PART I

FUNDAMENTALS OF THERMAL RADIATION

1

INTRODUCTION TO THERMAL RADIATION

In this chapter radiation heat transfer is defined as the heat interaction between a system and its surroundings that occurs in the absence of an intervening medium. Thermal radiation is then identified as electromagnetic radiation emitted solely due to the temperature of the emitter, and the dual wave–particle description of electromagnetic radiation is introduced. The chapter concludes with an atomic-scale description of emission of thermal radiation by a solid.

1.1 THE MODES OF HEAT TRANSFER

Students of thermodynamics learn that *heat* is the energy interaction that occurs solely due to the temperature difference between a system and its surroundings. Engineers commonly apply the somewhat redundant term "heat transfer" to this same phenomenon. The study of heat transfer is traditionally divided into three categories, or "modes": conduction, convection, and radiation. Some authorities include heat transfer with phase change, or "boiling heat transfer," as a fourth mode.

In electrical nonconductors *conduction heat transfer* occurs when thermal energy stored in vibratory modes of atoms is passed from atom to atom through the interatomic forces that maintain atomic spacing. The process is similar for electrical conductors except that now the migration of free electrons contributes significantly to or even dominates heat diffusion. A simple model for heat conduction in electrical insulators is illustrated in Figure 1.1. In this model the atoms are represented as point masses of mass M and the interatomic forces are represented as linear springs of modulus K. If one of the point masses (the "source") is set into vibration along one or more of its three independent axes, or degrees of freedom, its neighbors will be jostled. This will produce a wave that propagates through the lattice according to the

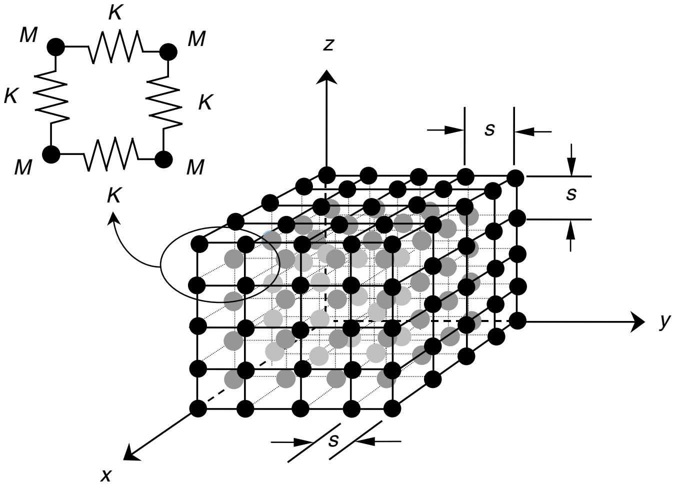

Figure 1.1 Model for heat conduction; an idealized crystal structure

laws of classical mechanics. The atoms behind the spreading wave front will be left to vibrate with amplitude that depends on the distance from the source, the effective mass of the atoms, the stiffness of the intermolecular bonds, and the manner in which the array of atoms is constrained at its boundaries. The amplitude and frequency of the vibrations are related to the physical quantity *temperature* such that heat is transferred from regions of higher temperature to regions of lower temperature in accordance with the second law of thermodynamics.

Convection heat transfer also requires the presence of an intervening physical medium between the system and its surroundings. The essential difference between convection and conduction is that in the case of the former the intervening medium must be a fluid that flows between the system being heated or cooled and its surroundings. Consider the heated flat plate shown in Figure 1.2. As the fluid flows over the plate it picks up "heat" (a student of thermodynamics would say "thermal en-

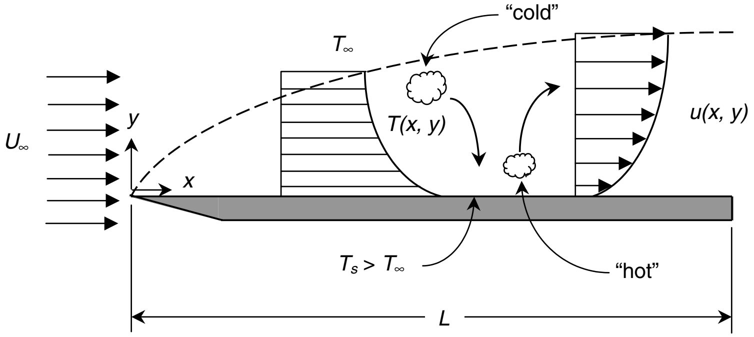

Figure 1.2 Convection heat transfer from a flat plate

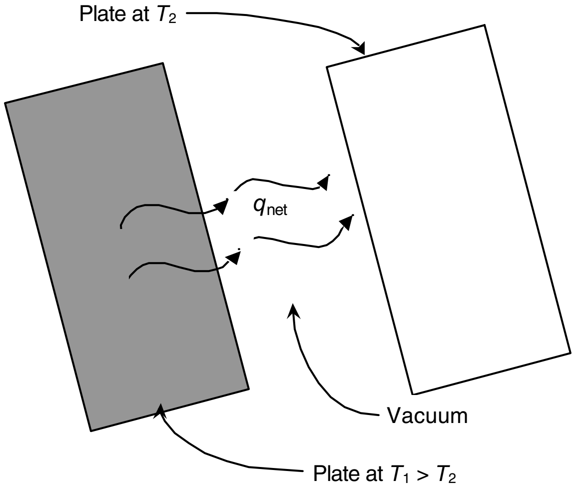

Figure 1.3 Radiation heat transfer between two flat plates

ergy") and stores it in its thermal capacity. It then carries this stored thermal energy to the cooler surroundings where some of it is deposited. Note that conduction heat transfer is the actual mechanism by which thermal energy is picked up from the heated plate and deposited in the cooler surroundings. Thus, heat conduction is an essential element of heat convection.

In contrast to conduction and convection heat transfer, *radiation heat transfer* does not require the presence of an intervening medium. Rather heat is transferred from the warmer body to the cooler body by *electromagnetic radiation,* as indicated in Figure 1.3. The nature of electromagnetic radiation and its emission and absorption by a material substance is the subject of this book. However, for the moment let us turn our attention to the distinguishing characteristics of the three modes of heat transfer.

1.2 CONDUCTION HEAT TRANSFER

The model describing conduction heat transfer is *Fourier's law of heat diffusion,* which states that the local heat flux $\mathbf{q}$ (W/m^2) is proportional to the local temperature gradient ∇T (°C/m); that is,

$$\mathbf{q} = -k \, \nabla T \tag{1.1}$$

The "constant" of proportionality k (W/m·°C) in Equation 1.1 is the *thermal conductivity,* and the minus sign is a consequence of the second law of thermodynamics, which requires that spontaneous heat transfer be directed from higher to lower temperatures, or down the temperature gradient. Originally the thermal conductivity was assumed to be a scalar quantity; however, the more modern view is that it is a tensor k_{ij} whose off-diagonal elements are usually but not always equal to zero.

Equation 1.1 may be thought of as the definition of the thermal conductivity. Then, if in a particular situation the relationship between the local heat flux and the local temperature gradient is observed to deviate from Fourier's law, this deviation may be attributed to a variation in the thermal conductivity with position, direction, or temperature, or with some combination of these three quantities. In other words, for a given local component of the heat flux vector, say q_x, the negative of the ratio of the heat flux component to the local temperature gradient in the same direction, $\partial T/\partial x$, is taken as the definition of the local value of the element k_{xx} of the thermal conductivity tensor. In Cartesian coordinates the thermal conductivity tensor is written

$$k_{ij} = \begin{vmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{vmatrix} \tag{1.2}$$

where, for example,

$$k_{xy} \equiv -\frac{q_x}{\partial T/\partial y} \tag{1.3}$$

The elements k_{xx}, k_{xy}, . . . of the thermal conductivity tensor are in general a function of both position and temperature, and the values of the off-diagonal elements of the thermal conductivity tensor are in most cases so small that they can be treated as zero.

When the thermal conductivity is independent of temperature it can be said that the local conduction heat transfer varies as the local temperature, assuming the surrounding temperature remains fixed. However, because of the actual temperature dependence of the thermal conductivity, the local heat transfer may vary as the local temperature to a power of as much as 1.1; that is,

$$q_{\text{cond}} \propto T^{1.0-1.1} \tag{1.4}$$

1.3 CONVECTION HEAT TRANSFER

The model for convection heat transfer is *Newton's law of cooling,* which, for external flow, states that the heat flux from a surface at temperature T_S to a surrounding fluid, whose temperature far from the surface is T_∞, is proportional to the temperature difference $T_S - T_\infty$; that is,

$$q_{\text{conv}} = h\,(T_S - T_\infty) \tag{1.5}$$

The *heat transfer coefficient h* (W/m^2·°C), defined by Equation 1.5, is an aerodynamic quantity. This means that its value depends on the details of the boundary layer flow across the surface being heated or cooled (laminar or turbulent, attached or separated, etc.). Now, the flow field surrounding a solid body is coupled to the temperature field through the kinematic viscosity and the mass density, whose values are generally temperature dependent. As a consequence of this and of the fact that the specific heat and thermal conductivity of the fluid are also temperature dependent, the heat transfer coefficient h in Newton's law of cooling is a function of temperature. When this is taken into account convection heat transfer from a surface may vary with surface temperature to a power as high as two; that is,

$$q_{\text{conv}} \propto T_S^{1-2} \tag{1.6}$$

1.4 RADIATION HEAT TRANSFER

We have seen that conduction and convection heat transfer are described by convenient *models* that define the thermal conductivity in the case of conduction and the heat transfer coefficient in the case of convection. The two models are referred to as "laws" because they are based on observation rather than derived from first principles. Their success is based on the wide latitude permitted by the adjustability of the coefficients involved. Radiation heat transfer, on the other hand, is described by a model subject to rigorous derivation, at least for one important special case (that of the blackbody, introduced in Chapter 2). For this reason it is perhaps inaccurate to refer to the model describing radiation heat transfer as a law. However, following common usage we will refer to this model as the *Stefan–Boltzmann law of radiation heat transfer,* which states that the heat flux emitted from a blackbody at absolute temperature T_b is

$$q_{\text{rad}} = \sigma T_b^4 \qquad \text{(blackbody)} \tag{1.7}$$

The coefficient σ in Equation 1.7 is the *Stefan–Boltzmann constant.*

The German physicist Josef Stefan (1835–1893) first suggested the form of Equation 1.7 in 1879 on the basis of data already available in the literature [1]. Stefan deduced that a straight line would result if the instantaneous cooling rate of a body suspended in a vacuum were plotted against the difference between its absolute temperature to the fourth power and that of its surroundings to the fourth power. Five years later Stefan's student, the Austrian physicist Ludwig Boltzmann (1844–1906), derived the form of Equation 1.7 on the basis of classical thermodynamics [2]. Boltzmann's development is described in Chapter 2. In contrast to Fourier's law and Newton's law of cooling, the coefficient in the Stefan–Boltzmann law is a physical constant whose value can be obtained exactly from well-established laws of physics.

Equation 1.7 must be modified when "real" surfaces and bodies, rather than the highly idealized blackbody, are considered. Under certain circumstances the modification may take the form of a factor ε, called the *emissivity,* which multiplies the Stefan–Boltzmann constant. Then under these circumstances the net heat flux from surface 1 to surface 2 for the arrangement of parallel flat plates shown in Figure 1.3 is well approximated by

$$q_{\text{rad, net}} = \frac{\varepsilon}{2-\varepsilon}\sigma(T_1^4 - T_2^4) \tag{1.8}$$

for sufficiently small spacing between the plates compared to their size. In writing Equation 1.8, the emissivity ε has been assumed to be the same for both surfaces.

We will learn in Chapter 3 that the emissivity is introduced to account for the fact that real surfaces generally emit and absorb less heat than a blackbody at the same temperature. In fact, under certain circumstances Equation 1.8 can be used to define the emissivity in much the same way that Fourier's law defines the thermal conductivity and Newton's law of cooling defines the heat transfer coefficient. As in the case of the thermal conductivity k and the heat transfer coefficient h, the emissivity can vary with temperature. Because of this variation the net radiation heat flux from a surface varies with the surface temperature to a power ranging between four and five; that is,

$$q_{\text{rad, net}} \propto T_S^{4-5} \tag{1.9}$$

The relations discussed in this section are summarized in Table 1.1. It can be concluded that radiation heat transfer becomes increasingly more important as other modes of heat transfer are suppressed (cryogenics, spacecraft thermal control, radiometry, thermal imaging) and when temperatures become high (combustion, electric arcs and thermal plasmas, atmospheric reentry).

1.5 THE ELECTROMAGNETIC SPECTRUM

It has already been stated that thermal radiation is an electromagnetic phenomenon. Electromagnetic waves, whose properties are explored elsewhere in this book, are capable of carrying energy from one location to another, even—indeed, especially—in

Table 1.1 Summary of the relationship between temperature and heat flux for the three modes of heat transfer

Heat Transfer Mode	Model, or "Law"	Coefficient Defined by Model	Range of Temperature Exponent
Conduction	Fourier's law $\mathbf{q} = -k\,\nabla T$	Thermal conductivity k	1.0–1.1
Convection	Newton's law of cooling $q = h\,\Delta T$	Heat transfer coefficient h	1.0–2.0
Radiation	The Stefan–Boltzmann law $q_e = \varepsilon\sigma T^4$	Emissivity ε	4.0–5.0

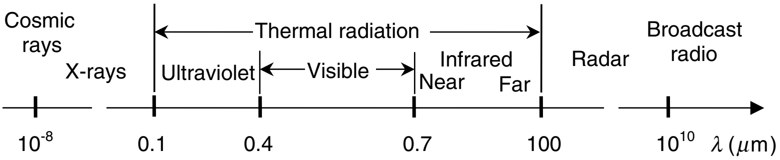

Figure 1.4 The electromagnetic spectrum

a vacuum. A familiar example is solar energy, which travels about 150 million kilometers through the vacuum of space to earth, where it is stockpiled in growing plants and acts as the power source that drives the earth's weather and climate. Other examples of electromagnetic energy carried by waves are radar and broadcast radio, microwaves, X-rays, cosmic rays, and the "light" signals received by our eyes. In fact, thermal radiation is but a very small segment of a broad electromagnetic spectrum that includes all of these phenomena and more. Some essential features of the electromagnetic spectrum are illustrated in Figure 1.4.

The frequency ν (Hz) of an electromagnetic wave is related to its wavelength λ (m) according to

$$\nu = \frac{c}{\lambda} = \frac{c_0}{\mathrm{n}\lambda} \tag{1.10}$$

where c (m/s) is the speed of light in the medium through which the radiation is passing, c_0 ($\approx 2.9979 \times 10^8$ m/s) is the speed of light in a vacuum, and $\mathrm{n} \equiv c_0/c$ ($-$) is the *index of refraction.*

Thermal radiation is electromagnetic radiation emitted by a material substance solely due to its temperature. The part of the electromagnetic spectrum considered to be "thermal" is not cleanly bounded; however, for the purposes of this book we will say that it extends from the short-wavelength limit of the ultraviolet (≈ 0.1 μm) to the long-wavelength limit of the far infrared (≈ 100 μm). Note that the part of the electromagnetic spectrum visible to the human eye occupies only about 0.3 μm out of a spectrum that ranges (for most practical purposes) over more than seventeen orders of magnitude!

In this book we will use the adjective "spectral" to indicate any quantity whose value varies with wavelength. Then the spectral distribution of radiation emitted from a source is the variation with wavelength (per unit wavelength) of the source strength. It is interesting to note that the spectral distribution of radiation emitted by the sun peaks at about 0.55 μm, or in the center of the visible spectrum. The human eye has evidently evolved to be optimally suited to exploit the earth's natural light source.

1.6 THE DUAL WAVE–PARTICLE NATURE OF THERMAL RADIATION

A young French doctoral student, Louis de Broglie (1892–1987), published a dissertation in 1924 that included an idea for which he was accorded the Nobel Prize in

1929. In his dissertation de Broglie (pronounced duh-BROY-ee) hypothesized that atomic particles could exhibit wave properties just as a photon can be assigned mass, thereby forever smearing the line separating waves and particles [3]. The de Broglie wavelength of a particle whose momentum is $p = mc$ is given by

$$\lambda = \frac{h}{p} \tag{1.11}$$

where h is *Planck's constant* ($h = 6.6237 \times 10^{-34}$ J·s). A discussion of the origin and significance of Planck's constant is deferred to Chapter 2. The German (later American) physicist Albert Einstein (1879–1955) had already established in 1905 the two principles underlying de Broglie's startling result: (1) the celebrated equivalence of mass and energy,

$$e = mc^2 \tag{1.12}$$

(special relativity) and (2) the photon view of light,

$$e = h\nu \tag{1.13}$$

(the photoelectric effect). It is left to the student to derive Equation 1.11 from Equations 1.10, 1.12, and 1.13.

The significance of Equation 1.11 is that it establishes the dual wave–particle nature of electromagnetic radiation. In some cases it is convenient to consider that light is emitted and propagates as a stream of energy packets called *photons.* Electrons can occupy only certain discrete energy levels within an atom. Thus, when an electron moves from one energy level to another within an atom, the principle of conservation of energy requires that a corresponding amount, or *quantum,* of energy be absorbed by or emitted from the atom. If the atomic transition occurs from energy level E_a to a lower energy level E_b, then, according to Equation 1.13, the frequency of the light emitted by the atom for this *bound–bound transition* is

$$\nu = \frac{E_a - E_b}{h} \tag{1.14}$$

In some cases a free electron with speed s will be captured by an atom and come to rest at some energy level, say E_a. In the case of such a *free–bound transition* the frequency of the light emitted will be

$$\nu = \frac{1}{h}\left(\frac{1}{2}m_e s^2 - E_a\right) \tag{1.15}$$

Because all speeds s are available to a free electron, Equation 1.15 permits a *continuum* of frequencies (or wavelengths), whereas Equation 1.14 permits emission only

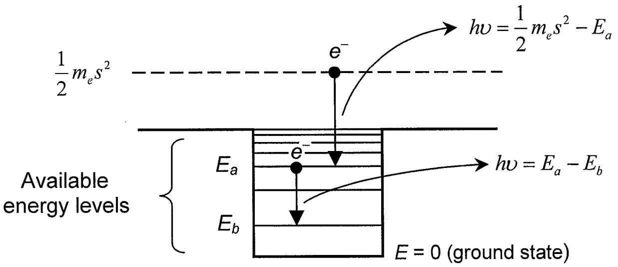

Figure 1.5 The atomic potential well illustrating a bound–bound and a free–bound transition

at discrete frequencies. In the case of high-temperature combustion processes, thermal plasmas, and electric arcs, a continuum is also produced by the acceleration of electrons as they follow curved paths. This latter phenomenon is sometimes referred to as *bremstrahlung* (from the German "burning radiation").

Equations 1.14 and 1.15 are the basis of the science of *spectroscopy,* which permits an atom or molecule to be identified by the spectral line structure of light emitted when it is excited. The transitions represented by Equations 1.14 and 1.15 are illustrated schematically in Figure 1.5. These transitions are reversible, with the reverse transitions corresponding to absorption rather than emission. Figure 1.6 shows most of the visible emission spectrum at the exit of an electric-arc-powered plasma generator using methane as the body gas. The CH and C_2 bands are associated with rotational and vibrational energy storage modes of these diatomic molecules (see Section 6.8), while the hydrogen line is due to a bound–bound electronic transition.

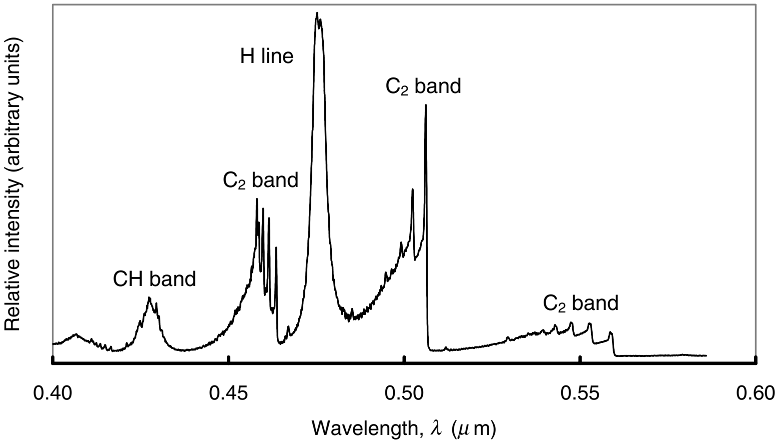

Figure 1.6 Visible portion of the emission spectrum from ionized methane [4]

The photon view of thermal radiation is the basis of the probablistic description of radiation heat transfer presented in Part III of this book, the *Monte Carlo ray-trace* (MCRT) method. In the MCRT method thermal radiation is described in terms of the mean behavior of a large number of energy bundles, or packets of photons, whose individual lives are governed by the laws of chance. Chapter 5 establishes a criterion for when we can model thermal radiation as a "raylike" phenomenon, and when we must resort to solution of the equations that describe its "wavelike" nature.

1.7 WAVE DESCRIPTION OF THERMAL RADIATION

In many cases of engineering interest it is more convenient to treat thermal radiation as a propagating wave than as a packet of photons. This is especially true when the effects of polarization, diffraction, and interference must be considered, as in certain optics and instrumentation applications. This view is also useful for modeling and understanding surface radiation properties and scattering from particles.

In 1864 the British physicist and mathematician James Clerk Maxwell (1831–1879) tied electricity and magnetism together and established that electromagnetic energy propagates through space as a wave at the speed of light [5]. Maxwell's equations for a homogeneous, isotropic medium in the absence of sources and sinks of electrical charge may be written

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{E}}{r_e} \tag{1.16a}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{1.16b}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon} \tag{1.16c}$$

and

$$\nabla \cdot \mathbf{H} = 0 \tag{1.16d}$$

In Equations 1.16, **H** (A/m) is the *magnetic field strength* (or *"intensity"*), **E** (V/m) is the *electric field strength,* ε ($C^2/N \cdot m^2$) is the *permittivity,* or *dielectric constant,* of the medium, r_e ($\Omega \cdot m$) is the *electrical resistivity* of the medium, μ (N/A^2) is the *permeability* of the medium, and ρ_e (C/m^3) is the *electric charge density.*

1.8 SOLUTION TO MAXWELL'S EQUATIONS FOR AN ELECTRICAL INSULATOR

For the case of an electrical insulator ($r_e \rightarrow \infty$, $\rho_e = 0$), Maxwell's equations describe an electromagnetic wave traveling through the medium at the speed of light. When the electromagnetic wave is propagating in the x direction, Maxwell's equations lead to the *wave equations*

$$\mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2} \tag{1.17a}$$

$$\mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} = \frac{\partial^2 E_z}{\partial x^2} \tag{1.17b}$$

$$\mu\varepsilon \frac{\partial^2 H_y}{\partial t^2} = \frac{\partial^2 H_y}{\partial x^2} \tag{1.17c}$$

and

$$\mu\varepsilon \frac{\partial^2 H_z}{\partial t^2} = \frac{\partial^2 H_z}{\partial x^2} \tag{1.17d}$$

where E_y, E_z, H_y, and H_z are the y and z components of the electromagnetic wave. In the special case where the magnitudes of the two components of the electric field and the two components of the magnetic field are equal, Equations 1.17 describe an *unpolarized,* or *randomly polarized,* wave.

1.9 POLARIZATION AND POWER FLUX

We say that an electromagnetic wave is *linearly polarized* if the electrical component is oriented in only a single direction. It can be shown that, in the free field far from the source, the magnetic component of an electromagnetic wave is in phase with and oriented in a direction at right angles to the electric component for linearly polarized radiation. A y-polarized electromagnetic wave freely propagating in the x direction is depicted in Figure 1.7. The power flux carried by an electromagnetic wave is given by the *Poynting vector,*

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \quad (\mathrm{W/m^2}) \tag{1.18}$$

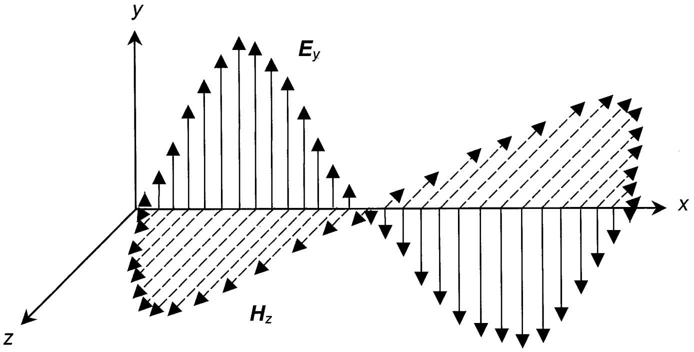

Figure 1.7 A y-polarized electromagnetic wave propagating in the x direction

In the case of the y-polarized wave depicted in Figure 1.7 we have

$$\mathbf{P} = P_x\mathbf{i} = E_y H_z\mathbf{i} \tag{1.19}$$

The quantity E_y in Figure 1.7 is the electric component of a *transverse-magnetic-polarized* electromagnetic wave, whose most general form is

$$E_y = f(x\sqrt{\mu\varepsilon} - t) + g(x\sqrt{\mu\varepsilon} + t) \tag{1.20}$$

where f and g are any single-valued, twice-differentiable functions of x and t. The function f describes a wave traveling in the positive x direction, and the function g describes a wave traveling in the negative x direction. The general right-running function f is shown at two different instants in time in Figure 1.8. From the figure it is clear that

$$f(x_1\sqrt{\mu\varepsilon} - t_1) = f[(x_1 + dx)\sqrt{\mu\varepsilon} - (t_1 + dt)] \tag{1.21}$$

Thus, because the function f is single valued it must be true that

$$x_1\sqrt{\mu\varepsilon} - t_1 = (x_1 + dx)\sqrt{\mu\varepsilon} - (t_1 + dt) \tag{1.22}$$

or

$$\frac{dx}{dt} \equiv c = \frac{1}{\sqrt{\mu\varepsilon}} \tag{1.23}$$

where c is the *speed of light* in the medium. To an accuracy of seven significant figures the speed of light in a vacuum is

$$c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 2.997925 \times 10^8 \text{ m/s} \tag{1.24}$$

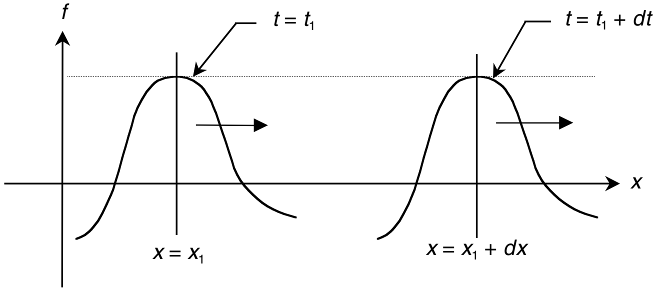

Figure 1.8 An "f wave" at time $t = t_1$ and at later time $t = t_1 + dt$

Now suppose E_y is a periodic electric wave whose Fourier series representation is

$$E_y\left(\frac{x}{c} - t\right) = E_0 + \sum_{m=1}^{\infty} |E_{y,\,m}| e^{2\pi i m\,(x/c-t)/T} \tag{1.25}$$

where $T = 1/\nu$, the period of the fundamental harmonic of $E_y\,(x/c - t)$. Then the mth harmonic of E_y is

$$E_{y,\,m} = |E_{y,\,m}|\; e^{i\omega_m\,(x/c-t)} \tag{1.26}$$

where

$$\omega_m = 2\pi\nu_m = \frac{2\pi m}{T} \quad \text{(r/s)} \tag{1.27}$$

In terms of the index of refraction, $\mathrm{n} \equiv c_0/c$, Equation 1.26 may be written

$$E_{y,m} = |E_{y,m}|\; e^{i\omega_m(\mathrm{n}x/c_0-t)} \tag{1.28}$$

Equation 1.28 describes a harmonic wave propagating without attenuation in the positive x direction. A version of this result will be used in Chapter 4 in an analysis aimed at understanding the phenomena of reflection and refraction that occur when radiation is incident to a plane interface separating two dielectrics having different indices of refraction. The mathematical description of the propagation of electromagnetic waves through electrically conducting media (finite r_e) is deferred until Chapter 4, when consideration of the interaction of electromagnetic waves with an interface between a vacuum (or air) and an electrically conducting medium is used as the basis for predicting trends in the surface radiation properties of metals.

1.10 DIFFRACTION AND INTERFERENCE

When two or more harmonic propagating electromagnetic waves share the same space, their instantaneous local electric fields are presumed to superimpose, i.e., to linearly combine. For example, if two harmonic waves having the same frequency are propagating in opposite directions along the same axis, as in the case of the f and g waves in Equation 1.20, a standing wave will result. However, in most radiation heat transfer applications a given space will be filled with randomly polarized waves arriving from all directions and at a wide range of frequencies. Furthermore, the components that do happen to have the same frequency are assumed to have random directions, polarizations, and phase relationships. Under these hypothetical circumstances it is reasonable to assume that the resulting radiation field will be "well mixed" so that the effects of diffraction and interference can be ignored. The justification for ignoring these effects is that if interference does occur at a given location and instant in time, on average both constructive and destructive interference will occur in equal amounts and so their effects will cancel. While this is a reasonable assumption in the usual radiation heat transfer context, it may lead to serious errors in certain optics and instrumentation applications. We return to this question in Chapter 5.

1.11 PHYSICS OF EMISSION AND ABSORPTION OF THERMAL RADIATION

Emission and absorption of photons by individual atoms is described above in terms of electronic transitions between energy levels within the atom or between the energy of a free electron and an atomic energy level. The probability that such a transition will lead to the emission or absorption of a photon by an atom is favored by high temperatures in gases but is relatively small in solids. For this reason when spectroscopy is used to identify the chemical makeup of a solid material, the sample is either burned or subjected to an electric arc to vaporize it and excite its atoms. While electronic transitions in atoms account for the absorption of relatively high-energy (short wavelength) radiation by cold gases and by cold electrically conducting solids (the photoelectric effect), the importance of this phenomenon as a heat transfer mechanism is negligible in most applications of engineering interest. We therefore seek a more efficient mechanism to explain emission from solids.

All matter is composed of atoms. Sometimes these atoms are organized into molecules, and often the resulting matter is a liquid or gas or an amorphous solid. However, for the purposes of the following discussion we will consider the idealized crystal structure of Figure 1.1 in which interatomic bonding forces maintain a regular array of atoms. The simplifications inherent in this model do not in any way limit the validity of the result it produces.

1.12 ELECTRICAL DIPOLE MOMENT

Each of the atoms in the crystal structure of Figure 1.1 acts as an oscillator whose stored energy is determined by its state of thermal equilibrium and certain laws of physics. Atoms are composed of an equal number of positively (protons) and negatively (electrons) charged particles, and, so, are electrically neutral. However, thermal "jostling" of the atom can lead to its mechanical distortion such that at a given instant it may have an *electrical dipole moment,* as shown in Figure 1.9. An atom having an electrical dipole moment, when placed in an electric field, will be subject to a torque that will tend to align its ± axis with the electric field.

The electron "cloud" (in this simple model) is assumed to have a uniform charge density, $-\rho_e$ (C/m^3), where C is the abbreviation for the unit of electrical charge, the *coulomb.* Thus, the charge q (C) of the electron cloud is given by

$$q = -\tfrac{4}{3}\pi R^3 \rho_e \tag{1.29}$$

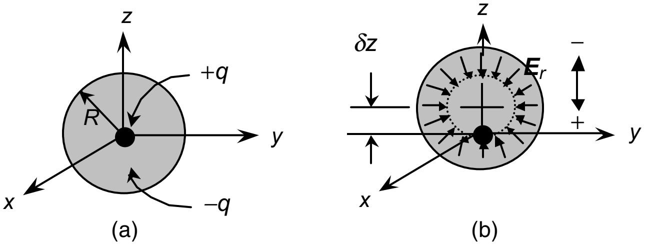

Figure 1.9 Atomic model (a) at equilibrium and (b) perturbed along the z axis

The local electric field E (V/m) on an imaginary closed surface S (m^2) is related to the electric charge Q (C) enclosed by the surface by the *Gauss flux theorem,*

$$\int_S \mathbf{E} \cdot \mathbf{n}\, dS = \frac{Q}{\varepsilon_0} \tag{1.30}$$

In Equation 1.30, **n** is the local outward-directed surface normal on surface S, and ε_0 ($= 8.854 \times 10^{-12}\ \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$) is the *permittivity of free space.*

For the idealized model of the jostled atom, the electric field $E_r = |\mathbf{E}_r|$, indicated by the radially inward-directed arrows in Figure 1.9b, is a constant on the spherical surface S of radius δz. The charge enclosed by S is

$$Q = \tfrac{4}{3}\pi\, (\delta z)^3\, \rho_e \tag{1.31}$$

Thus, applying the Gauss theorem, Equation 1.30,

$$4\pi(\delta z)^2\, E_r = -\frac{\tfrac{4}{3}\pi\, (\delta z)^3\, \rho_e}{\varepsilon_0} \tag{1.32}$$

or

$$E_r = -\frac{\rho_e}{3\varepsilon_0}\delta z \tag{1.33}$$

Eliminating the charge density ρ_e between Equations 1.29 and 1.33 yields

$$E_r = \frac{q}{4\pi\varepsilon_0 R^3}\delta z \tag{1.34}$$

We see from Figure 1.9b that the nucleus of the atom, whose charge is $+q$, lies in the spherical surface of radius δz where the magnitude of the radially inward-directed electric field is E_r. Therefore, the restoring force that tries to reestablish the equilibrium geometry of Figure 1.9a is

$$F = qE_r = \frac{q^2}{4\pi\varepsilon_0 R^3}\delta z \tag{1.35}$$

1.13 THE ATOMIC OSCILLATOR

For a linear spring–mass system such as the one shown in Figure 1.10, the restoring force of the spring is

$$F = K\,\delta z \tag{1.36}$$

Then, by analogy with the spring–mass system, the atom, once perturbed, acts as an oscillator with spring constant

$$K = \frac{q^2}{4\pi\varepsilon_0 R^3} \tag{1.37}$$

and mass

$$M = N m_e \tag{1.38}$$

where m_e is the mass of an individual electron and N is the number of electrons. The energy stored in each vibrational mode of the atomic oscillator, whose mechanical analog is shown in Figure 1.10, is

$$W = \frac{\mathcal{P}^2}{8\pi\varepsilon_0 R^3} \tag{1.39}$$

where $\mathcal{P} = q\,\delta z_{\max}$ is the *electric dipole moment* of the distorted atom. According to classical *equipartition of energy* applied to an atomic oscillator in thermal equilibrium with its surroundings at temperature T (K),

$$W = kT \qquad \text{(per mode)} \tag{1.40}$$

where k ($= 1.380 \times 10^{-23}$ J/K) is *Boltzmann's constant,* which may be thought of as the gas constant per molecule, or per atom in this case.

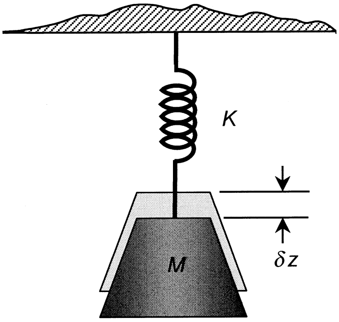

Figure 1.10 The atomic oscillator as a simple spring–mass system

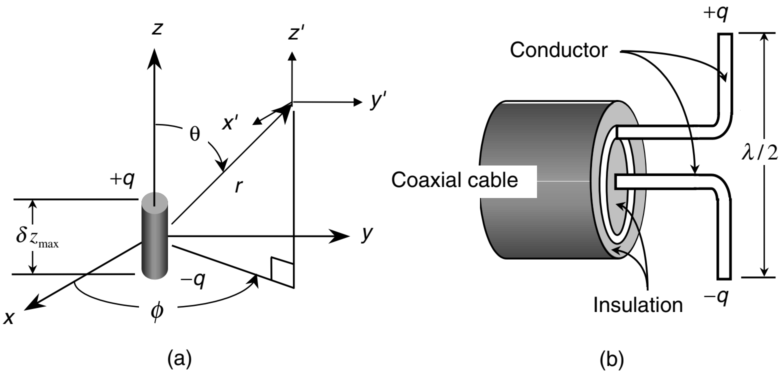

Figure 1.11 (a) The atomic oscillator as a dipole antenna and (b) a practical dipole antenna

1.14 THE ATOMIC OSCILLATOR AS A DIPOLE ANTENNA

The atomic oscillator can now be modeled as a *dipole antenna* carrying an alternating current

$$I = \frac{dQ}{dt} = \frac{d}{dt}(qe^{i\omega t}) = i\omega q e^{i\omega t} \tag{1.41}$$

where $i = \sqrt{-1}$ and $\omega = 2\pi\nu = 2\pi c_0/\lambda$. An elemental dipole antenna and its practical implementation are shown in Figure 1.11. The actual atomic oscillator has three vibrational degrees of freedom, one each along the x, y, and z axes. It is established in Appendix A that the total power radiated from three orthogonal, co-located dipole antenna elements like the one shown in Figure 1.11a is

$$P_r(\lambda, T) = \frac{\mathcal{P}^2}{\varepsilon_0} 6\pi^3 c_0 \lambda^{-4} \quad \text{(W)} \tag{1.42}$$

where now $\mathcal{P} = q\delta$ with $\delta = \delta x_{\max} = \delta y_{\max} = \delta z_{\max}$. Equating the two expressions for W, Equations 1.39 and 1.40, and rearranging, there results

$$\frac{\mathcal{P}^2}{\varepsilon_0} = 8\pi R^3 kT \tag{1.43}$$

Thus,

$$P_r(\lambda, T) = 48\pi^4 R^3 kT c_0 \lambda^{-4} \quad \text{(W)} \tag{1.44}$$

1.15 RADIATION DISTRIBUTION FUNCTION

The *radiation distribution function* of a source is defined such that, if it is multiplied by $d\lambda$ and the result integrated over some wavelength interval $[\lambda_1, \lambda_2]$, the total power per unit surface area emitted by the source over that wavelength interval is obtained. Thus, the power in watts radiated from our atomic oscillator in the wavelength interval $\Delta\lambda = \lambda_2 - \lambda_1$ is

$$P_r(\Delta\lambda, T) = \int_S \int_{\lambda_1}^{\lambda_2} p_{r\lambda}(\lambda, T)\, d\lambda\, dS \tag{1.45}$$

where $p_{r\lambda}(\lambda, T)$ is the radiation distribution function ($\mathrm{W/m^2 \cdot \mu m}$). Bearing in mind the rather gross assumptions already made about our oscillator (arrangement in a crystal matrix, uniform charge distribution of the "electron cloud," etc.), we are justified in dividing Equation 1.44 by a dimensional constant whose dimensions are length cubed, whose units are $\mathrm{m^2 \cdot \mu m}$, and whose order of magnitude is the same as $d\lambda\, dS$. Note that due to the small size of the quantities involved this is equivalent to taking the partial derivatives of Equation 1.45 with respect to λ and S. When we do this we obtain for our atomic oscillator,

$$p_{r\lambda}(\lambda, T) = 8\pi\, CkTc_0\lambda^{-4} \quad (\mathrm{W/m^2 \cdot \mu m}) \tag{1.46}$$

The factor C in Equation 1.46 accounts for all of the differences, including the step taking us from Equation 1.45 to Equation 1.46, between our rather simplified model of the atomic oscillator and reality. However, we shall see in Chapter 2 that if $C = 1$ Equation 1.46 is in agreement with a result for the same quantity derived directly from classical mechanics.

The analysis in the final five sections of this chapter verifies a principle often referred to as *Prevost's law,* which states that all material substances at a temperature above absolute zero continuously emit radiation, even when they are in thermal equilibrium with their surroundings.

Team Projects

TP 1.1 The following is a partial list of serials with which radiation heat transfer students and practitioners should be familiar:

1. *Journal of Heat Transfer* (ASME)
2. *International Journal of Heat and Mass Transfer*
3. *Journal of Quantitative Spectroscopy and Radiative Transfer*
4. *Applied Optics*
5. *Optical Engineering*
6. *AIAA Journal*
7. *Journal of the Optical Society of America* (Parts A and B)
8. *Numerical Heat Transfer*

Visit the library and find each of these journals on the shelves. You will find them in two locations: the recent numbers will be found unbound and stacked on shelves in one area of the library, and older numbers will be bound and in the stacks in another part of the library. (Note that very old numbers of some serials may be stored at an off-site location.)

1. Find and write down the call number for each of these journals.
2. Make a photocopy of the table of contents of the most recent unbound number for each of these journals. Use a marker to highlight articles treating radiation heat transfer.
3. Explore the stacks near the journals listed above. Are there other journals of a similar vein? If so, add them to the list and complete parts 1 and 2 above for them also. Submit a comprehensive word-processed list, with call numbers, of all journals you found.
4. Browse through the most recent numbers of these journals and find an article that you think you might like to read and share with the class.

TP 1.2 Technical papers reporting more recent developments in radiation heat transfer are published in *conference proceedings.* Conference proceedings are also typically archived in university libraries. Visit the library and locate the conference proceedings that include papers on radiation heat transfer. Such conferences are sponsored by the American Society of Mechanical Engineers (ASME), the American Institute of Aeronautics and Astronautics (AIAA), the Society of Photo-Optical Instrumentation Engineers (SPIE), the Optical Society of America (OSA), the American Society for Photogrammetry and Remote Sensing (ASPRS), the Resource Technology Institute (RTI), the National Geophysical Union (NGU), the American Meteorological Society (AMS), and the International Radiation Symposium (IRS), to name but a few.

1. Compile a complete list of the conference proceedings you were able to locate (bound and unbound) that treat aspects of radiation heat transfer. Limit your search to conference proceedings published in the last five years. Note the call number in each case.
2. Make a photocopy of the table of contents of the most recent of these conference proceedings.
3. Browse through the conference proceedings that you have found and find a paper that you think you might like to read and share with the class.

Discussion Points

DP 1.1 Figure 1.2 illustrates forced convection heat transfer from a flat plate. What can we say about the value of the Prandtl number in this case? Explain your answer.

DP 1.2 Equation 1.3 purports to define an off-axis element of the thermal conductivity matrix. Discuss the possibility of k_{xy} being nonzero in the context of the second law of thermodynamics and the definition of "heat" as the energy interaction between a body and its surroundings that occurs because of a temperature difference. You may also want to consider the idealized crystal structure model in Figure 1.1. Related topics: nonequilibrium thermodynamics and Onsager's relations.

DP 1.3 Consider Equation 1.8. What does the spacing between the two plates in Figure 1.3 have to do with the degree of approximation of the equation? How would Equation 1.8 be modified if the surroundings of the two plates were maintained at a temperature of absolute zero and the spacing between the two parallel plates was arbitrary?

DP 1.4 Referring to Equation 1.9, explain how the exponent can exceed 4 without violating the principle discovered by Stefan and explained by Boltzmann. Does this mean that a body can emit more radiation than a blackbody at the same temperature?

DP 1.5 In the last paragraph of Section 1.4 it is stated that radiation heat transfer becomes dominant, among other situations, in two extreme cases: very low temperatures (cryogenics) and very high temperatures (combustion). Is there a contradiction here? Please explain.

DP 1.6 Most vegetation appears green to the eye. We know that the solar spectrum peaks in the green part of the visible spectrum. Is this a coincidence? Speculate on how natural selection might have played a role in the evolution of chlorophyll.

DP 1.7 Show formally that the magnetic component of a linearly polarized electromagnetic wave is oriented at right angles to the electric component for free-field radiation.

DP 1.8 How are microwave antennas and infrared emitters alike and how are they different? In the context of this question, consider the difference in the way a microwave oven and a "convection" oven work.

DP 1.9 Two students should be designated to lead a debate of the proposition, "Resolved: It is important for an engineering graduate student to understand the material in Appendix A."

DP 1.10 Suppose you and your team set out to derive a *radiation distribution function,* that is, a mathematical relation that predicts the amount of radiation emitted from an ideal emitter as a function of wavelength and temperature. (No emitter can emit more radiation at a given wavelength than an ideal emitter at the same temperature.) How would you know if your eventual distribution function was correct? Among other things (such as experimental data) you would want to know in advance the asymptotic values of the function as temperature and wavelength approach the physical limits of zero and infinity. Turn-of-the-century physicists, when faced with this problem, immediately came up with the four asymptotic conditions—two in temperature and two in wavelength—to which their candidate distribution functions would have to adhere. Can you? In this context please criticize the radiation distribution function represented by Equation 1.46. Does it conform to your expectations? Explain.

DP 1.11 Discuss the atomic model of Figure 1.9b. Would not the electric field $\mathbf{E}_r$ be distorted by the presence of the positively charged nucleus at the outer rim of the electron cloud of radius δz? [*Hint:* What adjectives would you use to describe the equation identified as the Gauss flux theorem, Equation 1.30?]

DP 1.12 Criticize the model that is the basis of the last five sections of this chapter. Which assumptions and simplifications might intervene in determining the value of the constant C in Equation 1.46?

Problems

P 1.1 The data in Table 1.2 are from a classical article published by Dulong and Petit in 1817 [6]. In the experiment described in the article, bodies were suspended in a vacuum jar whose outer walls were surrounded by melting ice. The table gives the initial temperature of the bodies and the decrease in temperature after one minute. Stefan used these same data in 1879 (62 years later!) to establish the absolute-temperature-to-the-fourth-power dependence of radiative heat emission from a solid body. Can you reproduce Stefan's reasoning?

Table 1.2 Data from Dulong and Petit [6] used by Stefan [1] to establish the absolute temperature-to-the-fourth-power dependence of radiative heat emission

Initial Temperature of Body (°C)	Temperature Decrease After 1 min (°C)
240	10.69
220	8.92
200	7.40
180	6.10
160	4.89
140	3.88
120	3.02
100	2.30
80	1.74

P 1.2 Derive Equation 1.11, the expression for the de Broglie wavelength of a particle of mass m traveling at speed c with momentum $p = mc$, from Equations 1.10, 1.12 and 1.13.

P 1.3 Derive the wave equation, Equation 1.17a, from Maxwell's equations, Equations 1.16, for the case of an electrical insulator ($r_e \to \infty$, $\rho_e = 0$).

P 1.4 Demonstrate by direct substitution that Equation 1.20 is the most general solution to the wave equation for E_y, Equation 1.17a.

P 1.5 Resolve the units in Equation 1.30. That is, show that the units are the same on both sides of the equation.

P 1.6 Suppose that in the simple atomic model of Figure 1.9a the electric charge density ρ_e varies as the inverse square of the radial distance r from the center of the atom; that is,

$$\rho_e(r) = \rho_{e0}\left[1 - \left(\frac{r}{R}\right)^n\right] \tag{1.47}$$

where the radial position r is measured from the center of the electron cloud. What would be the charge q of the electron cloud in terms of ρ_{e0}, R, and n?

P 1.7 For sufficiently small perturbations ($\delta z \to 0$) it may be assumed that the electron cloud in Figure 1.9b remains spherical. Then for the electric charge density variation of Problem P 1.6 what is the electric charge Q contained within the spherical envelope of radius δz in terms of δz, ρ_{e0}, R, and n?

P 1.8 Derive Equation 1.39 by considering the energy stored in a spring, whose elastic constant is K, at its maximum displacement from equilibrium.

P 1.9 Suppose the atomic spacing s in Figure 1.1 is aR, where R is the atomic radius shown in Figure 1.9a and a is a dimensionless scaling factor. For what value of the scaling factor a is the constant C in Equation 1.46 equal to unity? What conclusion do you draw from this result? Can C reasonably be equal to unity in this model? [*Hint:* Assume $\lambda \sim R$ and let the surface S be that of a cubic volume of dimensions $aR \times aR \times aR$ surrounding the atomic oscillator.]

REFERENCES

1. Stefan, J., "Über die Beziehung zwischen der Wärmstrahlung und der Temperatur," *Sitzungberichte, Akademie der Wissenschaften,* Wien, Bd. 79, Pt. 2, 1879, pp. 391–428.
2. Boltzmann, L., "Ableitung des Stefan'schen Gesetzes, betreffend die Abhängigkeit der Wärmestrahlung von der Temperatur aus der electromagnetischen Lichttheorie," *Annalen der Physik und Chemie,* Bd. 22, 1884, pp. 291–294.
3. de Broglie, Prince Louis Victor Pierre Raymond, "Recherches sur la théorie des quanta," Thesis, University of Paris (Sorbonne), Paris, 1924 (published in *Annales de Physique,* 10^{e} série, Tome III, Masson et Cie, Editeurs, Paris, 1924, pp. 22–128).

4. Gallimore, S. D., *A study of plasma ignition enhancement for aeroramp injectors in supersonic combustion applications,* PhD Dissertation, Department of Mechanical Engineering, Virginia Tech, Blacksburg, VA, May 2001.
5. Maxwell, J. C., "A dynamical theory of the electromagnetic field," *Philosophical Transactions of the Royal Society of London,* 1865, pp. 459–512.
6. Dulong, P.-L., and A.-T. Petit, "Des recherches sur la mesure des températures et sur les lois de la communication de la chaleur (Seconde partie, des lois du refroidissement)," *Annales de Chimie et de Physique,* Tome 7, 1817, p. 247.

TEXTS FOR FURTHER REFERENCE

1. *The Theory of Heat Radiation,* M. Planck (trans. Morton Masius), Dover Publications, Inc., New York, 1959.
2. *Radiative Transfer,* S. Chandrasekhar, Dover Publications, Inc., New York, 1960.
3. *Engineering Radiation Heat Transfer,* J. A. Wiebelt, Holt, Rinehart and Winston, New York, 1966.
4. *Radiative Transfer,* H. C. Hottel and A. F. Sarofim, McGraw-Hill Book Company, New York, 1967.
5. *Radiative Heat Transfer,* T. J. Love, C. E. Merrill Publishing Company, Columbus, Ohio, 1968.
6. *Radiation Heat Transfer* (augmented edition), E. M. Sparrow and R. D. Cess, Hemisphere Publishing Corporation, Washington, 1978.
7. *Transfer Processes: An Introduction to Diffusion, Convection, and Radiation* (2nd edition), D. K. Edwards, V. E. Denny, and A. F. Mills, Hemisphere Publishing Corporation, Washington, 1979.
8. *Thermal Radiation Heat Transfer,* R. Siegel and J. R. Howell, Hemisphere Publishing Corporation, Washington, 1992.
9. *Thermal Radiative Transfer & Properties,* M. Quinn Brewster, John Wiley & Sons, New York, 1992.
10. *Radiative Heat Transfer,* M. F. Modest, McGraw-Hill, New York, 1993.