7

Three-Phase AC Circuits

While the previous chapter dealt with a single-phase (1-ϕ) AC that is transmitted through a transmission line (consisting of a pair of wires) to a load, attention now turns to a *three-phase* (3-ϕ) AC power system, in which three AC sources operate at the same frequency but with different phases. A 3-ϕ AC power system has the following advantages over a 1-ϕ AC power system:

1. The instantaneous power delivered to a load fluctuates much less in a polyphase AC power system than in a single-phase AC power system. Especially when it is used in rotating machinery like motors, the torque on the rotor pulsates much less than in a single-phase AC power system.
2. It can deliver the same power with appreciably less conductors and components than a single-phase AC power system. That is why almost all electric power in the world is generated, transmitted, and distributed in the form of three-phase AC (at 50 or 60 Hz) throughout the world.

In one example a 3-ϕ AC power system will be solved by using MATLAB and PSpice.

7.1 Balanced Three-Phase Voltages

It may be helpful in understanding three-phase AC circuits to see the rough structure and principle of a three-phase AC generator such as the ones illustrated in Figure 7.1 or Reference [W-9]. Both of the two three-phase generators consist of a rotor, a stator, and three separate armature coils with terminals a–a', b–b', and c–c' that are placed 120° apart around the rotor (Figure 7.1(a)) or the stator (Figure 7.1(b)). Since each armature coil has a flux linkage of $\lambda(t) = N\phi_{\mathrm{m}} \sin(\omega t + \theta)$ (N = the number of windings in an armature coil, ϕ_{m} = the flux produced by the magnet, ω = the angular velocity of the rotor, t = the time, and θ = the initial angular position of the rotor) depending on its angular position relative to the stator, the induced voltage between its terminals is

$$v(t) = \frac{\mathrm{d}}{\mathrm{d}t}\lambda(t) = \frac{\mathrm{d}}{\mathrm{d}t}[N\phi_{\mathrm{m}} \sin(\omega t + \theta)] = N\phi_{\mathrm{m}}\omega \cos(\omega t + \theta) = V_{\mathrm{m}} \cos(\omega t + \theta)$$

Thus, depending on the relative position of the three coils and the rotating direction of the rotor, the three induced voltages across the armature coils between terminals a–a', b–b', and c–c' can be written as follows:

Positive (abc) sequence

$$v_a(t) = V_{\mathrm{m}} \cos(\omega t), \quad v_b(t) = V_{\mathrm{m}} \cos(\omega t - 120^\circ), \quad v_c(t) = V_{\mathrm{m}} \cos(\omega t + 120^\circ) \tag{7.1a}$$

$$\mathbf{V}_a = V_{\mathrm{m}}\angle 0^\circ, \quad \mathbf{V}_b = V_{\mathrm{m}}\angle{-120^\circ}, \quad \mathbf{V}_c = V_{\mathrm{m}}\angle{+120^\circ} \tag{7.1b}$$

(a-phase voltage) (b-phase voltage) (c-phase voltage)

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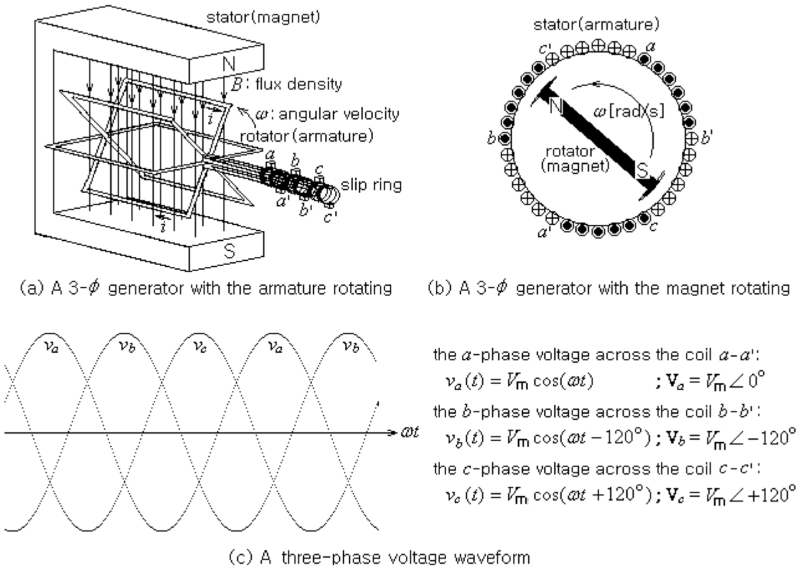


Figure 7.1 Basic structure and principle of an AC generator

Negative (acb) *sequence*

$$v_a(t) = V_m \cos(\omega t), \qquad v_b(t) = V_m \cos(\omega t + 120°), \qquad v_c(t) = V_m \cos(\omega t - 120°) \tag{7.2a}$$

$$\mathbf{V}_a = V_m \angle 0°, \qquad \mathbf{V}_b = V_m \angle {+120°}, \qquad \mathbf{V}_c = V_m \angle {-120°} \tag{7.2b}$$

Figure 7.1(c) shows a typical set of three-phase voltage waveforms in the positive (abc) sequence, which can be produced by a three-phase AC generator.

In order to operate more than one three-phase AC generators in parallel, their phase sequences should be the same, positive (abc) or negative (acb). Such a set of three AC voltages as these is said to be *balanced* because they all have the same frequency and magnitude but are out of phase with each other by 120°.

An important feature of balanced three-phase voltages is that their sum is zero:

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0 \tag{7.3}$$

This zero-sum property can also be shown in the time domain:

$$v_a(t) + v_b(t) + v_c(t) = 0 \tag{7.4}$$

$$\begin{aligned} &V_m \cos(\omega t) + V_m \cos(\omega t - 120°) + V_m \cos(\omega t + 120°) \\ &\overset{\text{(F.6)}}{=} V_m \cos(\omega t) + V_m[\cos(\omega t)\cos(120°) + \sin(\omega t)\sin(120°)] \\ &\quad + V_m[\cos(\omega t)\cos(120°) - \sin(\omega t)\sin(120°)] \\ &= V_m \cos(\omega t)[1 + 2\cos(120°)] = V_m \cos(\omega t)[1 + 2 \times (-1/2)] = 0 \end{aligned}$$

The output voltage/current level of a three-phase generator depends on which connection of the three armature coils is made, Y(*wye*)-*connection* or Δ(*delta*)-*connection*. The two connection diagrams of a

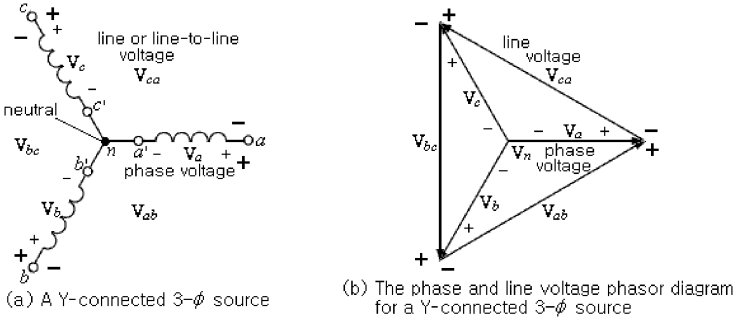


Figure 7.2 A Y-connected 3-ϕ source and its voltage phasor diagram

three-phase source together with the corresponding voltage/current phasor diagrams are depicted in Figures 7.2 and 7.3, respectively. Especially the Y-connected three-phase source (Figure 7.2(a)) has a common terminal of the three coils, labeled n, which is called the *neutral* (node or terminal) of the source.

Figure 7.2(b) shows the voltage phasor diagram of a Y-connected balanced three-phase source, where the a-phase voltage $\mathbf{V}_a$ is regarded as the reference with the phase angle of 0°. From this phasor diagram, the relationship between the *phase* (or *line-to-neutral*) *voltage*s and the *line* (or *line-to-line*) *voltage*s can be written as follows:

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = \sqrt{3}\, V_{\mathrm{Y}} \angle 30^\circ \tag{7.5a}$$

$$\mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = \sqrt{3}\, V_{\mathrm{Y}} \angle -90^\circ \tag{7.5b}$$

$$\mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = \sqrt{3}\, V_{\mathrm{Y}} \angle +150^\circ \tag{7.5c}$$

Note that for a Y-connected three-phase source, the amplitudes of line voltages are $\sqrt{3}$ times that of phase voltages and the line current is the same as the phase current:

$$V_l = \sqrt{3}\, V_{\mathrm{Y}} \qquad \text{and} \qquad \mathbf{I}_l = \mathbf{I}_{\mathrm{Y}} \tag{7.5d}$$

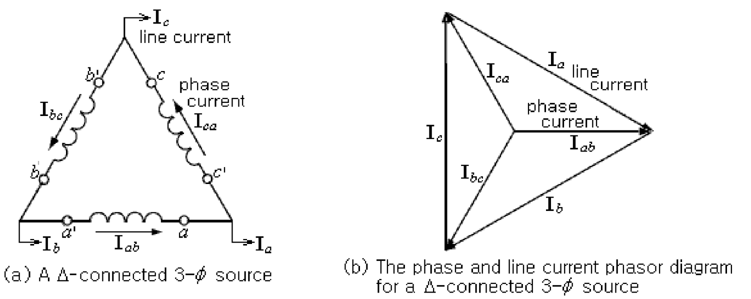


Figure 7.3 A Δ-connected 3-ϕ source and its current phasor diagram

Figure 7.3(b) shows the current phasor diagram of a Δ-connected balanced three-phase source, where the *a*-phase current ($\mathbf{I}_{ab}$) is regarded as the reference with the phase angle of 0°. From this phasor diagram, the relationship between the *phase currents* and the *line currents* can be written as follows:

$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca} = \sqrt{3}\, I_\Delta \angle -30^\circ \tag{7.6a}$$

$$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab} = \sqrt{3}\, I_\Delta \angle -150^\circ \tag{7.6b}$$

$$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc} = \sqrt{3}\, I_\Delta \angle +90^\circ \tag{7.6c}$$

Note that for a Δ-connected three-phase source, the amplitudes of line currents are $\sqrt{3}$ times that of phase currents and the line voltage is the same as the phase voltage:

$$I_l = \sqrt{3}\, I_\Delta \qquad \text{and} \qquad \mathbf{V}_l = \mathbf{V}_\Delta \tag{7.6d}$$

Since not only three-phase sources but also three-phase loads can be either Y-connected or Δ-connected, there are four possible configurations of a three-phase power system: Y-Y, Y-Δ, Δ-Y, and Δ-Δ. However, Δ-connected three-phase sources are seldom used in practice because they make a loop of voltage sources, which may have a large *circulating current* if the three-phase voltages are not exactly balanced and therefore do not sum to zero. For example, a slightly unbalanced three-phase voltage source of

$$\mathbf{V}_{ab} = 207.9\angle 29^\circ, \qquad \mathbf{V}_{bc} = 208.1\angle -90^\circ, \qquad \text{and} \qquad \mathbf{V}_{ca} = 208\angle +151^\circ$$

has an emf (electromotive force) around the Δ-loop inside the generator as

$$\begin{aligned}\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} &= 207.9(0.8746 + j0.4848) - j208.1 + 208(-0.8746 + j0.4848) \\ &= -0.0875 - j6.47\ \text{V}\end{aligned}$$

Since the impedance of the windings (coils) of a generator is very small, even this low emf may result in a large circulating current that will heat the Δ-connected generator, making its efficiency and life suffer.

7.2 Power of Balanced Three-Phase Loads

A three-phase load is said to be *balanced* if the three impedance legs are all the same. In this section we will find the power of a balanced three-phase load to which a balanced three-phase voltage source with an rms line voltage V_l and an rms line current I_l is applied:

Power of a Y-connected balanced three-phase load

$$\text{Active power:} \quad P_{\text{Y,Total}} = 3P_{\text{Y}} = 3\, V_{\text{Y}} I_{\text{Y}} \cos\theta \overset{(7.5\text{d})}{=} 3\frac{V_l}{\sqrt{3}} I_l \cos\theta = \sqrt{3}\, V_l I_l \cos\theta \tag{7.7a}$$

$$\text{Reactive power:} \quad Q_{\text{Y,Total}} = 3Q_{\text{Y}} = 3\, V_{\text{Y}} I_{\text{Y}} \sin\theta = \sqrt{3}\, V_l I_l \sin\theta \tag{7.7b}$$

$$\text{Complex power:} \quad S_{\text{Y,Total}} = 3S_{\text{Y}} = 3\,\mathbf{V}_{\text{Y}} \mathbf{I}_{\text{Y}}^* = \sqrt{3}\, V_l I_l \angle\theta = P_{\text{Y,Total}} + jQ_{\text{Y,Total}} \tag{7.7c}$$

with θ = phase angle or power factor angle of the load

Power of a Δ-connected balanced three-phase load

$$\text{Active power:} \quad P_{\Delta,\text{Total}} = 3P_\Delta = 3\, V_\Delta I_\Delta \cos\theta \overset{(7.6\text{d})}{=} 3\, V_l \frac{I_l}{\sqrt{3}} \cos\theta = \sqrt{3}\, V_l I_l \cos\theta \tag{7.8a}$$

$$\text{Reactive power:} \quad Q_{\Delta,\text{Total}} = 3Q_\Delta = 3\, V_\Delta I_\Delta \sin\theta = \sqrt{3}\, V_l I_l \sin\theta \tag{7.8b}$$

$$\text{Complex power:} \quad S_{\Delta,\text{Total}} = 3S_\Delta = 3\,\mathbf{V}_\Delta \mathbf{I}_\Delta^* = \sqrt{3}\, V_l I_l \angle\theta = P_{\text{Y,Total}} + jQ_{\text{Y,Total}} \tag{7.8c}$$

Note. It is implied that regardless of whether a balanced three-phase source/load is Y-connected or Δ-connected, the power transferred from the source to the load is determined by the line (or line-to-line) voltage and the line current.

[**Remark 7.1**] Instantaneous Power of a 1-ϕ System and a 3-ϕ System

Referring to Section 6.5, the instantaneous power of a load (with the impedance angle of θ) supplied by a single-phase AC source (with the rms values of its terminal voltage and current given as V and I, respectively) varies with time:

$$p(t) \overset{(6.25a)}{=} V\,I\,[\cos\theta + \cos(2\omega t + 2\theta_{\rm v} - \theta)] \tag{7.9}$$

By contrast, the instantaneous power of a balanced three-phase load supplied by a balanced three-phase AC source (with the rms values of its phase voltage and current given as V_ϕ and I_ϕ, respectively) does not vary with time, producing a constant torque for a three-phase AC motor:

$$\begin{aligned} p_{3\phi}(t) &= v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t) \\ &= V_{\phi\rm m}\cos(\omega t)I_{\phi\rm m}\cos(\omega t - \theta) + V_{\phi\rm m}\cos(\omega t - 120^\circ)I_{\phi\rm m}\cos(\omega t - 120^\circ - \theta) \\ &\quad + V_{\phi\rm m}\cos(\omega t + 120^\circ)I_{\phi\rm m}\cos(\omega t + 120^\circ - \theta) \\ &= V_\phi I_\phi\,[\cos\theta + \cos(2\omega t - \theta)] + V_\phi I_\phi\,[\cos\theta + \cos(2\omega t - 240^\circ - \theta)] \\ &\quad + V_\phi I_\phi\,[\cos\theta + \cos(2\omega t + 240^\circ - \theta)] \\ &= 3V_\phi I_\phi\cos\theta : \quad \text{constant} \end{aligned} \tag{7.10}$$

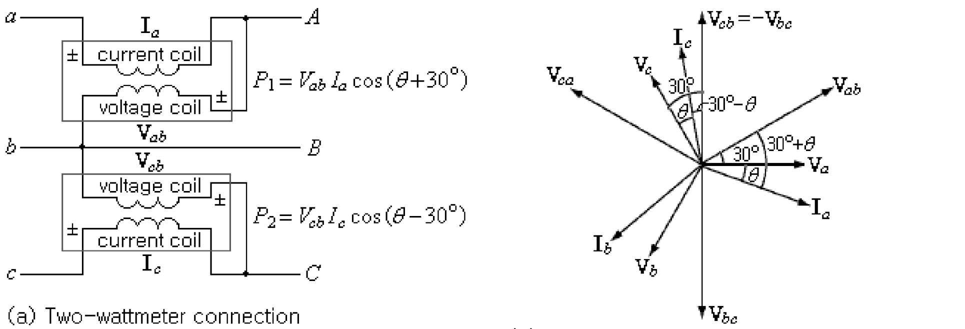
7.3 Measurement of Three-Phase Power

It is no wonder that three wattmeters are used to measure a three-phase power. However, it may be surprising that a three-phase power can be measured using just two wattmeters. Figure 7.4(a) shows how to connect two wattmeters for measuring a three-phase power, where their readings are written as

$$\begin{aligned} P_1 &= V_{ab}I_a\cos(\theta + 30^\circ) \\ P_2 &= V_{cb}I_c\cos(\theta - 30^\circ) \end{aligned} \tag{7.11}$$

from the phasor diagram in Figure 7.4(b). The sum of these two readings yields the three-phase active power:

$$\begin{aligned} P_1 + P_2 &= V_{ab}I_a\cos(\theta + 30^\circ) + V_{cb}I_c\cos(\theta - 30^\circ) \\ &= V_lI_l\cos(\theta + 30^\circ) + V_lI_l\cos(\theta - 30^\circ) \\ &\overset{(F.6)}{=} 2V_lI_l\cos\theta\cos 30^\circ = \sqrt{3}\,V_lI_l\cos\theta \overset{(7.7a),(7.8a)}{\equiv} P_{\rm Total} \end{aligned} \tag{7.12}$$



(a) Two-wattmeter connection to measure a three-phase AC power (b) The phasor diagrams for voltages/currents

Figure 7.4 A circuit and its phasor diagram for the two-wattmeter method to measure a 3-ϕ power

On the other hand, the difference between these two readings turns out to be

$$P_2 - P_1 = V_l I_l \cos(\theta - 30^\circ) - V_l I_l \cos(\theta + 30^\circ)$$
$$\overset{\text{(F.6)}}{=} 2V_l I_l \sin\theta \sin 30^\circ = V_l I_l \sin\theta \overset{\text{(7.7b),(7.8b)}}{\equiv} Q_{\text{Total}}/\sqrt{3} \tag{7.13}$$

Thus this can be multiplied by $\sqrt{3}$ to get the three-phase reactive power Q_{Total} and, furthermore, use can be made of the ratio of the reactive power to the active power to obtain the power factor as

$$PF = \cos\theta \overset{\text{Fig. 6.12}}{=} \cos\left(\tan^{-1}\frac{Q_{\text{Total}}}{P_{\text{Total}}}\right) \overset{\text{(7.12),(7.13)}}{=} \cos\left[\tan^{-1}\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right] \tag{7.14}$$

7.4 Three-Phase Power System

As discussed in Section 7.1, there are four possible configurations (Y-Y, Y-Δ, Δ-Y, and Δ-Δ) of three-phase power systems. In this section, only the Y-Y connection will be examined since Δ-connected three-phase sources are seldom used in practice due to the circulating current problem and Δ-connected loads can easily be converted into their equivalent Y-connected ones (see Table 6.1 in Section 6.4).

Figure 7.5 shows a Y-Y three-phase power system, where the neutral line connecting the two neutrals of the source and the load is shown as a dotted line to denote that it is dispensable in principle for a balanced system because no current will flow through it. The three-phase power system with no neutral line is denoted by '3ϕ-3w' and one having a neutral line by '3ϕ-4w'. To analyze this power system, the source-side neutral is set as the reference node (having zero potential) and KCL is applied to the load-side neutral N to write the node equation as

$$\frac{\mathbf{V}_a - \mathbf{V}_N}{Z_A} + \frac{\mathbf{V}_b - \mathbf{V}_N}{Z_B} + \frac{\mathbf{V}_c - \mathbf{V}_N}{Z_C} = \frac{\mathbf{V}_N}{Z_{nl}} \tag{7.15}$$

where $Z_A = Z_{al} + Z_{\text{AL}}$, $Z_B = Z_{bl} + Z_{\text{BL}}$, and $Z_C = Z_{cl} + Z_{\text{CL}}$ are the sums of each line impedance and load impedance per phase. This equation can be solved to obtain the load side neutral voltage as

$$\mathbf{V}_N = \frac{\mathbf{V}_a/Z_A + \mathbf{V}_b/Z_B + \mathbf{V}_c/Z_C}{1/Z_A + 1/Z_B + 1/Z_C + 1/Z_{nl}} \tag{7.16}$$

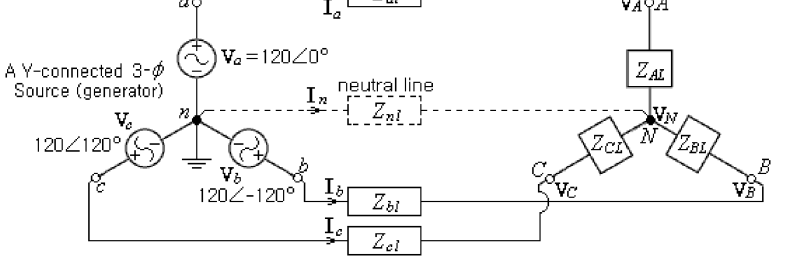


Figure 7.5 The Y-Y configuration of a three-phase power system

and, furthermore, the line currents (identical to the phase currents for a Y-connection) and the load-side (receiving end) voltages as

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{V}_N}{Z_A}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_b - \mathbf{V}_N}{Z_B}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_c - \mathbf{V}_N}{Z_C}, \qquad \mathbf{I}_n = -\frac{\mathbf{V}_N}{Z_{nl}} \tag{7.17}$$

$$\mathbf{V}_A = \mathbf{V}_a - Z_{al}\mathbf{I}_a, \qquad \mathbf{V}_B = \mathbf{V}_b - Z_{bl}\mathbf{I}_b, \qquad \mathbf{V}_C = \mathbf{V}_c - Z_{cl}\mathbf{I}_c \tag{7.18}$$

Note. With the mesh analysis, a set of two or three equations would have to be solved for $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

If the three-phase system is balanced with identical line impedances and load impedances

$$Z_A = Z_B = Z_C = Z \tag{7.19}$$

Equation (7.16) for the load-side neutral voltage becomes

$$\mathbf{V}_N = \frac{\mathbf{V}_a/Z + \mathbf{V}_b/Z + \mathbf{V}_c/Z}{1/Z + 1/Z + 1/Z + 1/Z_{nl}} = \frac{\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c}{3 + Z/Z_{nl}} \overset{(7.3)}{=} 0 \tag{7.20}$$

so that Equations (7.17) and (7.18) become

$$\mathbf{I}_a = \frac{\mathbf{V}_a}{Z}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_b}{Z} = \mathbf{I}_a\angle -120^\circ, \qquad \mathbf{I}_c = \frac{\mathbf{V}_c}{Z} = \mathbf{I}_a\angle +120^\circ, \qquad \mathbf{I}_n = -\frac{\mathbf{V}_N}{Z_{nl}} = 0 \tag{7.21}$$

$$\mathbf{V}_A = Z_{\mathrm{L}}\mathbf{I}_a, \qquad \mathbf{V}_B = Z_{\mathrm{L}}\mathbf{I}_b = \mathbf{V}_A\angle -120^\circ, \qquad \mathbf{V}_C = Z_{\mathrm{L}}\mathbf{I}_c = \mathbf{V}_A\angle +120^\circ \tag{7.22}$$

```
function [VN,VABC,Iabc,SABC] =y_y(Vabc,ZABCL,Zabcl)
% To solve a Y-Y connected 3-phase AC system (stored in an M-file "y_y.m")
%Input:  Vabc =[Va Vb Vc] : the three phase voltage sources
%        ZABCL=[ZAL ZBL ZCL] : the three phase load impedances
%        Zabcl=[Zal Zbl Zcl Znl] : the three or four line impedances
%        optionally with Znl= the neutral line impedance
%Output: VN= the load side neutral voltage
%        VABC=[VA;VB;VC] = the three load-side voltages
%        Iabc=[Ia;Ib;Ic] = the three line currents
%        SABC=[SA;SB;SC] = the 3-phase complex powers
% Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
Vabc=Vabc(:); ZABCL=ZABCL(:); Zabcl=Zabcl(:); ZABCN=Zabcl(1:3)+ZABCL;
tmp = sum(1./ZABCN);
if length(Zabcl)>3, tmp=tmp+1/Zabcl(4); end % with a neutral line if any
VN = sum(Vabc./ZABCN)/tmp; % Load side neutral voltage - Eq.(7.16)
Iabc = (Vabc-VN)./ZABCN; % 3 Line currents - Eq.(7.17)
VABC = Vabc - Zabcl(1:3).*Iabc; % 3 Load end voltages - Eq.(7.18)
% Complex power S=VI* based on the rms phasor voltages/currents
SABC = (VABC-VN).*conj(Iabc); % Complex power of load - Eq.(6.28):
disp('Neutral Voltage(Mag&Phase) at Load side=')
disp([abs(VN) angle(VN)*180/pi])
disp('Load end voltages(Mag&Phase) Line currents(Mag&Phase) Complex powers')
disp([abs(VABCN) angle(VABCN)*180/pi abs(Iabc) angle(Iabc)*180/pi SABC])
```

This result implies that the voltages/currents for the three phases have the same magnitude, but differing from each other in phase angle by 120° and, consequently, the three-phase system needs to be solved only for one phase in the same way as for a single-phase system. Another implication is that in the case of a balanced three-phase system, it makes no difference whether the neutral line exists or not since no current will flow through it. Even in the case of a (slightly) unbalanced three-phase system, the current through the neutral line is expected to be much smaller than the hot-line current. That is why a thinner and cheaper wire is used as the neutral line.

A three-phase system is efficient in the sense that it requires fewer conductors to handle the same power as three separate single-phase systems. However, the solution formulas (7.16) to (7.18) are difficult to compute by hand. That is why the following MATLAB routine `y_y()` is introduced, which can be used to solve a Y-Y connected three-phase system. A user is supposed to put the impedance of the neutral line as the fourth element of the third-input argument (Zabcl) only when it exists, as in the case of the three-phase four-wire (3ϕ-4w) power system.

Figure 7.6 shows a Y-Δ/Y configuration of 3ϕ-4w three-phase power system, where the load consists of a Y-connected one and a Δ-connected one. If there is no neutral line connected between the source-side neutral n and the load-side neutral N, virtually for a 3ϕ-3w system, the following steps could be taken to convert the power system into a Y-Y configuration:

1. Make the Y-Δ conversion of the Y-connected load to get the equivalent Δ-connection.
2. Make the parallel combination of the equivalent Δ-connection and the original Δ-connection.
3. Make the Δ-Y conversion of the composite Δ-connection to obtain the composite Y-connected load.

Note. Why not make a straight Δ-Y conversion of the Δ-connected one and then make a parallel combination of the two Y-connected loads? The neutral of the originally Y-connected one and the resulting neutral of the Y-connected one converted from the Δ-connected one do not generally match each other in the case of unbalanced Y-connected and Δ-connected loads. However, either will do in the case of balanced Y-connected and Δ-connected loads.

4. Apply the Y-Y system analysis implemented by the MATLAB routine `y_y()` as if there were only a Y-connected load.

This approach is, however, not applicable for a 3ϕ-4w system with a neutral line, for which a set of node equations should be written in the four unknown voltage variables $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$, and $\mathbf{V}_N$:

$$\begin{bmatrix} Y_{al}+Y_{AN}+Y_{AB}+Y_{CA} & -Y_{AB} & -Y_{CA} & -Y_{AN} \\ -Y_{AB} & Y_{bl}+Y_{BN}+Y_{AB}+Y_{BC} & -Y_{BC} & -Y_{BN} \\ -Y_{CA} & -Y_{BC} & Y_{cl}+Y_{CN}+Y_{CA}+Y_{BC} & -Y_{CN} \\ -Y_{AN} & -Y_{BN} & -Y_{CN} & Y_{AN}+Y_{BN}+Y_{CN}+Y_{nl} \end{bmatrix} \times \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_B \\ \mathbf{V}_C \\ \mathbf{V}_N \end{bmatrix} = \begin{bmatrix} Y_{al}\mathbf{V}_a \\ Y_{bl}\mathbf{V}_b \\ Y_{cl}\mathbf{V}_c \\ 0 \end{bmatrix} \tag{7.23}$$

After solving this set of equations for $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$, and $\mathbf{V}_N$, the line currents can be obtained as

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{V}_A}{Z_{al}}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_b - \mathbf{V}_B}{Z_{bl}}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_c - \mathbf{V}_C}{Z_{cl}}, \qquad \mathbf{I}_n = -\frac{\mathbf{V}_N}{Z_{nl}} \tag{7.24}$$

Note. With the mesh analysis, a set of six equations would need to be solved.

However, Equation (7.23) is formidable to compute by hand and thus the following MATLAB routine `y_dy()` is introduced, which can be used to solve a Y-Δ/Y-connected three-phase system like the one depicted in Figure 7.6.

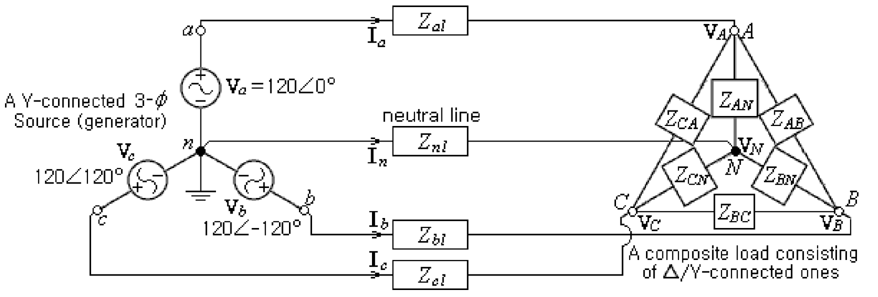


Figure 7.6 The Y-Δ/Y configuration of a three-phase power system

```
function[VN,VABC,Iabc,S_total] =y_dy(Vabc,ZABCN,ZABC,Zabcl)
% To solve a 3p-4w system with Delta/Y-connected loads
%Input:  Vabc=[Va Vb Vc] : the three phase voltage sources
%        ZABCN=[ZAN ZBN ZCN] : the Y-connected three phase load impedances
%        ZABC=[ZAB ZBC ZCA] : the Delta-connected three phase load impedances
%        Zabcl=[Zal Zbl Zcl Znl] : the three or four line impedances
%        optionally with Znl= the neutral line impedance
%Output: VN= the load side neutral voltage
%        VABC=[VA;VB;VC] = the load-side end voltages
%        Iabc=[Ia;Ib;Ic] = the three line currents
%        S_total= the total 3-phase complex power
% Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
YABCN=1./ZABCN; YABC=1./ZABC; Yabcl=1./Zabcl;
if length(Zabcl)>3, Ynl=Yabcl(4); Yabcl=Yabcl(1:3); else Ynl=0; end
Va=Vabc(1); Vb=Vabc(2); Vc=Vabc(3); % Voltages at the sending end
YAN=YABCN(1); YBN=YABCN(2); YCN=YABCN(3); % each of Y-connected admittances
YAB=YABC(1); YBC=YABC(2); YCA=YABC(3); % each of Delta-connected admittances
Yal=Yabcl(1); Ybl=Yabcl(2); Ycl=Yabcl(3); % each line admittance
Y=[Yal+YAN+YAB+YCA -YAB -YCA -YAN; -YAB Ybl+YBN+YAB+YBC -YBC -YBN;
   -YCA -YBC Ycl+YCN+YBC+YCA -YCN; -YAN -YBN -YCN YAN+YBN+YCN+Ynl];
VABCN = Y\[Yal*Va; Ybl*Vb; Ycl*Vc; 0]; % Eq.(7.23)
VABC= VABCN(1:3); VN= VABCN(4); Iabc = Yabc(:).*(Vabc(:)-VABC); % Eq.(7.24)
% Complex power S=VI* based on the rms phasor voltages/currents
S_Y = ((VABC-VN)./ZABCN(:))'*(VABC-VN) % Complex power of Y-load: Eq.(6.28)
V =[VABC(1)-VABC(2) VABC(2)-VABC(3) VABC(3)-VABC(1)]; % Line-to-line voltages
S_D = V*(V./ZABC)'  % Complex power of Delta-load
% The sum of complex powers for the Y-connected and Delta-connected loads
disp('Total complex power'), S_total = S_Y + S_D
disp('Neutral Voltage(Mag&Phase) at Load side=')
disp([abs(VN) angle(VN)*180/pi])
disp('Phase voltages(Mag&Phase) Line currents(Mag&Phase)')
disp([abs(VABC) angle(VABC)*180/pi abs(Iabc) angle(Iabc)*180/pi])
```

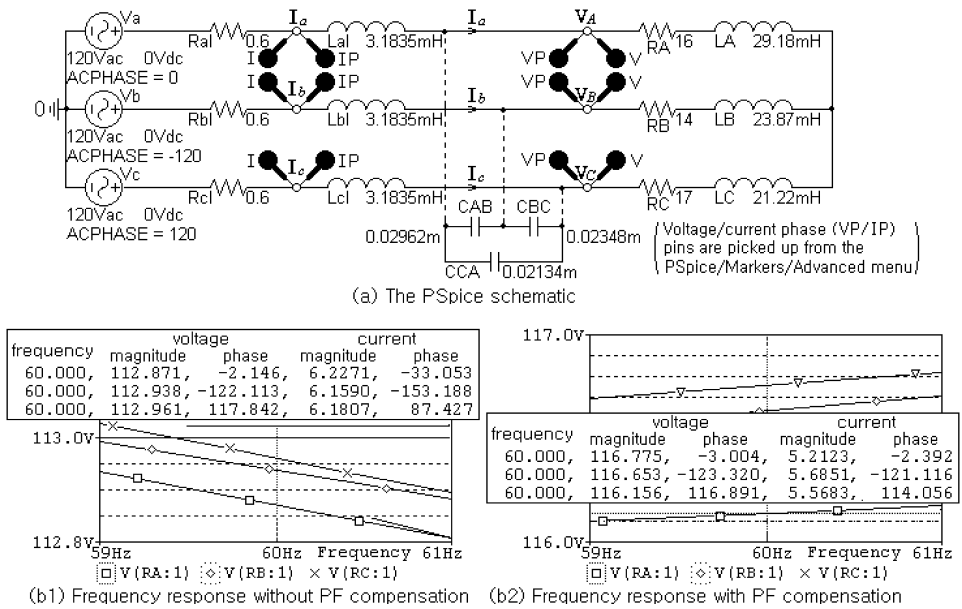


Figure 7.7 PSpice simulation of a three-phase power system

(Example 7.1) MATLAB Analysis and PSpice Simulation of a Three-Phase Power System

Consider the three-phase power system in Figure 7.7(a), where the voltage sources and the resistances/ inductances of the transmission lines and loads are

$$\text{Three voltage sources}: \quad \mathbf{V}_a = 120\angle 0^\circ, \quad \mathbf{V}_b = 120\angle{-120^\circ}, \quad \mathbf{V}_c = 120\angle{+120^\circ} \tag{E7.1.1}$$
$$(\omega = 2\pi f = 2\pi \times 60 \cong 377\,\text{rad/s})$$
$$\text{Three line impedances with } R_{al} = R_{bl} = R_{cl} = 0.6\,\Omega, \quad L_{al} = L_{bl} = L_{cl} = 3.1835\,\text{mH} \tag{E7.1.2}$$
$$\text{Three load impedances with } R_{AL} = 16\,\Omega, \quad R_{BL} = 14\,\Omega, \quad R_{CL} = 17\,\Omega \tag{E7.1.3}$$
$$L_{AL} = 29.18\,\text{mH}, \quad L_{BL} = 23.87\,\text{mH}, \quad L_{CL} = 21.22\,\text{mH}$$

The bank of three Δ-connected capacitors connected in dotted lines with the system will be installed to improve the power factor of the three-phase load to unity (100%). To find the values of the capacitances for PF correction, the Y-Δ conversion of the Y-connected load is made and the values of the capacitances are found such that each parallel combination of a capacitor and a load in Δ-connections will be purely resistive:

$$Z_{AL} = R_{AL} + j\omega L_{AL} = 16 + j11, \quad Z_{BL} = R_{BL} + j\omega L_{BL} = 14 + j9, \quad Z_{CL} = 17 + j8\,\Omega$$
$$\xrightarrow{\text{yd\_conversion( )}} Z_{AB} = 42.77 + j31.52, \quad Z_{BC} = 45.78 + j25.40, \quad Z_{CA} = 52.53 + j28.94\,\Omega \tag{E7.1.4}$$
$$C_{AB} = -\frac{\text{Im}\{1/Z_{AB}\}}{\omega} = 29.62\,\mu\text{F}, \quad C_{BC} = -\frac{\text{Im}\{1/Z_{BC}\}}{\omega} = 23.48\,\mu\text{F}, \quad C_{CA} = 21.34\,\mu\text{F} \tag{E7.1.5}$$

Note that in order to find the composite impedance of a Y-connected load and a Δ-connected load, as a general rule the Y-Δ conversion of the Y-connected one should be made, the parallel combination of the two Δ-connected loads is computed, and then, as needed, the Δ-Y conversion of the composite Δ-connected circuit is made to an overall Y-connected circuit.

In any case it would be very time consuming to do Y-Δ or Δ-Y conversions and, moreover, apply the formulas (7.16) to (7.18) with the above dirty values to compute the phase voltages and currents by hand. Thus the MATLAB routines `y_y( )` and `y_dy( )` will be used and the PSpice simulation will be performed to analyze this circuit.

(a) The program `cir07e01.m` is composed and run to perform the following jobs:

(1) It uses the routine `y_y( )` to solve the three-phase system without the bank of capacitors, where y_y( ) finds the load-side neutral voltage $\mathbf{V}_N$, the three-phase voltages $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ at the receiving ends, and the three line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.
(2) It uses the routine `yd_conversion( )` to get the equivalent Δ-connected loads.
(3) It finds the bank of Δ-connected capacitances C_{AB}, C_{BC}, and C_{CA} that should be connected in parallel with the Δ-connected loads to make them purely resistive so that the PF will be raised to unity (1).
(4) It uses the routine `y_dy( )` to solve the composite three-phase system consisting of the Y-connected loads and the Δ-connected bank of PF compensating capacitors.

Note. Alternatively, the Y-Δ conversion can be made of the Y-connected loads, the equivalent Δ-connected loads combined in parallel with the Δ-connected capacitors, the Δ-Y conversion made of the composite loads, and the routine `y_y( )` used to solve the equivalent Y-Y three-phase system with the bank of capacitors.

```
%cir07e01.m
clear
f=60; w= 2*pi*f; jw= j*w; % The source frequency
Vabc =[120 120*exp(-j*2*pi/3) 120*exp(j*2*pi/3)]; % Eq.(E7.1.1)
Zal =0.6+jw*3.1835e-3; Zabcl=[Zal Zal Zal]; % The line impedances (E7.1.2)
% Y-connected three-phase load impedances from Eq.(E7.1.3)
ZAL=16+jw*29.18e-3; ZBL=14+jw*23.87e-3; ZCL=17+jw*21.22e-3; % Eq.(E7.1.4)
% Analysis of 3-phase 3-wire power system without PF correction
ZABCL=[ZAL ZBL ZCL]; [VN,VABCN,Iabc,SABC] =y_y(Vabc,ZABCL,Zabcl);
[ZAB,ZBC,ZCA] = yd_conversion(ZAL,ZBL,ZCL); % Eq.(E7.1.4)
disp('Capacitances to be connected in parallel with the existing load')
CABC= -imag(1./[ZAB ZBC ZCA])/w % Eq.(E7.1.5)
PFc = 1; % desired Power Factor for correction
CABC= wC_for_PF_correction([ZAB ZBC ZCA],PFc)/w % Alternatively, Sec. 6.6
ZABC_C=1./(jw*CABC); %Impedances of Delta-connected compensating capacitors
disp('After PF correction')
% Analysis of 3-phase 3-wire power system with Delta-connected capacitors
[VN_c,VABCN_c,Iabc_c,SABC_c] =y_dy(Vabc,ZABCL,ZABC_C,Zabcl);
```

The above MATLAB program `cir07e01.m` is run to get the following result:

```
>>cir07e01
    Neutral Voltage(Mag&Phase) at Load side= 10.8893  -137.8994
    Load end voltages(Mag&Phase) Line currents(Mag&Phase) Complex powers
     1.0e+002 *
       1.1287  -0.0215          0.0623  -0.3305          6.2043+4.2657i
       1.1294  -1.2211          0.0616  -1.5319          5.3107+3.4135i
       1.1296   1.1784          0.0618   0.8743          6.4942+3.0560i
```

```
Capacitances to be connected in parallel with the existing load
  CABC = 1.0e-004 *   0.2962  0.2348  0.2134
After PF Correction
Neutral Voltage(Mag&Phase) at Load side= 5.9233    -170.3062
Load end voltages(Mag&Phase)  Line currents(Mag&Phase)  Complex powers
  116.7743    -3.0040             5.2124     -2.3953        1917.3+1143.9i
  116.6525  -123.3196             5.6847   -121.1170             0  -1143.9i
  116.1562   116.8903             5.5684    114.0588
```

This means that

$$\mathbf{V}_A = 112.87\angle{-2.15^\circ}, \quad \mathbf{V}_B = 112.94\angle{-121.11^\circ}, \quad \mathbf{V}_C = 112.96\angle 117.84^\circ$$
$$\mathbf{I}_a = 6.23\angle{-33.05^\circ}, \quad \mathbf{I}_b = 6.16\angle{-153.19^\circ}, \quad \mathbf{I}_c = 6.18\angle 87.43^\circ$$
$$\xrightarrow{\text{PF correction}} \mathbf{V}_A = 116.77\angle - 3.00^\circ, \quad \mathbf{V}_B = 116.65\angle - 123.32^\circ, \quad \mathbf{V}_C = 116.16\angle 116.89^\circ$$
$$\mathbf{I}_a = 5.21\angle - 2.40^\circ, \quad \mathbf{I}_b = 5.68\angle - 121.12^\circ, \quad \mathbf{I}_c = 5.57\angle 114.06^\circ$$

Note. Note the following:

1. The amplitudes of the voltages $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ at the receiving ends have become higher with the PF correction.
2. The complex powers of the Y-connected three-phase load and the Δ-connected capacitor bank are $1917.3 + j1143.9$ and $-j1143.9$, respectively. Thus the composite complex power is purely real, implying that the resulting power factor of 100 % has been achieved by the PF correction.

(b) Perform the PSpice simulation for the three-phase circuit in Figure 7.7(a).

- Draw the schematic as depicted in Figure 7.7(a), where the three VAC voltage sources are placed and their ACPHASE values are set to 0 or -120 or $+120$ in the Property Editor spreadsheet. Do not place the capacitors yet.
- In the Simulation Settings dialog box, set the Analysis type to 'AC Sweep' with the parameters as Start Frequency = 59, End Frequency = 61, and Points/Decade = 200.
- Place V/VP/I/IP Markers to measure the magnitudes/phases of $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$, $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ at the appropriate points as depicted in Figure 7.7(a).
- Click the Run button on the toolbar to make the PSpice A/D (Probe) window appear on the screen as depicted in Figure 7.7(b1).
- To get the numeric values of the measured variables, click the Toggle Cursor button on the toolbar, click the graphic symbol before each variable name at the bottom part of the Probe window by the left/right mouse button, and move the cross-type cursor to the 60 Hz position by pressing the left/right (Shift+)Arrow key or by using the left/right mouse button. Then you can read the numeric value of the measured variable from the Probe Cursor box (Figure 7.7(b1)).
- Modify the schematic by placing the capacitors as depicted in the dotted lines, click Run, and get the new numeric values of the measured variables (Figure 7.7(b2)).

Finally, compare the numeric values of $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$, $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ with those obtained from the MATLAB analysis in (a). If they turn out to be (almost) the same, you may celebrate your success.

7.5 Electric Shock and Grounding

DC circuits have been discussed in the first four chapters and AC circuits have also been studied. Equipped with basic knowledge about circuit theory and electrical terminology such as voltage and current, we may well relate the theory to the electrical devices and systems around us and begin to think about not only the usefulness but also the potential danger of electricity. Electricity quickly endangers our lives as well as meeting our convenience. But what use is all our knowledge if we happen to get injured or die as a result of an electrical accident? At this point, let us put aside the theoretical aspects for a moment and think about the electrical safety issue. However, while the safety issue may require several volumes for a comprehensive treatment, our discussion on this aspect will be very limited.

In the context of electrical safety, a question may arise:

'Which electricity endangers our life, high voltage or large current?'

Even if this question may sound absurd, it should be answered sincerely as follows:

'Both of them, but the former is dangerous as a cause, while so is the latter as a consequence.'

To be more specific, the fatality of an electrical shock depends on several factors such as how large the current is and how long and through which part of the human body the current flows, irrespective of the voltage causing it. The voltage is just a potential cause of a dangerous accident. Even though a person happens to be brought into contact with a conductor at high voltage, it would not be so dangerous as long as the resistance of the path via his/her body between the points of contact or the contact and the ground is large enough to keep the current less than a few milliamperes. Even an electrostatic voltage higher than 20 kV, which may damage some electronic devices, yields nothing more than a little discomfort to a human being because it usually causes the current to flow mainly over the body surface, and that for only a few microseconds. However, since the resistance of a human body with wet skin can be as small as a few hundred ohms, a person may be killed by 100 V AC or a much lower voltage of DC.

Before going into an example addressing the safety issue, note the following tips to avoid electrical shock when you are going to touch electrical/electronic appliances:

1. Turn off the electricity without assuming that the circuit is dead. If they have a capacitor of large capacitance, you should be very careful because it takes time to discharge after the power is off.
2. Noting that prevention is the best medicine, do not touch them when you are wet.
3. Respecting all voltage levels, use safety devices, wear suitable clothing (insulated shoes, gloves, etc.), and use just one (right) hand, especially when touching a high-voltage system.
4. Use a dry board, belt, clothing, or other available nonconductive material to free the victim from electrical shock. Do not touch the victim until the source of electricity is removed.
5. Make sure that there is a third wire on the plug for grounding in case of a short-circuit accident. The fault current should flow through the third wire to ground instead of through the operator's body to ground if an electric power apparatus is grounded or an insulation breakdown occurs.
6. The website <http://www.smud.org/safety/world/index.html> is worthwhile to visit for more information about electrical safety.

Note. How can birds sit on a power line without getting an electrical shock? It is because they are not touching the ground and so the electricity cannot find a path to flow to the ground. However, if one catches one power line with one leg and another line with the other leg, it will be killed instantly before realizing how serious the mistake is. Likewise, if your kite or balloon gets tangled in a power line when you touch the string, electricity could travel down the string and into your body on its way to the ground, causing a fatal shock.

(Example 7.2) Ground Fault Interrupter (GFI) with Grounding to Prevent an Electrical Hazard

Ground fault interrupters are designed to prevent an electrical shock by interrupting a household circuit when there is a difference between the currents in the hot and neutral lines. Such a difference indicates that an abnormal diversion of current occurs from the hot line, which might be flowing in the ground line.

(a) Figure 7.8(a) shows the connection diagram for a GFI that is used to prevent an electrical hazard against the case where the insulation of the motor winding inside the metal case fails and a user

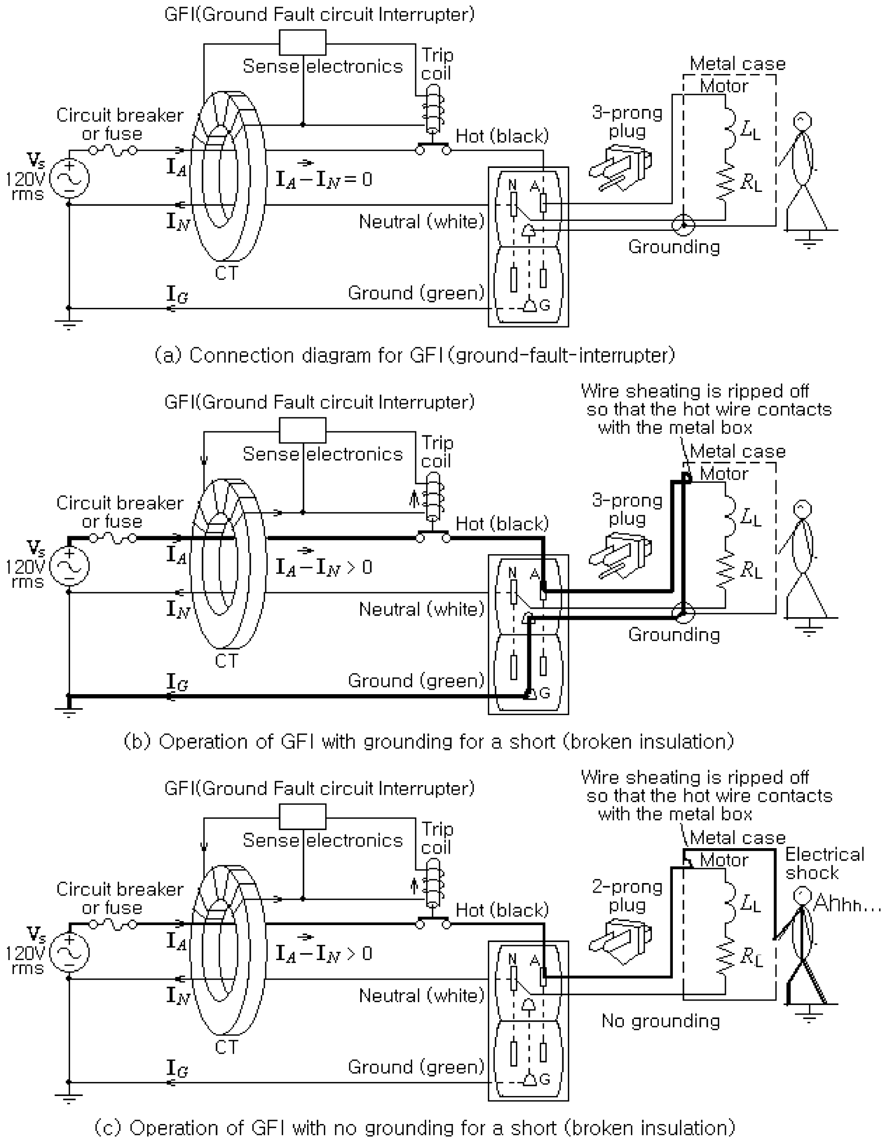


Figure 7.8 GFI process for preventing, detecting, and tripping a short-circuit, and the consequences of no grounding

touches the metal case. Note that in a normal situation with perfect insulation, the primary current of the current transformer (CT) (Problem 5.11) is $\mathbf{I}_A - \mathbf{I}_N = 0$ so that the secondary coil carries no current to produce a force needed to open the switch.

(b) Figure 7.8(b) shows how the GFI detects a short-circuit and produces a tripping signal to open the switch; i.e. in the case where the insulation of the motor winding inside the metal case fails, a large current flows from the fault position to the ground. This current will be $\mathbf{I}_A$, so that $\mathbf{I}_A - \mathbf{I}_N > 0$ and a nonzero current through the secondary coil produces a tripping signal to open the switch. Since

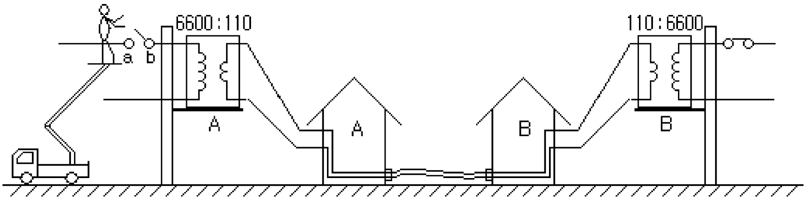


Figure 7.9 (From Reference [I-1]. Source: © Prentice-Hall)

the metal case is grounded, the user touching it will get no electrical shock regardless of whether the GFI works or not.

(c) Figure 7.8(c) shows the situation in which the metal case is not grounded. Everything is almost the same as in (b) except that the fault current flows to the ground not directly, but via the human body till the switch is opened by operation of the GFI so that the user might get an electrical shock before the circuit is interrupted. Besides, the fault current is less than that with grounding, possibly causing some delay in the tripping operation of the GFI. This makes us realize the importance of the grounding or 'chassis ground' for safety.

Note. Fuses and/or breakers are used to limit the current in most household applications. However, the typical limit of current to be interrupted by them is 20 A and their tripping operation is too slow to prevent electrocution. That is why GFIs are required by the electrical code for receptacles in bathrooms and kitchens, near swimming pools, and outside. The GFI is expected to detect currents of a few milliamperes and trip a breaker to remove the shock hazard.

(Example 7.3) Danger Hidden behind Help (Source: J. D. Irwin and C. H. Wu, *Basic Engineering Circuit analysis*, 6th edition, 1999, Example 11.12 with Figure 11.20. Source: © Prentice Hall)

Figure 7.9 describes the situation where the power line feeding house A is interrupted because of some fault and the person living in the house borrows electric power from his neighbor B (fed from another power line) by connecting a long extension cord between an outside receptacle in house A and another in house B. After the fault is recovered, a line technician from the utility company comes to reconnect the circuit breaker at the primary side installed on the utility pole. Not being informed of the fact that house A is fed from another power line and so the power transformer A is alive, he/she might touch contact b (at 6600 V) without wearing any nonconductive gloves and might never see his/her family again.

Problems

7.1 An Unbalanced 3ϕ-3w (Three-Phase Three-Wire) Power System

Figure P7.1 shows a Y-Y type of 3ϕ-3w power system operated at the source frequency of 60 Hz, where a bank of capacitors are to be installed for power factor (PF) correction.

(a) Find the voltages ($\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$) at the load end and the line currents ($\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$) with no capacitors in the polar form as $\mathbf{V}_A = 112\angle -1.58^\circ$ with three significant digits.

(b) Find the three capacitances needed to raise the power factor of the three-phase load to unity (1) in the form $C_{AB} = 49.3\ \mu\text{F}$ with three significant digits.

(c) Find the voltages ($\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$) at the load end and the line currents ($\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$) with the capacitors for PF correction in the polar form as $\mathbf{I}_a = 5.15\angle -3.33^\circ$ with three significant digits.

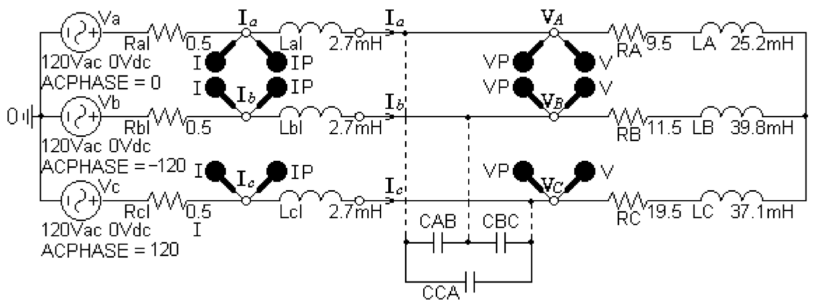


Figure P7.1

Hint. Referring to the MATLAB program `cir07e01.m` presented for solving Example 7.1 in Section 7.4, use the MATLAB routines `y_y()` and/or `y_dy()`.

(d) Perform the PSpice simulation (AC Sweep analysis for 200 frequency points/decade between 59 Hz and 61 Hz) two times, once without the PF compensating capacitors and once with them. Fill in the blanks of Table P7.1 with the PSpice simulation results and the theoretical analysis results obtained in (a) and (c).

Table P7.1 Results of the theoretical analysis and PSpice simulation

		$\mathbf{V}_A$	$\mathbf{V}_B$	$\mathbf{V}_C$	$\mathbf{I}_a$	$\mathbf{I}_b$	$\mathbf{I}_c$
Before PF	Theoretical	$112\angle -1.58°$					
correction	PSpice		$113\angle -120°$				$5.25\angle 85.5°$
After PF	Theoretical			$118\angle 118°$		$4.22\angle -128°$	
correction	PSpice				$5.15\angle -3.32°$		

7.2 An Unbalanced 3ϕ-4w (Three-Phase Four-Wire) Power System

Figure P7.2 shows a Y-Y type of 3ϕ-4w power system operated at the source frequency of 60 Hz. Perform the MATLAB analysis and PSpice simulation for the system two times, once with the bank of capacitors and once without it. Make a table similar to Table P7.1.

Hint. You can complete the following MATLAB program `cir07p02.m` and run it.

```
%cir07p02.m
f =60; w =2*pi*f; jw=j*w;
Vabc =[120 120*exp(-i*2*pi/3) 120*exp(i*2*pi/3)];
Zal=??? +jw*??????; Zbl=Zal; Zcl=Zal; Znl=? +jw*??????; Zabcl=[Za Zb Zcl];
ZAL=??? +j*???????; ZBL=???? +jw*???????; ZCL=???? +jw*???????;
ZABCL =[ZAL ZBL ZCL]; % The Y-connected load
[VN,VABCN,Iabc,SABC] =y_y(Vabc,ZABCL,[Zabcl Znl]);
CABC=[ ???????? ???????? ????????]; ZABC= -1./(jw*CABC); % D-connected load
disp('After PF correction')
[VN_c,VABCN_c,Iabc_c,SABC_c] =y_dy(Vabc,ZABCL,ZABC,[Zabcl Znl]);
```

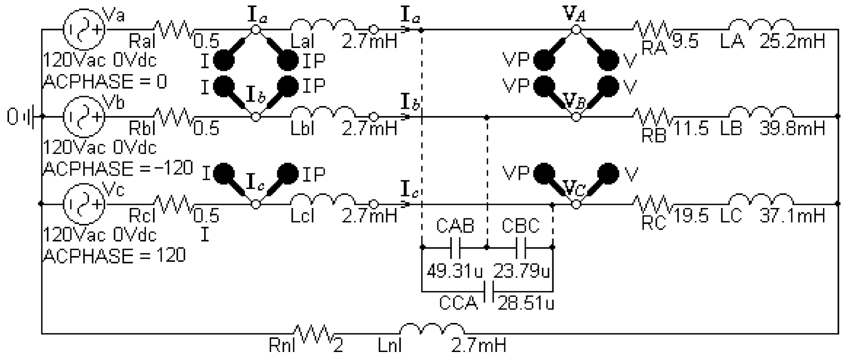


Figure P7.2

7.3 Parallel Combination of the Unbalanced Y-Connected Load and Δ-Connected Load

As mentioned in Section 7.4 and illustrated in Figure P7.3, the parallel connection of the Y-connected load and the Δ-connected load should be initiated by making the Y-Δ conversion of the Y-connected one rather than making the Δ-Y conversion of the Δ-connected one.

(a) To be assured of this assertion, solve the circuit with the capacitor bank in Figure P7.1 to find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ in the following two ways:

(1) Make the Δ-Y conversion of the Δ-connected capacitor bank and combine it with the Y-connected load in parallel (Figure P7.3(b1)–(b2)). Then use the MATLAB routine `y_y()` to solve the circuit and check if the results agree with those obtained in Problem 7.1(c).

(2) Make the Y-Δ conversion of the Y-connected load, combine it with the Δ-connected capacitor bank in parallel, and make the Δ-γ conversion (Figure P7.3(c1)–(c3)). Then use the MATLAB routine `y_y()` to solve the circuit and check if the results agree with those obtained in Problem 7.1(c).

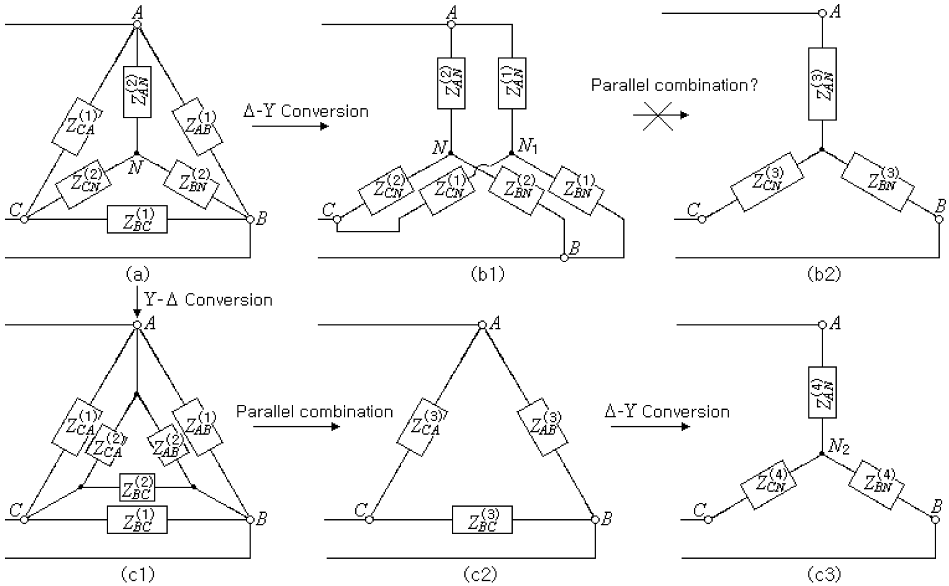


Figure P7.3 Parallel combination of the Y-connected load and the Δ-connected load

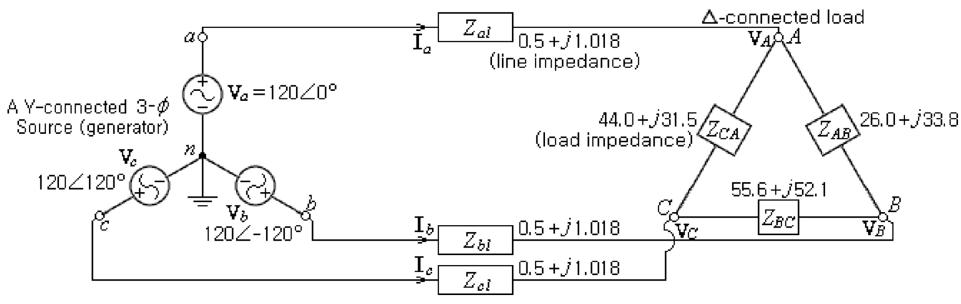


Figure P7.4 A Y-Δ connected 3ϕ-3w (three-phase three-wire) power system

(b) Does the parallel combination of a Y-connected load and a Δ-connected load work for a 3ϕ-4w power system like the one depicted in Figure P7.2?

7.4 An Unbalanced Y-Δ Connected 3ϕ-3w Power System

Figure P7.4 shows a Y-Δ type of 3ϕ-3w power system operated at the source frequency of 60 Hz. A set of node equations can be written in the three unknown node voltages $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ as follows:

$$\begin{bmatrix} Y_{al}+Y_{AB}+Y_{CA} & -Y_{AB} & -Y_{CA} \\ -Y_{AB} & Y_{bl}+Y_{AB}+Y_{BC} & -Y_{BC} \\ -Y_{CA} & -Y_{BC} & Y_{cl}+Y_{CA}+Y_{BC} \end{bmatrix} \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_B \\ \mathbf{V}_C \end{bmatrix} = \begin{bmatrix} Y_{al}\mathbf{V}_a \\ Y_{bl}\mathbf{V}_b \\ Y_{cl}\mathbf{V}_c \end{bmatrix} \tag{P7.4.1}$$

After solving this set of equations for $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$, the line currents can be obtained as

$$\mathbf{I}_a = \frac{\mathbf{V}_a - \mathbf{V}_A}{Z_{al}}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_b - \mathbf{V}_B}{Z_{bl}}, \qquad \mathbf{I}_c = \frac{\mathbf{V}_c - \mathbf{V}_C}{Z_{cl}} \tag{P7.4.2}$$

This solution procedure for the Y-Δ connected 3ϕ-3w power system is cast into the following MATLAB routine `y_d()`.

```
function[VABC,Iabc,SABC] =y_d(Vabc,ZABC,Zabcl)
% To solve a 3p-3w system with Delta-connected loads
%Input:  Vabc=[Va Vb Vc] : the three phase voltage sources
%        ZABC=[ZAB ZBC ZCA] : the Delta-connected three phase load impedances
%        Zabcl=[Zal Zbl Zcl] : the three line impedances
%Output: VABC= [VA;VB;VC] : the load-side end voltages
%        Iabc=[Ia;Ib;Ic] : the three line currents
%        SABC=[SAB;SBC;SCA] : the 3-phase complex power
% Copyleft: Won Y. Yang, wyyang53@hanmail.net, CAU for academic use only
Va=Vabc(1); Vb=Vabc(2); Vc=Vabc(3);
YABC=1./ZABC; Yabcl=1./Zabcl;
YAB=YABC(1); YBC=YABC(2); YCA=YABC(3); % each of Y-connected admittances
Yal=Yabcl(1); Ybl=Yabcl(2); Ycl=Yabcl(3); % each line admittance
Y=[Yal+YAB+YCA -YAB -YCA; -YAB Ybl+YAB+YBC -YBC; -YCA -YBC Ycl+YBC +YCA];
VABC = Y\[Yal*Va; Ybl*Vb; Ycl*Vc] ; % Solve Eq.(P7.4.1)
Iabc = Yabcl(:).*(Vabc(:)-VABC); % Eq.(P7.4.2)
VABC_Delta= VABC-VABC([2 3 1]); % Delta phase voltages
SABC = VABC_Delta.*conj(VABC_Delta./ZABC(:)); % Eq. (6.28)
disp('Load end voltages(Mag&Phase) Line currents(Mag&Phase) Complex powers')
disp([abs(VABC) angle(VABC)*180/pi abs(Iabc) angle(Iabc)*180/pi SABC])
```

(a) Make use of the MATLAB routine `y_d()` to solve the power system of Figure P7.4 for $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$, $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

(b) Make the Δ-Y conversion of the Δ-connected loads and make use of the MATLAB routine `y_y()` to solve the power system for $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$, $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$. Does the solution agree with that obtained in (a)?

7.5 Comparison of Various Power Transmission Schemes

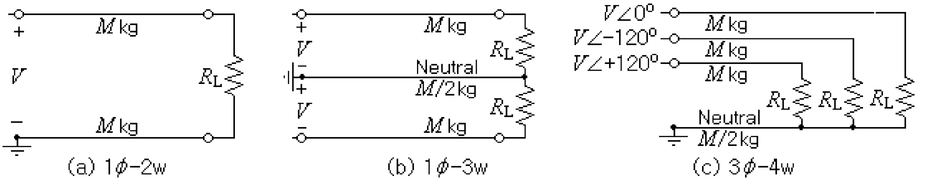


Figure P7.5

Figures P7.5(a), (b), and (c) show 1ϕ-2w, 1ϕ-3w, and 3ϕ-4w transmission schemes, respectively, where the mass of the neutral line is assumed to be half of that (M) of a hot line. Verify that the ratio of the power to the weight of power transmission lines for each of the three schemes is as listed in Table P7.5.

Table P7.5 Comparison of various power transmission schemes

Transmission scheme	1ϕ-2w	1ϕ-3w	3ϕ-4w
Transmitted power	$P_1 = V^2/R_L$	$P_2 = 2V^2/R_L = 2P_1$	$P_3 = 3V^2/R_L = 3P_1$
Weight of power lines	$2M$	$2.5M$	$3.5M$
Ratio of transmitted power to weight of lines	$(1/2)P_1/M$	$(4/5)P_1/M$	$(6/7)P_1/M$