the atomic size, $qa_B \gg 1$, the form-factor goes to zero $\sim (qa_B)^{-4}$, which points to the absence of any singularities or accumulation of charges at smaller scales: there are no substructures. In contrast, the *point-like* proton has its charge concentrated in a vanishingly small volume so that its form-factor does not depend on q at all,

$$\rho_p(\mathbf{r}) = e\delta(\mathbf{r}) \quad \rightsquigarrow \quad F_p(\mathbf{q}) = e. \tag{1.28}$$

In this approximation, the wavelength $\sim 1/q$ of the probe is always greater than the proton size and the probe sees the proton as a point. The full atomic form-factor is given by

$$F_H(q) = F_p(q) + F_e(q) = e\left\{1 - \frac{1}{[1 + (qa_B/2)^2]^2}\right\}. \tag{1.29}$$

At the wavelength greater than the Bohr radius, $qa_B \ll 1$, the electron cloud screens the proton, the atom looks as a neutral object and the form-factor vanishes. As the wavelength gets shorter, the form-factor grows $\sim e(qa_B)^2$. When $qa_B \gg 1$, only the charge of the point-like proton contributes to the form-factor; the electronic contribution disappears since at high momentum one is probing very small distances and only an infinitesimal part of the electron charge is found localized in ever-decreasing spatial region.

For a proton, viewed as a small sphere of radius r_p, instead of (1.28), the charge form-factor is

$$F_p(q) = \frac{e}{(4\pi/3)r_p^3} \int_{r \le r_p} d^3r \, e^{i(\mathbf{q}\cdot\mathbf{r})}, \tag{1.30}$$

or, after simple calculation,

$$F_p(q) = \frac{3e}{x^3}(\sin x - x\cos x), \quad x = qr_p. \tag{1.31}$$

This can also be expressed using the spherical Bessel function $j_1(x)$, which is derived from the expansion of the plane wave in (1.30) over spherical waves. The parameter x represents the ratio of the wavelength $\sim 1/q$ to the proton size r_p. For long wavelengths, $x \ll 1$, the point charge limit (1.28) is recovered since $\sin x \approx x - x^3/6$ and $\cos x \sim 1 - x^2/2$. For short wavelengths, $x \gg 1$, the form-factor (1.31) goes to zero as $\cos x/x^2$ with oscillations (again, there are no substructures inside the proton in this approximation). The momentum transfer for an experiment designed to see the proton size corresponds to the wavelength of the order of this size, $x \ge 1$, or at least

$$q \sim \frac{1}{r_p}, \quad \hbar q \sim \frac{\hbar}{r_p} = 250 \text{ MeV}/c. \tag{1.32}$$

We can note parenthetically that the quantity α, Eq. (1.7), naturally defines three fundamental length scales. The `Bohr radius`

$$a_B = \frac{\hbar^2}{me^2} = 0.529 \text{ Å}, \tag{1.33}$$

where m is the electron mass and $1 \text{ Å} = 10^{-8}$ cm, determines the typical atomic size: this is the mean radius of the lowest orbit in the hydrogen atom and at the same time the typical outer radius of neutral complex atoms because the outermost electron is moving in the screened field of the ion with the effective charge $+1$ as in the hydrogen atom. Multiplying a_B by α we come to the deeper scale, that of the *electron Compton wavelength*,

$$\lambda_e = \alpha a_B = \frac{\hbar}{mc} = 3.862 \times 10^{-11} \text{ cm}. \tag{1.34}$$

At this scale, similar to what was discussed earlier, the quantum relativistic effects restrict the possible localization of the particle, in this case of the electron. Finally, the next step leads to the *classical electron radius*

$$r_e = \alpha\lambda_e = \alpha^2 a_B = \frac{e^2}{mc^2} = 2.818 \times 10^{-13} \text{ cm}. \tag{1.35}$$

This length does not contain the Planck constant $\hbar$ and determines the intrinsic limit of validity of classical electrodynamics: the Coulomb self-energy of the electron as a small particle of radius r_e would reach its total rest energy mc^2.

Charged pions $\pi^\pm$ transfer electric charge from one nucleon to another as shown in Figure 1.1b,c. As a result of such transfer, proton and neutron interchange their roles. Consider for instance a process of $n{-}p$ scattering at small scattering angles in the center-of-mass frame. “Normally” the neutron would continue its motion after scattering close to the forward direction. However, after scattering that involves interaction mediated by a charged pion, the particle moving forward will be the proton. This is a reason to say that nuclear forces are of *exchange* character. It is easy to understand that in the exchange by neutral pions (Figure 1.1a) in the interaction of two neutrons (or two protons), there are two possibilities in final states. The crucial difference is that now we deal with *identical particles*, and these two outputs are in fact *indistinguishable*. According to quantum mechanical laws, one has to add in this case the *amplitudes* of two possible processes, direct and exchange. The presence of exchange interactions of identical particles is extremely important in all many-body systems. In particular, the exchange part of the Coulomb interaction between electrons is mostly responsible for the macroscopic magnetic ordering in ferromagnets. The existence of charged mesons as mediators of nuclear forces along with the isobaric symmetry extends the idea of exchange interactions to the nucleons that differ by their electric charge.

1.5 Magnetic Properties

Protons and neutrons have intrinsic *magnetic moments* μ_p and μ_n. Their interaction with electron magnetic moments is responsible for the *hyperfine* magnetic structure of atomic and molecular levels. The magnetic moments are known with high accuracy,

$$\mu_p = 2.792847351 \pm 0.000000028\ \mu_N, \quad \mu_n = -1.9130427 \pm 0.0000005\ \mu_N, \tag{1.36}$$

where the characteristic unit is the *nuclear magneton* (n.m.),

$$\mu_N = \frac{e\hbar}{2M_p c} = 1 \text{ n.m.} = 5.05 \times 10^{-24} \text{ erg Gs}^{-1} = 3.152 \times 10^{-14} \text{ MeV T}^{-1}. \tag{1.37}$$

These magnetic moments characterize the nucleons at rest and should be ascribed to spin and intrinsic quark structure rather than to orbital motion. Proton orbital motion creates orbital magnetism with the classical gyromagnetic ratio of the magnetic moment to the mechanical moment $\hbar\ell$ equal to $g^{(\ell)} = e/2M_p c$; this defines the magnetic moment of 1 nm. Because of internal strong interactions, the spin gyromagnetic ratios differ from the Dirac limit of 2 [QP, II, 13.5]. Instead of (1.36), if proton was a structureless Dirac particle, its magnetic moment would be 1 n.m. and the magnetic moment of a structureless neutron would vanish. The magnetic moment of the neutron reflects the distribution of charge and currents inside the neutron although the total charge is zero. The hyperfine structure splitting in atoms is small because the nuclear magneton is inversely proportional to the large nucleon mass $M \approx 1840\ m_e$.

Problem 1.3 Calculate the angular frequency of spin precession for electrons, protons, and neutrons in the magnetic field 0.1 T.

Solution
If the magnetic dipole $\boldsymbol{\mu}$ is not aligned along the magnetic field $\mathbf{B}$, it feels a torque

$$\mathbf{T} = [\boldsymbol{\mu} \times \mathbf{B}]. \tag{1.38}$$

In the presence of torque, the vector $\hbar\mathbf{J}$ of the angular momentum is not conserved. Its equation of motion takes the form

$$\hbar\frac{d\mathbf{J}}{dt} = \mathbf{T}. \tag{1.39}$$

The magnetic moment operator $\boldsymbol{\mu}$ is proportional to that of the total angular momentum (spin) of the particle,

$$\boldsymbol{\mu} = g\mathbf{J}, \tag{1.40}$$

where g is the gyromagnetic ratio in units of the corresponding magnetons $e\hbar/2mc$ for a particle of mass m (Bohr magneton μ_B for the electron and nuclear magneton μ_N for the nucleons). From those preliminaries, we find the equation of motion for the magnetic moment:

$$\frac{d\boldsymbol{\mu}}{dt} = \frac{g}{\hbar}[\boldsymbol{\mu} \times \mathbf{B}]. \tag{1.41}$$

As follows from (1.41), the magnetic moment precesses around the field direction, and the vector of the precession frequency is given by

$$\boldsymbol{\omega} = \frac{g\mathbf{B}}{\hbar}. \tag{1.42}$$

For electrons, protons, and neutrons $J = s = 1/2$, and

$$g_e = 2\mu_B, \quad g_p = 5.58\mu_N, \quad g_n = -3.83\mu_N. \tag{1.43}$$

This determines

$$\omega_e = 1.76 \times 10^{10}\ \mathrm{s}^{-1}, \quad \omega_p = 2.68 \times 10^{7}\ \mathrm{s}^{-1}, \quad \omega_n = 1.83 \times 10^{7}\ \mathrm{s}^{-1}.$$

The methods of *nuclear spectroscopy* are almost exclusively based on electromagnetic interactions. Stationary states of any quantum system form a discrete spectrum. The gamma-transitions between the discrete levels carry away energy and angular momentum. The measurements of intensity, branching, lifetime of excited states, angular distribution, and polarization of gamma-rays allow the experimentalists to recreate the nuclear level scheme with the corresponding quantum numbers.

1.6 Weak Interactions

The weak processes, Figure 1.2, are mediated by *intermediate vector bosons* – very short-lived and massive particles with spin 1 – two charged $W^{\pm}$ bosons with mass $m_W = 80.4$ GeV and a neutral Z^0 boson with mass $m_Z = 91.2$ GeV. Together with the massless photon γ, these four intermediate bosons are responsible for all phenomena of electromagnetic and weak interactions that are naturally combined in the *electroweak* theory. Similar to electromagnetism, in the combined electroweak theory we have the interaction of *currents*, for example, of the nucleon current with the electron-neutrino (*lepton*) current in the examples of Figure 1.2.

At energies much lower than $m_W c^2$, the manifestations of electromagnetic and weak physics are quite different. The range of forces is inversely related, Eq. (1.5), to the mass

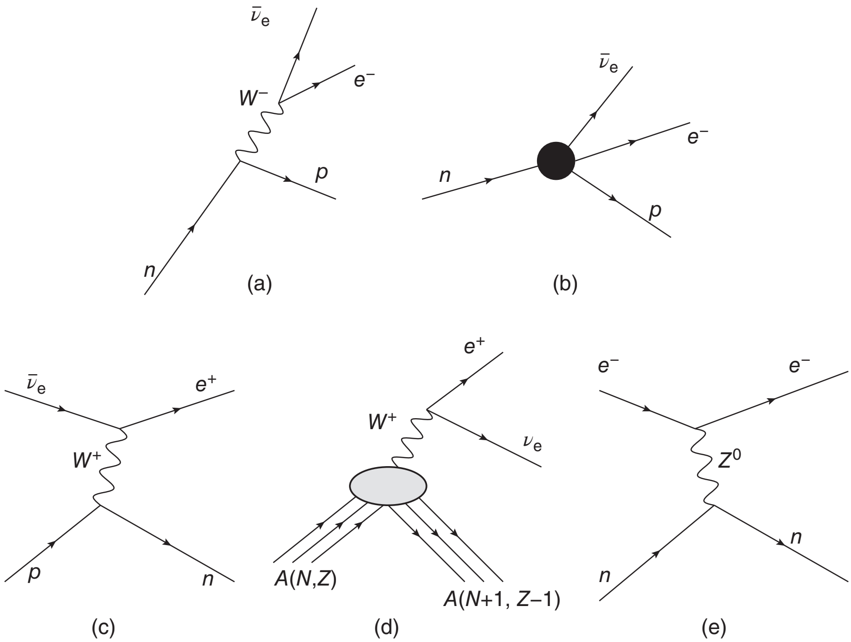

Figure 1.2 Examples of weak processes: (a) neutron beta-decay and (b) its contact image, (c) neutrino proton scattering, (d) positron decay of a nucleus, and (e) electroweak electron scattering.

of the agent mediating the interaction. In contrast to electromagnetic forces of infinite range ($m_\gamma = 0$), the weak interactions have the range

$$\Delta R_{\rm weak} \simeq \frac{\hbar}{m_W c} \simeq 2 \times 10^{-16}\ {\rm cm}. \tag{1.44}$$

Such distances are much smaller than those of importance in nuclear phenomena at not very high energies. Therefore, in many applications, the weak processes can be considered as resulting from a *contact*, point-like interaction (Figure 1.2b). The corresponding interaction strength is much weaker than that due to strong and electromagnetic forces. At distances of the order of the proton Compton wavelength (1.4) used in earlier estimates, Eq. (1.6), the intensity of the weak interaction between two nucleons is of the order of $10^{-5}Mc^2$. Typical *cross sections* of reactions due to the weak interaction are smaller than 10^{-40} cm^2, well below normal nuclear cross sections of the order of millibarns or even barns (1 barn = 10^{-24} cm^2).

Thus, we can definitely ignore the contribution of the weak interactions to the total energy of a nuclear system. But the weak interaction is extremely important because it (i) makes possible some processes that would not be allowed with strong and electromagnetic forces only and (ii) violates some fundamental symmetries. The whole nuclear and chemical evolution of the Universe at some stages could only proceed via weak processes. Weak interactions can be studied both in elementary processes and in complex nuclei. There are physical mechanisms that may enhance weak effects in the many-body nuclear environment. Such cases are especially interesting as amplifiers of the weak interactions, and, on the other hand, as bright signatures of those many-body mechanisms at work. The main manifestation of weak interaction is in weak nuclear decays that are very slow compared to characteristic nuclear times. As weak interactions violate specific symmetry properties, they can be observed by the characteristic effects of such violations.

Finally, we need to mention *gravitational forces*. They are extremely weak for two nucleons at typical nuclear distances but become exceedingly important on an astrophysical scale because of their long-range character that covers huge masses of matter in stars and galaxies.

1.7 Neutron Decay

From the single-nucleon point of view, the most prominent effect of the weak interaction is instability of the free neutron. The excess of mass, Eq. (1.1), of the neutron compared to the proton makes the neutron unstable. Therefore, the decay $n \to p+$(something) is energetically allowed. This "something" should carry the negative electric charge, $e = -1$. The only available light negative charge carrier is the electron. But then the angular momentum conservation requires the presence of another electrically neutral particle of half-integer spin. This particle has to be light since the decay energy is only $(M_n - M_p - m_e)c^2 = 783$ keV. This role is played by the *neutrino* (ν). The neutron decay is the simplest example of the *β-decay*,

$$n \to p + e^- + \overline{\nu}_e. \tag{1.45}$$

The β-decay (1.45), Figure 1.2a, is studied in detail; the mean life time $\tau_n \approx 15$ min of the neutron, huge on a nuclear scale, is due to the weakness of the underlying interaction and very limited energy release in the process.

As reflected in the notations for the neutrino, historically it was assumed that in weak interactions the *leptonic charge* is conserved. The class of particles called leptons includes electron ($m_e c^2 = 0.511$ MeV) and its heavy analogs, *muon* μ, $m_\mu c^2 = 105.658$ MeV, and *τ-lepton*, $m_\tau = 1784$ MeV, together with their antiparticles, and the corresponding neutrinos and antineutrinos. Each lepton is coupled to its own neutrino, such as $\overline{\nu}_e$ in the case of (1.45), where it appears along with the electron. For each *generation* of leptons, e, μ, or τ, the specific leptonic charge is assigned, +1 for a lepton and −1 for a corresponding antilepton. Hadrons have zero leptonic charge. To conserve the leptonic charge, the particle accompanying the electron in (1.45) has to be an antilepton, that is, *electron antineutrino* (antiparticles can be denoted by the same symbols as particles with the bar added).

Apparently the neutrinos have no magnetic moments; for example, for the magnetic moment of the electronic neutrino, the experimental upper boundary is $|\mu_{\nu_e}/\mu_e| < 4 \times 10^{-10}$. Thus, neutrinos participate in weak and gravitational interactions only. Therefore, they have an astronomical mean free path in matter. The first direct observation of the interaction with matter of electron antineutrinos, produced from the processes like (1.45) in the nuclear reactor, was carried out [3], with the reaction, Figure 1.2c,

$$\overline{\nu}_e + p \to n + e^+. \tag{1.46}$$

Here, as compared to the beta-decay in (1.46), we reversed the direction of the processes and transferred the electron to the opposite side of the equation substituting it by the positron. Such substitutions are always allowed from the viewpoint of charge conservation, although not all possibilities are energetically open. The cross section of the reaction (1.46) is very low, $\approx 10^{-43}$ cm^2.

The similar process

$$\overline{\nu}_e + n \rightarrow p + e^- \tag{1.47}$$

turned out [4] to be forbidden; actually the reaction was sought for not on a free neutron (such a target did not exist) but on the nucleus ^{37}Cl ($Z = 17, N = 20$) that was expected to be transformed into ^{37}Ar ($Z = 18, N = 19$). From Eq. (1.45) we see that the possible process with the lepton number conservation should be initiated by the electron neutrino rather than by the antineutrino,

$$\nu_e + n \rightarrow p + e^-. \tag{1.48}$$

The absence of the reaction (1.47) was interpreted as a fact that neutrino and antineutrino are different particles and the lepton number is a convenient tool to differentiate between them.

Later it turned out that this interpretation might be doubtful. The process (1.48) can be forbidden by the property of the weak interaction based on *left currents*: neutrinos in such processes are always left-polarized while antineutrinos are right-polarized. Here polarization is the sign of conserved *helicity*, spin projection on the linear momentum. The left-polarized particle cannot induce the reaction (1.48). The distinction would be absolute if the neutrinos were *massless*. However, later discovered *neutrino oscillations* show that electronic, muonic, and tau neutrino appearing in weak interactions are not stationary being linear combinations of stationary neutrino mass eigenstates. Then neutrinos have nonzero mass, their helicities are not absolute, and the idea of the conserved leptonic quantum number is of a limited applicability. A massive neutrino can still be identical to its antiparticle (the so-called *Majorana particles*) but this can be established only by an experiment. The electronic neutrino mass is very low, not greater than 1 eV. Its precise measurement is a difficult experimental task. For almost all practical purposes in ordinary nuclear physics, we can set $m_{\nu_e} = 0$. The problem of the neutrino mass, extremely important both theoretically and for the search of missing *dark matter* in the Universe, attracts many researchers (another direction in the problem of dark matter is related to the search for the so-called weakly interacting massive particles, WIMPs).

Similar to (1.45) weak decays of the *proton* are energetically forbidden. If the electric and baryonic charges are strictly conserved, the *lightest* particles with nonzero values of those charges – the electron and the proton, respectively, – must be absolutely stable. In some theories, however, the conservation of baryonic charge can be violated, so that the proton can decay, for example, into a positron and neutral pion π^0. In the models combining strong, electromagnetic, and weak interactions (*grand unification*), the baryon and lepton numbers are not necessarily separately conserved, and the proton decay processes such as $p \rightarrow e^+ + \pi^0$ are allowed. Until now, in spite of continuing efforts, there is no experimental indication for the existence of such decays, and it was possible only to establish a lower boundary: the lifetime of the proton is longer than 10^{34} years. The proton decay could be a missing ingredient for the understanding of the above-mentioned *baryonic asymmetry* of the Universe (absence of antimatter); other ingredients are violation of $\mathcal{CP}$ symmetry (combined charge conjugation $\mathcal{C}$, transforming particles into antiparticles, and spatial inversion $\mathcal{P}$) and a nonstationary situation that was present at the early stages of the expanding Universe [5]. Other ideas of explaining the matter–antimatter asymmetry of the Universe are related to neutrino physics.

1.8 Nuclear World

Every nuclide can be conveniently placed as a small cell on a *nuclear chart*, Figure 1.3, with its electric charge Z as the vertical coordinate and the neutron number $N = A - Z = B - Z$ as the horizontal coordinate. The symmetric nuclei $N = Z$ find themselves along the diagonal, while the perpendicular diagonal combines the nuclei with the same A (isobars); all isotopes of the same chemical element are located on the same horizontal line and all isotones on a vertical line.

A lot of detailed information about nuclei can be found in nuclear databases [6]. The majority of nuclear systems are *unstable*. They widely differ by their lifetimes. Usually only *particle-stable* nuclides, even if they undergo slow radioactive decay, are included in the nuclear charts. Particle-unstable species can be formed in nuclear reactions and usually emit neutrons during time $\sim 10^{-(21\div23)}$ s after the formation. Proton-unstable nuclei (*proton emitters*) sometimes can live longer if the energy of the proton is low and its path outside requires a long penetration through the Coulomb barrier of the residual nucleus. The spectroscopy of long-lived proton emitters can be studied experimentally. The lines on the nuclear chart showing the boundary of particle-unstable nuclei are called *drip lines*. The proton drip line is measured approximately up to $Z = 20$, and the neutron drip line is measured up to exotic oxygen isotope ^{26}O (the last particle-stable oxygen isotope is ^{24}O). At the drip lines, the particle *separation energy*, the difference of the ground-state binding energies of the parent and daughter nuclei is zero or negative (in stable nuclei, one needs to invest positive energy to extract a particle, which is similar to the atomic ionization potential).

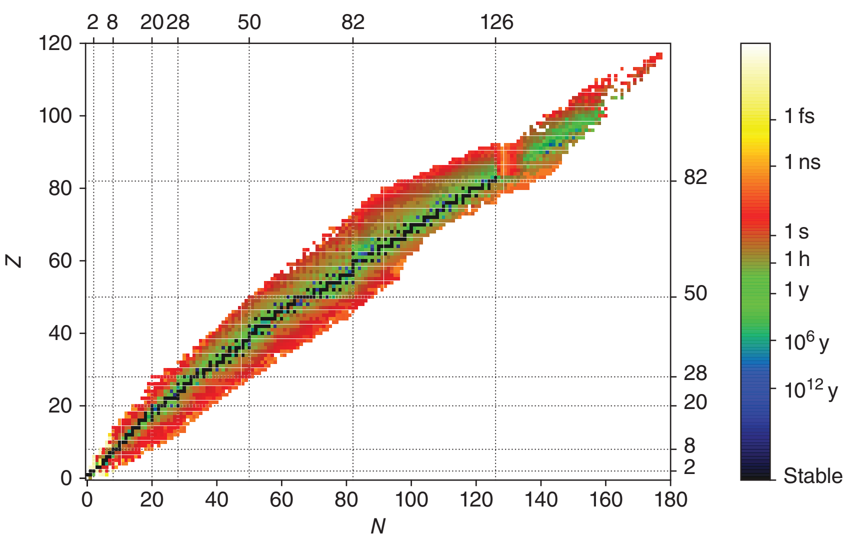

Figure 1.3 Nuclear chart. Colors represent ground-state lifetimes.

Estimates and extrapolations from the properties of well-known nuclei show that probably there can exist up to 8000 particle-stable nuclides. Only about 3300 of them were experimentally observed; this number steadily grows due to the efforts of many devoted laboratories. We should note that it has been an unfortunate tradition that only the discovery of a new chemical element – which means the progress along the Z-coordinate of the nuclear chart – was celebrated in the past as a real discovery repeatedly awarded by Nobel Prizes in chemistry; in contrast, the motion along the N-coordinate – related to the discovery of new isotopes – has been less known but often it requires great experimental efforts [7].

Less than 300 nuclides can be called stable. In fact, many of them are energetically allowed to undergo *fission* into two smaller but more tightly bound nuclear fragments. Similar to proton decay, the fission rate is determined by the probability of Coulomb barrier penetration. For nuclei in the middle of the chart, beyond the iron–nickel region, the fission probability is extremely small and often can be ignored. At the same time, *alpha-decay* with the emission of the *alpha-particle*, the nucleus of ^{4}He, and fission define the finite lifetime of very heavy elements with large electric charge. This determines the end at the right upper corner of the nuclear chart; the heaviest observed nuclide is $^{294}_{118}(\mathrm{Og})_{176}$ recently named *Oganesson* to recognize the Dubna physicist Yu. Oganessian. At the left lower end, we can start with the nucleons p and n, or with the lightest $A = 2$ system, the *deuteron* $^2_1\mathrm{H}_1$.

The region of studied nuclei occupies a band around the *valley of stability*. This valley deviates down from the $N = Z$ diagonal. This means that the growing Coulomb repulsion establishes the most energetically favorable ratio $N/Z > 1$ for the stable nuclei. Nuclei on both sides of the valley of stability improve the value of this ratio by *beta-decay* that keeps the same A but moves the nuclei closer to the stability line. This is a typical route of the *nucleosynthesis* in the stellar environment. The lifetimes for various types of beta-decay are large compared to the typical nuclear times, recall the neutron beta-decay (1.48), so that it is possible to study the spectroscopy of both, initial and final, nuclei.

Among observed nuclei, we can distinguish three classes: about one-quarter of nuclei are *even–even* with respect to both N and Z; one-half have an *odd mass number A* when *N or Z* is odd; and the rest are *odd–odd*. The even–even nuclei are the most stable, while there are only very few stable nuclei among the odd–odd ones. This reveals the *pairing* effect that amounts to extra binding existing mainly between two identical nucleons. As we will see later, this effect is similar to the Cooper pairing of electrons responsible for superconductivity in condensed matter physics; sometimes heavy nuclei are termed *superfluid*.

One of the main tools of nuclear spectroscopy is the study of *gamma-rays* emitted in the transitions (with no change in A or Z) of a nucleus between its energy levels. Various combinations of possible initial and final states in a nucleus produce many gamma-rays. The typical experimental problem is that of combining the observed transitions into a consistent *level scheme*, whereas the theoretical task is to interpret this scheme from the viewpoint of underlying many-body structure. The highest number of gamma-transitions, 1319, is known in $^{53}_{25}\mathrm{Mn}_{28}$. The largest number of energy levels, 578, is established for $^{40}_{20}\mathrm{Ca}_{20}$. At the same time, for approximately 1100 nuclides no gamma-rays were measured, and for about 800 nuclides only the ground state is known.

The loosely bound nuclei, for example, the *halo nucleus* ^{11}Li with two outer neutrons at relatively large distances from the center, have no excited states and therefore decay after any excitation exceeds the binding energy. This nucleus also gives an example of a *Borromean system* as the neighboring ^{10}Li is unbound and the excitation releases a pair of neutrons.

Medium and heavy stable nuclei on average display a simple mass dependent on the size: the mean square radius of nuclear density on average depends on the mass number A as

$$R = r_0 A^{1/3}, \quad r_0 \approx 1.2 \text{ fm}. \tag{1.49}$$

This means that the nuclear volume is roughly proportional to the mass number, that is, the nuclear matter in equilibrium is *incompressible* being characterized by a typical bulk density. The parameter $A^{1/3}$ does not exceed 7 in the heaviest nuclei so that the sizes of nuclei are between 1 and 10 fm. The simple rule (1.49) is violated in loosely bound nuclei, like ^{11}Li, where the outer nucleons can form nuclear *skins* and *halos*. The study of the nuclei away from the valley of stability is currently considered as the main goal of nuclear research.

Typical velocities of the nucleons inside the nucleus are about 0.2 of the speed of light. Therefore, the periods of nucleonic motion inside the nucleus are of the order of 10^{-22} s. The timescale for neutron emission from a particle-unstable nucleus should be on average of the same order.

Problem 1.4 Estimate kinetic energy of electrons necessary to reveal the diffraction on the nuclear boundary of the alpha-particle and the uranium nucleus.

Solution
With $R = 1.2A^{1/3}$ fm, the wavelength $2\pi\hbar/p$ is of the order of the nuclear radius at electron kinetic energy equal to $\approx$ 660 and 170 MeV for ^{4}He and ^{238}U, respectively. In both cases, the electron is ultrarelativistic and it is possible to neglect its rest mass.

Nuclear transformations (beta-decay, alpha-decay, gamma-emission, fission) that proceed relatively slowly compared to typical periods of internal motion probe the structure of the nucleus as if it would be stable. For gamma-emission, usually the lifetime τ is greater than 10^{-18} s; strong (intensive, *collective*) transitions proceed in picoseconds. Some exceptionally long-lived excited states are called *isomers*. The record here belongs to the nucleus ^{180}Ta, where the isomeric state at excitation energy 77 keV has an amazingly long lifetime greater than 10^{15} years, while the beta-decaying ground state of the same isotope has a half-life time of only 8 h. Usually, one can understand specific structural reasons for the hindered transitions. Beta-decay is typically slow, $\tau > 10^{-8}$ s. All slow nuclear transformations are conventionally combined under the name of *radioactivity*, in distinction to faster *nuclear reactions* or gamma-radiation.

Nuclear quantum states are characterized by certain energy defined within the limits of the uncertainty relation, $\Delta E \sim \hbar/\tau$, where τ is the mean lifetime. The states also have exactly conserved quantum numbers of total angular momentum J (called usually *nuclear spin*) and its projection $J_z = M$ onto a quantization axis. If one can ignore the effects of the weak interactions, parity Π is also conserved. Angular momentum and parity are conserved in electromagnetic interactions. Therefore, the quantum numbers of the initial and final states restrict the character of the possible gamma-radiation between these states: the triangle rule should be fulfilled,

$$\mathbf{J} = \mathbf{J}' + \boldsymbol{\lambda}, \tag{1.50}$$

where $\mathbf{J}$ and $\mathbf{J}'$ refer to the initial and final nuclear states, whereas $\boldsymbol{\lambda}$ is the angular moment carried away by the emitted photon (*multipolarity*). The radiation of high multipoles is strongly suppressed, and the main observed modes are electric and magnetic dipole, $\lambda = 1$, and electric quadrupole, $\lambda = 2$. The highest experimentally observed multipolarity of nuclear gamma-radiation is that of the electric radiation in ^{53}Fe with $\lambda = 6$ (half-life of about 2.6 min). In the previously mentioned isotope ^{180}Ta, the difference of spins between the excited ($J^{\Pi} = 9^-$) and ground ($J^{\Pi} = 1^+$) states is too large, which makes the gamma-radiation highly improbable; this isomeric state decays through a weak process.

Problem 1.5 The ratio of intensities of two components of the hyperfine structure of the spectral line $(^2P_{1/2}) \to (^2S_{1/2})$ in the sodium atom is close to 10 : 6. Find the ground-state spin of the nucleus ^{23}Na.[1]

Solution

The hyperfine splitting of the upper term $^2P_{1/2}$ is negligibly small (the valence electron in the p-state has zero probability to be in the nuclear volume). The lower term $^2S_{1/2}$ is split into two hyperfine components with total atomic angular momenta $F_> = J + 1/2$ and $F_< = J - 1/2$ where J is the nuclear spin. The number of magnetic sublevels is, respectively, $2F_> + 1 = 2J + 2$ and $2F_< + 1 = 2J$. These sublevels are populated statistically, that is, the intensities are proportional to the statistical weights (number of sublevels). Therefore, the ratio of intensities is

$$\frac{10}{6} = \frac{2J+2}{2J} = \frac{J+1}{J} \quad \Rightarrow \quad J = \frac{3}{2}. \tag{1.51}$$

Problem 1.6 In the Stern–Gerlach experiment, an atomic beam traverses a weak but strongly nonuniform transverse magnetic field. For atoms of the isotopes ^{13}C and ^{133}Cs, the beam was split into 2 and 16 components, respectively. The atomic terms are correspondingly $(^3P_0)$ and $(^2S_{1/2})$. Determine nuclear spins.

Solution

Each subset of atoms with a given value of total angular momentum F, $\mathbf{F} = \mathbf{J}_e + \mathbf{J}$, where $\mathbf{J}_e$ is the total angular momentum of atomic electrons and $\mathbf{J}$ is the nuclear spin, is split into $2F + 1$ deflected beams. The total number of hyperfine components is

$$\sum_F (2F+1) = (2J_e + 1)(2J + 1). \tag{1.52}$$

[1] Atomic levels given in parentheses are labeled in terms of their total spin S, total orbital momentum L, and total angular momentum J as $(^{2S+1}L_J)$.

From here, we find

$$^{13}\mathrm{C}: \quad J_e = 0,\ 2J+1 = 2,\ J = \frac{1}{2};$$
$$^{133}\mathrm{Cs}: \quad J_e = \frac{1}{2},\ 2J+1 = 8,\ J = \frac{7}{2}.$$

Problem 1.7 Molecules LiF have the total angular momentum of electron shells $J_e = 0$. In the magnetic resonance measurement with the static magnetic field $\mathcal{B} = 0.5$ T, two resonance peaks at the frequencies of the time-dependent transverse magnetic field $\nu_{\mathrm{Li}} = 8.3$ MHz and $\nu_{\mathrm{F}} = 20.0$ MHz are observed corresponding to the ^{7}Li and ^{19}F nuclides, respectively. Determine the magnetic moments of these nuclei.[2]

Solution
Nuclear spins are $J = 3/2$ and $J = 1/2$ for ^{7}Li and ^{19}F, respectively. From (1.40), we find

$$\mu = gJ = \frac{\hbar\omega}{\mathcal{B}}J = \frac{2\pi\hbar\nu}{\mathcal{B}}J, \tag{1.53}$$

or

$$\mu = \frac{4\pi\nu McJ}{e\mathcal{B}}\ \mu_N \tag{1.54}$$

with the proton mass M. This gives

$$\mu(^{7}\mathrm{Li}) = 3.26\ \mu_N, \quad \mu(^{19}\mathrm{F}) = 2.62\ \mu_N.$$

References

1 A.T. Yue *et al.*, Phys. Rev. Lett. **111**, 222501 (2013).
2 A.P. Serebrov *et al.*, Phys. Lett. B **605**, 72 (2005).
3 F. Reines and C.L. Cowan, Phys. Rev. **113**, 273 (1959).
4 R. Davis, Phys. Rev. **97**, 766 (1955).
5 A.D. Sakharov, JETP Lett. **5**, 24 (1967); republished as A.D. Sakharov, Sov. Phys. Usp. **34**, 392 (1991).
6 National Nuclear Data Center – Brookhaven National Laboratory, www.nndc.bnl.gov.
7 M. Thoennessen, Rep. Prog. Phys. **75**, 056301 (2013).

[2] Take the nuclear ground-state spin values from the isotope tables.