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Elementary Steady-State Heat Transfer

From a thermal point of view a building can be looked upon as a modifier of the climate. For enjoyment and for many forms of manual and intellectual activity, a dry, draught-free and quiet space is needed with a much more restricted range of temperature than may be encountered out of doors. The fabric of the building serves as a filter or buffer generally and to this passive function is added that of its ability to impose a bias on ambient temperature through provision of heating or cooling. Much of this book will be taken up with internal temperature in relation to the fabric and supply of heat but it is convenient to present some brief and elementary account of the factors that lead to choice of room temperature, measures of severity of ambient temperature as it affects provision of comfort conditions, and the exchange of heat by ventilation and conduction between the internal and external environments: these factors determine the heating or cooling load.

1.1 HUMAN THERMAL COMFORT

Carefully devised heating appliances have been evident from early times: the Romans used warm-air heating in villas; one may note the flues built into the towers of the late thirteenth-century castles in Wales and the improvement to flue design urged by Count Rumford before 1800. (Rumford also installed a steam heating system in the Royal Institution in London.) The book by Roberts (1997) provides illustrations of heating, ventilating and refrigeration devices of earlier times. An article by Yunnie (1995) describes other early examples of climate control in the UK (including the system in St George's Hall, Liverpool, completed in 1845) and articles by Lewis (1995) and Greenberg (1995) give interesting accounts of the evolution of HVAC systems over 150 years in the United States.

Study of the relation between temperature, an objective measure, and perception of thermal comfort, a subjective measure, dates from the 1920s. There have been broadly two lines of enquiry: field observations and laboratory measurements. de Dear (1998) remarks: 'Both methods have their strengths and weaknesses. In the case of the climate chamber, precise measurements of, and control over, the main parameters in the comfort matrix are maximised. For example, the effects of inter-individual variations in clothing is typically eliminated by dressing subjects in a standard uniform, but the penalty

Building Heat Transfer Morris G. Davies
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has been a reliance on small and possibly unrepresentative samples of human subjects, usually college students. Furthermore, the highly contrived setting of the climate chamber usually bears little resemblance to the complex environments within real buildings, raising doubts about the validity of generalizing from laboratory to "real" indoor environments. Field methods, on the other hand, involve real buildings, in use by large numbers of occupants going about their normal daily routine, and so retain the integrity of the person-environment relationship ... field study research ... seems most appropriate to the task of validating thermal comfort standards and models.' He goes on to describe a very large database of raw data on thermal comfort found from cross-sectional and longitudinal studies.

Here the response of the occupant is sought to temperature (or other physical variable); a good example is provided by the Bedford seven-point comfort scale (Bedford 1964: 94), where the subject in some location (e.g. a boot factory) is invited to say whether they are much too warm (coded as 1), too warm (2), comfortably warm (3), comfortable (4), comfortably cool (5), too cool (6) or much too cool (7). The wording was variable and a scoring scheme from $+3$ to -3 was also used.[1] A linear regression equation could then be evaluated, relating the ordinal measure with some measure of temperature; its success could be expressed through a correlation coefficient (values of around 0.5 were reported); additional variables (e.g. measures of humidity or air speed) could be included to improve the predictive value of the equation. This form of approach is still used; see for example Newsham and Tiller (1997).

In parallel with this empirical approach went enquiries into the physical and physiological mechanisms that might lead to perception of thermal comfort. This perception depends on control of deep body temperature, skin temperature and rate of loss of moisture by perspiration. The principal environmental factors affecting body heat loss are the room air temperature encountered, the radiant field the body intercepts, and the local humidity level and air speed. Heat loss is restrained by clothing, the amount and type of which may be determined by free choice or by custom. The body itself supplies two physiological mechanisms: vasoconstriction/vasodilation and perspiration or sweating. The involuntary aim of the mechanisms is to keep deep body temperature constant at around 36–37°C; at room temperatures lower than around 23°C most of the metabolically generated heat is lost by convection and radiation, but as the surrounding temperature increases, the loss by evaporation increases. In surroundings above 37°C the body *gains* heat by radiation or convection and sweating remains the only means of preventing a rise in deep body temperature. To counter conditions of extreme cold, the body has a further involuntary mechanism – shivering.

The interplay between the four physical heat flow mechanisms (convection, radiation, air speed and humidity), the voluntary choices of activity level and amount of clothing worn, together with the involuntary moisture loss mechanism, ensure that we can achieve thermal neutrality in a wide range of circumstances. Shivering is excluded since it is associated with discomfort. *Thermal comfort* itself has been defined as 'the condition of mind that expresses satisfaction with the thermal environment'.

These factors had been much studied since the 1920s (Bedford reviews them) but Fanger (1970) appears to have been the first worker to develop a comprehensive model to include them. Since then Fanger's model has been influential in the field of HVAC. It

[1]The ASHRAE wording is hot, warm, slightly warm, neutral, slightly cool, cool, cold.

is based on a steady-state continuity equation:

$$\begin{bmatrix} \text{Internal heat production} \\ \text{in the body} \\ - \text{ loss by water diffusion} \\ \text{through the skin;} \\ - \text{ loss by evaporation of sweat;} \\ - \text{ latent respiration loss;} \\ - \text{ dry respiration loss.} \end{bmatrix} = \begin{bmatrix} \text{Transfer from} \\ \text{skin to clothing} \\ \text{outer surface} \end{bmatrix} = \begin{bmatrix} \text{Radiant loss from} \\ \text{clothing surface} \\ + \\ \text{Convective loss from} \\ \text{clothing surface} \end{bmatrix}.$$

To this are added two empirical relations based on observations of subjects: mean skin temperature as a function of metabolic heat production per unit area (which decreases with increase in production) and evaporative heat loss from the body surface as a function of metabolic heat production; (the loss increases with metabolic rate). The pattern of exchanges can be represented approximately by a thermal circuit (Figure 1.1).

Fanger produced a series of charts indicating circumstances leading to thermal neutrality (Figure 1.2). The principal variables are air and radiant temperature. The parameters are the activity level and amount of clothing worn. A 'sedentary' activity describes for example a subject sitting quietly and corresponds to a metabolic heat output of about 60 W/m^2, 'medium activity' (120 W/m^2) corresponds to walking on the level at 3.2 kph and 'high activity' (175 W/m^2) corresponds to walking up a 5% slope at 3.2 kph. The thermal resistance of clothing is conventionally expressed in clo units and represents the resistance between the skin and the outer surface of the clothing. It is the resistance of the convective and radiative links between the skin and the clothing inner surface plus the resistance of the clothing itself. 1 clo $\equiv$ 0.155 m^{2}K/W. A light clothing ensemble of 0.5 clo might consist of long, lightweight trousers and an open-necked shirt with short sleeves. One clo unit is the resistance of a typical American business suit. The sloping solid lines in the figures relate to the surrounding air speed, however induced. The figures relate to a relative humidity (RH) of 50%.

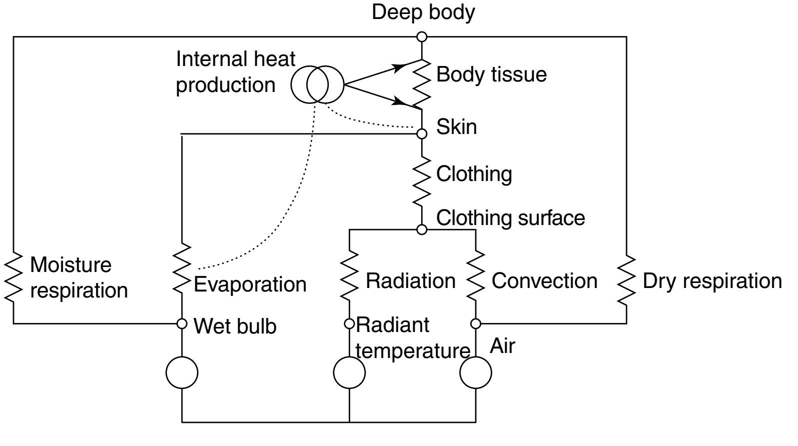

Figure 1.1 Simplified thermal circuit of Fanger's comfort model. The dotted lines indicate an imposed relation between the quantities linked

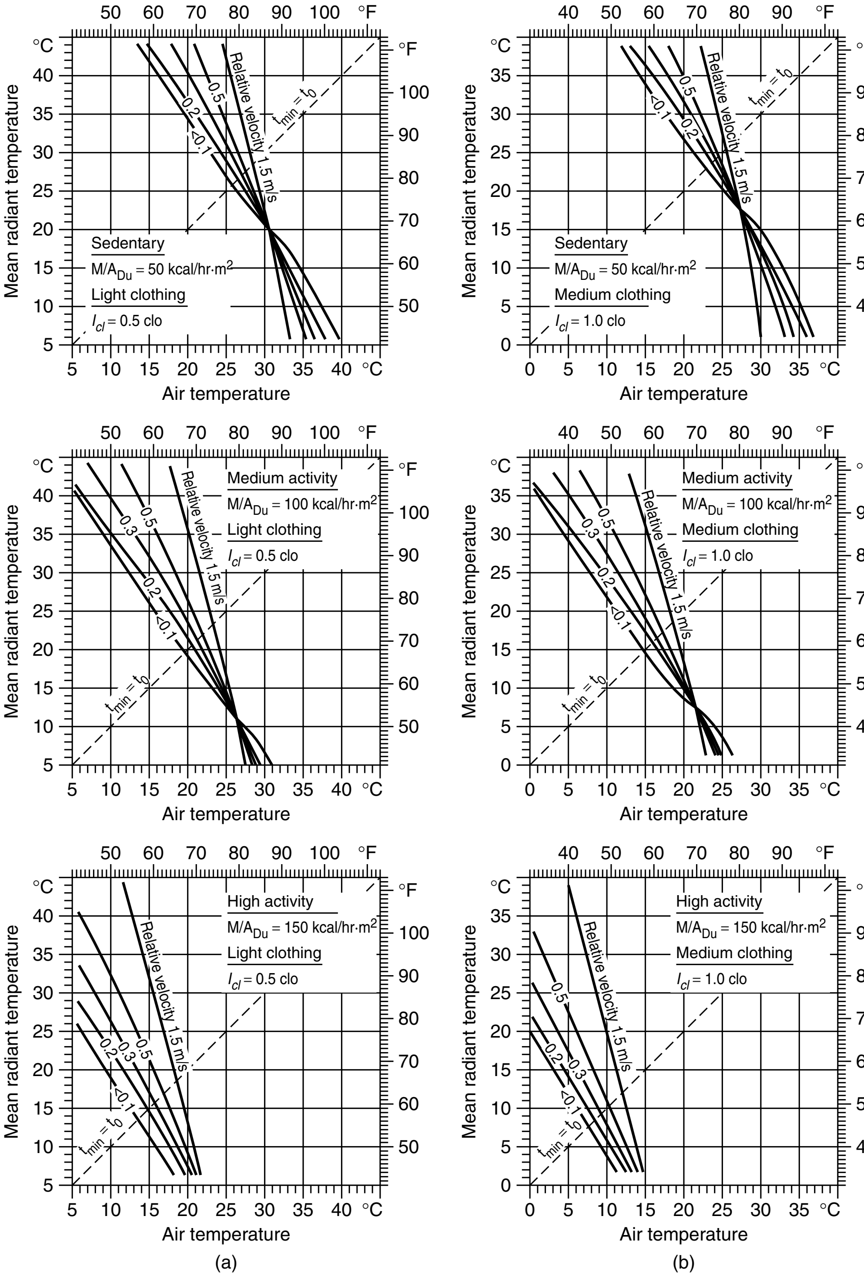

Figure 1.2 Comfort lines: air temperature versus mean radiant temperature with relative air velocity as parameter (a) for persons with light clothing ($I_{cl} = 0.5\,\text{clo}$, $f_{cl} = 1.1$) and (b) for persons with medium clothing ($I_{cl} = 1.0\,\text{clo}$, $f_{cl} = 1.15$). The relative humidity is 50% and plots are shown for three different activity levels (P.O. Fanger, *Thermal Comfort: Analysis and Applications in Environmental Engineering*, © 1972 McGraw-Hill, New York. Reproduced with permission of The McGraw-Hill Companies)

According to the Fanger model, if a lightly clothed subject engaged in medium activity is in a space at 50% RH and an airstream of 0.2 m/s, he/she will be in a thermally neutral state, neither wishing to be warmer or cooler anywhere between radiant temperature 35°C and air temperature 10°C and radiant temperature 11°C and air temperature 27°C. The dotted diagonal indicates the situation when radiant and air temperatures are equal. In this case a lightly clothed subject in still air conditions will be thermally neutral when T is about 26°C, 19°C and 13°C for sedentary, medium and high activity levels, respectively. With medium clothing the values are 23°C, 15°C and 7°C, respectively.

The results afford some justification for the preference of a temperature in the low 20s for office and domestic purposes where modest activity levels are the norm. When radiant and air temperatures are equal, RH = 50% and in still air conditions, the charts indicate the following values of temperature for thermal neutrality:

sedentary, light clothing	26°C	sedentary, heavy clothing	20°C	difference	6 K
high activity, light clothing	13°C	high activity, heavy clothing	2°C	difference	11 K

At a modest activity level in the 20s, one may be near thermal neutrality for a range of clothing levels; engaged in high activity, one's preferred temperature depends much more on the amount of clothing worn. The results also explain the common observation that while you may clothe yourself to withstand very low temperatures, you have to maintain an appropriate level of activity. If you pause, you will need extra clothing. A further factor favouring room temperatures in the 20s relates to manual dexterity; at low temperatures it becomes impaired and only coarser tasks may be undertaken. Gloves have limited utility.

Fanger's analysis is structured from the computational viewpoint. Skin temperature and moisture loss are not causally related to metabolic rate as Figure 1.1 suggests: all are controlled by the hypothalamus. The real independent variables are the activity level of the subject and the choice of clothing. Although the analysis has proved very valuable to building services engineers, it is a simplification of a very complicated process. See for example Griffiths and McIntyre (1974) and McIntyre (1980). We may sweat for reasons unrelated to thermal stress. The use of mean skin temperature obscures the fact that the face and hands are normally uncovered while other parts of the body may be under three layers of fabric. There is significant variation over the body surface in skin temperature and in the tendency to produce sweat. The ventilation action of clothing in motion makes it dubious to assess appropriate clo values.[2] The wide scatter of points relating body evaporative loss to metabolic rate is such that its representation as a straight line, while optimum, is not well founded. The same applies to the relation between skin temperature and metabolic rate, although there is less scatter. These empirical results were obtained from American college-age subjects. It is not clear whether they are also valid for an elderly population for whom matters of thermal comfort are more important.[3] Collins

[2]Ghali *et al.* (1995) have studied the modelling of heat and mass transfer in fabrics. There are four different forms of energy transport in the wicking of unevaporated sweat through a fabric that comes in contact with the skin. The main one is evaporation of the moisture to the atmosphere surrounding the fabric. Conduction, diffusion of moisture in the plane of the fabric, and convection of liquid in the plane of the fabric are less important.

[3]It is easy to find young subjects willing and able to participate in tests and surveys. It is much harder to gain access to elderly subjects and arrange that they undergo laboratory investigations.

and Hoinville (1980) describe a study to compare the thermal responses of elderly and young subjects with the following ages in years: 8 males 79 ± 7, 8 females 77 ± 6, 8 males 28 ± 5 and 8 females 27 ± 8. The elderly group wore clothing of about 1 clo and the young group wore clothing of around 0.87 clo. Subjects spent an initial period of 30 min at 22°C and then a further period of 2 h in a room controlled to 12, 15, 18, 21 and 24°C. Deep body temperature (urine temperature) decreased very little in the cool environment, but marked decreases occurred in mouth and ear temperatures, in the main with bigger falls for the elderly than for the young. Similar changes were noted for mean skin temperature, especially for the feet, but there was only a marginal difference for hand temperature. An ambient temperature of 21.1°C appeared to be satisfactory for old and young. The authors note how the vulnerability of elderly people in cold environments is essentially due to a lifestyle that involves a relatively low level of activity and an increased risk in the cold because of poor thermoregulatory responses and blunted perception of temperature changes.

Recent studies have taken account of much more physiological detail. The Fanger model (Figure 1.1) was based on single measures for skin and for clothing surfaces, but the model of Huizenga *et al.* (2001) includes a large range of detail. The body can be divided into an indefinitely large number of segments (typically 16). For each segment there are nodes corresponding to muscle, fat and skin temperature. Four types of link between the skin and the external environment are possible: bare skin in contact with solid surfaces (conduction loss), bare skin to the surrounding space (convective and radiant loss), clothed skin with conductive loss to solid surfaces, clothed skin with convective and radiant loss. Also included are mechanisms representing heat transfer by blood flow and the thermal and moisture capacity of clothing. Murakami *et al.* (2000) have used computational fluid dynamics and the k-ε turbulence model to examine sensible and latent heat transfer from the human body.

Analyses of thermal neutrality are based on a static model of thermal comfort: it views occupants as 'passive recipients of thermal stimuli driven by the physics of the body's thermal balance with its immediate environment and mediated by autonomic physiological responses' (de Dear and Brager 1998). It is taken to 'prescribe relatively constant indoor design temperatures... [which] have come to be regarded as universally applicable across all building types, climate zones and populations'. The model ignores the important cultural, climatic and social aspects of comfort and in treating the occupant as a passive element, it ignores his/her capacity to adapt to the environment. This includes behavioural adjustments – choice of clothing, local control of heating, cooling and ventilation, taking a siesta – psychological adaptation in that repeated exposure to a stimulus diminishes the response it evokes, and physiological adaptation whether intergenerational or by personal acclimatisation. Busch (1992) reported a study of comfort ratings of office workers in Bangkok, Thailand, in air-conditioned buildings and naturally ventilated buildings. According to widely adopted standards of thermal comfort, the upper limit for comfort is 26.1°C but Busch found values of 28°C for workers in cooled buildings and as much as 31°C for workers in naturally ventilated spaces. Imposition of American standards[4] results in a waste of energy.[5] See Santamouris and Wouters (1994). The adaptive approach to thermal comfort has been developed by Humphreys and Nicol (1998), also Nicol and

[4]See for example Chapter 8 of the 1993 *ASHRAE Handbook of Fundamentals*.

[5]Prins (1992) has written a provocative and highly critical discourse on the American rush to cooling. It is followed by a series of spirited rebuttals.

Humphreys (2002): *If a change occurs such as to produce discomfort, people react in ways which tend to restore their comfort*.[6] Humphreys and Nicol (2000: Figure 4) have summarised the results of empirical studies on how preferred comfort temperatures depend upon the mean outdoor temperature. For buildings that are heated or cooled, comfort temperatures lie in a band between about 18 and 24°C; for free-running buildings, numerous studies indicate preferred temperature of 26–29°C when ambient temperature is 25–29°C. Recognition of traditional preferences may lead to much reduced energy demand for comfort control.

The extensive literature on thermal adaptation in the built environment has been reviewed by Brager and de Dear (1998) and it is of interest to cite some of their conclusions:

> The adaptive approach to modeling thermal comfort acknowledges that thermal perception in 'real world' settings is influenced by the complexities of past thermal history, non-thermal factors and thermal expectations. Thermal adaptation in the built environment can be attributed to three different processes – behavioral adjustment, physiological acclimatization and psychological habituation or expectation. Evidence reviewed [in their article] indicates that the slower physiological process of acclimatization appears not to be so relevant to thermal adaptation in the relatively moderate conditions found in buildings, whereas behavioral adjustment and expectation have a much greater influence and should therefore be the focus of future research and development in this area.
>
> One of the most important findings from our review of field evidence was the distinction between thermal comfort responses in air-conditioned vs. naturally ventilated buildings. Analysis suggested that behavioral adaptation incorporated in conventional heat balance models could only partially explain these differences and that comfort was significantly influenced by people's expectations of the thermal environment. Occupants in naturally ventilated buildings had more relaxed expectations and were more tolerant of temperature swings, while also preferring temperatures that tracked the outdoor climatic trends. In contrast, occupants in closely controlled air-conditioned buildings had much more rigid expectations for a cool, uniform, thermal environment and were more sensitive to conditions that deviated from these constant setpoints.

Thus it appears that where cooling plant is used to achieve some value, it is desirable that it should indeed achieve it. Federspiel (1998) reports a survey of unsolicited complaints made by 23,500 occupants in 690 commercial buildings in the US (millions of square metres area) and noted that the overwhelming majority of environmental complaints related to thermal sensation, mostly due to poor control performance and HVAC system faults. The neutral temperature was close to 23°C with minor variations for summer and

[6]Davies and Davies (1987VI) provide a compact illustration of adaptive response, found from the response of children in a passively solar-heated school to their thermal environment and expressed as correlation coefficients between various variables. There was a very high correlation between ambient and globe temperature, as indeed there must be in a passive building. There were moderately high correlations between globe temperature and the position of the windows, whether or not the lights were on (the lights were explicitly used for their heating potential), and the amount of clothing worn: the children had control over these factors. But the correlations between these control variables and the reported state of thermal comfort were low, showing that the controls had been used so as to reduce discomfort. Windows were more likely to be opened in warm weather, as is very clearly shown by Table 1 of Davies and Davies (1987VII).

winter, and for men and women. Countless studies have of course been conducted in which the respondent has been asked whether they are too warm, too cool, etc., in circumstances over which they had no control by way of personal action or by requesting some action. In this study, complaints were lodged and each complainant was assured that their complaint would be investigated. This might lead to a sharper sense of perception of the environment than passive respondents might have had.

1.2 AMBIENT TEMPERATURE

The heat loss from a building depends on ambient temperature, T_e. T_e can be recorded continuously but it is often reported as an average value over a period of an hour. T_e values over a period of months are needed to provide accurate (computer-based) estimates of the energy need for heating over the cold season or heating and cooling during warm or hot periods (Chapter 19). From the hourly values, other mean values may be derived such as daily mean and monthly mean values of T_e, which can be used to evaluate further measures that are suitable for manual use. Two will be presented briefly here.

1.2.1 Design Temperature

Consider the daily mean value $\overline{T}_e$ of T_e. Its frequency distribution over a year (365 values) is roughly binomial. A cumulative frequency distribution $\sum n(\overline{T}_e)$ starting from a value of T_e below the lowest value of $\overline{T}_e$ varies from 0 to 365 at the highest $\overline{T}_e$ value. In this way a value $\overline{T}'_e$ can be found below which $\overline{T}_e$ falls on some small number of occasions, say two per year. This is called a design temperature. The heating system is sized to cope with an ambient temperature of this value; the risk that it will not cope with the coldest day actually encountered is taken to be acceptable. Thus at some UK location, $\overline{T}_e$ may fall below $-5.0°C$ just once per year on average and this serves as the design temperature for a lightweight building, one where the thermal capacity of the fabric is insufficient to prevent a rapid response to changing conditions. When the building is more massive, a less severe measure may be adopted based on the value of T_e averaged over a 48 h period, $-3.5°C$ for the location concerned. Thanks to the Gulf Stream or, as has recently been suggested, the Rocky Mountains, the UK is much milder than most countries north of latitude 50°N. Much lower values than this are common.

1.2.2 Degree-Day Value

The heat requirement of a building may be taken as proportional to the difference between the comfort temperature T_c required and the external temperature T_e when lower than T_c. Assuming that a constant value of T_c is maintained, the total heat requirement over some period (a day, a month, the winter season) is proportional to $\int(T_c - T_e)\,\mathrm{d}t$ or $\sum(T_c - T_e)\,\delta t$, where δt is one hour or one day depending on the interval over which T_e is averaged. Some of the heat is supplied though casual gains – the sun, occupants, lighting and other equipment – and these gains are sufficient to maintain comfort conditions when T_e is above some value, say T_{base}. T_{base} rather than T_c serves as the temperature with which to estimate the plant output.

Neglecting the thermal capacity of the building and assuming steady conditions,

$$Q_{plant} + Q_{casual} = (T_c - T_e)K, \tag{1.1}$$

where K is the sum of the ventilation and conduction conductances (see below). The casual gains alone lead to an increment ΔT, equal to Q_{casual}/K. So

$$Q_{plant} = ((T_c - \Delta T) - T_e)K = (T_{base} - T_e)K, \tag{1.2a}$$

where

$$T_{base} = T_c - Q_{casual}/K. \tag{1.2b}$$

Karlsson *et al.* (2003) refer to T_{base} as the 'balance temperature' of a building. They are concerned with the solar contribution to Q_{casual}.

The heat load – the heat to be supplied by the heating system – is therefore proportional to $\sum(T_{base} - T_e)\,\delta t$, summed when positive. This is called the degree-hour or degree-day value for the location. It provides a compact means to summarise ambient temperature over a period of time as it relates to the need for heating in a building. Since Q_{casual} varies considerably from building to building, as does K, the value of T_c is a matter of choice; the value of T_{base} is arbitrary. A value of 15.5°C is taken in the UK, 18.3°C in the US and 18.0°C in parts of Europe. The degree-day value is accordingly

$$\mathrm{DD} = \sum(T_{base} - T_e)_+ \times (1\ \mathrm{day}),\ \ \text{units K day}. \tag{1.3}$$

The subscript + denotes that only positive values are summed. Values for the heating season lie between about 1900 and 2900 K day in the UK and between 1000 and 5000 K day in the US. See for example Hitchin (1981). Thom (1954), Erbs *et al.* (1983), Hitchin (1983) and Schoenau and Kehrig (1990) provide means of converting values from to one base to another.

The quantity

$$\mathrm{DH} = \sum(T_{base} - T_e)_+ \times 1\ \mathrm{hour}, \tag{1.4}$$

where T_e is the mean value of ambient temperature over a period of an hour, provides the most rigorous measure of severity, since little is gained through a finer time division. Waide and Norton (1995) discuss the degree-hour value as an index. DD is then simply DH/24. Degree-day values have been used since the 1930s. In the early days, data were most conveniently collected using a maximum-minimum thermometer to record T_e, reset daily, and DD values were evaluated from daily extremes of T_e rather than its continuous variation. For details of UK and US practice, see Day and Karayiannis (1998).

Degree-day values provide a satisfactory means of comparing temperature aspects of the severity of the weather on different sites; see Eto (1988). Hitchin (1990) has noted some possible improvement to their formulation but as noted below, they cannot give close estimates of the heat need for a particular building.

Attention should be drawn to the phenomenon of the urban heat island which is formed as urban areas expand and create their own climates. Air temperatures are higher than in the surrounding rural areas and this leads to increased cooling energy needs and accelerated smog formation in summer. See for example Meier (1997) and subsequent symposium papers.

See also the set of articles edited by Levermore (2002) which discuss the consequences of global warming for energy use in buildings as well as heat islands.

1.3 THE TRADITIONAL BUILDING HEATING MODEL

The total heat need in a room according to the traditional model is

$$Q = (T_i - T_e)\left(V + \sum AU\right). \tag{1.5}$$

T_i is the room index temperature, serving as the measure that drives the steady state heat loss to ambient T_e by the mechanisms of ventilation and conduction, and the temperature at which heat from the heating appliance and other sources is delivered. It also served as the measure of thermal comfort.[7] Since heat is input to a room and then distributed around the room by convection and radiation, two unlike mechanisms, this model provides a much simplified description of room internal heat transfer and the issue will be examined in more detail later on. It was, however, the main means of sizing heating plant up to about 1970 and may be expected to provide adequate estimates in simple situations. T_i and T_e values were discussed in Sections 1.1 and 1.2, respectively; we have to examine the ventilation and conduction loss terms, V and $\sum AU$.

The quantity $V + \sum AU$ is known as the heat loss factor or loss coefficient and is sometimes denoted by the single term UA. It is simple to measure: electric heaters are placed inside the building and room temperature is kept almost constant. Observations are made in stable conditions of ambient temperature and by night to avoid solar gains. UA is the ratio of heat input to temperature difference. Simmonds (1992) compares the details of its implementation in four European codes of practice.

1.3.1 Ventilation Loss

It is normally assumed that air at ambient temperature T_e enters a room, immediately becomes fully mixed with the room air and is lost again at room temperature T_i. The term 'natural ventilation' is often used to denote the exchange of air between the room and spaces external to it through architecturally designed openings such as open windows, vents and doorways. Infiltration is the uncontrolled movement of air through cracks of various kinds. Each is driven by a combination: wind forces bring about cross-ventilation due mainly to horizontal differences in pressure, and further flow may be generated by the thermal stack effect, which causes vertical differences in pressure. Forced ventilation implies an airflow driven by a fan, either simply installed in a wall or supplied through ductwork.

> Liddament (1998) has summarised the status of ventilation in buildings: Ventilation and air infiltration into buildings represents a substantial energy demand that can account for between 25–50% of a building's total space heating (or cooling) demand. As buildings become more thermally efficient, airborne energy loss

[7] T_i was an ad hoc index, not formally related to measurable temperatures although it was taken as corresponding roughly to the value found from a centrally placed thermometer. Figure 7.3 shows the relation between a formally defined index temperature and observed values.

is expected to become the dominant thermal transport mechanism. Unnecessary or excessive air change, therefore, can have an important impact on global energy loss. On the other hand, insufficient ventilation may result in poor indoor air quality and consequential health problems. Designing for optimum ventilation is therefore a vital part of building design to ensure energy efficiency and a healthy indoor environment. This task is made especially difficult, however, by the complexities of air flow behaviour, climatic influences, occupant characteristics and pollutant emission characteristics.

If v is the volume flow into the room (m^3/s) and s the volumetric specific heat of air (about 1200 J/m^3K), the difference in internal energy, $vs(T_i - T_e)$ must equal the heat gain Q_c to the air. Thus, ignoring the small decrease in density, the ventilation loss conductance V (W/K) is given as

$$V = Q_c/(T_i - T_e) = vs. \tag{1.6a}$$

It is common to express a required ventilation rate in terms of the supposed number of complete air changes per hour, so

$$\begin{aligned} V &= ((\text{number } N \text{ of air changes per hour}/3600)[\mathrm{s}^{-1}]) \times (\text{room volume } V_r[\mathrm{m}^3])(1200[\mathrm{J/m^3K}]) \\ &= \tfrac{1}{3} N V_r [\mathrm{W/K}]. \end{aligned} \tag{1.6b}$$

Recommended values of N for many classes of room lie between 1/2 and 1 volume air change per hour (CIBSE 1999: Table A4.10), although this can lead to excessive values for large spaces.

T_i is an ill-defined quantity and will be replaced later by T_a or T_{av} (6.55), the air temperature averaged over three-dimensional space and so a 3D construct. Strictly speaking, it should be written $T_{a,exit}$, which is the mean temperature of the air over the cross-sectional area of the duct or other opening through which the air leaves, and is therefore a 2D construct. The two are the same if the air is fully mixed but they may differ if there is significant short-circuiting between the points of entry and exit of the airflow.

1.3.2 Conduction Loss

The convective exchange between air and a solid surface is described by its convective heat transfer coefficient h_c, which the traditional model takes to be about 3 W/m^2K. The radiative exchange per unit area of a room surface, such as the floor, emissivity ε and the enveloping surfaces, supposedly black body, is εh_r, equal to about $0.9 \times 5.7 \approx 5$ W/m^2K.[8] The model merges these values to give an internal film coefficient of

[8]Most building materials have an emissivity ε of around 0.9. Dust collection, moisture condensation and corrosion lead to this value even though a clean new surface may have a lower value. But as noted by Goss and Miller (1989), it has been known since the 1930s that aluminium retains a high reflectivity (low emissivity) for radiant heat transfer due to a protective layer of transparent oxide. For the radiant exchange between surfaces, see equation (6.53).

$h_i = h_c + \varepsilon h_r \approx 8\,\text{W/m}^2\text{K}$. Thus the heat flow from T_i to a surface bounding the room of area A and at T_n is

$$Q = (T_i - T_n)Ah_i, \tag{1.7}$$

where n denotes the nth layer in the wall, counting from outside. Similarly, the loss of heat by convection and radiation to ambient is

$$Q = (T_0 - T_e)Ah_e. \tag{1.8}$$

Subscript 0 denotes the interface between layers 0 and 1, where layer 0 here is the outer film and layer 1 is the outermost layer of the wall. Like h_i, the outer film coefficient has radiative and convective components but h_e is largely determined by the forced convection due to wind speed and is very variable; a value of $h_e = 18\,\text{W/m}^2\text{K}$ is often assumed.

The one-dimensional heat flow by conduction through a slab of thickness X_1 and conductivity λ_1 and face temperatures T_0 and T_1 is

$$Q = (T_1 - T_0)A\lambda_1/X_1. \tag{1.9}$$

In steady-state conditions, the flow from inside at T_i through two such layers to ambient at T_e is

$$Q/A = \underset{\text{outer film}}{(T_0 - T_e)h_e} = \underset{\text{layer 1}}{(T_1 - T_0)\lambda_1/X_1} = \underset{\text{layer 2}}{(T_2 - T_1)\lambda_2/X_2} = \underset{\text{inner film}}{(T_i - T_2)h_i} \tag{1.10a}$$

$$= \frac{T_0 - T_e}{1/h_e} = \frac{T_1 - T_0}{X_1/\lambda_1} = \frac{T_2 - T_1}{X_2/\lambda_2} = \frac{T_i - T_2}{1/h_i} \tag{1.10b}$$

$$= \frac{T_i - T_e}{1/h_e + X_1/\lambda_1 + X_2/\lambda_2 + 1/h_i} \tag{1.10c}$$

$$= \frac{\text{temperature difference}}{\text{sum of the thermal resistances}}. \tag{1.10d}$$

The U value[9] or thermal transmittance of the wall is defined as

$$U = \frac{\text{heat flow per unit area in steady conditions}}{\text{temperature difference}} = \frac{Q/A}{T_i - T_e}, \tag{1.11}$$

so

$$1/U = 1/h_e + X_1/\lambda_1 + X_2/\lambda_2 + 1/h_i = \sum(\text{thermal resistances}). \tag{1.12}$$

If the wall includes a cavity, its resistance (around $0.18\,\text{m}^2\text{K/W}$) must be included.

The overall behaviour of the wall can also be found by multiplication of the separate layer transmission matrices. Consider the flow through layer 1, the wall outer layer. Taking

[9] U is also called the U factor. The performance of the wall is also described by its resistance $R = 1/U$, with units $\text{m}^2\text{K/W}$ or $\text{h ft}^2\,{}^\circ\text{F/Btu}$. This has the merit that a high value of R denotes a well-insulated wall. The designation R-3, for example, denotes a resistance of $3\,\text{h ft}^2\,{}^\circ\text{F/Btu}$ or $0.53\,\text{m}^2\text{K/W}$.

T_e to be situated to the left of T_i and T_i to be higher than T_e, the heat flow q_0 at the left surface is $(T_1 - T_0)\lambda_1/X_1$ and in the negative x direction, so

$$-q_0 = (T_1 - T_0)\lambda_1/X_1 \quad \text{and} \quad q_1 = q_0. \tag{1.13a}$$

In matrix form this is

$$\begin{bmatrix} T_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} 1 & X_1/\lambda_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_i \\ q_i \end{bmatrix}. \tag{1.13b}$$

Since temperature and heat flux are continuous across the interface between two layers, $[\, T_1 \quad q_1 \,]^T$ is given by a similar matrix involving the resistance X_2/λ_2, and similarly for the outside and inside films. Thus we can write

$$\begin{bmatrix} T_e \\ q_e \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1/h_e \\ 0 & 1 \end{bmatrix}}_{\text{outer film}} \underbrace{\begin{bmatrix} 1 & X_1/\lambda_1 \\ 0 & 1 \end{bmatrix}}_{\text{layer 1}} \underbrace{\begin{bmatrix} 1 & X_2/\lambda_2 \\ 0 & 1 \end{bmatrix}}_{\text{layer 2}} \underbrace{\begin{bmatrix} 1 & 1/h_i \\ 0 & 1 \end{bmatrix}}_{\text{inner film}} \begin{bmatrix} T_i \\ q_i \end{bmatrix} \tag{1.14a}$$

$$= \begin{bmatrix} 1 & 1/h_e + X_1/\lambda_1 + X_2/\lambda_2 + 1/h_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_i \\ q_i \end{bmatrix}. \tag{1.14b}$$

The only significant term in the product matrix is the sum of the resistances, hence $U = q/(T_i - T_e)$ as before. Both these methods of arriving at U are trivial, but the matrix approach becomes essential in time-varying conditions when we must also take account of the thermal capacity of solid layers and all the elements become significant.

In the calculation it is assumed that the conductivity in some layer is constant and the temperature gradient is then uniform. However, λ values of some building materials increase with moisture content and in masonry materials λ may increase toward the exterior surface, either because moisture diffuses to cooler places or through wetting by rain. In this case the gradient decreases toward the outer surface.

A sheet of glass is so thin that its thermal resistance is negligible[10] and the U value for a window depends on the films alone. For single glazing (and any very thin wall), $U \approx (1/8 + 1/18)^{-1} = 5.5\,\text{W/m}^2\text{K}$; for double glazing, $U \approx (1/8 + 0.18 + 1/18)^{-1} = 2.8\,\text{W/m}^2\text{K}$. A value of $0.18\,\text{m}^2\text{K/W}$ is usually taken for the resistance of an air cavity. Argon, krypton and xenon can replace air, and by using multiple glazing and low-emissivity coatings, transmission coefficients down to $0.5\,\text{W/m}^2\text{K}$ can be achieved; see Muneer and Han (1996).

Bricks and blocks are sometimes provided with slots arranged in various ways which increase their face-to-face resistance and improve the thermal insulation they provide. Anderson (1981) shows how the resulting two-dimensional flow pattern can be analysed.

The ordering of the layers, in particular the position at which insulation is placed in the wall, does not affect the steady-state transmittance.[11] However, it becomes relevant for the dynamic behaviour of the wall: the combination of insulation inside/mass outside

[10] The resistance of 6 mm glass is $X/\lambda = 0.006/1.05 = 0.006\,\text{m}^2\text{K/W}$, negligible compared with the inner film resistance of $0.12\,\text{m}^2\text{K/W}$.

[11] At corners, a given thickness of insulation is most effective to reduce heat loss when placed inside, but then the structure is colder than a plane wall, with the risk of freezing.

results in a rapid response to heat input. This may be the desired outcome, but if the input is due to solar gain, it may lead to high room temperatures or a large cooling plant to restrain them. With mass inside/insulation outside the room is thermally more stable and solar gain may contribute usefully to the heat need, but the space may then require a large heat input to reach a comfortable temperature in reasonable time if the space has previously been unheated in cool conditions. Furthermore, the arrangement with insulation inside/mass outside may lead to interstitial condensation in the predominantly cold external structure. Sonderegger (1977) reaches these conclusions using the method of harmonic analysis presented in Chapter 15. Boji'c and Loveday (1997) describe a study comparing two wall constructions with the same U value, one of form masonry, insulation, masonry (MIM) and the other of form insulation, masonry, insulation (IMI). They confirm that if the building is to be intermittently heated, the IMI form is better but for intermittent cooling, the MIM form is better. For continuous cooling, the structure does not matter. The differences in energy needed are of order 30%.

Although much of this book is devoted to a study of wall behaviour in non-steady conditions, the simple U value or U factor of a wall remains its most important thermal descriptor. Methods to find the transmittance of building elements composed of bridged layers are given in Section 3.3.11 in Book A of the 1999 *CIBSE Guide*. Maximum permitted values are specified by the building regulations in many countries. Following the increased awareness in the 1970s of the amount of energy needed to heat and cool buildings, maximum permitted values have been progressively reduced, especially in Scandinavia.[12] By incorporating 300 mm of rock wool insulation, U values of around 0.1 W/m^{2}K are reached; a value of 0.09 W/m^{2}K has been reported in Finland.

A simple expression allows us to estimate the thickness X of insulation, conductivity λ, that might on economic grounds be added to a wall of basic U value U_0. X will increase with

- F, the cost of fuel, \$/J say;
- N degree-days per year, a measure of the severity of the climate;
- f, the proportion of the 24 h period during which comfort conditions are to be maintained.

X will decrease with

- P, the interest rate on the capital borrowed to purchase the insulating material;
- z, the cost of the insulating material, \$/m^3.

The optimal value of X is $(FNf\lambda/(Pz))^{1/2} - \lambda/U_0$. In effect, the optimal wall resistance is $(FNf/(Pz'))^{1/2}$, where z' is the cost of insulating material expressed as \$/m^2 per unit of added resistance. A closely similar expression is given in equation 13 of Hasan (1999). One would suppose that to conserve energy, a 'hot' surface requires thicker insulation than a 'cool' surface. Bejan's initial analysis (Bejan 1993) does not support this view. He considers insulating a surface whose temperature varies linearly from ambient to some high value using a certain fixed volume of insulation. The loss turns out to be the same when the insulation is applied uniformly and when its thickness is proportional to the

[12] Values for 2001 in the UK are walls 0.35, roofs 0.20, floors 0.30, glazing 2.20 W/m^{2}K.

temperature difference but Bejan's argument supposes that the insulation provides the only resistance to heat loss; it does not consider the outside film resistance.

Wall insulation can be viewed from a strictly economic standpoint: the saving in running costs. It can also be seen in relation to environmental pollution: the saving of running costs is concomitant with reduced pollution but the manufacture of insulating material together with its transport and installation entails increased pollution. Erlandsson *et al.* (1997) have made a life-cycle assessment for additional external wall insulation for Scandinavia; for economy, insulation thicknesses between 100 and 170 mm are appropriate but environmental considerations favour the greater thickness.

In steady-state conditions, the temperature gradient $\mathrm{d}T/\mathrm{d}x$ through any one layer is constant but differs from layer to layer. If however temperature is plotted as a function of progressive resistance, the gradient $\mathrm{d}T/\mathrm{d}(x/\lambda)$ is uniform through the wall and the construction can be extended to include the surface films. (Strictly speaking, a profile cannot be traced through a film; the part associated with convection is unchanged in the bulk air and only changes within the boundary layer. The radiant component however cannot be displayed in this way.) In unsteady conditions, the temperature profile in any layer is curved, but when plotted against resistance, the gradient at an interface remains continuous.

Much work over a long period has been devoted to find experimental U values for a large range of wall types. This lies outside the scope of the book but observational values of wall and roof U are usually higher (i.e. worse) than the values computed from assumed h, λ and X values would suggest. Siviour (1982) reports that measurement of heat flow through a wall insulated with urea-formaldehyde corresponded to a U value of 0.65 W/m^{2}K while the calculated value was about 0.5 W/m^{2}K. Reasons for this include higher values of λ and h in practice than tabulated ones (since λ depends strongly on moisture content), evaporation of rain, thermal bridging due to wall ties or debris lodged in the cavity, ventilation of the cavity, thermal bridging at window frames and additional losses at corners. Errors may be made in the measurement of temperature itself; Bénard *et al.* (1990) report a detailed study of possible errors in measurement of surface temperature by a thermocouple.

Because of these factors and possible omission of insulation, one might suppose that the observed heat loss coefficient $UA = V + \sum AU$ should be larger than its calculated value, but this is not necessarily the case. Liu and Claridge (1995) summarise studies from the 1980s onwards which showed that the calculated value of UA could be double its observed value. They attributed this to neglect of air infiltration heat recovery and neglect of the heat discharge from thermal storage during the night.

Heat loss from a solid floor cast on earth is a three-dimensional flow problem which is much more complicated than that for simple wall losses. An estimate has been provided in the past by Macey's formula (Macey 1949). Consider a solid floor of length L (the major dimension) and breadth B, surrounded by a solid wall of thickness W (so that the external breadth is $B + 2W$). If the floor surface and surrounding land surfaces have temperatures T_{in} and T_{out}, the steady state flow outward is $(T_{in} - T_{out})UBL$, where

$$U = \frac{4\lambda}{\pi B} \operatorname{arctanh}\left(\frac{B}{B+W}\right) \exp\left(\frac{B}{2L}\right) \tag{1.15}$$

and λ is the soil conductivity. U is a *surface-to-surface* conductance, not an *index-to-index* conductance as usually defined. Although not misleading, Macey's expression has some logical flaws (see later).

Transparent insulating materials act as thermal insulation but simultaneously permit the transmission of solar energy. See Braun *et al.* (1992) and other articles in this issue. Wood and Jesch (1993) present a detailed account of transparent insulating materials. Affixed to the exterior of a massive wall with a dark exterior surface, a transparent insulating material acts as an insulator in the usual way but allows incident radiation to be absorbed by the wall. Most of the energy is transmitted with some phase lag to the space behind the wall. An analysis of the mechanism is somewhat like that indicated in Figure 9.14 (although these materials have optical properties which depend on the angle of incidence and on temperature.) Gorgolewski (1996) reports that in the Scottish climate (latitudes above about 55°N) a south-facing wall of this kind can reduce the annual heating load by 200 kWh/m^2. The material requires external protection and a movable blind must be supplied to prevent excessive gain in summer.

Hens (Hens and Fatin 1995) has listed a number of checks that relate to the performance of a cavity wall. Of these, the U value and thermal bridging have already been mentioned. Steady-state aspects of hygric stress and moisture balance are discussed in Chapter 8 with some mention of dynamic effects in Chapter 17 and Chapters 15 to 19 deal with unsteady heat conduction. Hens also discusses the permeance of a wall to airflow due to a pressure difference Δp_a (Pa) across it. He cites values such as $2.5 \times 10^{-3} \Delta {p_a}^{-0.5}$ kg/(m^{2}s Pa) and $10^{-6} \Delta {p_a}^{-0.28}$ kg/(m^{2}s Pa) but this consideration lies outside the scope of the book.

1.3.3 Loss from a Cylinder

If inside and outside temperatures remain constant, the effect of adding a layer of material to a plane wall is to reduce the heat loss. This is not necessarily the case if a layer of material is added to a cylinder at a fixed temperature. Consider a cylinder of radius R_0 and length L at temperature T_0. It loses heat Q (W) to the surroundings at T_2. The loss is $2\pi R_0 Lh\ (T_0 - T_2)$ where h (W/m^{2}K) is the combined convective/radiative film coefficient. Suppose now that a layer of material of conductivity λ is added to form a cylinder of radius R_1 with temperature T_1. From (3.17) and continuity of heat flow,

$$\frac{Q}{L} = \frac{2\pi\lambda(T_0 - T_1)}{\ln(R_1/R_0)} = 2\pi R_1 h(T_1 - T_2). \tag{1.16}$$

It follows that

$$\frac{R_1 h/\lambda}{2\pi(T_0 - T_2)L}\left(\ln\frac{R_1}{R_0} + \frac{\lambda}{R_1 h}\right)^2 \frac{\mathrm{d}Q}{\mathrm{d}R_1} = h - \frac{\lambda}{R_1}. \tag{1.17}$$

The expression is valid if $R_1 = R_0$. It shows that if material is added to the cylinder, it will lead to an *increase* in heat loss if $h > \lambda/R_0$. $R_0 h/\lambda$ has the form of a Biot number B (Chapter 13).[13] The heat loss is maximised when the perimeter of the insulation is $2\pi\lambda/h$ and Hsieh and Yang (1984) have shown that this is true too for a square section.

[13]Note that both the Biot number B and the Nusselt number Nu have the form, Xh/λ, where X is some characteristic length, h is a film coefficient (W/m^{2}K) and λ is conductivity (W/m K). The Biot number features in conduction-dominated problems; X is a layer thickness aligned in the direction of heat flow and λ is the conductivity of the layer material. The Nusselt number is used in convection-dominated problems; X can be chosen to be parallel or perpendicular to the flow direction and λ is the conductivity of the fluid.

1.4 SEASONAL HEAT NEED

By assuming some value for the hourly air change rate and summing over the various wall, window and roof elements of a building, the term $V + \sum AU$ can be found. According to the simple model, the energy to be supplied by the heating plant is

$$Q_{plant} = (T_{base} - T_e)\left(V + \sum AU\right), \tag{1.18}$$

so the energy need (J) is over some fixed period, a month say:

$$E = \mathrm{DD} \times 86\,400\left(V + \sum AU\right), \tag{1.19}$$

where DD is the degree-day value (equation 1.3) for the site for the relevant period and 86 400 is the number of seconds in a day; to convert to kilowatt-hours divide by 3.6×10^6.

This quantity is easy to evaluate but it represents a simplified approach to the problem of energy supply. The incremental temperature ΔT at some time t in fact depends on time-varying heat inputs. Further, the ventilation rate may well be higher by day than by night and when the internal temperature varies, the conduction loss conductance L (having the value $\sum AU$ in steady conditions but now including conduction into all bounding surfaces) in effect becomes a varying quantity. We have to write

$$\Delta T(t) = \frac{Q_{solar}(t) + Q_{occupants}(t) + Q_{lighting}(t) + Q_{equipment}(t)}{V(t) + L(t)}. \tag{1.20}$$

These elements have in varying degrees steady, cyclic and transient components and the sequence of values of ΔT at hourly intervals may be expected to show large variation and to differ from the sequence in a nearby room or building. Clearly, a degree-hour value

$$\mathrm{DH} = \sum (T_c - \Delta T(t) - T_e(t)) \times 1\,\text{hour}, \tag{1.21}$$

in which $T_e(t)$ too takes on hourly values, provides a coarse measure to estimate seasonal energy needs. Some indication of reliability is provided in a study by Day and Karayiannis (1999). They considered a particular model building with specified thermal capacity, fabric conductance, glazed area, infiltration rate, occupancy, casual gains and ten year weather data and they used advanced means to find its energy needs. With this as the 'truth' value, they found values based on various simplifications. They took a fixed value for the inside temperature T_i (the setpoint or T_c value), and also hourly values of T_i and its daily and monthly mean values. Similarly, hourly plus daily and monthly averaged values were taken for the casual gains. These assumptions lead to a series of base temperatures of form

$$T_{base} = T_i - Q_{casual}/\left(V + \sum AU\right). \tag{1.22}$$

See equation 1.2b. According to the definition of T_{base}, DD values ranged from 1117 to 2090 K days. The worst energy estimate was found with a combination of a fixed setpoint value and hourly gains, when the energy need was some 90% larger than its true value.

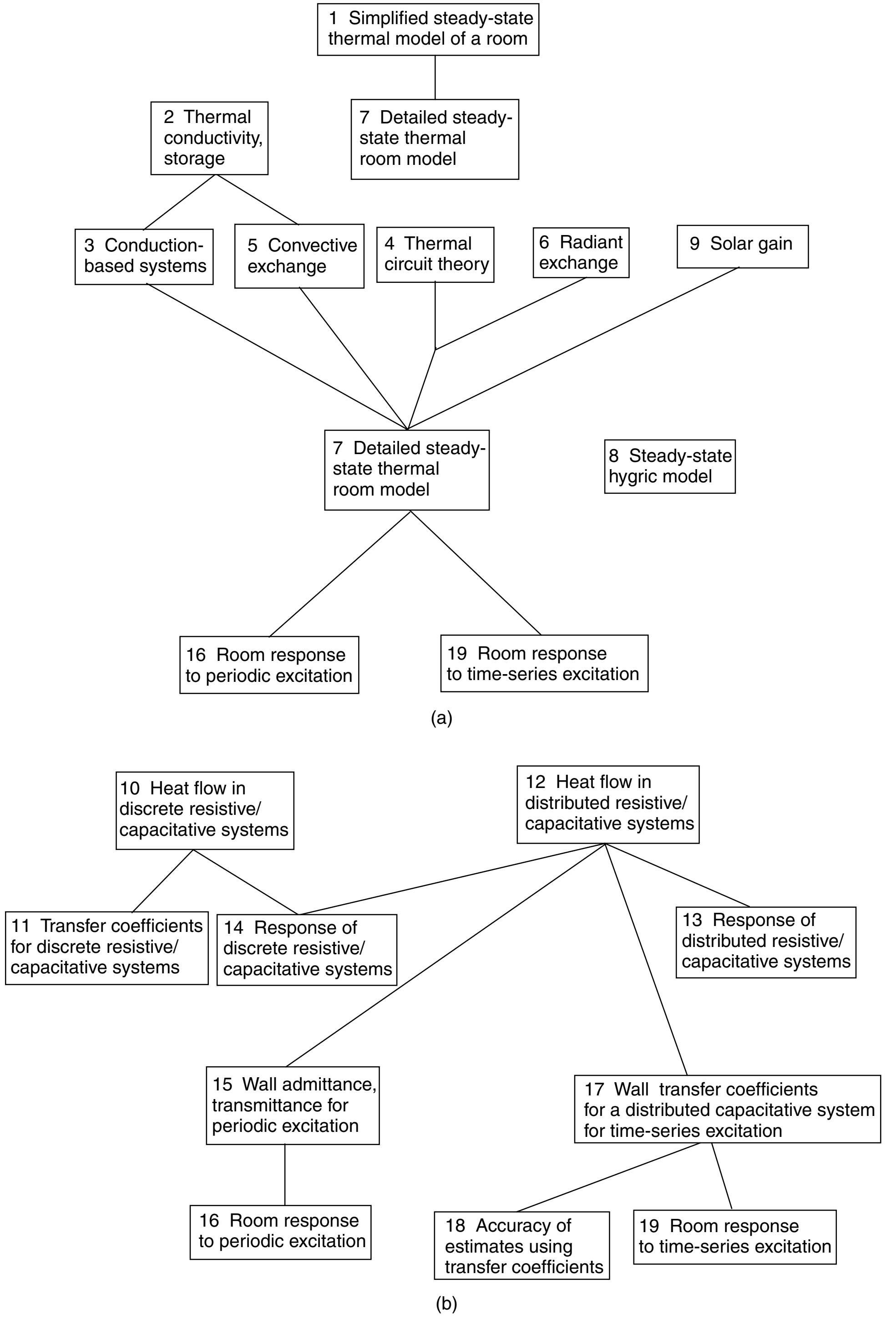

Figure 1.3 Plan of the book: (a) basic mechanisms and (b) wall conduction/storage and room thermal models

With actual values for T_i (rather than a supposed value T_c) and averaged over a day or a month, with similarly averaged values for the gains, the overestimate was reduced to less than 4%. The authors' worst case is based on hourly gains, and not even this information may be available to the building designer at an early stage of design. Their results therefore suggest that a seasonal energy estimate for a building based simply on its loss coefficient and the local degree-day value – the only information the designer may have early on – may be seriously in error. Better estimates involve more effort. The approach using transfer coefficients is given in Chapter 19.

1.5 PLAN OF THE BOOK

The principal question for building heat transfer studies is to find the amount of heating or cooling that the plant must deliver to maintain some specified level of temperature (and humidity), or the daily profile of temperature in a space if it responds to heat inputs in an uncontrolled manner, or perhaps some combination of fixed and floating conditions. Other texts, handbooks and technical literature cover details of plant design, operation and control, so the means of heating or cooling will be assumed without further discussion. The exception is the heating effect of the sun, discussed in Chapter 9. The theory of convection and conduction uses the conductivity λ, the density ρ and the mass specific heat at constant pressure c_p of the materials concerned. For gases these quantities can be found semiquantitatively by elementary kinetic theory, and this is outlined in Chapter 2. Chapter 3 considers the conduction-dominated three-dimensional heat flow from a floor slab. Convective and radiative studies can be treated separately (Chapters 5 and 6), and in combination they lead to a more detailed model for steady-state room heat transfer (Chapter 7). They are combined using some thermal circuit theory given in Chapter 4. The room elementary humidity model (Chapter 8) is formally similar to the thermal model.

In unsteady heat flow we must take account of the storage potential of wall materials in addition to their conductivities and this considerably complicates the calculation of wall response. A useful simplification is to suppose that the continuous distribution of storage and resistance can be represented as localised or discretised elements and Chapters 10 and 11 show how the conventional dynamic wall parameters, developments of the simple U value, can be found. Chapter 12 presents various forms of solution to the Fourier continuity equation then uses them to find the dynamic parameters when storage and resistance are considered as distributed properties, eventually to be used in Chapters 15 and 17. Before that, some classical solutions are presented for cases where a wall or a room is subjected to a step temperature excitation (Chapter 13) and for some room models so simplified that their thermal capacity is represented as one, two or possibly three lumped capacities (Chapter 14). Room models using period-based parameters are discussed in Chapter 16 and models using time-series parameters are covered in Chapter 19. The plan is illustrated in Figure 1.3.

Chapter 1 of Clarke (2001) gives a detailed justification for a study of these processes in the context of energy flow simulation in a building.